October 10, 2018

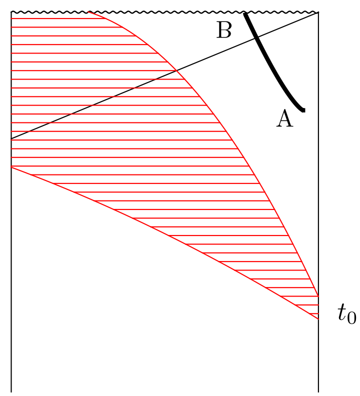
We argued that the black hole information paradox in flat space is closely connected to the question of smoothness of the black hole horizon

Does a big black hole in AdS have a smooth horizon? Can the CFT describe the interior?

Arguments against smooth interior (firewall argument for big AdS BHs)

Proposal for reconstructing the interior (with S. Raju)

Connection with traversable wormholes (with J.de Boer, R. van Breukelen, S. Lokhande, E. Verlinde)



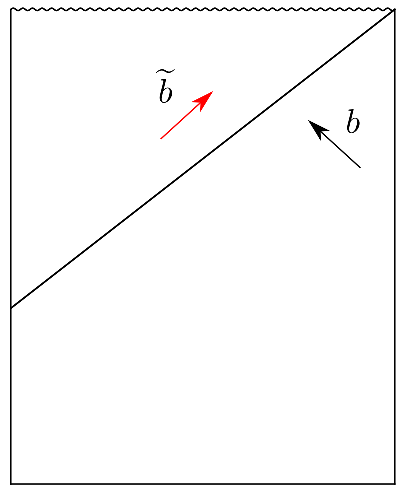
Local bulk field outside horizon of AdS black hole

$$\phi_{\mathrm{CFT}}(t,\Omega,z) = \sum_m \int_0^\infty d\omega\, \mathcal{O}_{\omega,m}\, f^{\beta}_{\omega,m}(t,\Omega,z) + \mathrm{h.c.}$$

At large N (and late times) the correlators

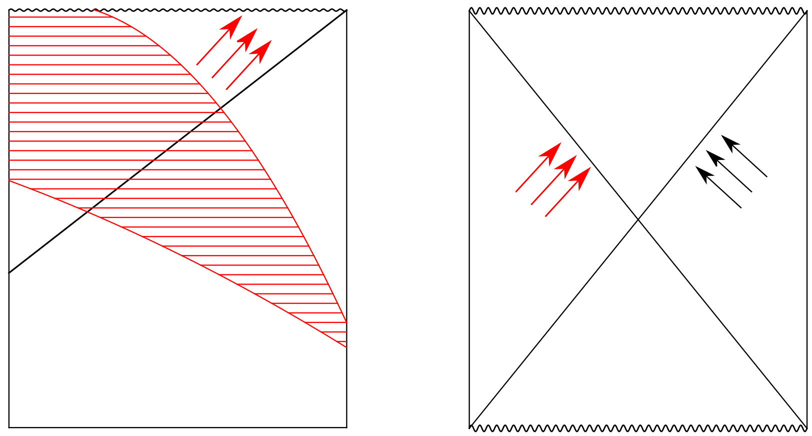
$$\langle\Psi|\phi_{\mathrm{CFT}}(t_1,\Omega_1,z_1)...\phi_{\mathrm{CFT}}(t_n,\Omega_n,z_n)|\Psi\rangle$$

reproduce those of semiclassical QFT on the BH background



We identified b with modes of $\mathcal{O}$ in CFT.

Which CFT operators correspond to $\widetilde{b}$? — whatever these operators are, we denote them as $\widetilde{\mathcal{O}}$.



$$\phi_{\mathrm{CFT}}(t,\Omega,z)=\sum_m\int_0^\infty d\omega\Big[\mathcal{O}_{\omega,m}\,e^{-i\omega t}Y_m(\Omega)g^{(1)}_{\omega,m}(z)+\mathrm{h.c.}$$

$$+\boxed{\widetilde{\mathcal{O}}_{\omega,m}}\,e^{-i\omega t}\,Y_m(\Omega)\,g^{(2)}_{\omega,m}(z)+\mathrm{h.c.}\Big]$$

The operators $\widetilde{\mathcal{O}}_{\omega,m}$ must obey the following conditions, in order for the BH to have a smooth horizon:

