UNIVERSITY OF ICELAND

Stockholm University

Introduction to Black Hole Informatics

- Lecture 3 -

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2018 Arnold Sommerfeld School on
“Black Holes and Quantum Information”

Arnold Sommerfeld Center for Theoretical Physics
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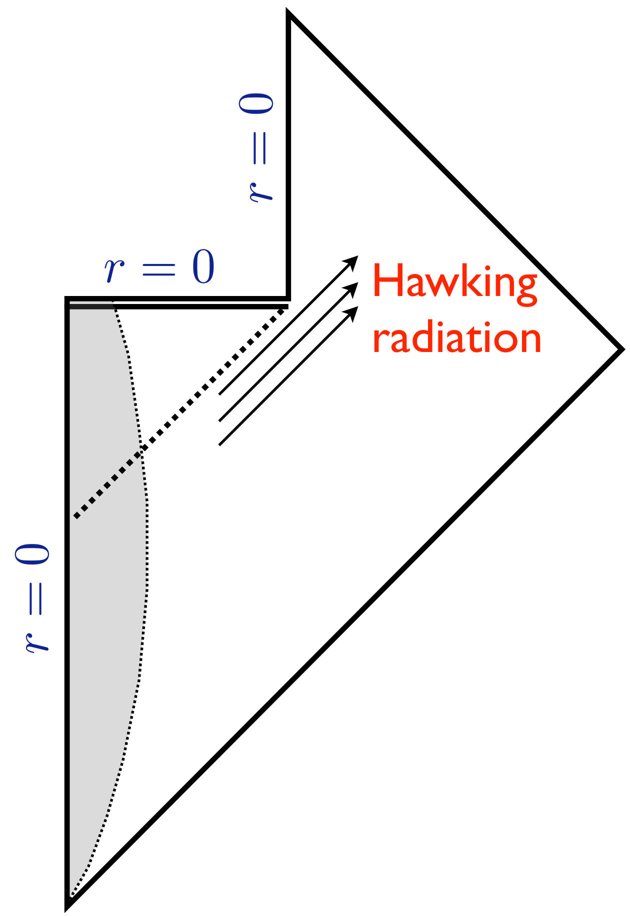
Semi-classical black hole



Formulation of the information paradox - v1

Quantum mechanical unitarity

- if a system starts out in a pure initial state $|\psi_{\rm in}\rangle$
- then the final state $|\psi_{\rm out}\rangle$ is also a pure state

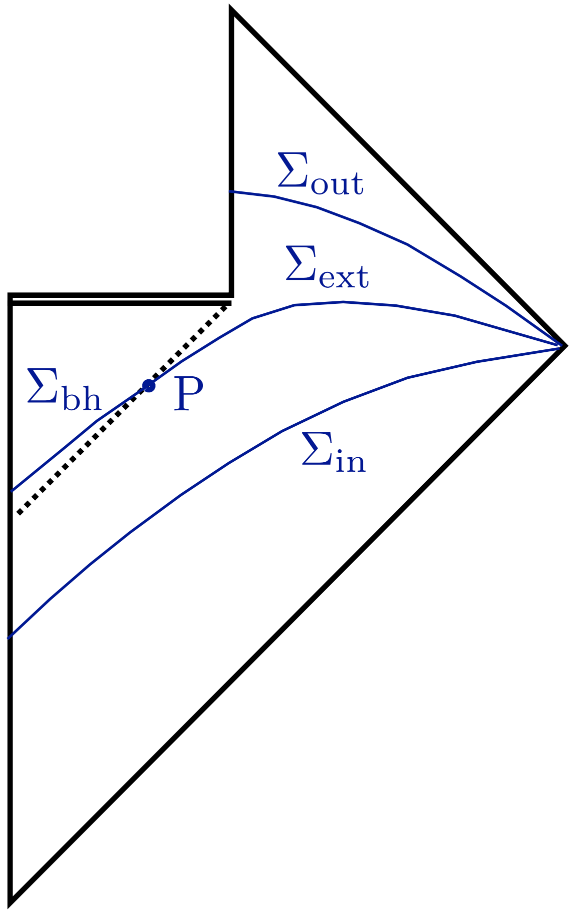
$$|\psi_{\rm out}\rangle = S\,|\psi_{\rm in}\rangle$$

- the S matrix is unitary $\quad S\,S^\dagger \;=\; S^\dagger S \;=\; \mathbf{1}$
- it follows that $\quad |\psi_{\rm in}\rangle = S^\dagger\,|\psi_{\rm out}\rangle$

The final state carries all information about the initial state

Effective field theory

- assume that local effective field theory can be applied in regions of weak curvature, away from black hole singularity
- the explicit form of the effective field theory is not needed
- construct a convenient set of Cauchy surfaces -- ‘nice’ time slices
- effective field theory Hamiltonian generates unitary evolution of states



Assume a pure initial state $|\psi(\Sigma_{\rm in})\rangle$

It evolves into another pure state $|\psi(\Sigma_{\rm P})\rangle$ which has support partially inside and partially outside the horizon on $\Sigma_{\rm bh}$ and $\Sigma_{\rm ext}$

Observables with support on $\Sigma_{\rm ext}$ commute with observables on $\Sigma_{\rm bh}$

The state on $\Sigma_{\rm P}$ is therefore an element of a tensor product Hilbert space

$$|\psi(\Sigma_{\rm P})\rangle \in \mathcal{H}_{\rm bh} \otimes \mathcal{H}_{\rm ext}$$

