The fuzzball paradigm

Lecture III

Samir D. Mathur

The Ohio State University

Previous lecture

The size of bound states grows with the number of branes



Fractionation generates low tension objects that can stretch far

2-charge NSI-P extremal hole



covering space

A NSI string carrying the momentum P in the form of travelling waves



We do not find a horizon

The states are not spherically symmetric

Generic states will have structure at the string scale, but we can estimate the size of the region over which the metric deformations are nontrivial







Simple fuzzball state

More complicated fuzzball state

Modes evolve like in a piece of coal, so there is no information problem





DI-D5-P states

Avery, Balasubramanian, Bena, Carson, Chowdhury, de Boer, Gimon, Giusto, Guo, Hampton, Keski-Vakkuri, Levi, Lunin, Maldacena, Maoz, Martinec, Niehoff, Park, Peet, Potvin, Puhm, Ross, Ruef, Russo, Saxena, Shigemori, Simon, Skenderis, Srivastava, Taylor, Turton, Vasilakis, Virmani, Warner ...



NSI-P ←→ DI-D5



(a)
$$n_1 \rightarrow n_5'$$

 $n_p \rightarrow n_1'$

(b) A mode α^{i}_{-k} maps to a loop with winding k and spin in the direction i

(c) NSI-P:







Dualize the geometries



(Rhychkov 06)

Elementary objects in IIB string theory



Any one of these objects can be mapped to any other by S,T dualities, which are exact symmetries of the theory

General states:





DI-D5-P extremal

DI-D5-P near-extremal





$$[J_{-2n+1}^+ \dots J_{-1}^+ | 0^+ \rangle_R]^{n_1 n_5}$$

(Giusto, SDM, Saxena 04)



(using the pp-wave formalism)





$$|\Psi\rangle = \bar{J}_{-n_R}^+ \dots \bar{J}_{-1}^+ J_{-n_L}^+ \dots J_{-1}^+ [\sigma_k]^{\frac{N}{k}} |0\rangle_R$$



(Jejalla, Madden, Ross Titchener '05)



General program pioneered by Bena+Warner: find large families of solutions.

Several different innovative techniques used

A toy model

Start with the 3+1 dimensional Schwarzschild metric

$$ds^{2} = -(1 - \frac{r_{0}}{r})dt^{2} + \frac{dr^{2}}{1 - \frac{r_{0}}{r}} + r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2})$$

Make the analytic continuation $t \rightarrow -i\tau$

$$ds^{2} = (1 - \frac{r_{0}}{r})d\tau^{2} + \frac{dr^{2}}{1 - \frac{r_{0}}{r}} + r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2})$$

Let the au direction be a circle $0 \le au < 4\pi r_0$



This gives the 4-d Euclidean Schwarzschild geometry

Dimensional reduction



Dimensional reduction of the τ circle gives a scalar Φ in the 3+1 spacetime

$$g_{\tau\tau} = e^{\frac{2}{\sqrt{3}}\Phi}$$

$$\Phi = \frac{\sqrt{3}}{2}\ln(1 - \frac{r_0}{r})$$

The 3+1 metric is defined as

$$g^E_{\mu\nu} = e^{\frac{1}{\sqrt{3}}\Phi}g_{\mu\nu}$$

$$ds_E^2 = -(1 - \frac{r_0}{r})^{\frac{1}{2}}dt^2 + \frac{dr^2}{(1 - \frac{r_0}{r})^{\frac{1}{2}}} + r^2(1 - \frac{r_0}{r})^{\frac{1}{2}}(d\theta^2 + \sin^2\theta d\phi^2)$$

The action is

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left(R_E - \frac{1}{2} \Phi_{,\mu} \Phi^{,\mu} \right)$$



The stress tensor is the standard one for a scalar field

$$T_{\mu\nu} = \Phi_{,\mu}\Phi_{,\nu} - \frac{1}{2}g^E_{\mu\nu}\Phi_{,\lambda}\Phi^{,\lambda}$$

which turns out to be

$$T^{\mu}{}_{\nu} = \text{diag}\{-\rho, p_r, p_{\theta}, p_{\phi}\} = \text{diag}\{-f, f, -f, -f\}$$
$$f = \frac{3r_0^2}{8r^4(1 - \frac{r_0}{r})^{\frac{3}{2}}}$$

(a) We see that the energy density and radial pressure are positive. The tangential pressures are negative

(b) All these quantities diverge as we reach the tip of the cigar.