1. For every $\mathcal{O}$ there is a $\widetilde{\mathcal{O}}$
2. The algebra of $\widetilde{\mathcal{O}}$'s is isomorphic to that of the $\mathcal{O}$'s
3. The $\widetilde{\mathcal{O}}$'s commute with the $\mathcal{O}$'s
4. The $\widetilde{\mathcal{O}}$'s are "correctly entangled" with the $\mathcal{O}$'s

Equivalently:

Correlators of all these operators on $|\Psi\rangle$ must reproduce (at large N) those of the thermofield-double state

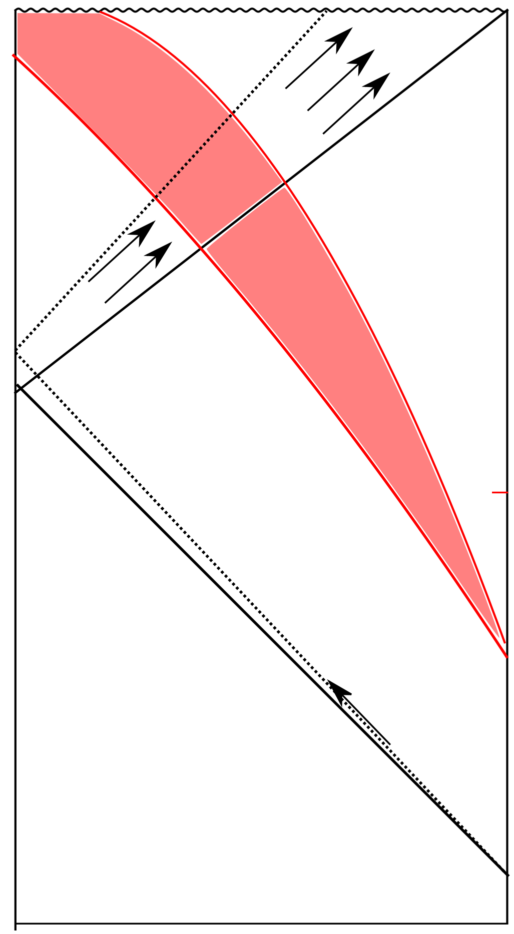
$$|TFD\rangle = \sum_i \frac{e^{-\beta E_i/2}}{\sqrt{Z}} |E_i, \widetilde{E}_i\rangle$$

$$\langle\Psi|\mathcal{O}(t_1)...\widetilde{\mathcal{O}}(t_k)..\mathcal{O}(t_n)|\Psi\rangle \approx \frac{1}{Z}\mathrm{Tr}\big[\mathcal{O}(t_1)...\mathcal{O}(t_n)\mathcal{O}(t_k+i\frac{\beta}{2})...\mathcal{O}(t_m+i\frac{\beta}{2})\big]$$

Main Question: Does the CFT contain the operators $\widetilde{\mathcal{O}}$ with the desired properties?

If so, then the CFT can describe the interior of the black hole and we have free infall through the horizon.

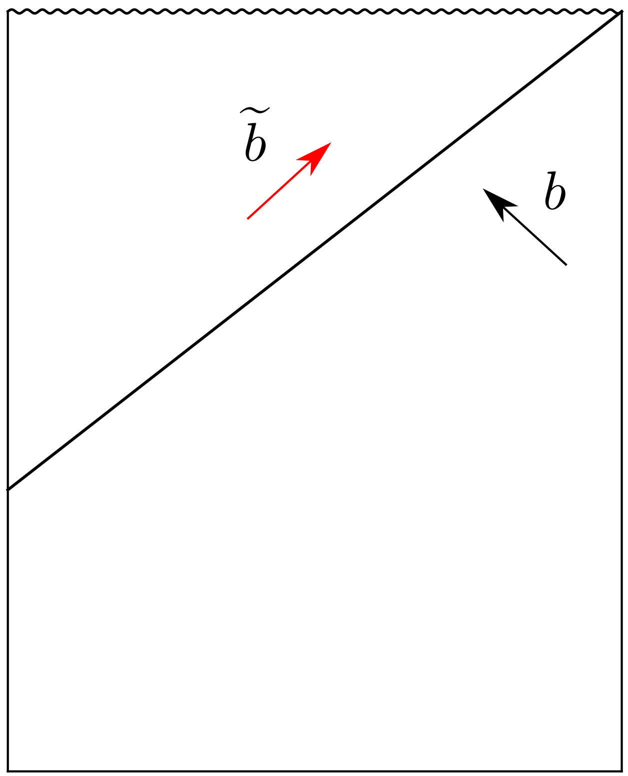
How do we find these operators?