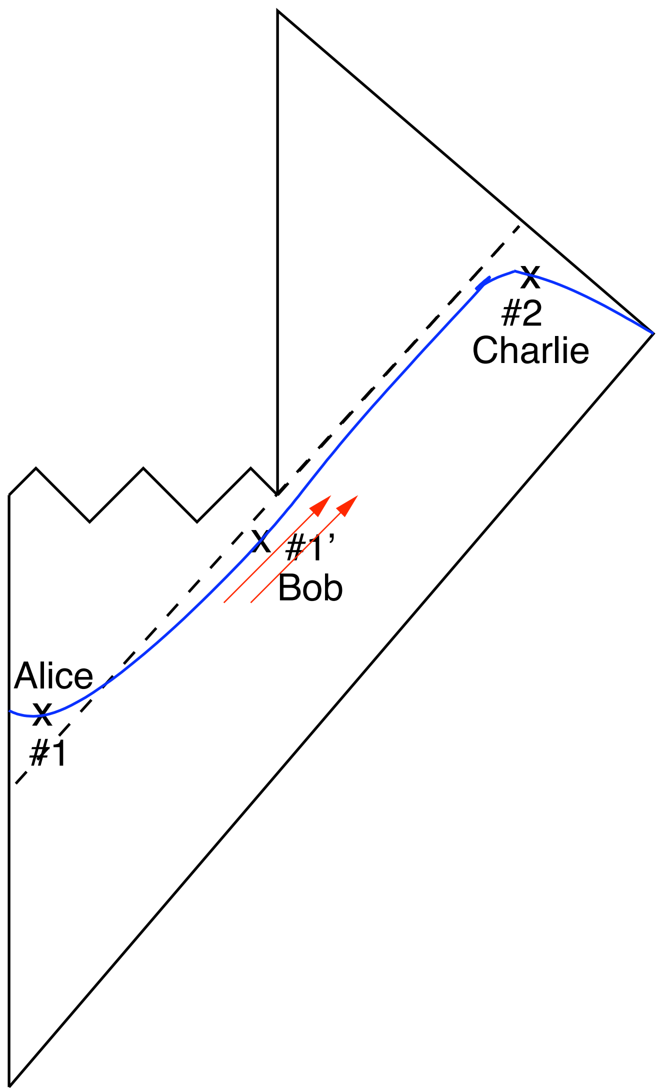
Taking a trace over $\mathcal{H}_{\rm bh}$ results in a mixed state density matrix on $\mathcal{H}_{\rm ext}$

which will then evolve to another density matrix on $\mathcal{H}_{\rm out}$

The net result is that a pure initial state on $\Sigma_{\rm in}$ has evolved into a mixed state on $\Sigma_{\rm out}$

Formulation of the paradox - v2

Assume EFT is valid on nice slices and carry out a gedanken experiment



- prepare singlet pair (#1,#2)

- Charlie takes #2 far away from BH. At an agreed upon time he flips a coin and depending on the outcome he measures #2 along x or z axis

- Alice carries #1 into BH at time of Charlie's measurement and promptly measures #1 along z and broadcasts result

- Bob keeps track of Hawking radiation and measures #1' (quantum clone of #1) along z

- local EFT : independent measurements by Alice and Bob

- Bob enters black hole and receives message from Alice

- if their measurements disagree Bob discovers that Charlie measured #2 along x axis

$\Rightarrow$ acausal signal from Charlie to Bob

Some suggested resolutions

- Non-unitary evolution Hawking ’76
  - generalized quantum mechanics Hawking ’82
- Black hole remnants
  - Planck scale Aharonov, Casher, Nussinov ’87 Banks, O’Loughlin ’93
  - Macroscopic Giddings ’92; Almheiri, Marolf, Polchinski, Sully ’12
- Information returned in Hawking radiation Page ’80, ’t Hooft ’91
  - black hole complementarity Susskind, LT, Uglum ’93; Kiem, Verlinde, Verlinde ’93
  - eternal AdS black holes Maldacena ’01 see Papadodimas lectures
  - final state projection Horowitz, Maldacena ’03
- Non-singular quantum geometry see Mathur lectures
  - supergravity fuzzballs Mathur, Saxena, Srivastava ’03
- Non-violent unitarization Giddings ’17 see Giddings colloquium
- Soft hair on black holes Hawking Perry, Strominger ’16, ’18

Information loss

Purely thermal Hawking radiation implies non-unitary evolution

Hawking '76

Generalized quantum mechanics Hawking '82

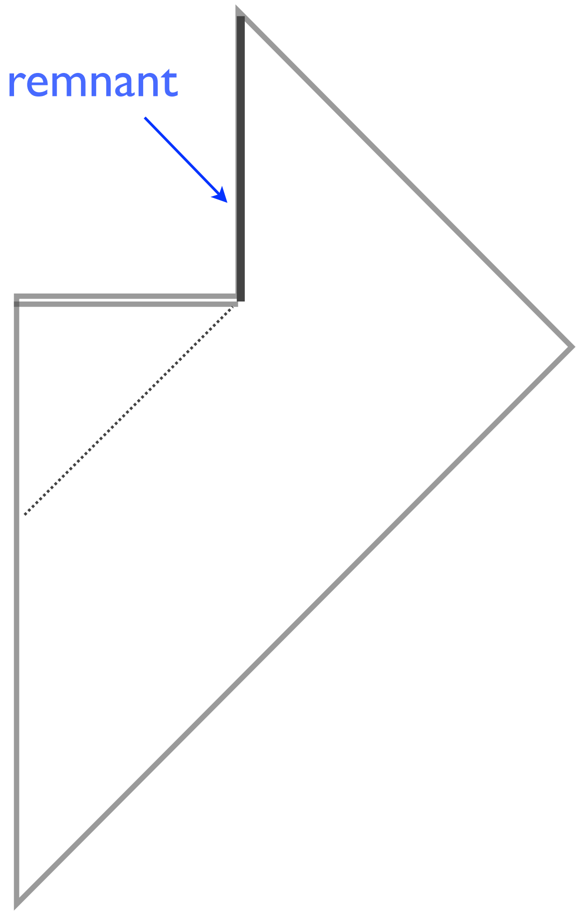
- replace states by density matrices
- replace S matrix by super-scattering operator $

Energy not conserved - vacuum heats up to Planck temperature

Banks, Susskind, Peskin '84
Ellis, Hagelin, Nanopoulos, Srednicki '84

Decoherence without dissipation Unruh, Wald '95; Unruh '12

Black hole remnants

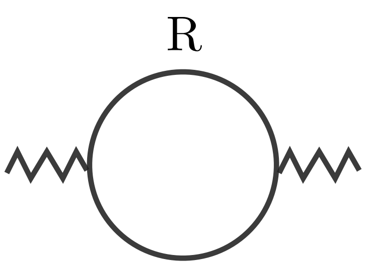


Information about initial state stored in a stable remnant Aharonov, Casher, Nussinov '87

