The fuzzball paradigm

Lecture II

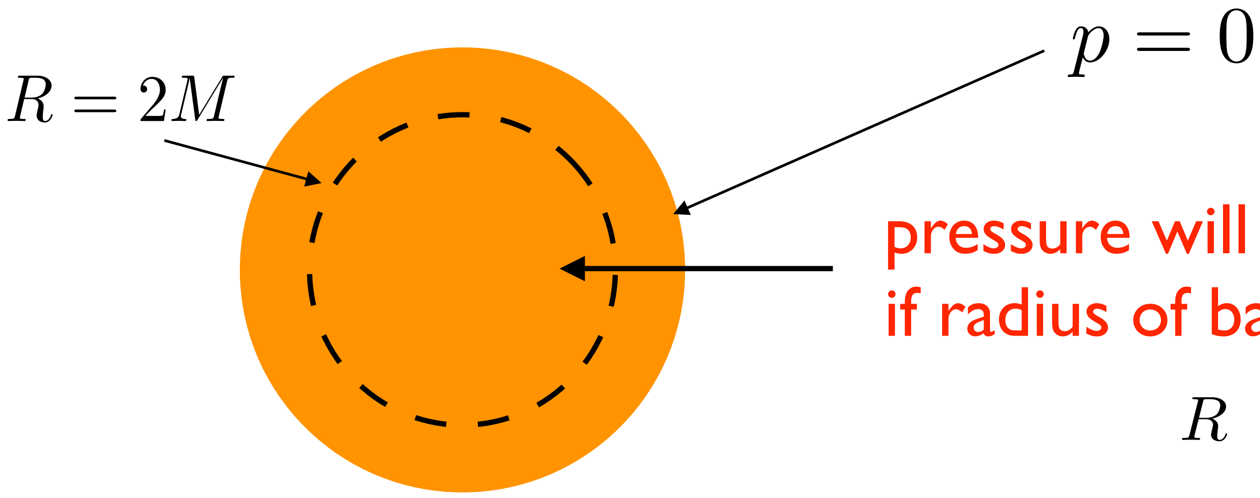
Samir D. Mathur

The Ohio State University

Summary so far

Summary so far:

(a) A sufficiently squeezed star will collapse to a black hole



pressure will diverge somewhere if radius of ball is

$$R < \frac{9}{4}M$$

(b) Once we have a horizon, we cannot modify the solution by adding hair.

Thus the quantum state at the horizon is the local vacuum

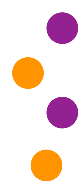
(c) The vacuum state around the horizon will create entangled pairs.

This is a low energy process, not needing details of quantum gravity

(d) Two possibilities:

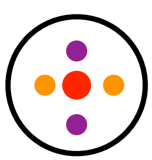
(a) Black hole evaporates away completely

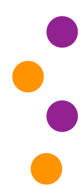
vacuum



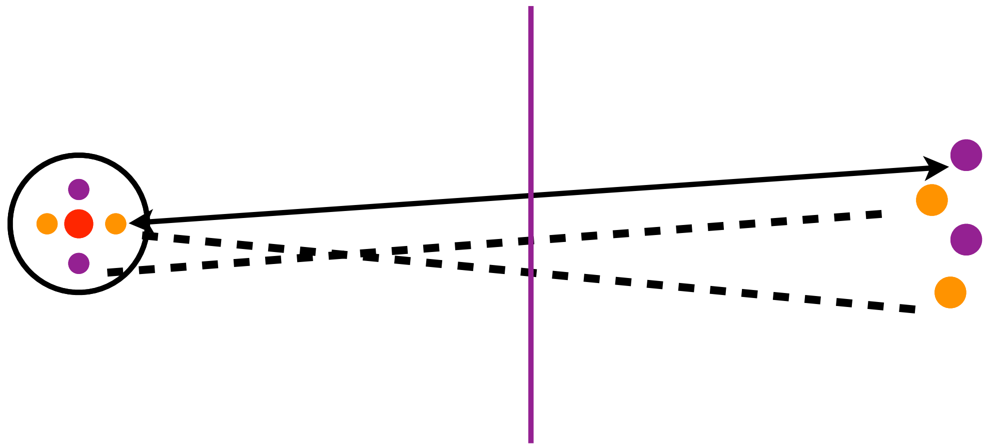
Violates quantum theory

(b) A planck sized remnant is left



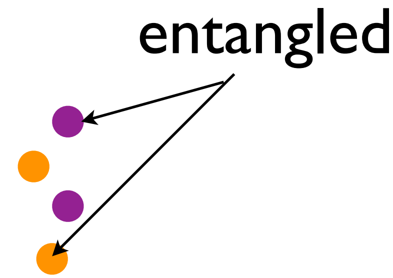


Difficult to understand,
also not consistent with
AdS/CFT

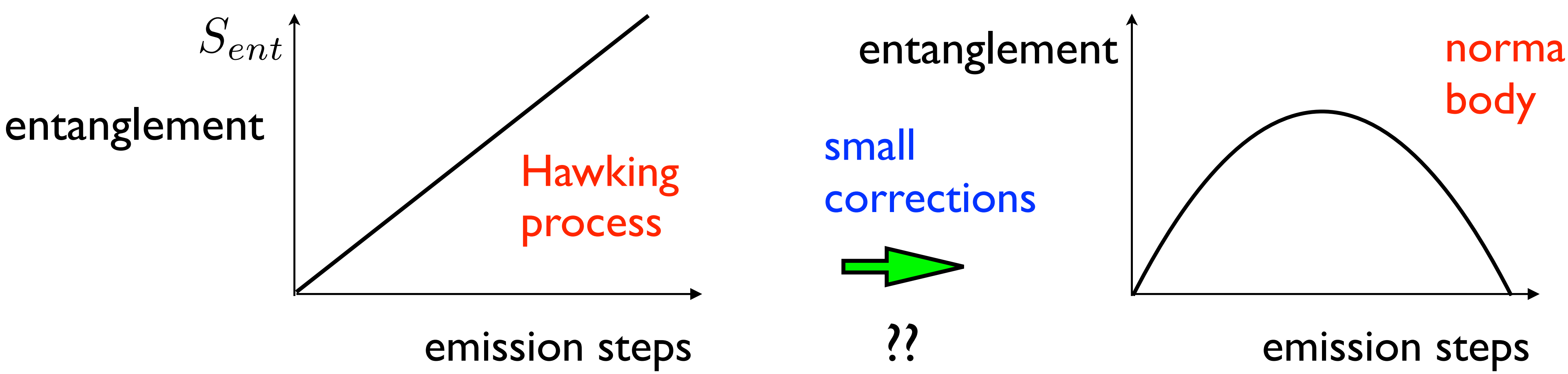


Can small corrections
may disentangle the radiation
from the remnant ?

vacuum



??



Suppose that at the first step of emission we have no change

$$\frac{1}{\sqrt{2}}(|0\rangle_{b_1}|0\rangle_{c_1} + |1\rangle_{b_1}|1\rangle_{c_1})$$

At the second step of emission, suppose that if we had 00 at the first step then a 00 is slightly more likely, and if we had a 11 at the first step, then a 11 is slightly more likely

Overall state after two emissions

$$\frac{1}{2}\Big(|0\rangle_{b_1}|0\rangle_{c_1}[(1+\epsilon_1)|0\rangle_{b_2}|0\rangle_{c_2} + (1-\epsilon_1)|1\rangle_{b_2}|1\rangle_{c_2}]$$
$$+|1\rangle_{b_1}|1\rangle_{c_1}[(1+\epsilon_1')|0\rangle_{b_2}|0\rangle_{c_2} + (1-\epsilon_1')|1\rangle_{b_2}|1\rangle_{c_2}]\Big)$$

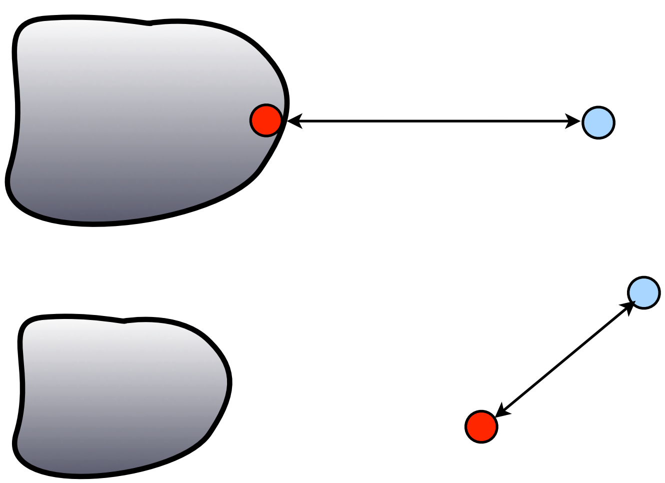
After N steps of emission, there are $\sim 2^N$ correction terms

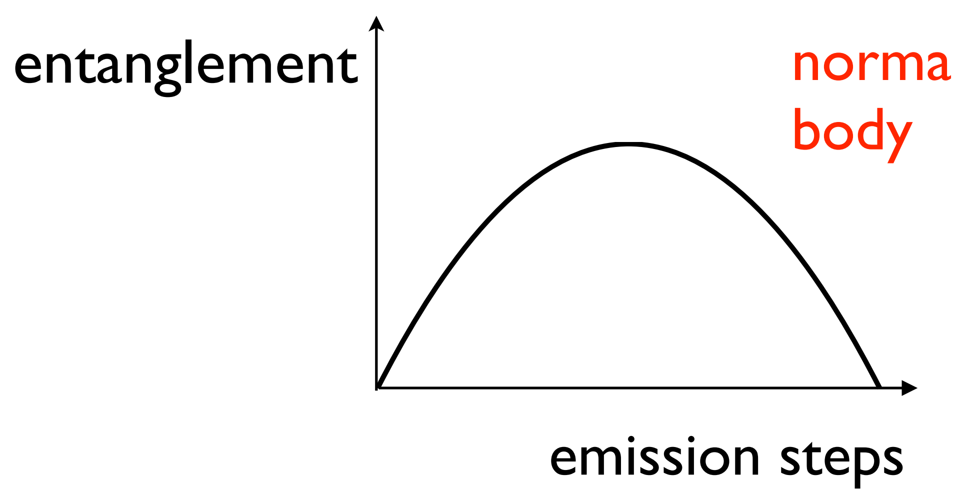
Small corrections theorem: No, the smallness of ϵ cannot be traded against the largeness of N to resolve the problem

This result should be distinguished from the theorem by Page
(In a sense the two theorems address opposite issues)

Page 93: For generic states of a system, any subsystem (of size less than half the system) is almost maximally entangled with the remainder

$$S(A) \approx \log M - \frac{M}{2N}$$

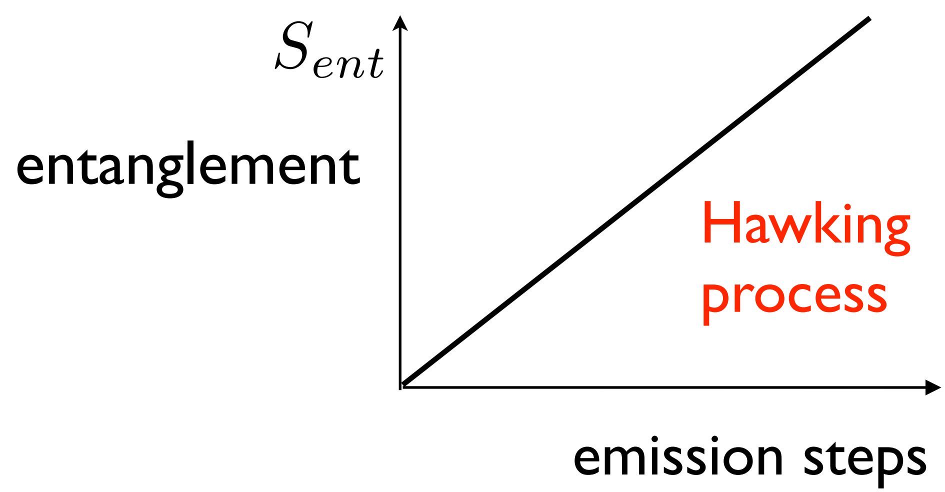




Hawking radiation creates a particular (nongeneric) state

$$|\Psi\rangle = |\psi\rangle_M \otimes \Big(\frac{1}{\sqrt{2}}|0\rangle_{c_1}|0\rangle_{b_1} + \frac{1}{\sqrt{2}}|1\rangle_{c_1}|1\rangle_{b_1}\Big) \otimes \Big(\frac{1}{\sqrt{2}}|0\rangle_{c_2}|0\rangle_{b_2} + \frac{1}{\sqrt{2}}|1\rangle_{c_2}|1\rangle_{b_2}\Big) \ldots$$

For this state, the entanglement keeps growing
(so not like the generic case of Page)



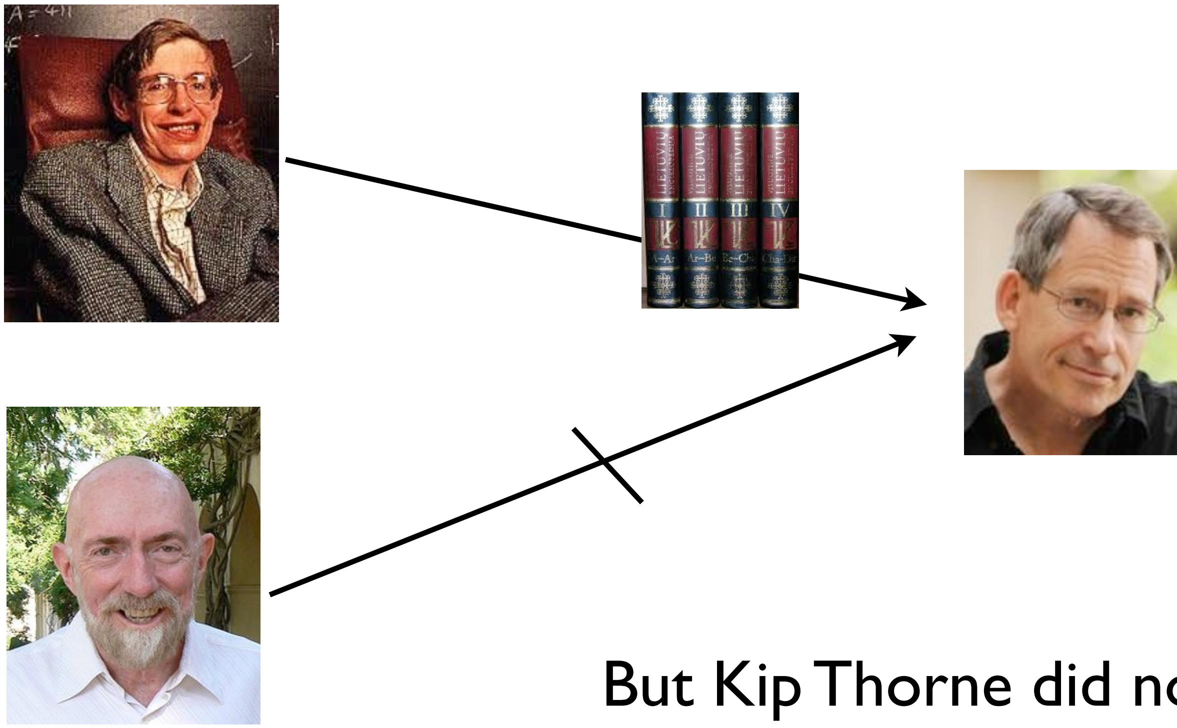
The small corrections theorem says that this result is stable against small corrections to the Hawking radiation process

$$S_{N+1} > S_N + \ln 2 - (\epsilon_1 + \epsilon_2)$$

In 2004, Stephen Hawking surrendered his bet to John Preskill ...