(c) g_{tt} never changes sign, so there is no horizon

(SDM 16)

So what happened to Buchdahl's theorem?



Because the radial pressure diverged, Buchdahl would have discarded this solution as unphysical.

But we see that the problem is with the dimensional reduction: the full spacetime is completely smooth



Hawking radiation from fuzzballs

Recall our difficulty with the information paradox ...



The average rate of radiation is the same in both cases, but the detailed mechanism of radiation is very different

(a) We take a simple CFT state at weak coupling



(b) We know its metric at strong coupling ... it does not have a horizon as naively expected, but a 'cap'



$$\begin{aligned} \mathrm{d}s^{2} &= -\frac{f}{\sqrt{\tilde{H}_{1}\tilde{H}_{5}}}(\mathrm{d}t^{2}-\mathrm{d}y^{2}) + \frac{M}{\sqrt{\tilde{H}_{1}\tilde{H}_{5}}}(s_{p}\mathrm{d}y-c_{p}\mathrm{d}t)^{2} \\ &+ \sqrt{\tilde{H}_{1}\tilde{H}_{5}}\left(\frac{r^{2}\mathrm{d}r^{2}}{(r^{2}+a_{1}^{2})(r^{2}+a_{2}^{2})-Mr^{2}} + \mathrm{d}\theta^{2}\right) \\ &+ \left(\sqrt{\tilde{H}_{1}\tilde{H}_{5}} - (a_{2}^{2}-a_{1}^{2})\frac{(\tilde{H}_{1}+\tilde{H}_{5}-f)\cos^{2}\theta}{\sqrt{\tilde{H}_{1}\tilde{H}_{5}}}\right)\cos^{2}\theta\mathrm{d}\psi^{2} \\ &+ \left(\sqrt{\tilde{H}_{1}\tilde{H}_{5}} + (a_{2}^{2}-a_{1}^{2})\frac{(\tilde{H}_{1}+\tilde{H}_{5}-f)\sin^{2}\theta}{\sqrt{\tilde{H}_{1}\tilde{H}_{5}}}\right)\sin^{2}\theta\mathrm{d}\phi^{2} \\ &+ \frac{M}{\sqrt{\tilde{H}_{1}\tilde{H}_{5}}}(a_{1}\cos^{2}\theta\mathrm{d}\psi + a_{2}\sin^{2}\theta\mathrm{d}\phi)^{2} \\ &+ \frac{2M\cos^{2}\theta}{\sqrt{\tilde{H}_{1}\tilde{H}_{5}}}[(a_{1}c_{1}c_{5}c_{p}-a_{2}s_{1}s_{5}s_{p})\mathrm{d}t + (a_{2}s_{1}s_{5}c_{p}-a_{1}c_{1}c_{5}s_{p})\mathrm{d}y]\mathrm{d}\psi \\ &+ \frac{2M\sin^{2}\theta}{\sqrt{\tilde{H}_{1}\tilde{H}_{5}}}[(a_{2}c_{1}c_{5}c_{p}-a_{1}s_{1}s_{5}s_{p})\mathrm{d}t + (a_{1}s_{1}s_{5}c_{p}-a_{2}c_{1}c_{5}s_{p})\mathrm{d}y]\mathrm{d}\phi \\ &+ \sqrt{\frac{\tilde{H}_{1}}{\tilde{H}_{5}}}\sum_{i=1}^{4}\mathrm{d}z_{i}^{2} \end{aligned}$$