Need a Planck scale remnant for every possible initial black hole

- infinite degeneracy of states

- divergent contribution to quantum loops



$$\text{amplitude} \sim G \sum_R \frac{1}{M_R^2} = \infty$$

Possible loophole: Remnants with large intrinsic geometry Banks, O'Loughlin '92
Hossenfelder, Smolin '09

Information return

Postulates: 't Hooft '90
Susskind, LT, Uglum '93
Kiem, Verlinde, Verlinde '93

1. Black hole evolution, as viewed by a distant observer, is described by quantum theory with a unitary S-matrix relating the state of infalling matter to that of outgoing radiation.
2. Outside the stretched horizon of a massive black hole, physics can be described to good approximation by a set of semi-classical field equations.
3. To a distant observer, a black hole appears to be a quantum system with discrete energy levels. The dimension of the subspace of states that describe a black hole of mass M is

$$\exp\left(\frac{A}{4}\right) = \exp\left(4\pi M^2\right)$$

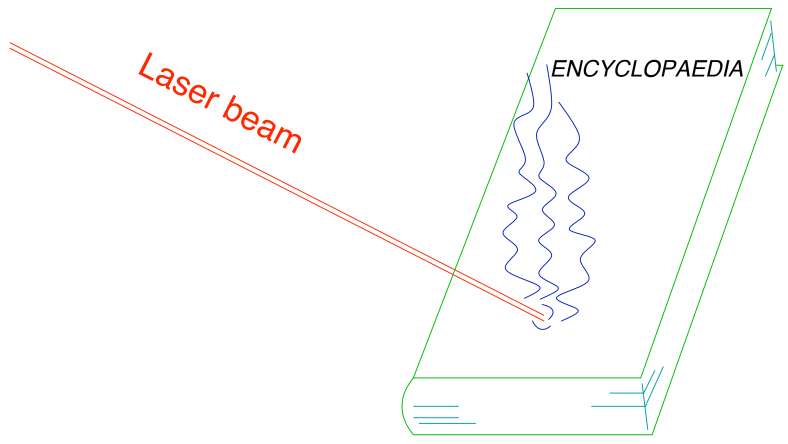
Black hole complementarity

Susskind, LT, Uglum '93

There is no contradiction between outside observers finding information encoded in Hawking radiation and infalling observers entering a black hole unharmed.

- Apparent violation of no-cloning theorem of QM
- Low energy observers in any single reference frame cannot detect duplication of information
- Contradictions only arise when descriptions in very different reference frames are compared
- BHC is consistent with known low-energy physics but implies non-locality and a new degree of relativity in spacetime physics

Information lost to black holes vs. traditional information loss



Emitted radiation appears thermal

For practical purposes information is lost in process

b u t

Final state with outgoing radiation (+ remaining ashes) contains all the information **in principle**

There are subtle correlations between early and late-time radiation

Information carried in the outgoing radiation has been removed from the book

Input from string theory

Black hole entropy Strominger, Vafa ’96

String theory provides a microphysical basis for the entropy of a certain class of (supersymmetric) black holes

$$S_{\rm bh} = \frac{A}{4} = \log\left(\#\ \text{of microstates}\right)$$

-- leaves no room for black hole remnants

Gauge theory / gravity correspondence Maldacena ’97

Non-perturbative string theory defined in terms of unitary quantum field theory

-- bulk reconstruction inside BH horizon remains a challenge

see lectures by Papadodimas

-- bounds on non-local effects in unitary black hole evolution in AdS/CFT Lowe, LT ’99 & ’06

Tests of black hole complementarity

Membrane paradigm   Thorne, Price, MacDonald '82-'86

Replace black hole by a stretched horizon -- a membrane 'near' the event horizon

In astrophysical applications 'near' means close compared to f.ex. distance to companion in a binary system

Quantum mechanical stretched horizon Susskind, LT, Uglum'93

Minimal stretching: $A_{\mathrm{sh}} = A_{\mathrm{eh}} + 1$

Unspecified microphysics with $\# \text{ of states } = \exp(A/4)$

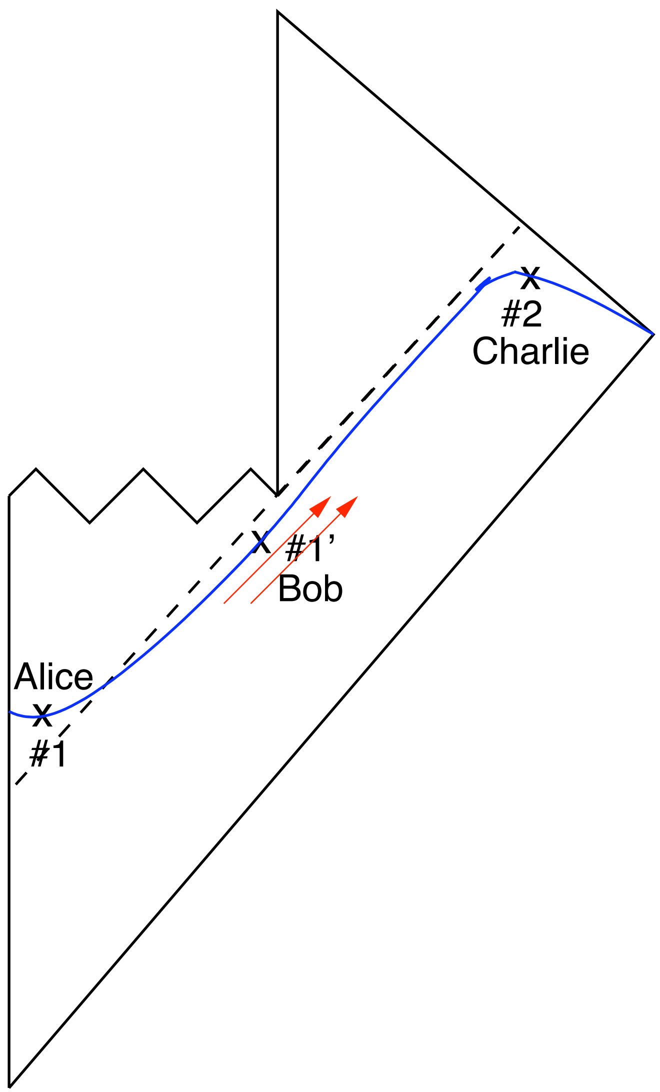
Gedanken experiments   Susskind, LT '93

Apparent violations of BHC can be traced to assumptions about physics at Planck energy (or higher)

Information paradox involves Planck scale in subtle ways

Firewall for infalling observers?