Stephen Hawking

Kip Thorne



John Preskill

But Kip Thorne did not agree to surrender the bet ...

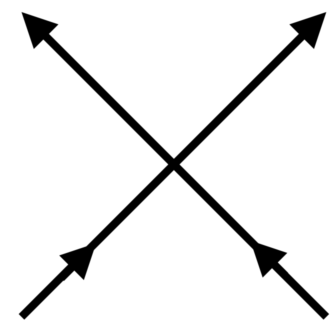
The small correction theorem says that Kip Thorne was correct here .... we need order unity corrections to the state of the emitted pair to resolve the problem

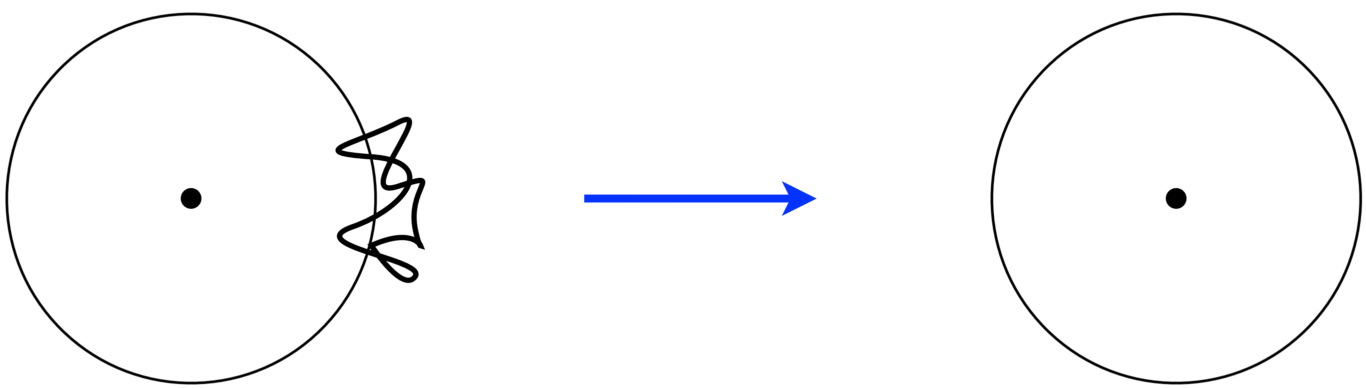
Black holes in string theory

How can strings help to solve our puzzle ?

Hawking's paradox is so strong because it does not use any details of quantum gravity

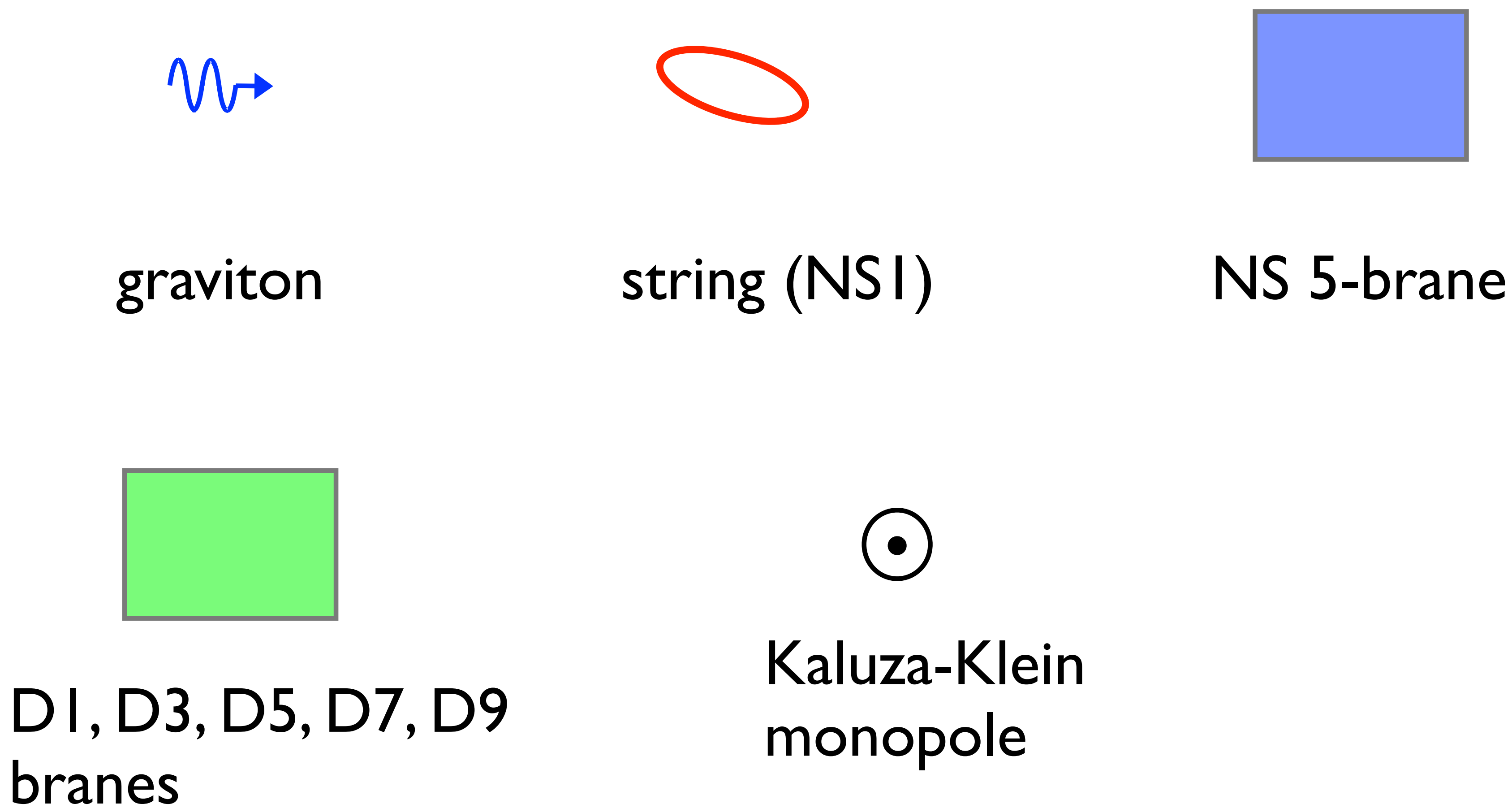
In graviton-graviton scattering, there is no sign of string behavior if energies are below the string scale





Q: How does string theory manage to violate the semiclassical approximation that is expected to hold around the horizon?

(a) Elementary objects in IIB string theory



Any one of these objects can be mapped to any other by S,T dualities, which are exact symmetries of the theory

(b) The string coupling is a field $g = e^{\phi}$

Thus we can take the coupling to be small or large

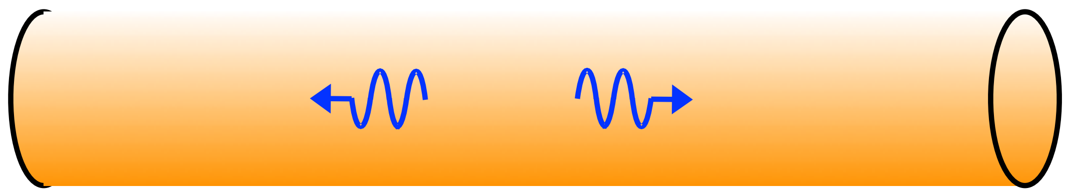
(c) The theory lives in 9+1 dimensions.

So we have to compactify some directions to get black holes in lower dimensions

(d) The theory is unique: The set of elementary objects, their tensions, interactions etc. are all fixed

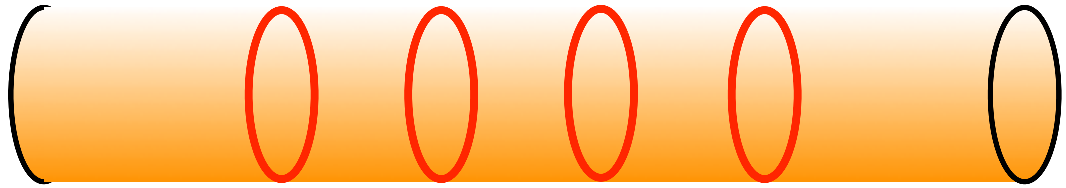
We have to make a black hole using the objects in the theory

If we just use many gravitons, they will disperse, at least at weak coupling



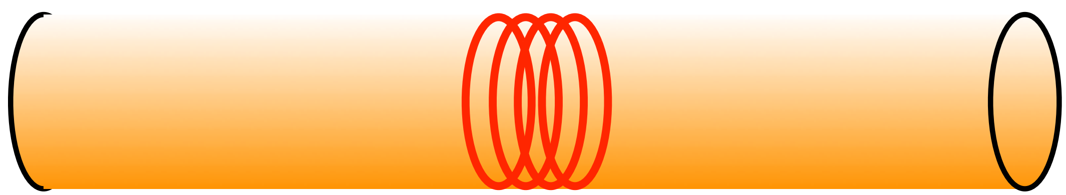
We can wrap a string around a compact circle

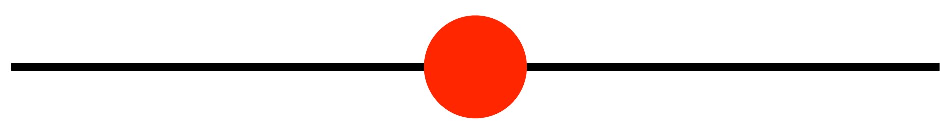
To get a large mass, we should take many strings



For an observer who
cannot resolve the
compact dimension:

This suggests many separate tiny black holes .... we should instead look for one heavy object

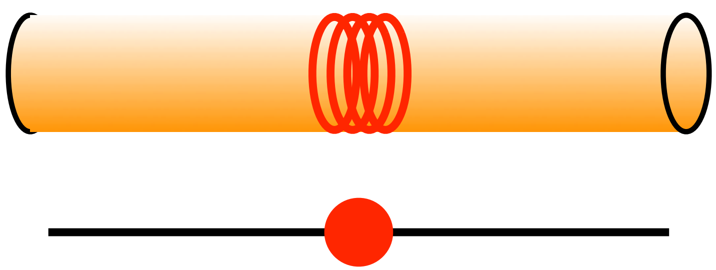


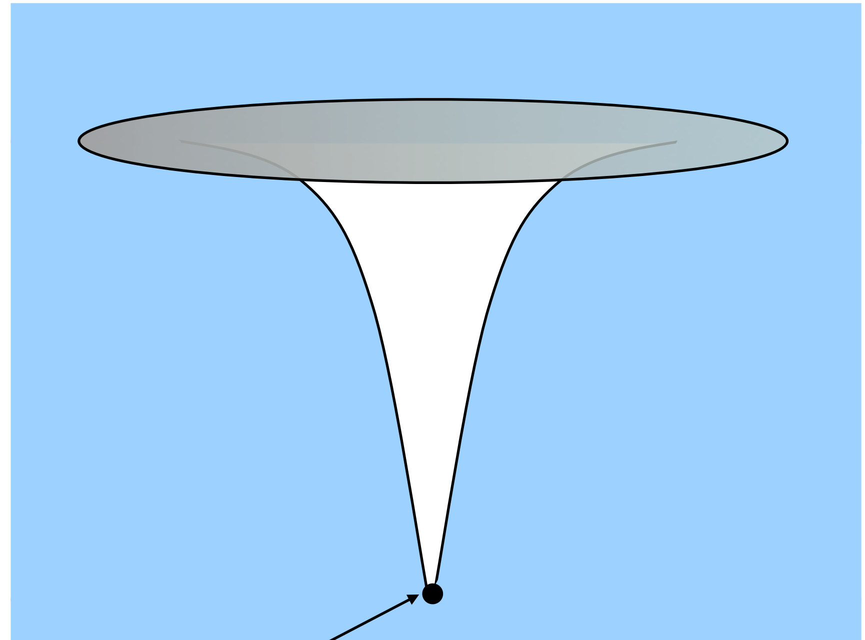


Mass M Winding charge Q $M = |Q|$

What geometry does this object create?