Using bulk EFT evolution to find the $\widetilde{\mathcal{O}}$? $\Rightarrow$ Trans-planckian problem

Typical states vs states formed by collapse

Counting argument, against existence of $\widetilde{b}$ operators in CFT (AMPSS)



$$[b, b^\dagger] = 1$$
$$[H, b^\dagger] = \omega b^\dagger$$

$$[\widetilde{b}, \widetilde{b}^\dagger] = 1$$
$$[H, \widetilde{b}^\dagger] = -\omega \widetilde{b}^\dagger$$

The required algebra between $\widetilde{b}, \widetilde{b}^\dagger, H$ is inconsistent with spectrum of states in CFT

$$[\widetilde{b}, \widetilde{b}^\dagger] = 1 \qquad \Rightarrow \qquad \widetilde{b}^\dagger = \text{"creation operator"}$$

$\Rightarrow \widetilde{b}^\dagger$ should not annihilate (typical) states of the CFT $(*)$.

On the other hand

$$[H, \widetilde{b}^\dagger] = -\omega \widetilde{b}^\dagger$$

implies that $\widetilde{b}^\dagger$ lowers the energy so it maps CFT states of energy E to $E - \omega$.

But in CFT, we have $S(E) > S(E - \omega)$.

$\Rightarrow$ if $\widetilde{b}^\dagger$ is an ordinary linear operator, it must have a nontrivial kernel.

Inconsistent with statement (*).

$\Rightarrow$ The CFT does not contain $\widetilde{b}$ operators and cannot describe the BH interior (?)

Previous counting argument can be made somewhat more precise. The previous algebra implies

$$\mathrm{Tr}[\frac{e^{-\beta H}}{Z}\widetilde{b}^\dagger\widetilde{b}] < 0$$

Related argument $Tr[N_a] \neq 0$ (Bousso, Marolf-Polchinski)

Additional general argument: if $\widetilde{b}$ is a fixed, linear operator, it is hard to understand how $\mathbf{typical}$ CFT states can have the $\mathbf{particular, special}$ entanglement between $b, \widetilde{b}$ needed for smooth interior

These arguments against the existence of a smooth interior for a big black hole in AdS provide a very precise version of the firewall paradox

A proposal for the interior operators

(KP, S. Raju)

If we take a CFT state $|\Psi\rangle$ of $O(N^2)$ energy, we expect that at late times it will thermalize.

$$\langle\Psi|\mathcal{O}_1(x_1)...\mathcal{O}_n(x_n)|\Psi\rangle \approx Z^{-1}\mathrm{Tr}(e^{-\beta H}\mathcal{O}_1(x_1)...\mathcal{O}_n(x_n))$$

This is true only for simple observables $n \ll N$

Thermalization of pure state $\Rightarrow$ must have the notion of a small algebra of observables

In a large N gauge theory, natural small "algebra" $=$ products of few, single trace operators

Even though we are in a single CFT in a pure state, the small algebra of single trace operators probes the pure state $|\Psi\rangle$ as if it were an entangled state

$$\langle\Psi|..|\Psi\rangle \approx Tr[e^{-\beta H}...] \quad \leftrightarrow \quad |TFD\rangle = \sum_E \frac{e^{-\beta E/2}}{\sqrt{Z}}|E\rangle \otimes |E\rangle$$

The $O(N^2)$ d.o.f. of the CFT play the role of the "heat bath" for the small algebra

Whatever operators the single trace operators are entangled with, will play the role of $\widetilde{\mathcal{O}}$ behind the horizon.

How do we identify these operators concretely?

Key algebraic property: the small algebra cannot annihilate the pure state $|\Psi\rangle$

Suppose we have a typical BH microstate $|\Psi\rangle$ and bulk observer at $t = 0$.

Consider possible simple experiments the observer can perform within EFT.

To describe those, we do not need the entire Hilbert space of the CFT, but rather a smaller subspace.

If $\phi(x)$ is a bulk field, the states we need to use are

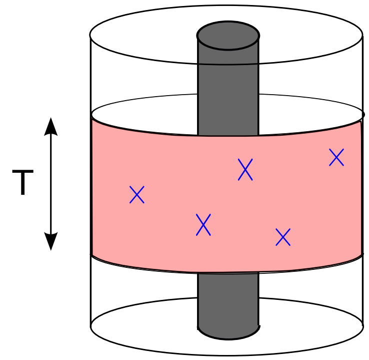
$$\phi(x)|\Psi\rangle$$

$$\phi(x_1)\phi(x_2)|\Psi\rangle, ...$$

$$\phi(x_1)...\phi(x_n)|\Psi\rangle, ...$$

and their linear combinations, where the number of insertions n does not scale with N and the points x_i are not too spread-out in time.