Revisit gedanken experiment



Bob must wait before information can be extracted from Hawking radiation

Young BH: $t \sim r_s S_{bh}$ Page 1993

Old BH: $t \sim r_s \log r_s$ Hayden & Preskill 2007

Alice has short time for spin measurement

Young BH: $\Delta t \sim e^{-S_{bh}}$

Old BH: $\Delta t \sim r_s^{-1}$

$\Rightarrow$ limited measurement accuracy

Charlie can measure state of Hawking radiation to arbitrary accuracy and projects BH state into eigenstate of his measurement operators

Observation of Hawking radiation burns infalling observer at horizon D.Lowe, LT '06

A. Almheiri, D. Marolf, J. Polchinski, J. Sully '12

S.L. Braunstein, S. Pirandola, K. Zyczkowski '12

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A holographic view of the black hole interior

D. Lowe & L.T. - JHEP **1801** (2018) 049
JHEP **1612** (2016) 024
JHEP **1512** (2015) 096
Phys. Lett. B **737** (2014) 320

BHC in a holographic setting

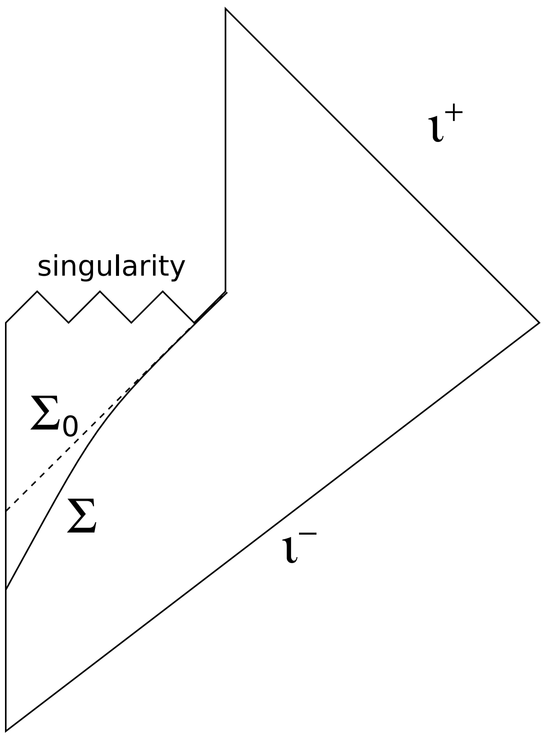
- In a quantum theory, general covariance leads to a conflict between unitarity and locality.
- In holographic models unitarity is preserved at the expense of bulk locality.
- How does the non-locality avoid infecting observations made by low-energy local observers?
- Soft violation of general covariance at finite N in holographic models

  — symmetry is restored in $N \to \infty$ limit.
- Hawking emission is a 1/N effect

  — information paradox cannot be posed in the strict $N \to \infty$ limit.
- The breaking of general covariance is implemented via the holographic reconstruction of the bulk radial direction.
- We model this “holographic regulator” by discretising radial direction.

Modeling the exterior region

D.Lowe & LT '13



The effective field theory of Postulate 2 applies outside stretched horizon Σ

This effective field theory can in principle be obtained from the dual boundary theory.

Model slowly evaporating black hole by static Schwarzschild solution.

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Modeling the black hole interior

L.Susskind '13; D.Lowe, LT '14, '16

#1 : Stretched horizon theory

Black hole interior is encoded in outside dof's

Description of the interior is non-local and employs finite # of dof's

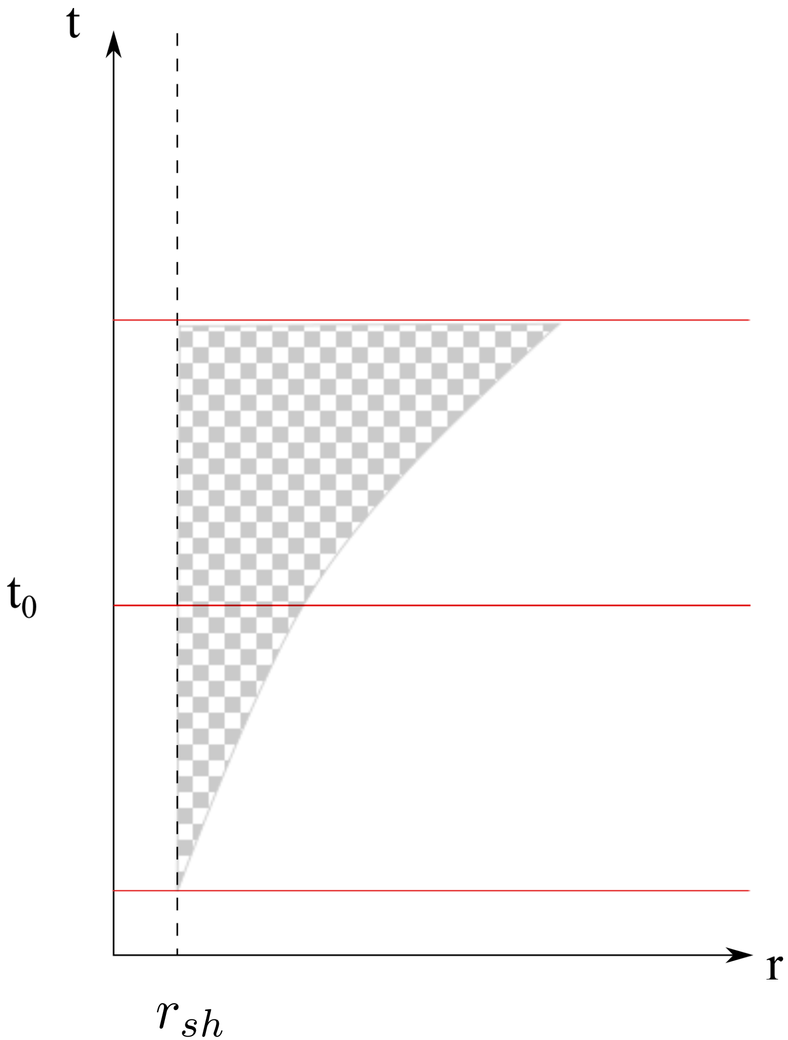
#2 : Local effective field theory (extended inside horizon)