n_1





Weak coupling

Strong coupling

Horizon area vanishes

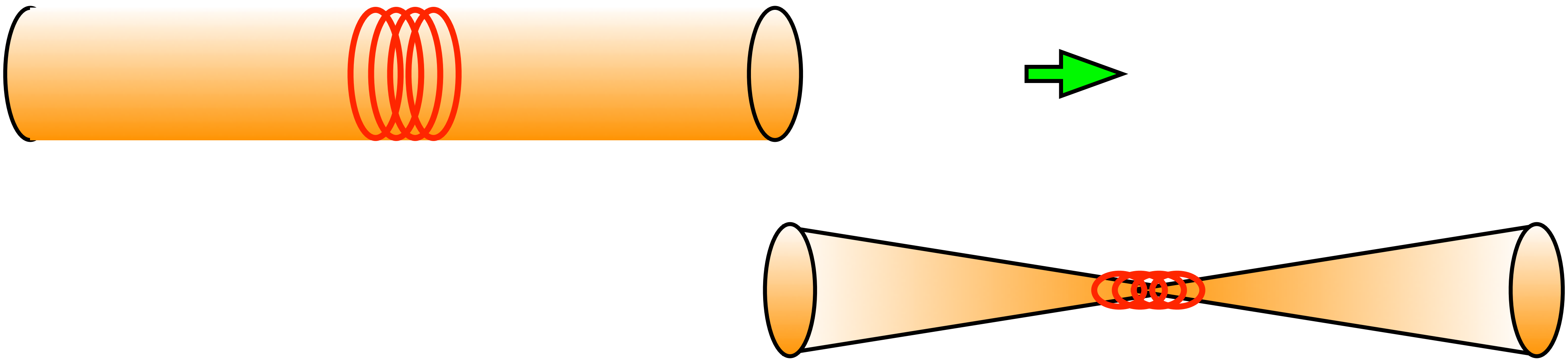
Unique state (actually 256 dimensional supermultiplet)

$$S_{bek} = \frac{A}{4G} = 0$$

$$S_{micro} = \log[256] \approx 0$$

Entropy does not grow with winding number n_1

The energy of the string is lowered when the length of the radius of the circle is reduced



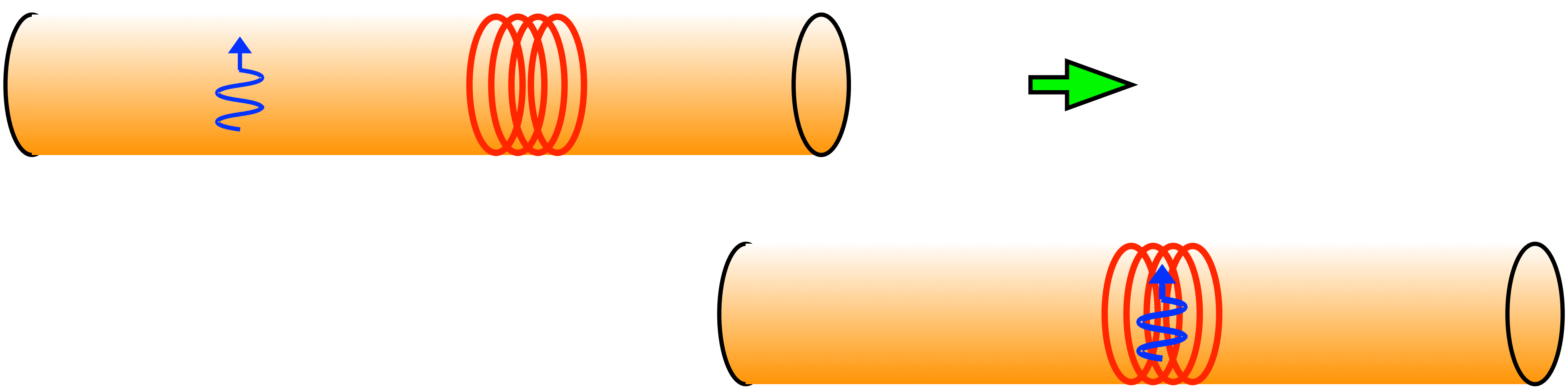
Thus the circle gets pinched, and the horizon area goes to zero

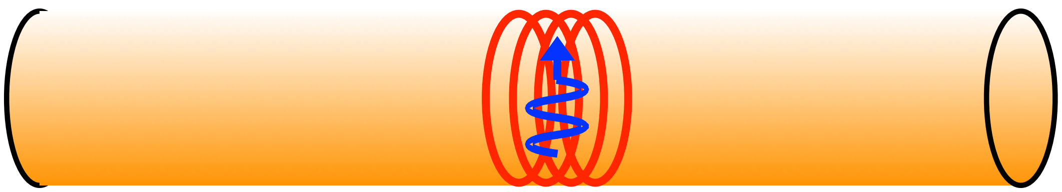
To fix this problem, we should add something whose energy *grows* when the radius of the circle is reduced

We take a graviton running around the compact circle

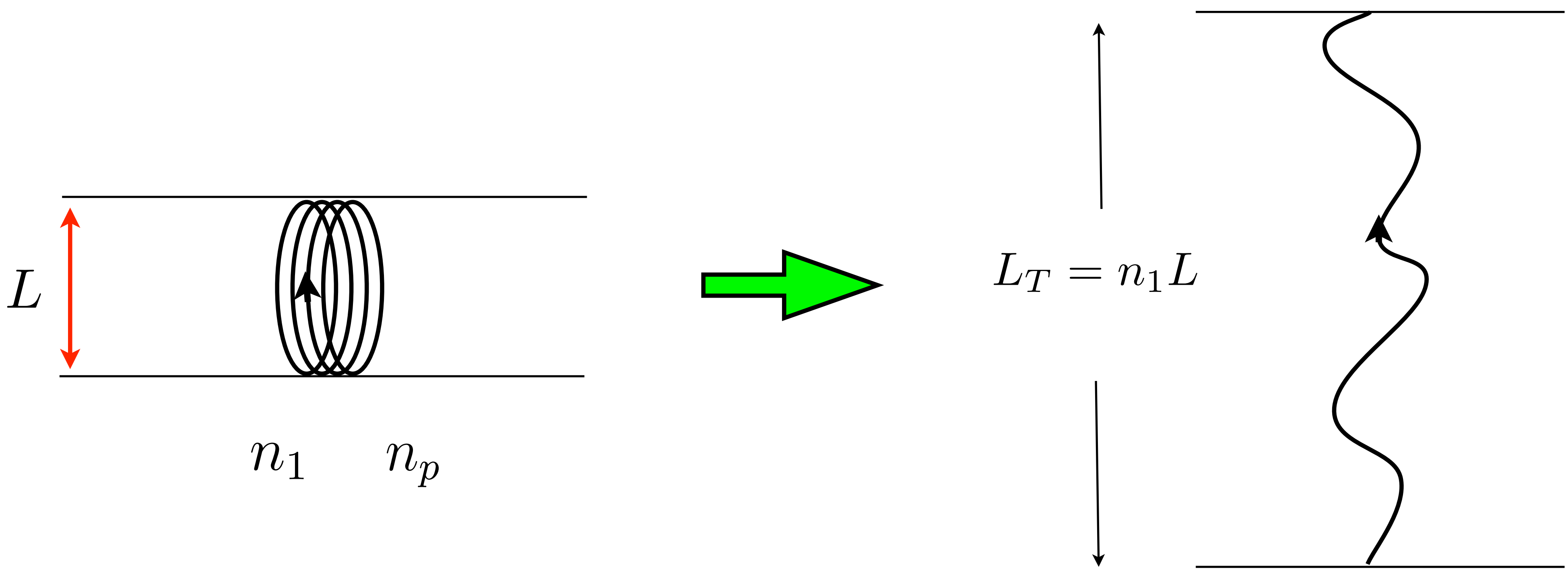
$$E = P = \frac{2\pi n_p}{L}$$

We want a bound state of n_1 units of string winding charge and n_p units of momentum charge

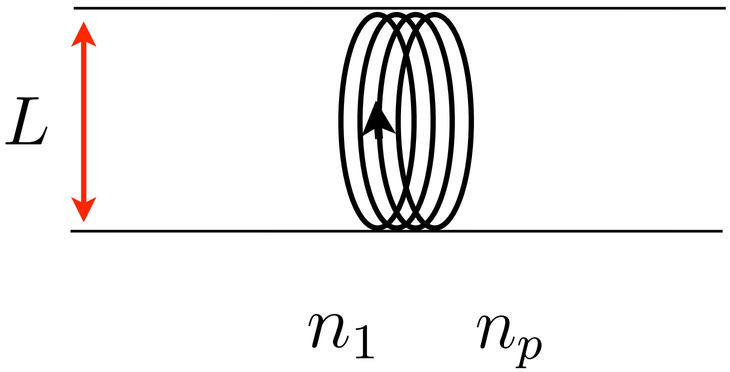


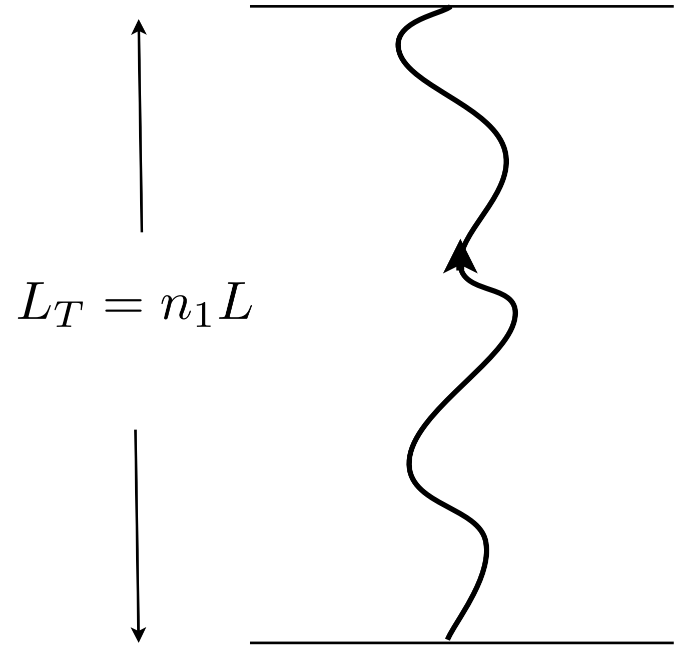


The bound state of a string and momentum is described by a string carrying travelling waves



But there are many ways to partition the total momentum among different harmonics .... entropy !





A quantum of the harmonic k has momentum $\dfrac{2\pi k}{L_T}$

We can write the momentum as $P = \dfrac{2\pi n_p}{L} = \dfrac{2\pi(n_1 n_p)}{L_T}$

Thus there are $n_1 n_p$ units of momentum on the string of length L_T

We can partition these $n_1 n_p$ units among different harmonics in different ways.

Let there be n_k units of excitations in harmonic k

Then we need $n_1 + 2n_2 + 3n_3 + \cdots = n_1 n_p$

$$\sum k \, n_k = n_1 n_p$$

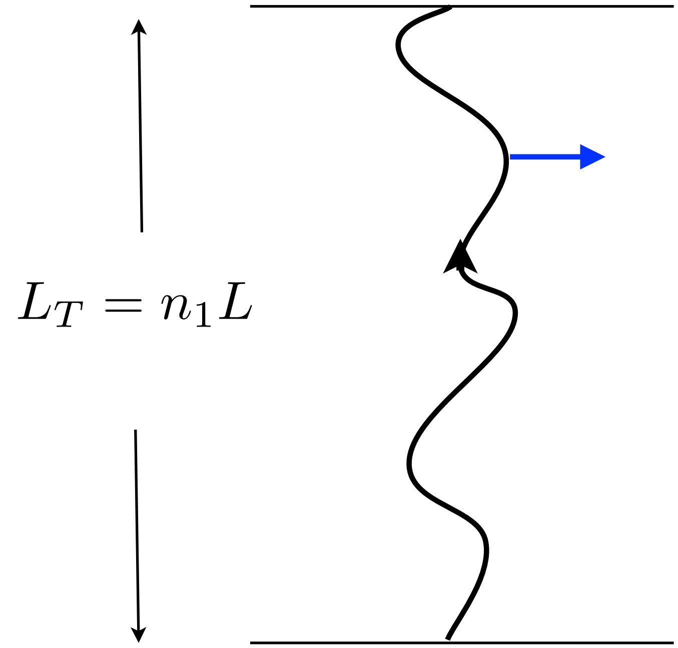
The number of solutions to this relation is called the Partitions of the number $n_1 n_p$

$$N \sim e^{2\pi\sqrt{n_1 n_p}}$$

Then we can write $S_{micro} = \log N$

We have to also take into account the 8 different transverse directions of vibration that are allowed

Also, there are as many fermions as bosons, and 2 fermions are equivalent to 1 boson

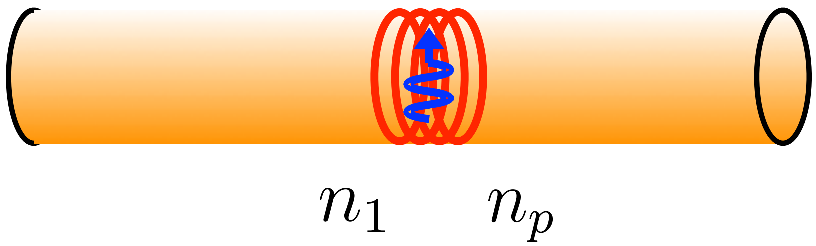


$$S_{micro} = 2\pi\sqrt{2}\sqrt{n_1 n_p} \qquad \text{(IIB)}$$

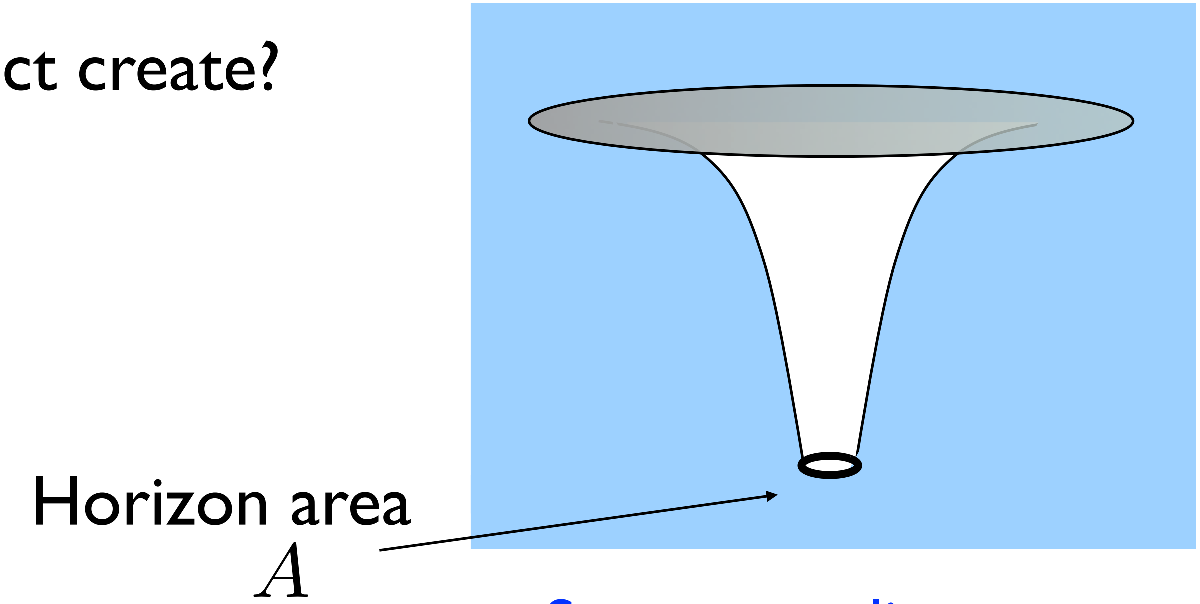
$$S_{micro} = 4\pi\sqrt{n_1 n_p} \qquad \text{(Heterotic)} \qquad \text{(Sen 95)}$$

We now have an entropy that grows with the charges in the bound state …

What geometry does this object create?