Defining the "small algebra"



In the CFT BH microstate $\rightarrow$ typical QGP state $|\Psi\rangle$

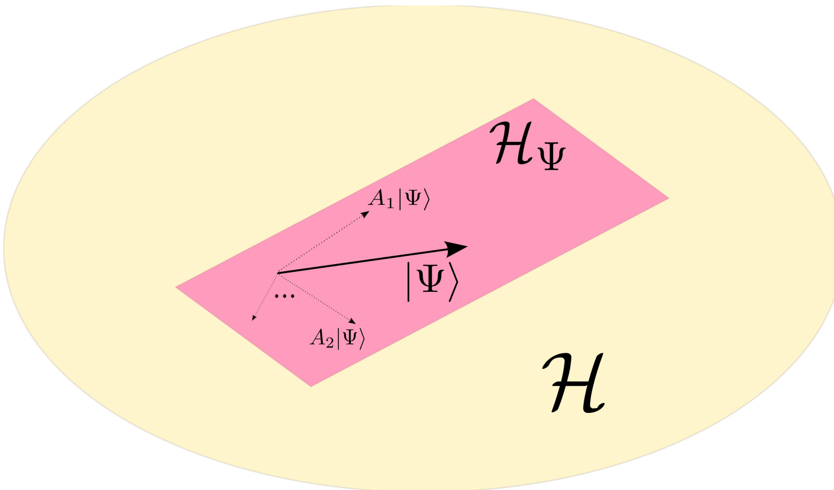
Bulk field ϕ related to boundary single-trace operator $\mathcal{O}$

$\mathcal{A}=$ "algebra" of small products of single-trace operators

$$\mathcal{A} = \text{span of}\{\mathcal{O}(t_1, \vec{x}_1),\ \mathcal{O}(t_1, \vec{x}_1)\mathcal{O}(t_2, \vec{x}_2), ...\}$$

Here T is a long time scale and we also need some UV regularization.

Defining the "small Hilbert space $\mathcal{H}_\Psi$"



We define

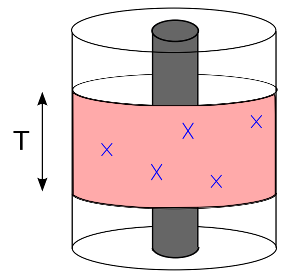
$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle = \{\text{span of} : \mathcal{O}(t_1, \vec{x}_1)...\mathcal{O}(t_n, \vec{x}_n))|\Psi\rangle\}$$

Simple EFT experiments in the bulk, around BH $|\Psi\rangle$ take place within $\mathcal{H}_\Psi$

The interior operators $\widetilde{b}$ will be defined to act only on this subspace.

$\mathcal{H}_\Psi$ is similar to "code subspace"

Important point:



$$\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle = \{\text{span of} : \mathcal{O}(t_1, \vec{x}_1) ... \mathcal{O}(t_n, \vec{x}_n))|\Psi\rangle\}$$

already contains the states describing the BH interior! (i.e. states we would get in bulk EFT by acting with $\widetilde{b}$)

entanglement, compare with Reeh-Schlieder theorem in QFT

The CFT operators that will correspond to $\widetilde{b}$, will act within the subspace $\mathcal{H}_\Psi$

We will call them **mirror operators** and denote them by $\widetilde{\mathcal{O}}$. Notice that $\mathcal{O}, \widetilde{\mathcal{O}}$ must commute.

What is special when $|\Psi\rangle$ is a BH microstate, which allows the "small Hilbert space" $\mathcal{H}_{\Psi} = \mathcal{A}|\Psi\rangle$ to be big enough to accomodate the action of operators $\widetilde{\mathcal{O}}$ which commute with $\mathcal{O}$?

A typical BH microstate $|\Psi\rangle$ cannot be annihilated by (nonvanishing) elements of the small algebra $\mathcal{A}$

This implies that the representation of $\mathcal{A}$ on $\mathcal{H}_\Psi$ has qualitative differences when $|\Psi\rangle$ is a BH microstate, compared to -say- when $|\Psi\rangle$ is the vacuum.

Physical interpretation:

The state $|\Psi\rangle$ appears to be entangled when probed by the algebra $\mathcal{A}$.

Algebras and Representations

We have the small algebra $\mathcal{A}$ acting on $\mathcal{H}_\Psi = \mathcal{A}|\Psi\rangle$, with the property that it cannot annihilate the state.