Approximately describes measurements made by a typical observer who falls inside black hole

Applies on a restricted set of time slices with a radial cutoff in place

Fails for an atypical observer who has measured state of BH

Overlap region



Theories #1 and #2 both describe observations in shaded region

Q: Can measurements of Hawking particles emitted during [t_0 - t_{scr} , $t_0 + t_{scr}$] affect O inside the black hole?

A: O will burn at $t \sim t_0 + t_{scr}$, but that is also when O runs into the singularity.

No drama for infalling observer at horizon

Theories #1 and #2 need to have the following properties:

(1) The time required for outside observers to extract quantum information from the black hole (in theory #1) has a lower bound of order the scrambling time.

P. Hayden, J.Preskill '07
D.Lowe, LT '16

(2) From the viewpoint of an infalling observer, who enters the black hole, any quantum information that entered more than a scrambling time earlier has been erased.

Property (2) holds in infalling lattice model D.Lowe, LT '15

Breakdown of bulk description

We want to model a laboratory that falls into a black hole.

Early on the lab is well described by the bulk effective Hamiltonian of theory #2.

The lab has a complementary description in terms of theory #1 and must eventually decohere with respect to the exact Hamiltonian.

This will appear highly non-local from the viewpoint of theory #2.

In a toy model we find that the decoherence time matches the scrambling time, which is also when lab approaches the singularity.

Results support the idea that singularity approach is complementary to decoherence of the infalling state.

Toy model for theory #1

D. Lowe and LT '16
N. Lashkari et al. '13

Spin model with non-local pairwise interactions

$$\mathcal{H} = \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_N$$

$$H = \sum_{\langle x,y \rangle} H_{\langle x,y \rangle}$$

Graph *G(V,E)* with N vertices and E edges corresponding to non-zero $H_{\langle x,y \rangle}$

Degree of vertices: $D = O(N)$

$$\sum_{\langle x,y \rangle} |H_{\langle x,y \rangle}| < \text{ c} \quad \text{at large } N$$

We want to use this model to study the evolution of infalling degrees of freedom.
We conjecture that evolution with respect to the bulk effective Hamiltonian of theory #2 is dual to mean field evolution in the holographic model.

Mean field evolution

Begin with an initial pure state of product form

$$|\Psi(0)\rangle = |\psi_1\rangle_{\mathcal{H}_1} \otimes \cdots \otimes |\psi_N\rangle_{\mathcal{H}_N}$$

Then one may build a state dependent mean field Hamiltonian

$$H^{MF} = \sum_x H_x^{MF}(t)$$
$$H_x^{MF} = \sum_y \mathrm{tr}_y \left(H_{\langle x,y \rangle} \Psi_y^{MF}(t) \right)$$

where Ψ^{MF} evolves according to H^{MF} starting from the same initial state $|\Psi(0)\rangle$. A key point is that with these definitions, and choice of initial state, the mean field Hamiltonian never generates entanglement between different sites, remains in the same product form as the initial state.

It is important to note that not all states yield sensible mean field evolutions. Moreover, the mean field Hamiltonian depends on the state. We conjecture that states close to smooth bulk spacetimes do have useful mean field descriptions, and that the mean field evolution is dual to the usual time evolution with respect to the bulk Hamiltonian.

One then wishes to calculate the timescale for which the trace norm distance between $\Psi_x(t)$ and $\Psi_x^{MF}(t)$ is small.

This problem was solved in [10] via careful application of Lieb-Robinson bounds

$$t \sim \log N$$ N. Lashkari et al. '13

Mean field evolution for infalling laboratory

The observables we will be interested in correspond to experiments conducted in freely falling laboratories, which we may then represent as a site on the lattice where the state is of the form

$$|\psi\rangle = |\psi_{\mathrm{lab}}\rangle \otimes |\psi_{\mathrm{bh}}\rangle, \tag{1.3}$$

where $|\psi_{\mathrm{lab}}\rangle$ is a pure state of a spin on a lattice site. Here the term "spin" is used to represent any finite-dimensional Hilbert subspace, which encodes the full quantum state of the laboratory, and its onsite Hamiltonian can be arbitrary. For a young black hole $|\psi_{\mathrm{bh}}\rangle$ is a pure maximally entangled state on the remaining lattice sites. An old black hole on the other hand is maximally entangled with previously emitted Hawking radiation whose degrees of freedom are not included in the spin model and in this case the state $|\psi_{\mathrm{bh}}\rangle$ should be replaced by a density matrix describing a maximally mixed state.