Weak coupling



Strong coupling

IIB: $A = 0$

Heterotic: With first order stringy corrections, for 3+1 dims noncompact

$$S_{bek-wald} = \frac{A}{4G} + \frac{A}{4G} = \frac{A}{2G} 4\pi\sqrt{n_1 n_5} = S_{micro}$$

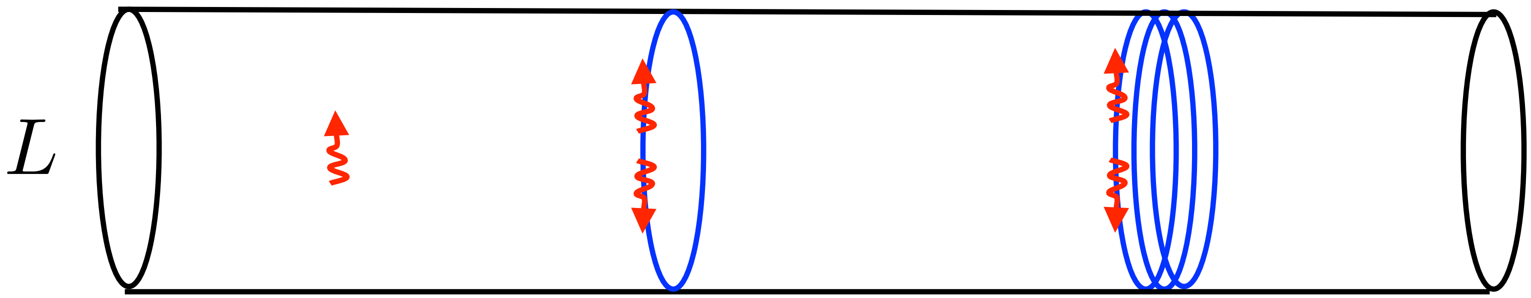
(Dabholkar, Kallosh, Maloney 04)

But higher level stringy corrections are of the same order …

Also, numerical coefficient does not match for 4+1 dimensions

(Cano, Ramirez, Ruiperez 18)

3-charge states



A graviton has energy $$E = \frac{2\pi k}{L}$$

Thus the minimum energy excitation with no net charge is

$$\Delta E = \frac{2\pi}{L} + \frac{2\pi}{L} = \frac{4\pi}{L}$$

If we excite a singly wound string we get the same gap $$\Delta E = \frac{4\pi k}{L}$$

But for a string of winding n_1 the gap is

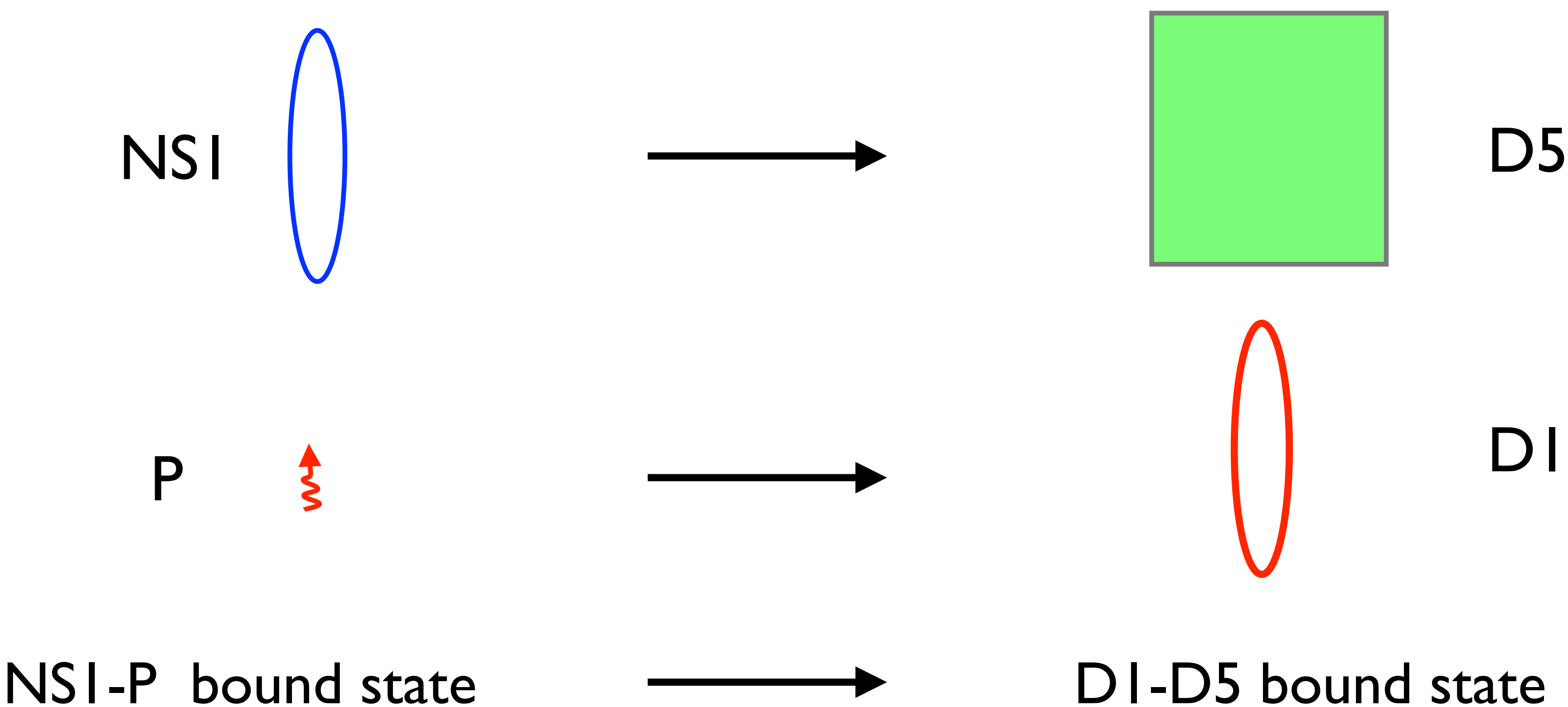
$$\Delta E = \frac{2\pi}{n_1 L} + \frac{2\pi}{n_1 L} = \frac{4\pi}{n_1 L}$$

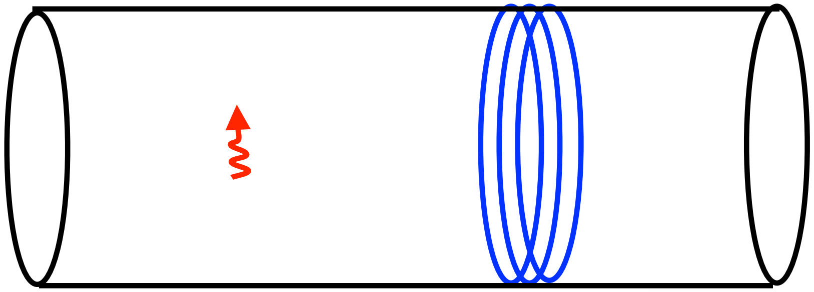
A graviton by itself comes in integral units of $\dfrac{2\pi}{L}$

But when it is bound to n_1 strings it gets 'fractionated'.

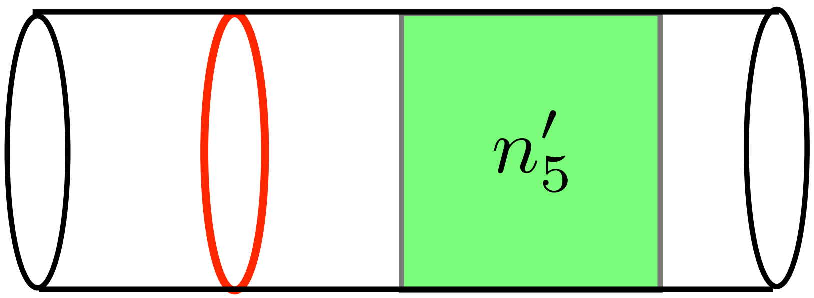
We get n_1 'fractional gravitons' of energy $\dfrac{2\pi}{n_1 L}$ each

This looks like a normal phenomenon, but what makes it interesting in string theory is that we can apply dualities

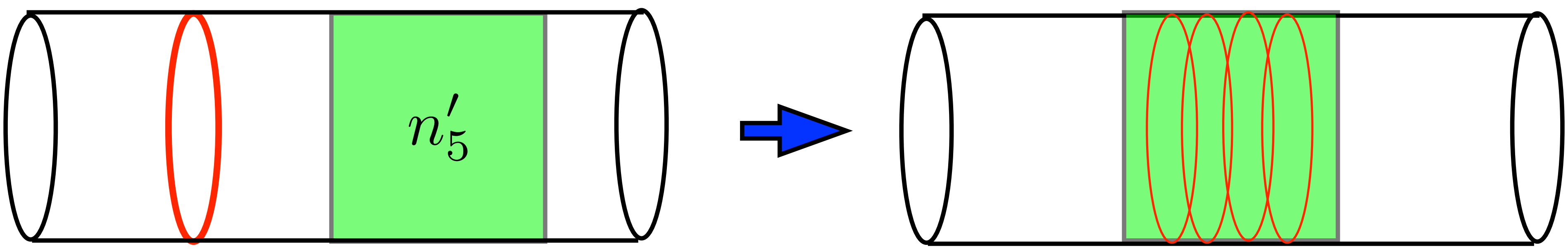




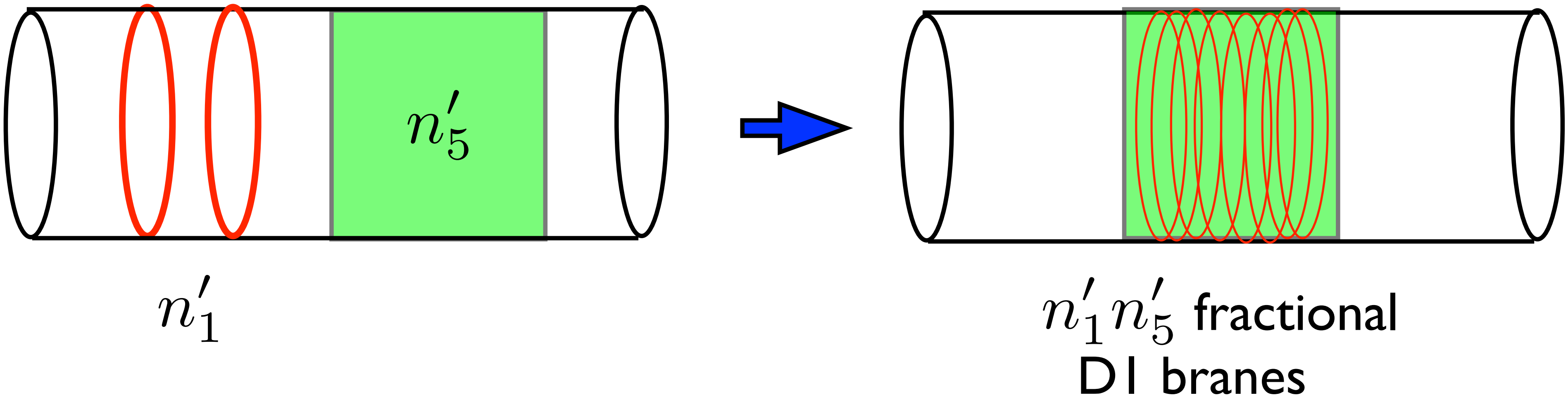
When graviton binds to n_1 strings, it breaks into n_1 fractional units