Ψ is a *cyclic* and *separating* vector.

Tomita-Takesaki theorem $\Rightarrow$ the representation of the algebra is reducible, and the algebra has a nontrivial commutant acting on the same space.

Tomita-Takesaki construction

Define an antilinear map acting on $\mathcal{H}_\Psi$ by

$$SA|\Psi\rangle = A^\dagger|\Psi\rangle \qquad A \in \mathcal{A}$$

we then define

$$\Delta = S^\dagger S, \qquad , \qquad J = S\Delta^{-1/2}$$

where J is (anti)-unitary. Then the operators in the commutant are

$$\boxed{\widetilde{O} = JOJ}$$

The operator Δ is a positive, hermitian operator and can be written as

$$\Delta = e^{-K}$$

where

$$K = \text{modular Hamiltonian}$$

For entangled bipartite system $A \times B$ this construction would give $K_A \sim \log(\rho_A)$ i.e. the usual modular Hamiltonian for A.

In the large N gauge theory and using the KMS condition for correlators of single-trace operators we find that for equilibrium states

$$K = \beta(H_{CFT} - E_0)$$

Constructing the mirror operators

Putting everything together we find that the mirror operators are given by the following set of linear equations

$$\widetilde{\mathcal{O}}_\omega|\Psi\rangle = e^{-\frac{\beta\omega}{2}}\mathcal{O}^\dagger_\omega|\Psi\rangle$$

and

$$\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle = \mathcal{O}...\mathcal{O}\widetilde{\mathcal{O}}_\omega|\Psi\rangle$$

$$[H,\widetilde{\mathcal{O}}_\omega]\mathcal{O}....\mathcal{O}|\Psi\rangle = \omega\widetilde{\mathcal{O}}_\omega\mathcal{O}....\mathcal{O}|\Psi\rangle$$

These conditions are self-consistent because $A|\Psi\rangle \neq 0$, which in turns relies on

1. The algebra $\mathcal{A}$ is not too large
2. The state $|\Psi\rangle$ is complicated (this definition would not work around the ground state of CFT)

Reconstructing the interior

Using the $\mathcal{O}_\omega$'s and $\widetilde{\mathcal{O}}_\omega$'s we can reconstruct the black hole interior by operators of the form

$$\phi(t,r,\Omega) = \sum_m \int_0^\infty d\omega \Big[\mathcal{O}_{\omega,m}\, e^{-i\omega t} Y_m(\Omega) g^{(1)}_{\omega,m}(r) + \mathrm{h.c.}$$

$$+\widetilde{\mathcal{O}}_{\omega,m}\, e^{-i\omega t}\, Y_m(\Omega)\, g^{(2)}_{\omega,m}(r) + \mathrm{h.c.}\Big]$$

Low point functions of these operators reproduce those of effective field theory in the interior of the black hole

$\Rightarrow$

Smooth interior

Nothing dramatic when crossing the horizon

State dependence of construction

The operators $\widetilde{\mathcal{O}}$ are defined as linear operators acting only on the "small Hilbert space" around any given state.

Different microstate — different "small Hilbert space"— different linear operators $\widetilde{\mathcal{O}}$

Can we stitch them together into globally defined (linear) operators?

NO , e^S states, overlaps between $\mathcal{H}_\Psi$'s too large, remember previous counting arguments and paradoxes

(but generically can be done for small subsets of states)

How state dependence resolves counting paradoxes

1. Counting argumen about $\widetilde{b}^\dagger$ lowering energy

2. $\mathrm{Tr}(N_a) \neq 0$ argument

3. Explains how we get correct entanglement for typical states since $\widetilde{b}$ operators (partly) "selected by entanglement"

Realization of Complementarity

The operators $\widetilde{\mathcal{O}}$ seem to commute with the $\mathcal{O}$'s

This is only approximate: the commutator $[\mathcal{O}, \widetilde{\mathcal{O}}] = 0$ only inside low-point functions i.e. in the "small Hilbert space" $\mathcal{H}_\Psi$

If we consider N^2-point functions, then we find that the construction cannot be performed since we will violate