Breakdown of the mean field evolution

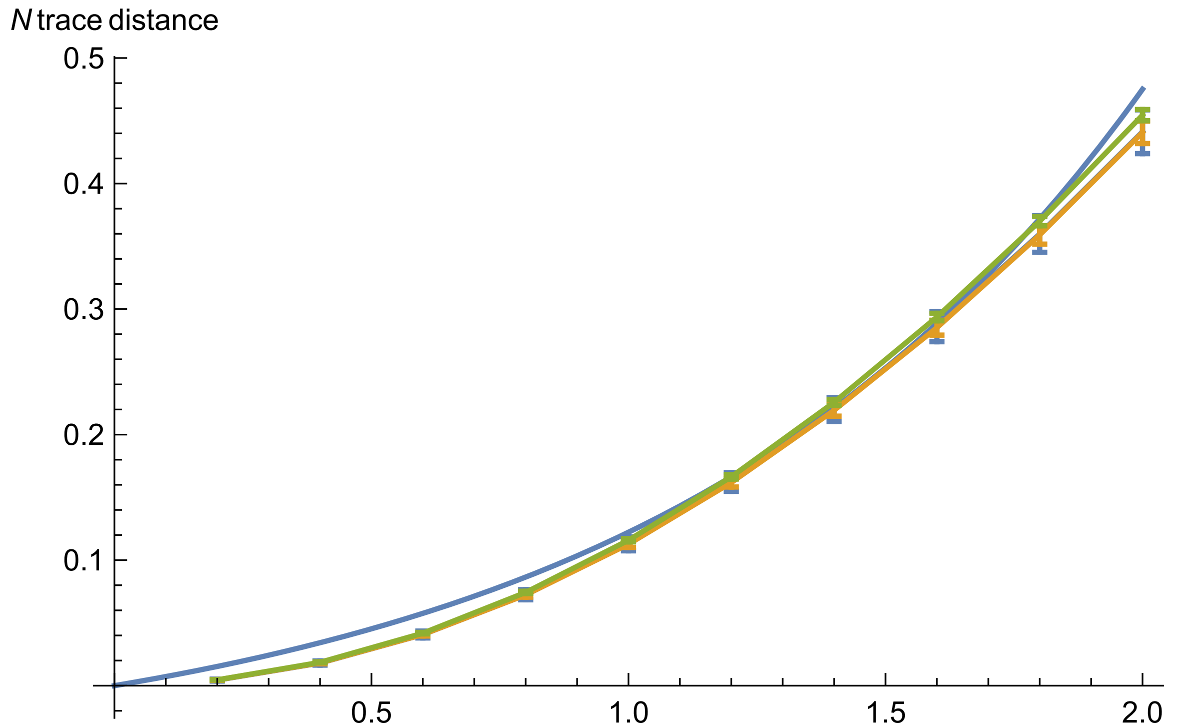


Figure 1. $N \left\| \rho_{\text{lab}}(t) - \rho_{\text{lab}}^{\text{MF}}(t) \right\|_1$ is plotted for various values of $N \leq 14$. The error bars indicate an uncertainty due to picking different initial states. The numerical data is bounded by the curve $0.065\,(\exp(1.06t) - 1)$ shown in blue in the plot.

2.3 Summary

We have provided evidence that the early time behavior of the trace distance takes a universal form as N becomes large. This comes close to saturating a bound of the form $\frac{a}{N}(exp\,(bt) - 1)$ where a, b are N independent constants. The scrambling time $t \sim \log N$ then naturally emerges from this construction.

Summary

The information paradox highlights the incompatibility between general relativity (locality + equivalence principle) and quantum physics (unitarity).

Gauge theory - gravity correspondence implies unitary black hole evolution.

Black hole complementarity provides a "phenomenological" description, which preserves unitarity and the equivalence principle, but requires giving up locality.

Typical infalling observers do not see drama on their way towards a black hole formed from a generic pure state. (Special pure states, as well as special observers, exist for which this is not true.)

An approximate description of observers in the black hole interior can be given in terms of an effective field theory, defined on a limited set of time slices, such that no drama is seen until near the singularity.

A spin system with a nonlocal pairwise interaction provides a toy model for the holographic description of the black hole interior.

Evolution with respect to the bulk effective Hamiltonian is dual to mean field evolution in the holographic model.

Decoherence time for an infalling lab matches black hole scrambling time.

Infalling lattice model

S.Corley & T.Jacobson '97; D.Lowe, LT '15

Coordinate system for infalling observer

$$ds^2 = -dt^2 + v^2(r)dy^2 + r^2 d\Omega^2$$

$$v(r) = -\sqrt{\frac{2M}{r}}$$

$$r(y,t) = 2M\left(1 + \frac{3}{4M}(y-t)\right)^{2/3}$$

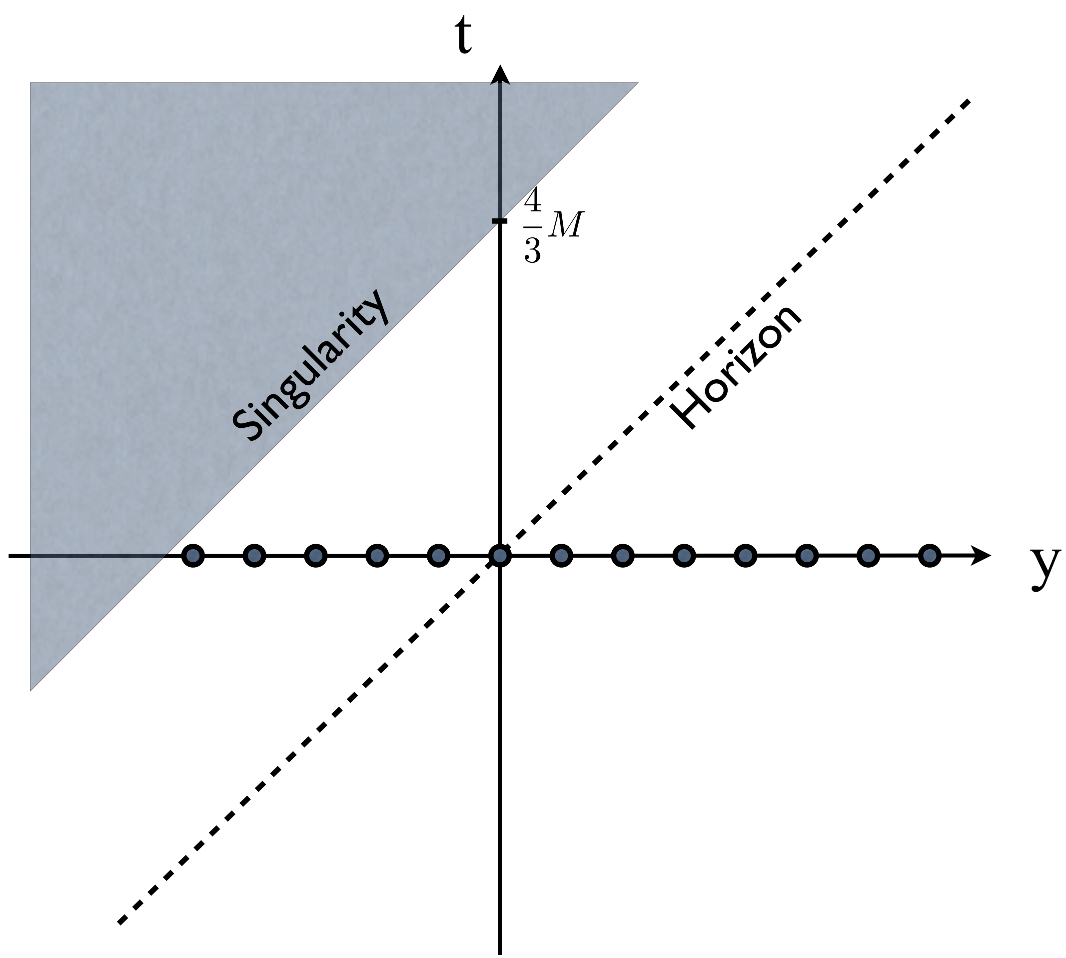
Observer in free fall near horizon: t = proper time, y = constant