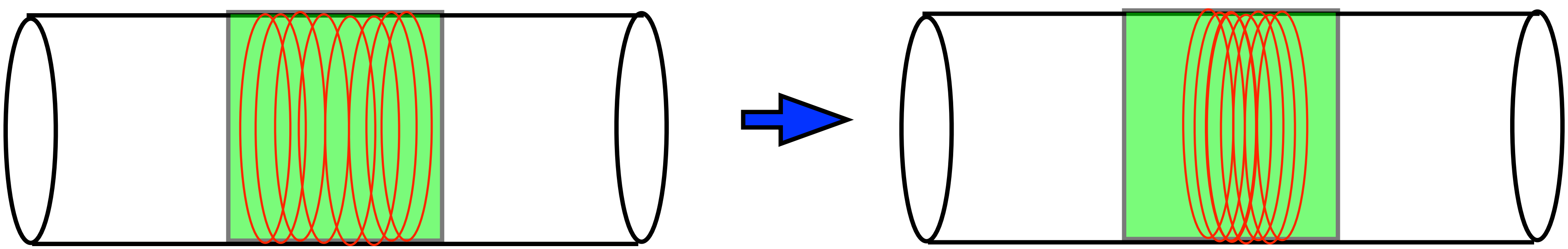


When a D1 binds to n_5' D5 branes, it should break into n_5' fractional units



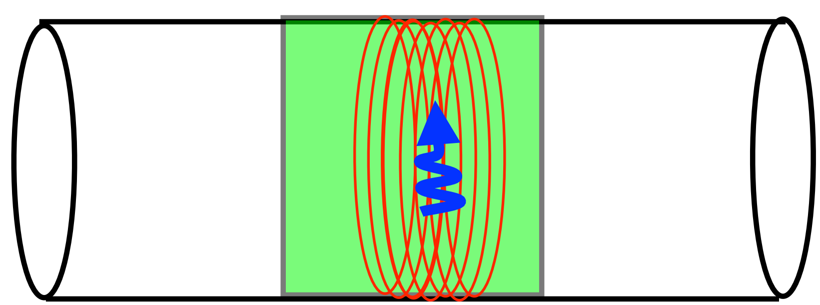
n_5' fractional D1 branes



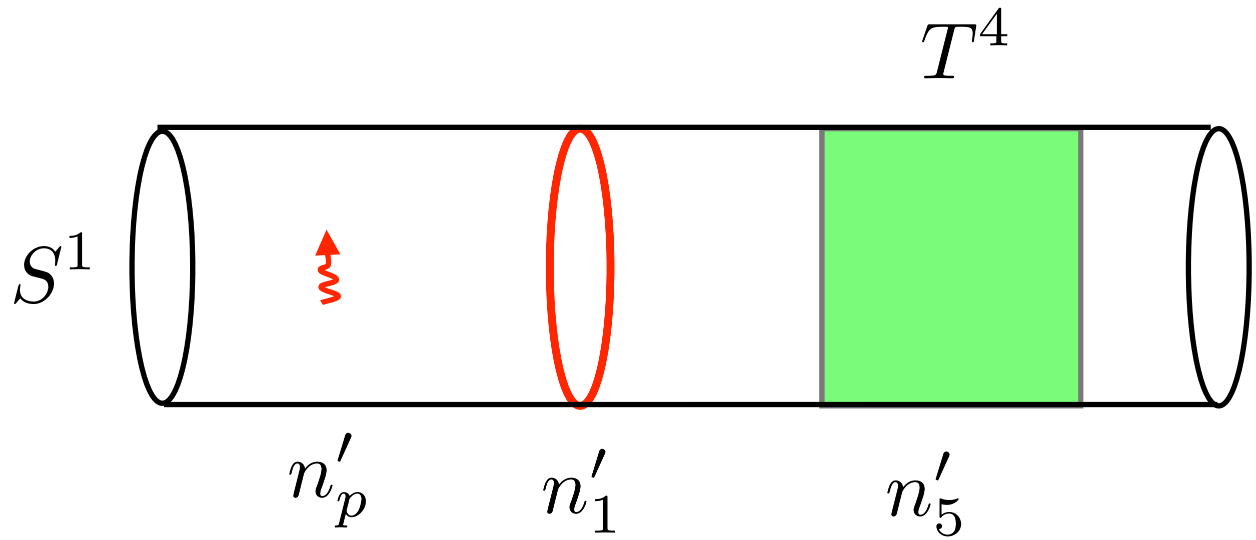


We can join these $n_1' n_5'$ fractional D1 branes into one multiwound 'effective string'

We can add a momentum P along this effective string



3-charge D1-D5-P extremal hole



We have n'_1 D1 branes wrapped on an S^1 of length L

We have n'_p units of momentum P along the S^1

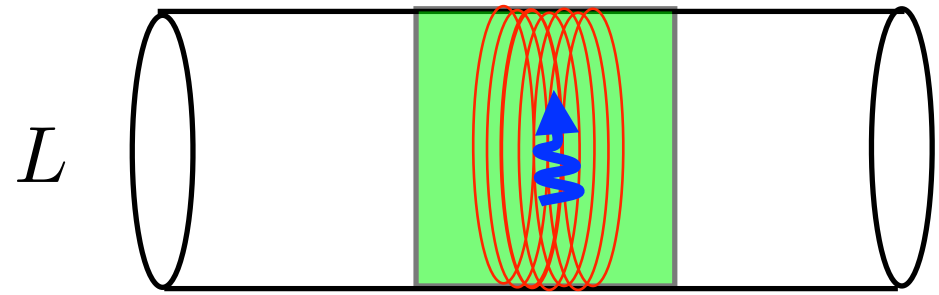
We have n'_5 D5 branes wrapped on S^1 times a T^4 of volume V

The string coupling is g

The string tension is $T = \frac{1}{2\pi\alpha'}$

Noncompact dimensions 4+1

Microscopic entropy



The effective string has winding number $n_1' n_5'$

Thus its total length is $L_T = n_1' n_5' L$

A quantum of the harmonic k has momentum $\frac{2\pi k}{L_T}$

The momentum is $P = \frac{2\pi n_p'}{L} = \frac{2\pi n_1' n_5' n_p'}{L_T}$

Let there be n_k units of excitations in harmonic k

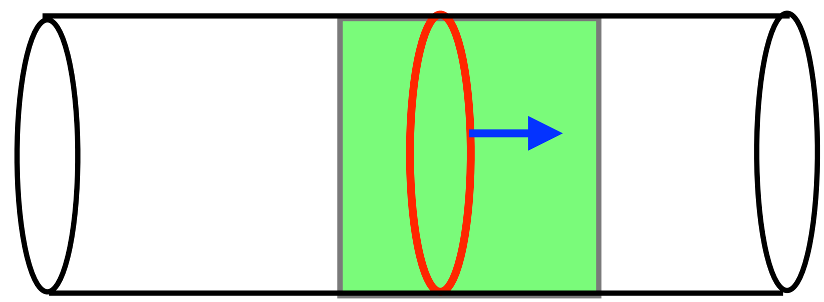
Count partitions $\sum_k k\, n_k = n_1' n_5' n_p'$

Counting partitions

$$\text{Partitions}\,[N] \;\sim\; e^{2\pi\sqrt{\frac{c}{6}N}}$$

Here c is the 'central charge', which is the number of effective directions of vibration of the 'effective string'

A D1 string bound to the D5 brane can vibrate only inside the D5 brane, so it has 4 transverse directions of vibration



By supersymmetry, there are 4 fermionic directions, so

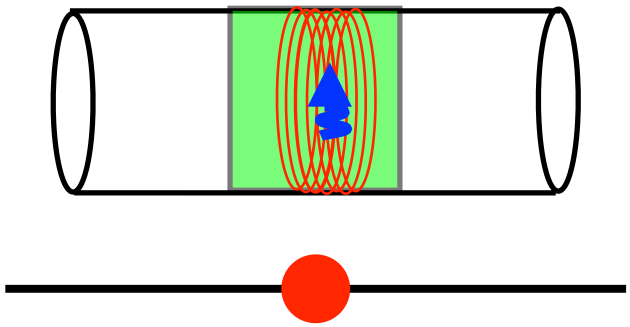
$$c = 4 + \frac{4}{2} = 6$$

Number of different possible vibrations on effective string

$$\mathcal{N} \sim e^{2\pi\sqrt{n_1' n_5' n_p'}}$$

Microscopic entropy

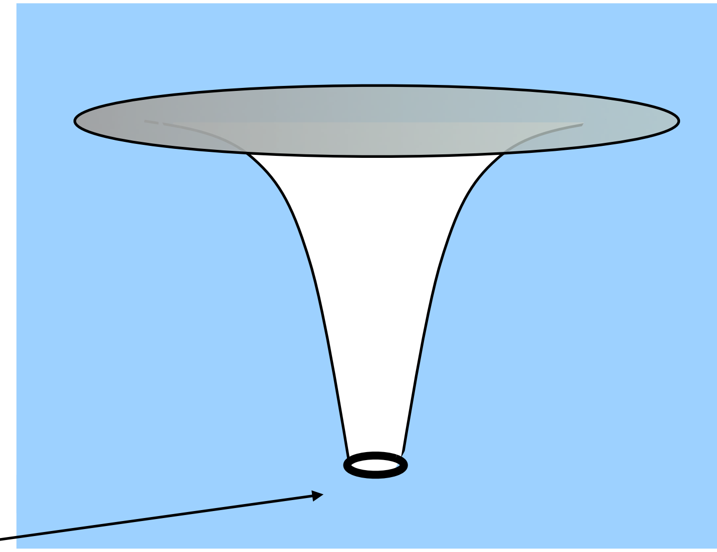
$$S_{micro} = \log[\mathcal{N}] = 2\pi\sqrt{n_1' n_5' n_p'}$$



n_1', n_5', n_p'

Weak coupling

Horizon area
A



Strong coupling

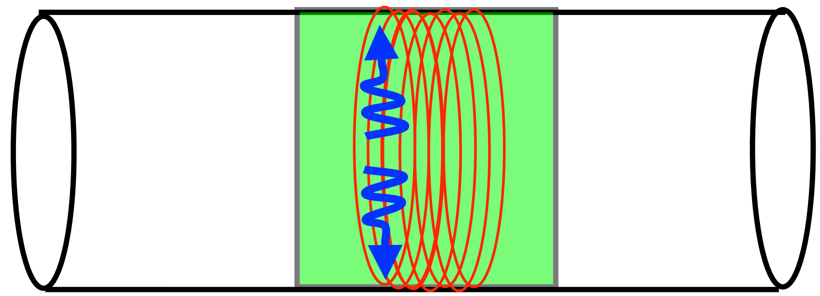
$$S_{bek} = \frac{A}{4G} = 2\pi\sqrt{n_1' n_5' n_p'} \;=\; S_{micro}$$

(Strominger and Vafa 96)

Extremal black holes (Mass=Charge) do not radiate

Near-extremal black holes

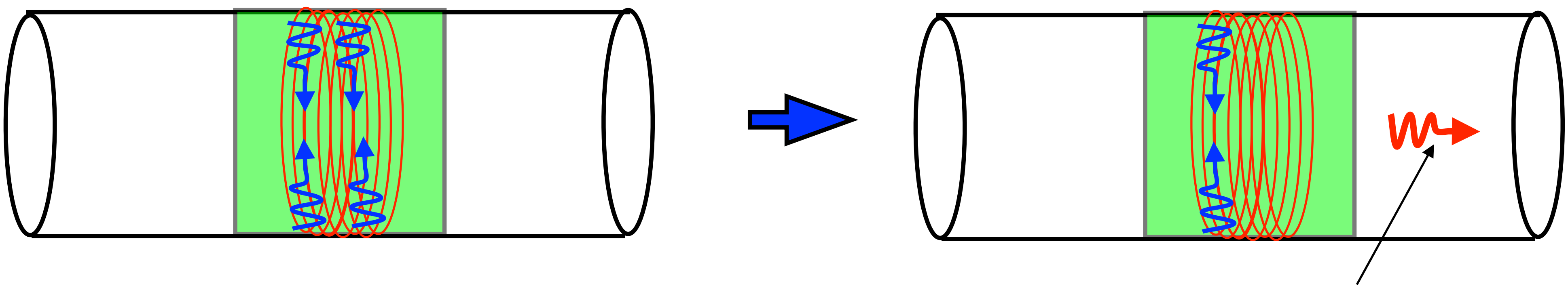
$$D1 - D5 - P\bar{P}$$



We again find

$$S_{micro} = 2\pi\sqrt{n_1 n_5}(\sqrt{n_p} + \sqrt{\bar{n}_p}) = S_{bek}$$

Radiation from near-extremal holes



(weak coupling)

radiated graviton

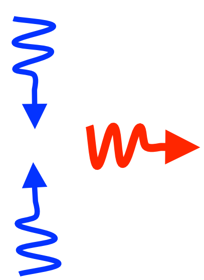
Number of gravitons emitted per unit time

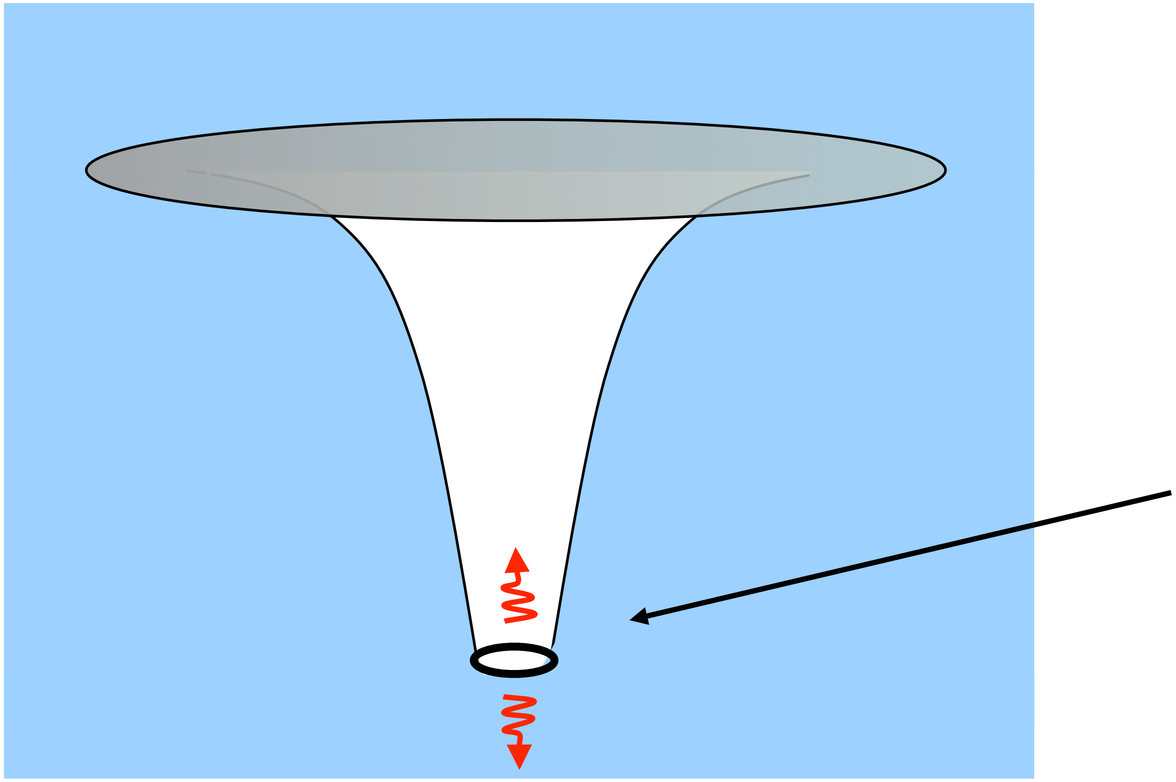
$$\Gamma_{micro} = |V|^2 \rho_L \, \rho_R$$

coupling

flux of left movers

flux of right movers





Compute Hawking radiation
from the black hole with
the same mass and charges

(Strong coupling)