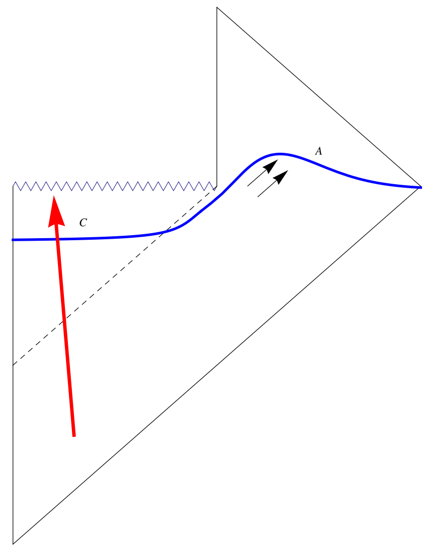
$$A|\Psi\rangle \neq 0, \qquad \text{for} \qquad A \neq 0$$

or in a sense we will find that $[\mathcal{O}, \widetilde{\mathcal{O}}] \neq 0$ inside complicated correlators.

Relatedly, we can express the $\widetilde{\mathcal{O}}$'s as very complicated combination of $\mathcal{O}$'s.

Simple vs Complicated experiments and decomposition of Hilbert space in interior$\times$ exterior

Black Hole interior is not independent Hilbert space, but highly scrambled version of part of exterior



In this construction:

Exterior of black hole $\Rightarrow$ local operators ϕ

Interior of black hole $\Rightarrow$ local operators $\widetilde{\phi}$

In low-point correlators ϕ, $\widetilde{\phi}$ seem to be independent

If we act with too many (order S_{BH}) of ϕ's we can "reconstruct" the $\widetilde{\phi}$'s

Complementarity can be realized consistently with locality in effective field theory, non-locality is acceptably small for simple experiments.

Quantum cloning/strong subadditivity paradox of Mathur/AMPS avoided, as interior and exterior are not independent Hilbert spaces

AMPS thought experiment: If observer extracts scrambled qubit (complicated experiment), the interior is (non-locally) modified and infalling observer does not see cloning

We described a scenario where black hole evaporation is unitary while we only have small corrections to simple experiments in effective field theory.

Mathur's theorem

" Small corrections to Hawking's computation cannot restore unitarity"

This theorem assumes:
1) Locality/Independence of Hilbert spaces (it uses strong subadditivity)
2) State independence

The previous construction does not satisfy these assumptions.

Questions

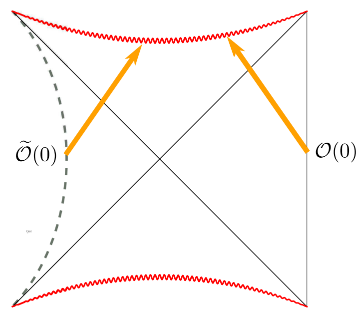
1. What is the dynamical principle that selects the state-dependent operators for smooth infall? Not only enantanglement, otherwise "frozen vacuum" [Bousso]. Quantum Mechanics for the infalling observer, decoherence....

2. Technical question: According to previous proposal, does bulk observer detect any deviations from linearity of QM?

3. More technical questions (details of coarse-graining, precise identification of equilibrium states, $1/N$ corrections,...)

Creating negative energy shockwaves for 1-sided black hole

[J. de Boer, R. van Breukelen, S. Lokhande, KP, E. Verlinde]

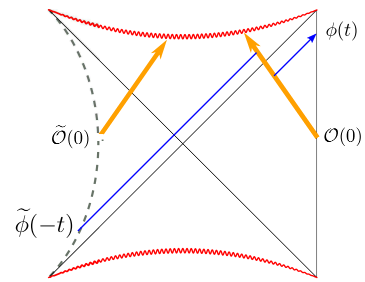


At $t = 0$ we perturb CFT Hamiltonian by

$$g\mathcal{O}\widetilde{\mathcal{O}}(0)$$

Compute effect on bulk correlators $\Rightarrow$ generates negative energy shockwaves for appropriate choice of g. Computation of $\langle T_{\mu\nu}\rangle_{\text{bulk}}$ similar to that of Gao-Jafferis-Wall

The experiment



We create a probe in the left region of the black hole by acting with $\widetilde{\phi}(-t)$.

Then at $t = 0$ we perturb the CFT by $g\mathcal{O}(0)\widetilde{\mathcal{O}}(0)$. Finally we detect the probe by measuring $\phi(t)$.

The conjectured Penrose diagram makes a prediction about CFT correlators (signal around $t = \beta \log S$)

$$\langle \Psi_0 | [\widetilde{\phi}(-t), e^{-ig\widetilde{\mathcal{O}}\mathcal{O}(0)} \phi(t) e^{ig\widetilde{\mathcal{O}}\mathcal{O}(0)}] | \Psi_0 \rangle$$