 $= \frac{g\alpha'^3}{V}n_1$

 $= g \alpha' n_5$

 $= \frac{g^2 \alpha'^4}{V R^2} n_p$

$$\tilde{H}_i = f + M \sinh^2 \delta_i, \quad f = r^2 + a_1^2 \sin^2 \theta + a_2^2 \cos^2 \theta$$

 $Q_1 = M \sinh \delta_1 \cosh \delta_1, \quad Q_5 = M \sinh \delta_5 \cosh \delta_5, \quad Q_p = M \sinh \delta_p \cosh \delta_p$

As in any statistical system, each microstate radiates a little differently

$$\Gamma_{CFT} = V \ \rho_L \ \rho_R$$

Emission vertex

Occupation numbers of left, right excitations Bose, Fermi distributions for generic state



$$\Gamma_{CFT} = V \ \bar{\rho}_L \ \bar{\rho}_R$$

Occupation numbers for this particular microstate

Emission from the special microstate is peaked at definite frequencies and grows exponentially, like a laser

Radiation from the special microstate'



(A) The emitted frequencies are peaked at

$$\omega \simeq \omega_R = \frac{1}{R} \left(-l - m_{\psi} m + m_{\phi} n - | -\lambda - m_{\psi} n + m_{\phi} m | -2(N+1) \right)$$

(B) Emission rate grows exponentially with time because after n de-excited strings have been created, the probability for creating the next one is Bose enhanced by (n+1) $\hat{a}^{\dagger}|n\rangle = \sqrt{n+1}|n+1\rangle$



Emission rate grows as

 $\exp[\omega_{\tau}^{CF'T}t]$





Ergoregions produce particle pairs



A quantum rotating opposite to the ergoregion slows down rotation, and so decreases energy: Net negative energy as seen from infinity

Ergoregion instability: the produced waves grow exponentially

Radiation: The gravity calculation

$$M_{9,1} \to M_{4,1} \times T^4 \times S^1$$

Graviton with indices on the torus is a scalar in 6-d

$$h_{12} \equiv \Psi$$

 $\Box \Psi = 0$



$$M_{4,1} \to t, r, \theta, \psi, \phi$$

$$S^{1} \to y \quad y: \ (0, 2\pi R)$$

$$\Psi = exp(-i\omega t + i\lambda \frac{y}{R} + im_{\psi}\psi + im_{\phi}\phi)\chi(\theta)h(r)$$

(Solve by matching inner and outer region solutions)



$$\omega \simeq \omega_R = \frac{1}{R} \left(-l - m_{\psi} m + m_{\phi} n - | -\lambda - m_{\psi} n + m_{\phi} m | -2(N+1) \right)$$

$$\omega_I = \frac{1}{R} \left(\frac{2\pi}{[l!]^2} \left[(\omega^2 - \frac{\lambda^2}{R^2}) \frac{Q_1 Q_5}{4R^2} \right]^{l+1} \quad {}^{l+1+N} C_{l+1} \quad {}^{l+1+N+|\zeta|} C_{l+1} \right)$$

These exact match the CFT values

(Chowdhury+SDM '07)









Thus the microstate radiates like a piece of coal; there is no information problem



How is the semiclassical approximation violated at the horizon?



In 1972, Bekenstein taught us that black holes have an entropy

$$S = \frac{c^3}{\hbar} \frac{A}{4G} \sim \frac{A}{l_p^2}$$

This means that a solar mass black hole has $\ \sim 10^{10^{144}}$ states

This is far larger than the number of states of normal matter with the same energy

Consider a collapsing shell



As it approaches the horizon radius, there is a small amplitude for it to tunnel into a fuzzball state


Amplitude for tunneling

$$\mathcal{A} \sim e^{-S_{tunnel}}$$
 $S_{tunnel} \sim \frac{1}{16\pi G} \int d^4x \sqrt{-g} \mathcal{R}$