We find

$$\Gamma_{Hawking} = \Gamma_{micro}$$

(Das+SDM 96,
Maldacena and Strominger 96)

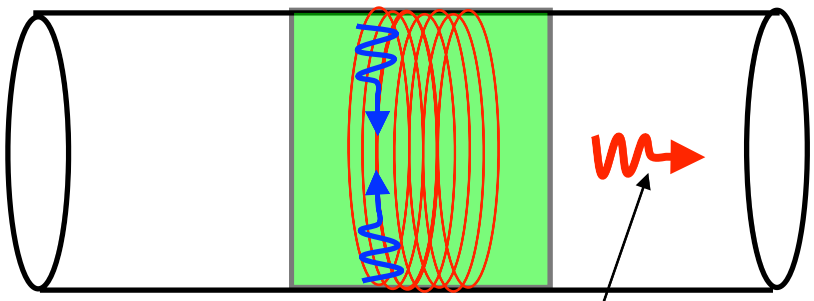
All black holes have similar expressions for entropy

2-charges	$S = 2\sqrt{2}\pi\sqrt{n_1 n_2}$
3-charges	$S = 2\pi\sqrt{n_1 n_2 n_3}$
4-charges	$S = 2\pi\sqrt{n_1 n_2 n_3 n_4}$
2 charges + nonextremality	$S = 2\pi\sqrt{n_1 n_2}(\sqrt{n_3} + \sqrt{\bar{n}_3})$
3-charges + nonextremality	$S = 2\pi\sqrt{n_1 n_2 n_3}(\sqrt{n_4} + \sqrt{\bar{n}_4})$

The radiation agreement also works out for different holes ...

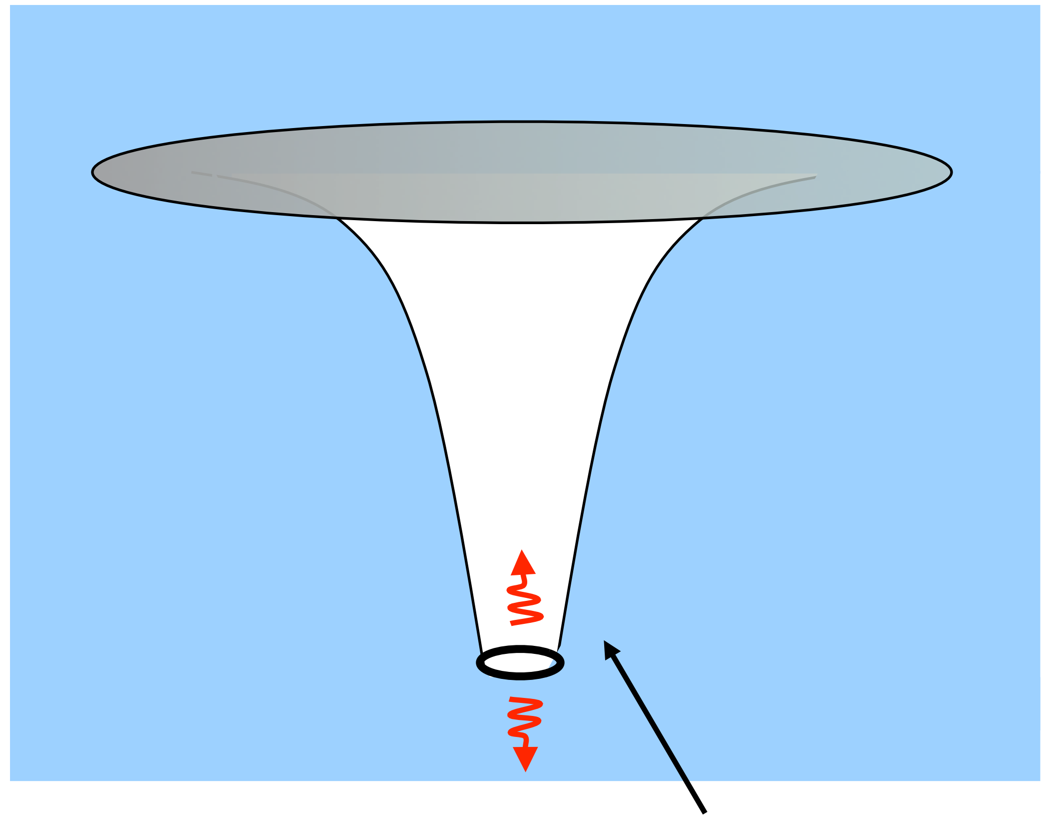
$$\Gamma_{Hawking} = \Gamma_{micro}$$

But all this does not help us with the information paradox …



(weak coupling)

Radiates like a
normal body;
no problem of
growing entanglement

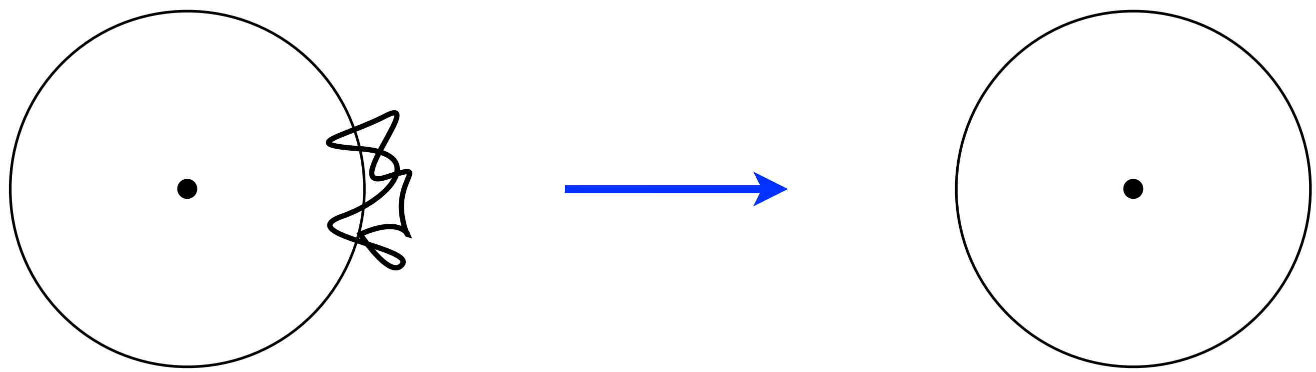


(Strong coupling)

Entangled pairs;
Entanglement keeps
growing

The average rate of radiation is the same in both cases, but the detailed mechanism of radiation is very different

Q: If string theory changes physics at the string scale, what can it do at the horizon?



The size of a bound state

Fractionation

The minimum energy excitation (with no net charge) is

$$\Delta E = \frac{2\pi}{L} + \frac{2\pi}{L} = \frac{4\pi}{L}$$

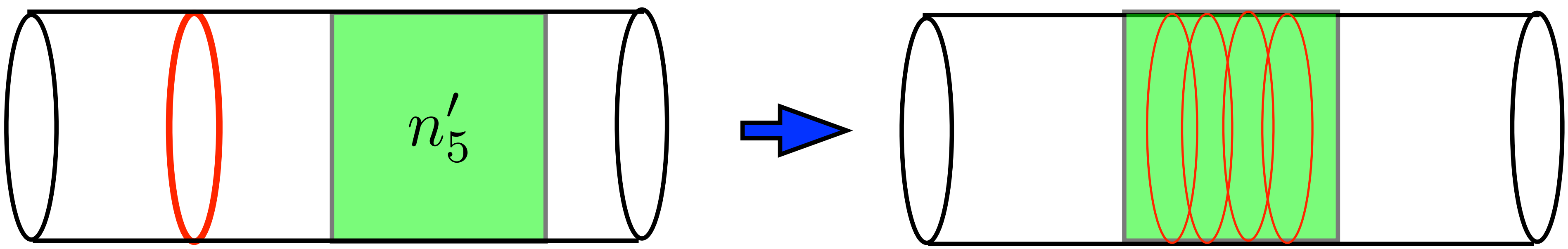
But when the gravitons are bound to a string of winding n_1 they come in fractional units $1/n_1$

$$\Delta E = \frac{2\pi}{n_1 L} + \frac{2\pi}{n_1 L} = \frac{4\pi}{n_1 L}$$ (single fractionation)

(The total momentum must be an integer multiple of $\frac{2\pi}{L}$) (Das+SDM 96)

Duality

n'_5 fractional D1 branes

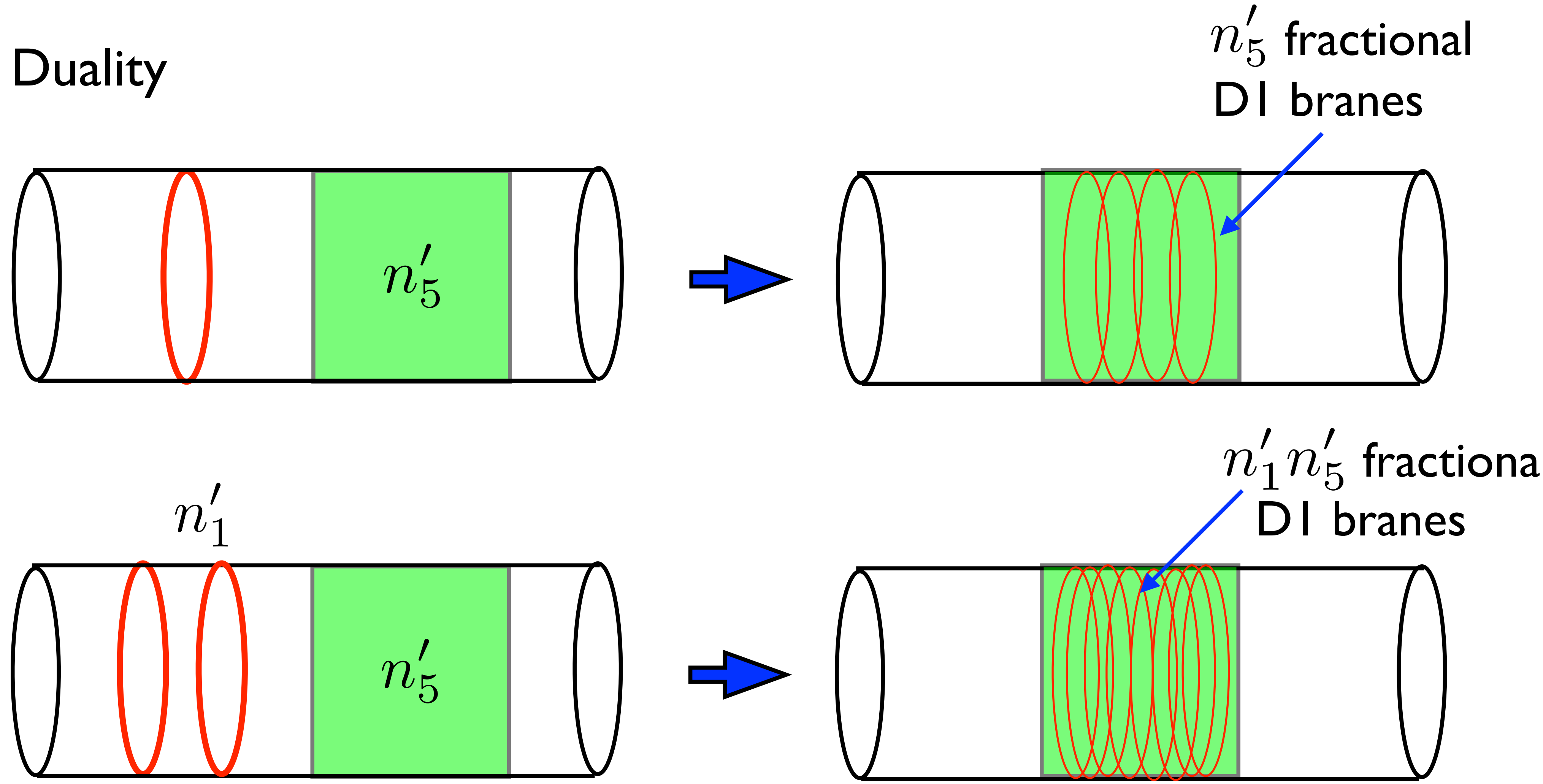


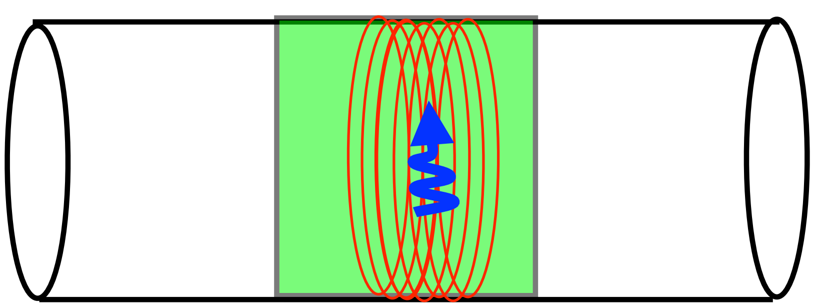
$$T = \frac{1}{2\pi\alpha' g}$$

$$T = \frac{1}{2\pi\alpha' g}\frac{1}{n'_5}$$

Low tension objects can stretch easily …. thus they generate physics at longer length scales