Horizon is at $y = t$. Observer enters black hole at $t = y = 0$.

Curvature singularity is at $y = t - 4M/3$.

Lattice model: Discretise y coordinate

Infalling lattice (continued)



Infalling lattice (continued)

Lattice action: $$S = \frac{1}{2}\sum_y \int dt \left(|v(r(y,t))| \left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{2\,(D_y\phi)^2}{|v(r(y+1,t)) + v(r(y,t))|} \right)$$

Killing symmetry: $$(y,t) \to (y+1, t+1)$$

Mode functions: $$\phi(y,t) = e^{-i\omega t} e^{ik(r)(y-t)}$$

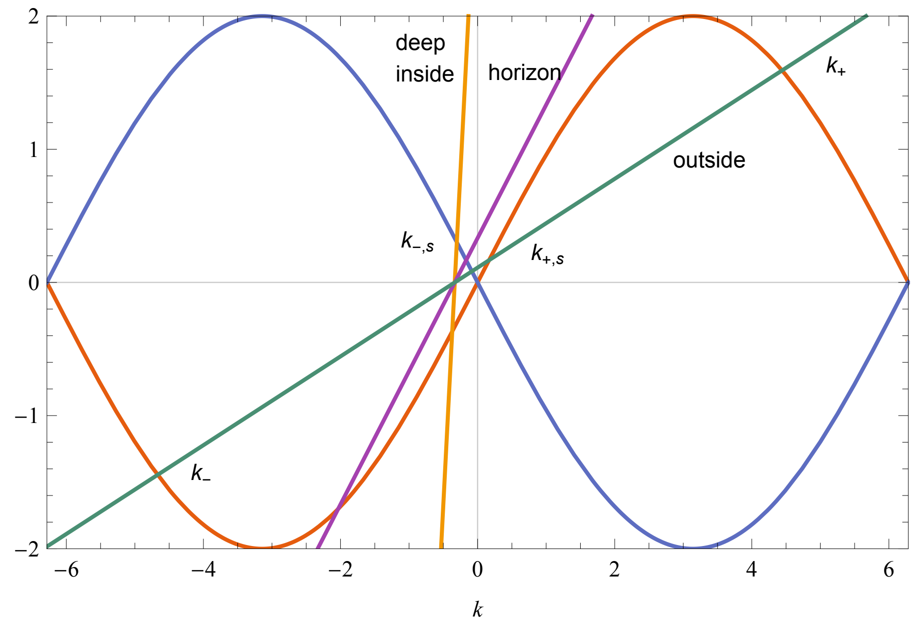
Free fall frequency: $$\omega_{ff} = \omega + k$$

Dispersion relation: $$|v(r)|(\omega + k) = \pm 2\sin(k/2)$$

Group velocity: $$v_g = \frac{d\omega}{dk} = \pm\frac{\cos(k/2)}{|v|} - 1$$

Dispersion relation

$$|v(r)|(\omega + k) = \pm 2\sin(k/2) \qquad v(r) = -\sqrt{\frac{2M}{r}}$$



S.Corley & T.Jacobson '97:

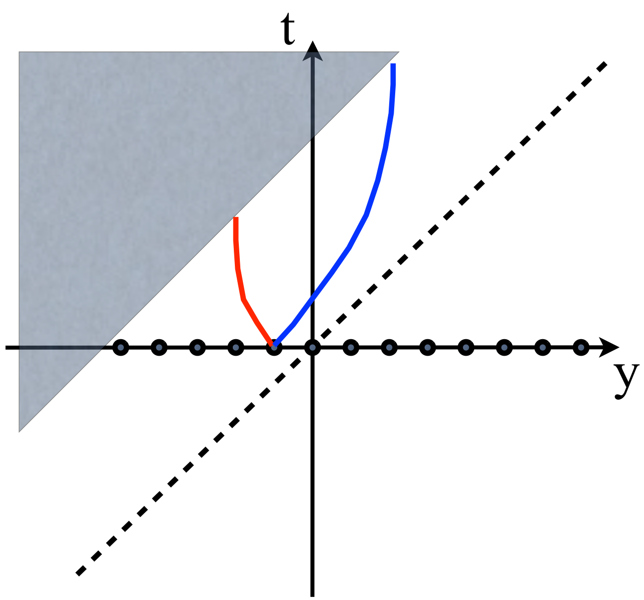
Free-fall vacuum initial state at $t = 0$ gives rise to outgoing thermal flux far outside the black hole.

Follows from WKB analysis of wavepackets outside black hole.