Let us set all length scales to $\ L \sim GM$

$$\int d^4x \sqrt{-g} \sim (GM)^4 \qquad \qquad \mathcal{R} \sim \frac{1}{(GM)^2}$$

$$\implies S_{tunnel} \sim GM^2$$

The probability of tunneling into the chosen fuzzball state is

$$P_{tunnel} = |\mathcal{A}_{tunnel}|^2 \sim e^{-2S_{tunnel}}$$

Using our estimate

$$P_{tunnel} \sim e^{-\alpha GM^2}$$

We should now multiply thus by the number of fuzzball states we can tunnel to

$$\mathcal{N} \sim e^{S_{bek}} \sim e^{\frac{A}{4G}} \sim e^{\frac{4\pi (GM)^2}{G}} \sim e^{4\pi GM^2}$$

We thus see that it is possible for the total probability for tunneling into fuzzballs can be order unity

$$\mathcal{PN} \sim 1$$
 (SDM 0805.3716)

Toy model

Small amplitude to tunnel to a neighboring well, but there are a correspondingly large number of adjacent wells



In a time of order unity, the wavefunction in the central well becomes a linear combination of states in all wells

We call this phenomenon 'Entropy enhanced tunneling'

An argument can be made using Kraus-Wilczek tunneling from black holes that the exponentials exactly cancel

(Kraus+SDM 15)

For simple families of fuzzball microstates the entropy enhanced tunneling has been explicitly calculated

(Bena, Mayerson, Puhm, Vernocke 15)



For traditional macroscopic objects the measure is order \hbar while the action is order unity

But for black holes the entropy is so large that the two are comparable ...

Thus the black hole is not a semiclassical object

A pictorial description of 'entropy enhanced tunneling'

A pictorial description of 'entropy enhanced tunneling'





(1) Shell far outside horizon, semiclassical collapse

(2) As shell approaches its horizon, there is a nucleation of Euclidean Schwarzschild 'bubbles' just outside the shell



(3) The bubbles cost energy, which is drawn from the energy of the shell. The shell now has a lower energy, which corresponds to a horizon radius that is smaller. The shell thus moves inwards without forming a horizon



(4) As the shell reaches close to its new horizon, more bubbles nucleate, and so on.



(5) Instead of a black hole with horizon, we end up with a horizon sized structure which has no horizon or singularity

The causality paradox

The causality problem



- So information cannot come out
- When the hole evaporates away, what happens to the information in the shell?

In quantum gravity do we have to stay inside the light cone?

(A) Perturbative quantum gravity





Light cones in flat spacetime $\eta_{\mu
u}$

Light cones fluctuate under $\eta_{\mu\nu} + h_{\mu\nu}$

Can we have small violations of causality?

No: We can quantize $h_{\mu\nu}$ and we will find that its causal propagators vanish outside the light cone

(B) Nonperturbative quantum gravity:

Bubble nucleation: false vacuum (red) changes to true vacuum (brown)



bubble surface does not move faster than light

General states



 $\Psi[^{(3)}g]$

A general state is a superposition of many 3-manifolds

So which light cones should we choose to determine causality ?

For a general state there is no well defined notion of causality

But consider a space with a maximal symmetry group like Minkowki space, de-Sitter space etc.

Assume that the quantum vacuum state |0
angle also satisfies these symmetries

Then we conjecture that in our full quantum gravity theory there is a definition of local operators, such that causality is maintained using the light cones of the maximally symmetric background



We also assume that for gently curved space, we get 'approximate locality'

The region used in the Hawking argument is gently curved

Thus the leakage of the wavefunction outside the light cone must be small

The small corrections theorem then tells us that these small violations of nonlocality will not help

If we assume order unity effect of nonlocal physics, then there was no black hole puzzle in the first place ...