Duality



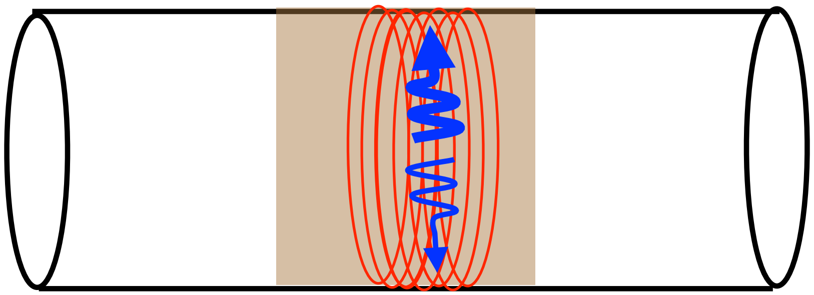


Momentum comes in fractional units: (double fractionation)

$$\frac{2\pi}{n_1' n_5' L}$$

(Maldacena+Susskind 96)

4-charge hole in 3+1 noncompact directions: D1-D5-KK-P charges



Momentum comes in fractional units: (triple fractionation) $$\frac{2\pi}{n'_1 n'_5 n'_{KK} L}$$

General heuristic picture:

(a) Make a bound state of any set of objects that are mutually supersymmetric

(b) Let the number of these objects be $n_1, n_2, \ldots n_k$

(c) Add some extra energy

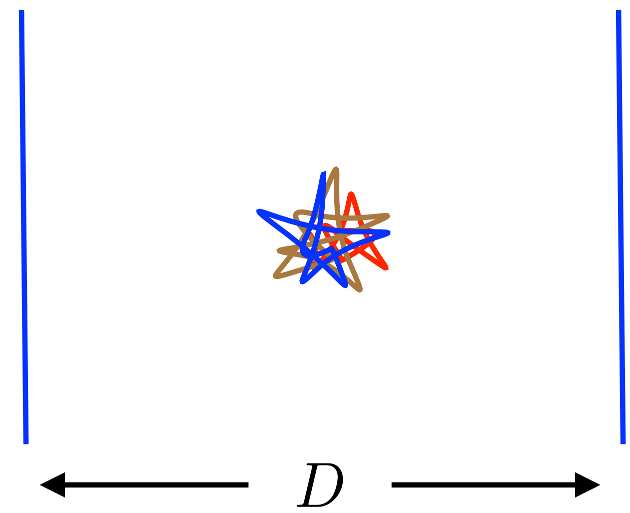
(d) This will create pairs of branes in fractional units $$\frac{1}{n_1 n_2 \ldots n_{k-1}}$$

The brane pairs created correspond to the lightest fractional branes

What is the size of a 3-charge bound state (say D1-D5-P)

What is the distance to which the fractional branes stretch (in the noncompact directions)?

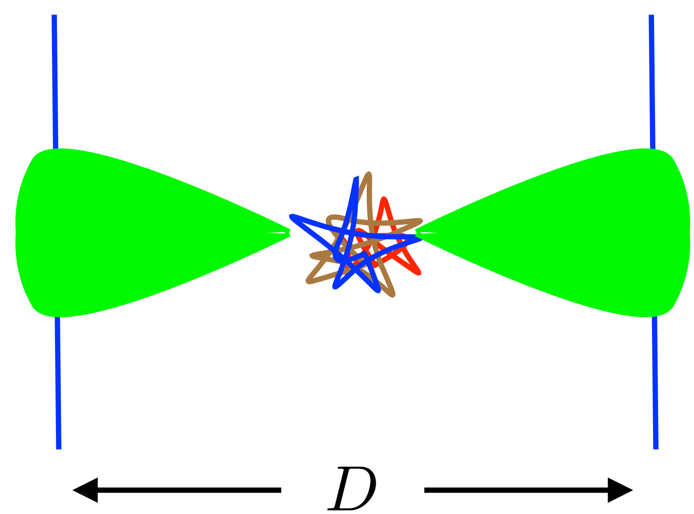
(a) Start with a 3-charge bound state



(b) Compactify one transverse direction to a circle of radius D

(c) Add the minimum allowed energy for the box

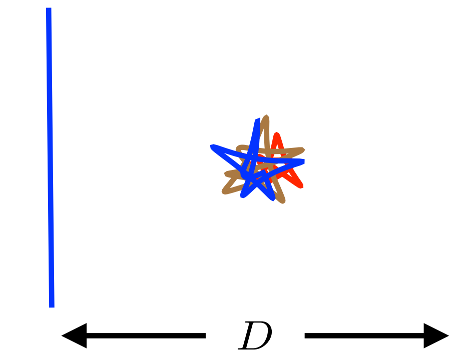
$$\Delta E \sim \frac{1}{D}$$



(d) Ask if brane pairs can use thus energy to wrap around the box.

The 3-charge extremal bound state had an entropy

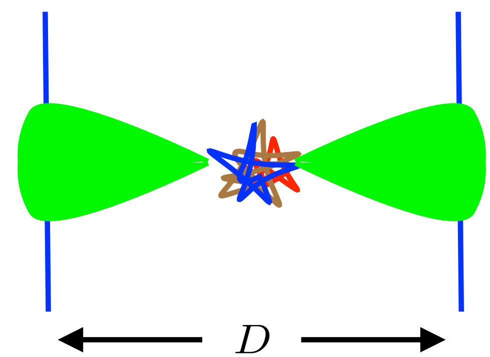
$$S = 2\pi\sqrt{n_1 n_2 n_3}$$



Adding the extra energy $\Delta E \sim \dfrac{1}{D}$ creates brane pairs so that the entropy becomes

$$S = 2\pi\sqrt{n_1 n_2 n_3} + \Delta S$$

$$= 2\pi\sqrt{n_1 n_2 n_3}(\sqrt{n_4} + \sqrt{\bar{n}_4})$$



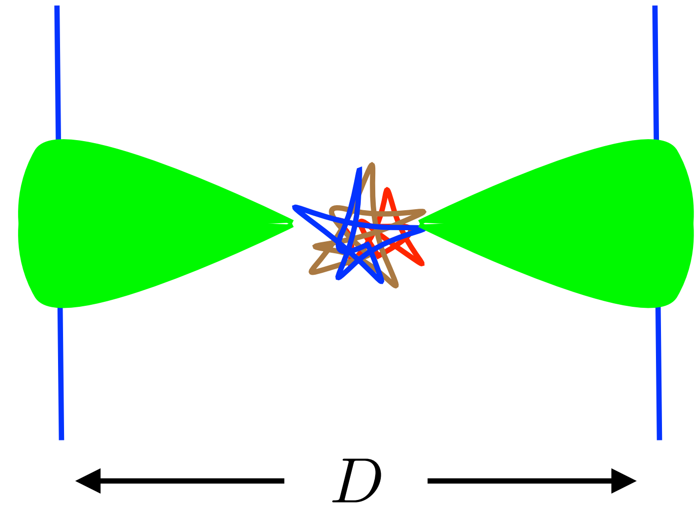
We want the brane pair creation to be not just possible, but probable

We require $\Delta S = 1$

Phase space with brane pairs is 2.7 times larger than phase space without brane pairs

We find

$$D \sim \left[\frac{\sqrt{n_1' n_5' n_p'} g^2 \alpha'^4}{VL}\right]^{\frac{1}{3}}$$



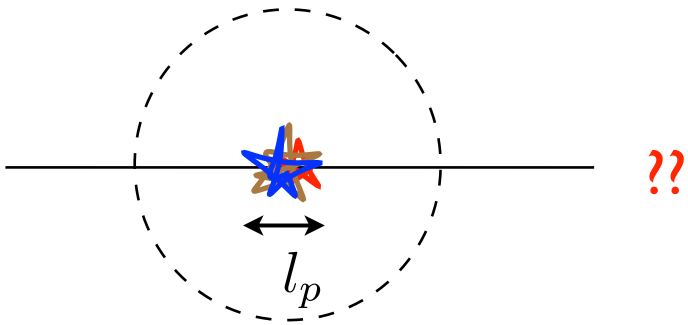
The horizon radius of the D1-D5-P hole is

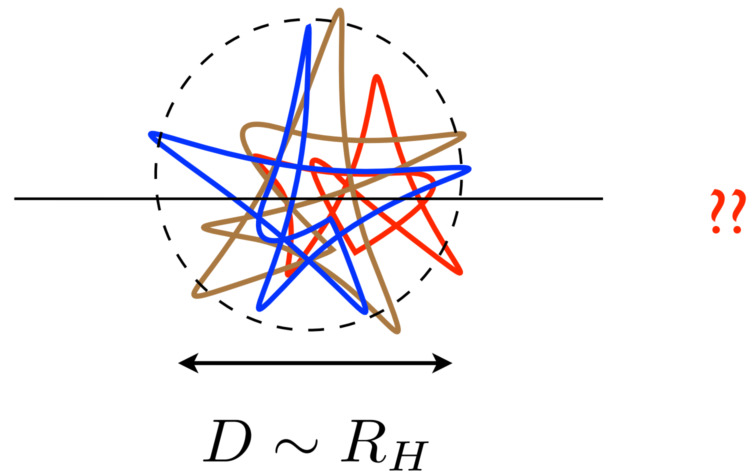
$$R_H = \left[\frac{(2\pi)^5 \sqrt{n_1' n_5' n_p'} g^2 \alpha'^4}{VL}\right]^{\frac{1}{3}} \quad \Rightarrow \quad D \sim R_H$$

(SDM 97
SDM 0510180)

Suppose we make a bound state with a large number of branes

Does the size of the bound state grow with the number of branes?





In this case bound states in string theory will behave like normal bodies, that radiate from their surface (rather than through pair creation at a horizon).

Then there will be no information paradox

The fuzzball construction

All black holes have similar expressions for entropy

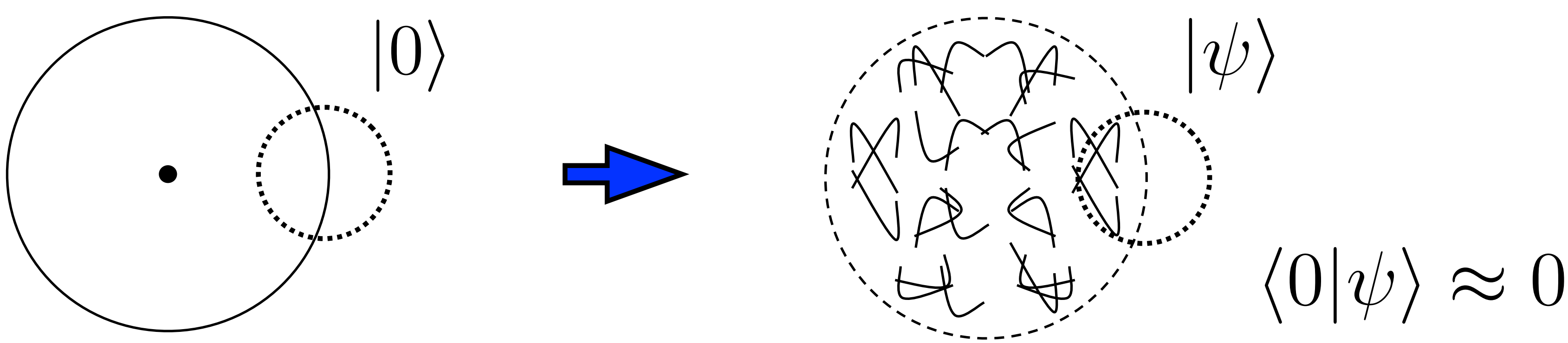
2-charges $$S = 2\sqrt{2}\pi\sqrt{n_1 n_2}$$

3-charges $$S = 2\pi\sqrt{n_1 n_2 n_3}$$

4-charges $$S = 2\pi\sqrt{n_1 n_2 n_3 n_4}$$

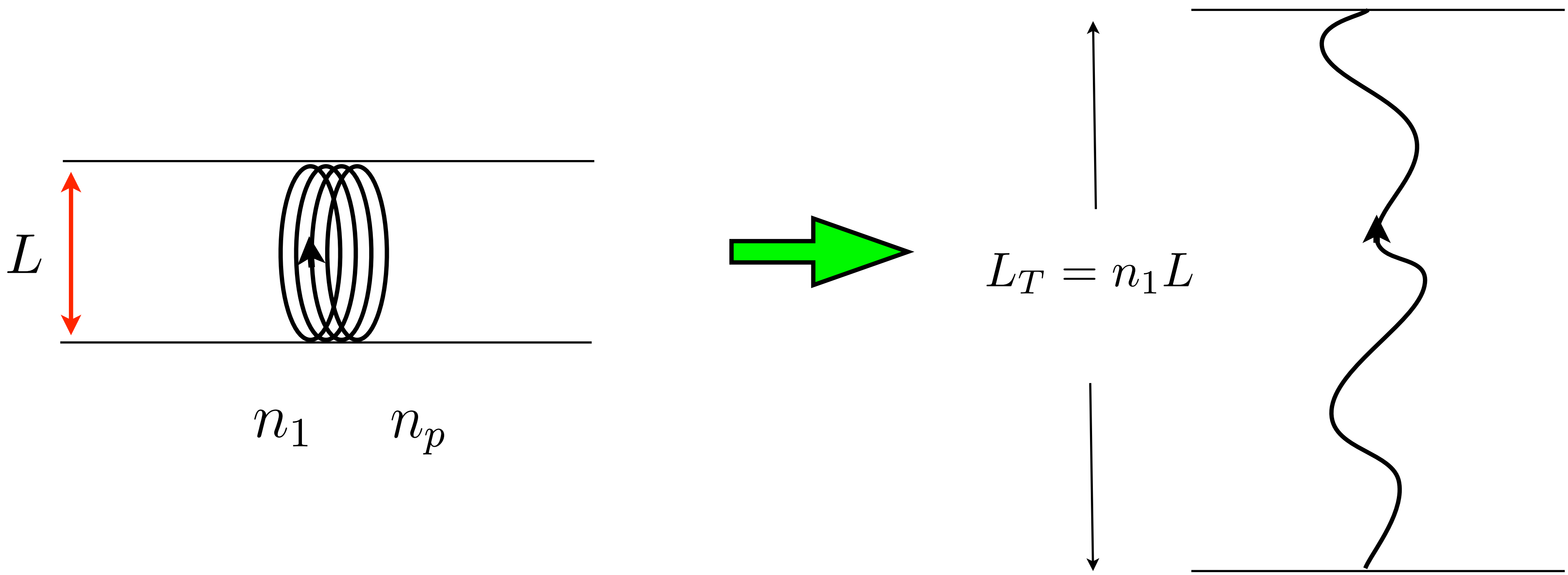
2 charges
\+ nonextremality $$S = 2\pi\sqrt{n_1 n_2}(\sqrt{n_3} + \sqrt{\bar{n}_3})$$