Interior wavepacket trajectories

D.Lowe, LT '15

Left- and right-moving wavepackets start at $t = 0$, $y = -1$

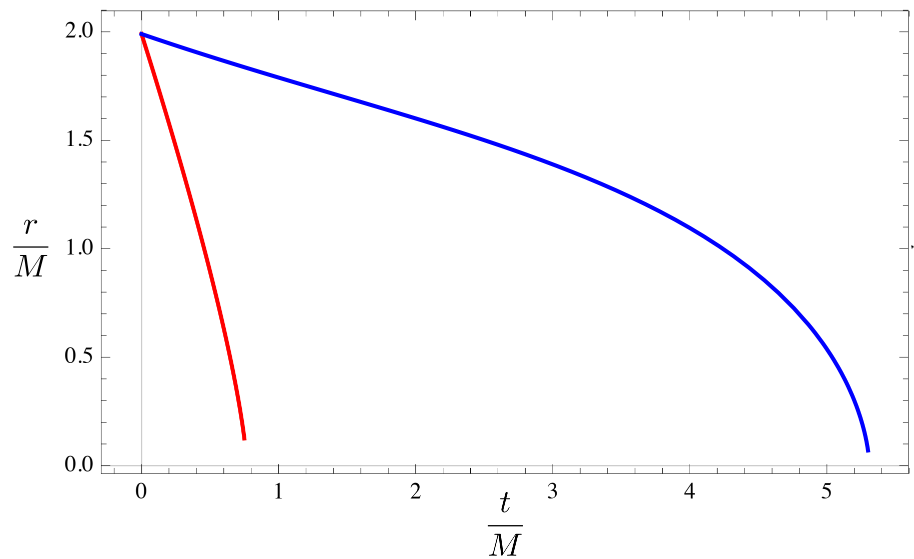


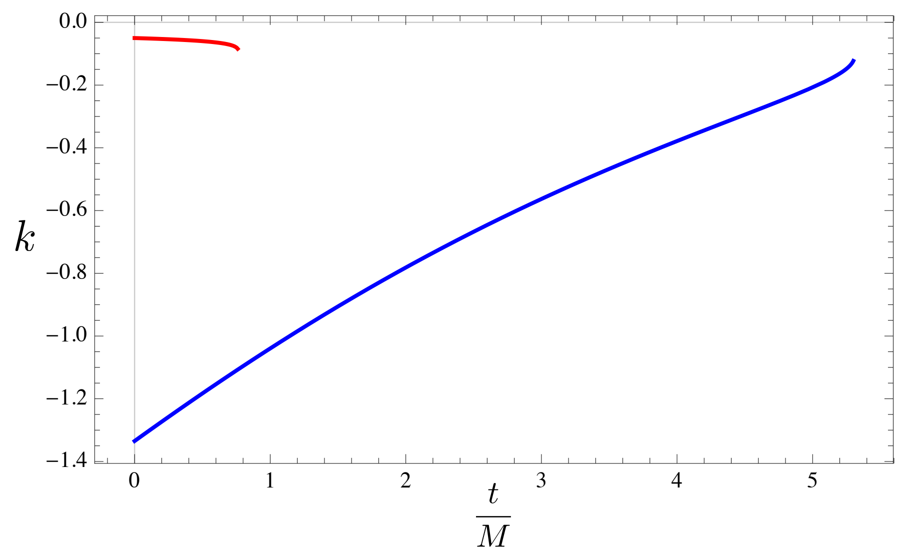
The wavepackets reach the singularity at different times:

Left: $t < 4M/3$

Right: $t < t_{scr} \equiv 4M \log(4M)$

WKB approximation

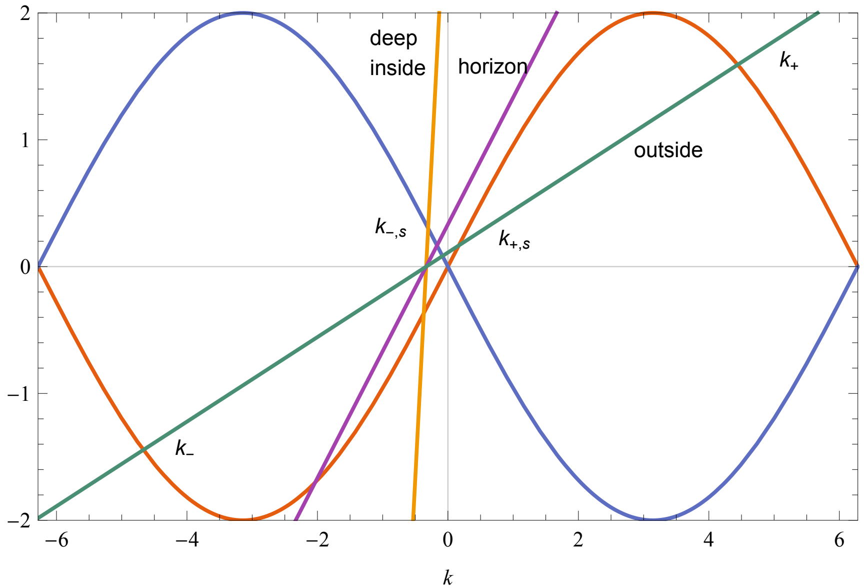




All information about interior quantum state at $t = 0$ is erased by $t = t_{scr}$

Occupation numbers

Solutions to lattice dispersion relation outside b.h.: $\psi_+,\ \psi_-,\ \psi_{+,s}$ and $\psi_{-,s}$



Free-fall frequency: $\omega_{ff} = \omega + k$

$\psi_+,\ \psi_-^*,\ \psi_{+,s}$ and $\psi_{-,s}$ annihilate free-fall vacuum at early time

Use WKB to evolve modes with time

Late time occupation numbers in free-fall vacuum:

$$N_{-,s}(\omega) \approx 0 \qquad \boxed{N_-(\omega) \approx 0} \qquad \boxed{N_{+,s}(\omega) = \frac{1}{e^{\omega/kT} - 1}}$$

D. Lowe & LT '15

S.Corley & T.Jacobson '97

$\boxed{\text{AMPS firewall corresponds to } N_-(\omega) \approx 1}$

D. Marolf & J. Polchinski '13

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