We can just take the stuff inside the hole and place it outside, and then there is no paradox Many of the alternatives to fuzzballs invoke some kind of nonlocal physics:

(A) Nonlocality on scale M for low energy modes (Giddings)

(B) Nonlocal effects on scales M^3

Wormholes between the hole and its radiation (Maldacena and Susskind 2013)

Bits describing the radiation are not independent of bits describing the remaining hole (Papadodimas and Raju)



(C) Nonlocal effects on infinitely long length scales

Gauge modes arising from diffeomorphisms at infinity (Hawking, Perry Strominger 2015)

Q: Can we show a concrete computation in string theory which gives effects outside the light cone?

We will assume that there is no significant leakage outside the light cone ...

Resolving the causality paradox

We can ask how the causality problem is avoided in the fuzzball paradigm ...



Shell falls in at speed of light

Sees only normal physics when far



 (\mathbf{I})

(2)

When shell approaches horizon, it tunnels into fuzzballs ...

We have seen that at this location a tunneling into fuzzballs becomes possible ...

But we can still ask: What local property tells the infalling shell that it should start tunneling into fuzzballs at this location?

If the local spacetime is the vacuum (or close to the vacuum), then one might try to use the equivalence principle and say that the shell can feel nothing as it passes this location



Is there a picture of spacetime where low energy matter sees nothing special, but matter with energy more than a given threshold sees significantly altered physics ?

Toy model: The c = 1 Matrix model

The essential point is that spacetime is not just a manifold, but has an additional property that we can call the "depth" or "thickness"

Is there a picture of spacetime where low energy matter sees nothing special, but matter with energy more than a given threshold sees significantly altered physics ?

Toy model: The c = 1 Matrix model (Das+Jevicki)

 $M : N \times N$ matrix

$$L = Tr \left(\frac{1}{2}\dot{M}(t)^{2} - V(M)\right) \qquad \qquad Z = \int \prod_{i,j} dM_{ij} \ dM_{ij}^{*} \ e^{iL}$$

Eigenvalues behave like fermions, so the lowest energy state has energy levels filled upto a fermi surface ...



Small deformation on the fermi sea travel as massless bosons



But large deformation on the fermi sea suffer a distortion



The waveform can 'fold' over, after which it is no longer described by a classical scalar field ... (Das+SDM 95)

In fact the matrix model gives a fermi sea of varying depth



The matrix model does not actually have a black hole ...

Also, in our actual fuzzball paradigm, what effect provides the analogue of the varying depth fermi sea?

Conjecture: The Rindler region outside the hole has a different set of quantum fluctuations from those in a patch of empty Minkowski space ('pseudo-Rindler')



Quantum fluctuations will be different near the surface of the fuzzball since there is a nontrivial structure there ...

(a) What is the nature of these fluctuations?

(b) Why should they be important ?

(a) The fluctuations are the fluctuations to larger fuzzballs



 $M \rightarrow M + \Delta M$

Our energy is still M so this is a virtual excitations (vacuum fluctuation)

(b) The reason these fluctuations are important is because they are 'entropy enhanced' (there are a lot of fuzzballs with that larger mass)

$Exp\left[S_{bek}(M + \Delta M)\right]$ states







pseudo-Rindler space

(Quantum fluctuations are different from empty space)

At a location depending on the energy of the quantum, there will be a tunneling into fuzzballs ...

This resolves the causality paradox

Complementarity and fuzzball complementarity

't Hooft, Susskind

Is it possible that the interior of a black hole is a manifestation of some new physics (different from the physics outside) ?



information reflects from stretched horizon for the purpose of the outside observer

A second copy of the information continues to fall in

Normally we cannot do 'quantum cloning'. But here it is allowed since we cannot compare the two copies easily (a) But what reflects the information off the horizon?

(b) Also, if we look at good slices, what special physics separates the interior from the exterior?





The Firewall argument tries to gives a rigorous proof that this idea does not work

But with fuzzballs things are quite different



We have a surface rather than a smooth horizon, so there is no difficulty in radiating the information back

There is no interior, so can we have any notion of smooth infall?

Should we not already say that the surface of a fuzzball will have to behave like a firewall?

No, because there is a possibility which we will call Fuzzball complementarity

A collapsing shell tunnels into a linear combination of fuzzball states



There is a vast space of fuzzball states

$$\mathcal{N} \sim e^{S_{bek}(M)}$$

When the collapsing shell tunnels into fuzzballs, then its state keeps evolving in this large space of fuzzball states ...



Thus we must study dynamics is SUPERSPACE, the space of all gravity configurations





Superspace, the space of all fuzzball configurations

The full quantum gravity state is a wavefunctional over superspace **Conjecture:** This evolution in superspace can be approximately mapped to infall in the classical black hole



This may seem strange, but something like this happened with AdS/CFT duality (Maldacena 1997) ...



Create random excitations

A complicated set of gluon excitations spreads on the D-branes We can map this complicated evolution to free infall of the graviton

The difference is that AdS/CFT duality is exact, while fuzzball complementarity is an approximate map

Fuzzball complementarity





Different fuzzballs radiate different at energies $E \sim T$



For these hard impact processes the evolution in the space of fuzzballs map to the 'vibrations' of empty space

Causality and the firewall argument

What is the firewall argument?

Hawking (1975) showed that if we have the vacuum at the horizon (no hair) then there will be a problem with growing entanglement

This is equivalent to the statement: If we assume that

Ass: 1 The entanglement does not keep rising, but instead drops down after some point like a normal body

Then the horizon cannot be a vacuum region.

AMPS use the same argument of bits and strong subadditivity that we used to make the Hawking argument rigorous (the small corrections theorem)

So what is the difference between the Hawking paradox and the firewall argument?

AMPS tried to make the a stronger statement, by adding an extra assumption

Ass2: Outside the stretched horizon, we have 'normal physics' (Effective field theory). In particular, a shell coming in at the speed of light will encounter no new physics till it hits the stretched horizon (causality)

AMPS claim: Given assumptions Ass: I and Ass:2, an infalling object will encounter quanta with energies reaching planck scale as it approaches the horizon (Firewall)



The intuition behind the AMPS argument is simple:



In the Hawking computation with vacuum horizon, the particles do not actually materialize until they are long wavelength (low energy) excitations



If we replace the black hole by a normal hot body, then we will have no entanglement problem (by definition)

But we can now follow the radiation quanta back to the emitting surface, where they will be high energy real quanta

high energy quanta

But there is a problem with the AMPS argument:

The two assumptions Amp: I and Amp:2 are in conflict with each other ...





Sees only normal physics (Ass:2)



(2)

Shell passes through its own horizon without drama, since by causality it has not seen the hole



Light cones point inwards inside the new horizon, so the information of the shell cannot be sent to infinity without violating causality.

This contradicts assumption Ass: I

Thus there is a conflict between Assumptions Ass: I and Ass: 2 made in the firewall argument.

If we drop the new assumption Ass:2, then we cannot argue that there is a firewall: in fact we can construct a bit model where high energy quanta feel 'no drama' at the horizon (fuzzball complementarity)

(SDM+Turton 2013, SDM 2015)



Suppose an object of energy E>>kT falls in

Now there are $e^{S(M+E)}$ possible states of the hole

$$\frac{N_f}{N_i} = \frac{e^{S(M+E)}}{e^{S(M)}} = \frac{e^{S(M)+\Delta S}}{e^{S(M)}} = e^{\Delta S} \approx e^{\frac{E}{kT}} \gg 1$$

So most of the new states created after impact are not entangled with the radiation at infinity

(This is just like the entanglement before the halfway evaporation point)

Complementarity is the dynamics of these newly created degrees of freedom, and says that this dynamics is captured by the physics of the black hole interior

AMPS worry only about experiments with Hawking modes b, c, but these have $E{\sim}kT$

SUMMARY



In string theory we find the fuzzball paradigm, where black holes do not have the traditional structure, and radiate like normal bodies.

The lesson: The scale of quantum gravity excitations increases with the number of particles involved, and always prevents horizon formation

$$l_p \rightarrow N^{\alpha} l_p$$