3-charges
\+ nonextremality $$S = 2\pi\sqrt{n_1 n_2 n_3}(\sqrt{n_4} + \sqrt{\bar{n}_4})$$



2-charge NS1-P extremal hole

IIB: $M_{9,1} \rightarrow M_{4,1} \times S^1 \times T^4$



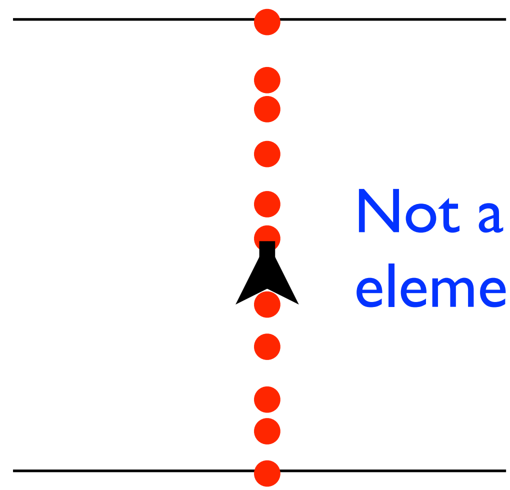
covering space

A NS1 string carrying the momentum P in the form of travelling waves

A key point

The elementary string (NS1) does not have any LONGITUDINAL vibration modes

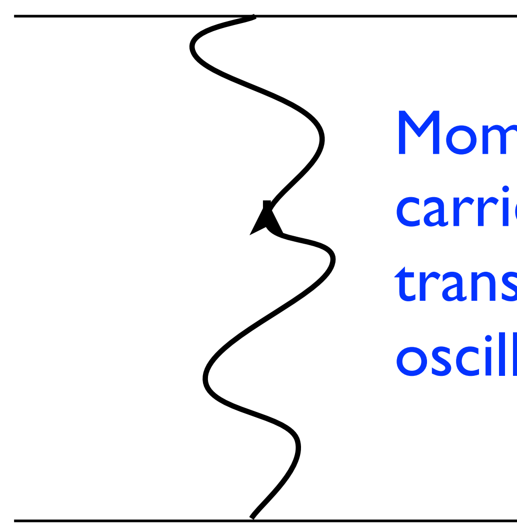
This is because it is not made up of ‘more elementary particles’



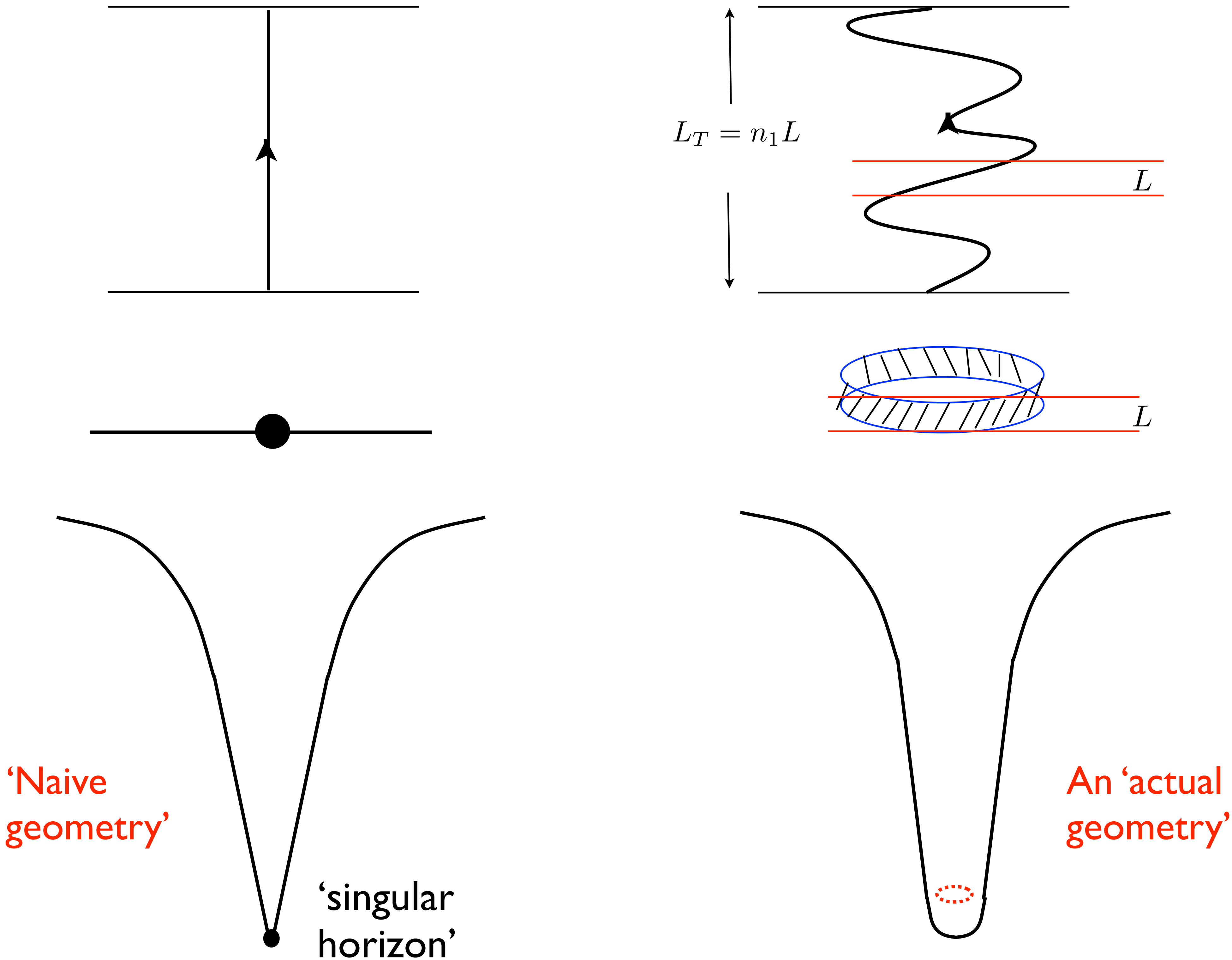
Not a mode for the elementary string

Thus only transverse oscillations are permitted

This causes the string to spread over a nonzero transverse area



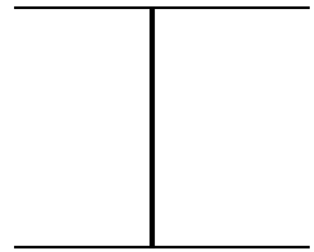
Momentum is carried by transverse oscillations



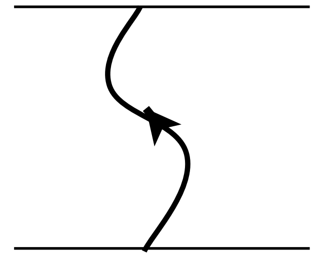
$L_T = n_1 L$
L
L
‘Naive geometry’
‘singular horizon’
An ‘actual geometry’

Making the geometry

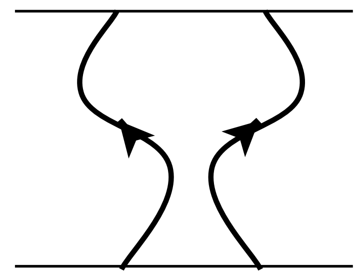
We know the metric of one straight strand of string



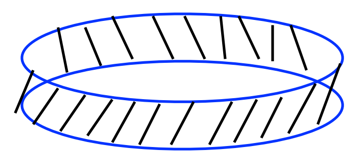
We know the metric of a string carrying a wave -- ‘Vachaspati transform’



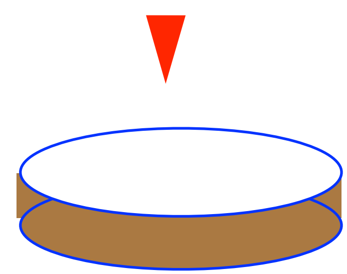
We get the metric for many strands by superposing harmonic functions from each strand



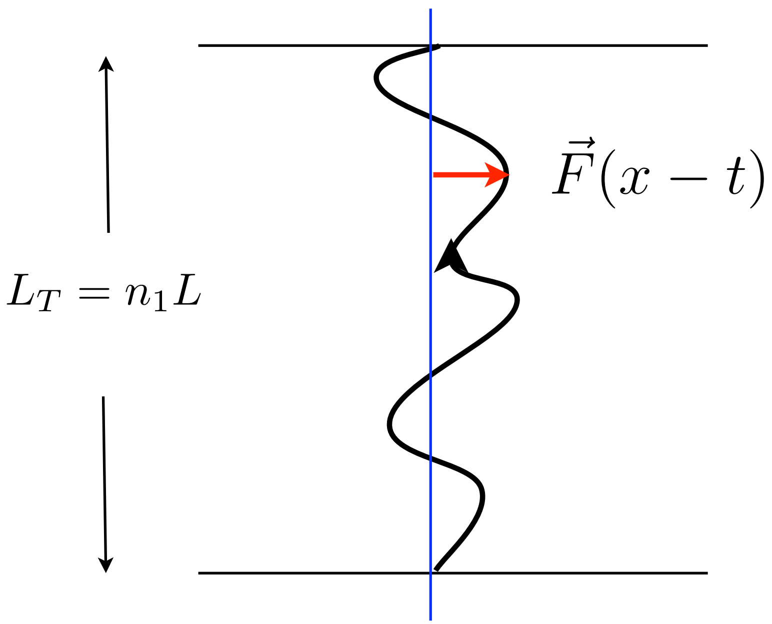
(Dabholkar, Gauntlett, Harvey, Waldram ’95, Callan, Maldacena, Peet ’95)



In our present case, we have a large number of strands, so we ‘smear over them to make a continuous ‘strip’ (Lunin+SDM ’01)



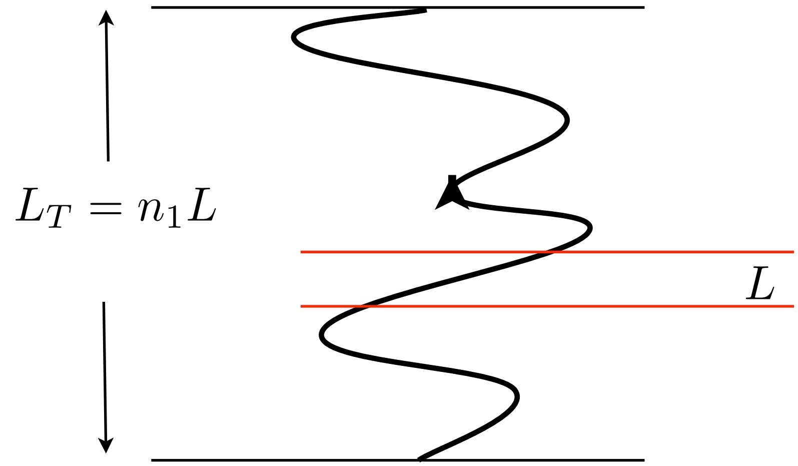
$$
\begin{aligned}
ds^2_{string} &= H[-du dv + K dv^2 + 2A_i dx_i dv] + \sum_{i=1}^{4} dx_i dx_i + \sum_{a=1}^{4} dz_a dz_a \\
B_{uv} &= \frac{1}{2}[H-1], \qquad B_{vi} = HA_i \\
e^{2\phi} &= H
\end{aligned}
$$



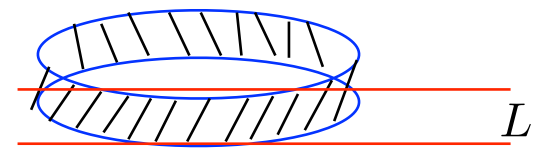
$$
\begin{aligned}
H^{-1} &= 1 + \frac{Q_1}{L_T} \int_0^{L_T} \frac{dv}{|\vec{x} - \vec{F}(v)|^2} \\
K &= \frac{Q_1}{L_T} \int_0^{L_T} \frac{dv (\dot{\vec{F}}(v))^2}{|\vec{x} - \vec{F}(v)|^2} \\
A_i &= -\frac{Q_1}{L_T} \int_0^{L_T} \frac{dv \dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|^2}
\end{aligned}
$$

Lessons:

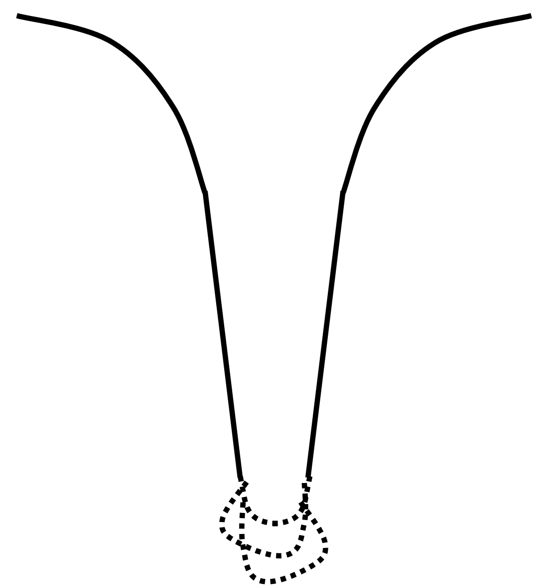
(a) We do not find a horizon; we just get a distributed string source



(b) The traditional approach to black holes looked for metric with a spherically symmetric ansatz. But the actual microstates are not spherically symmetric



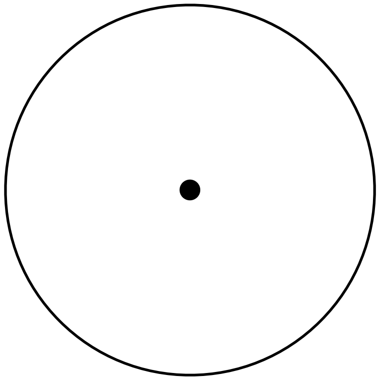
(c) Different microstates have different structures. Thus they carry the information of the state (hair).



Two useful tools:

In general black hole microstates are expected to have structure at the planck scale

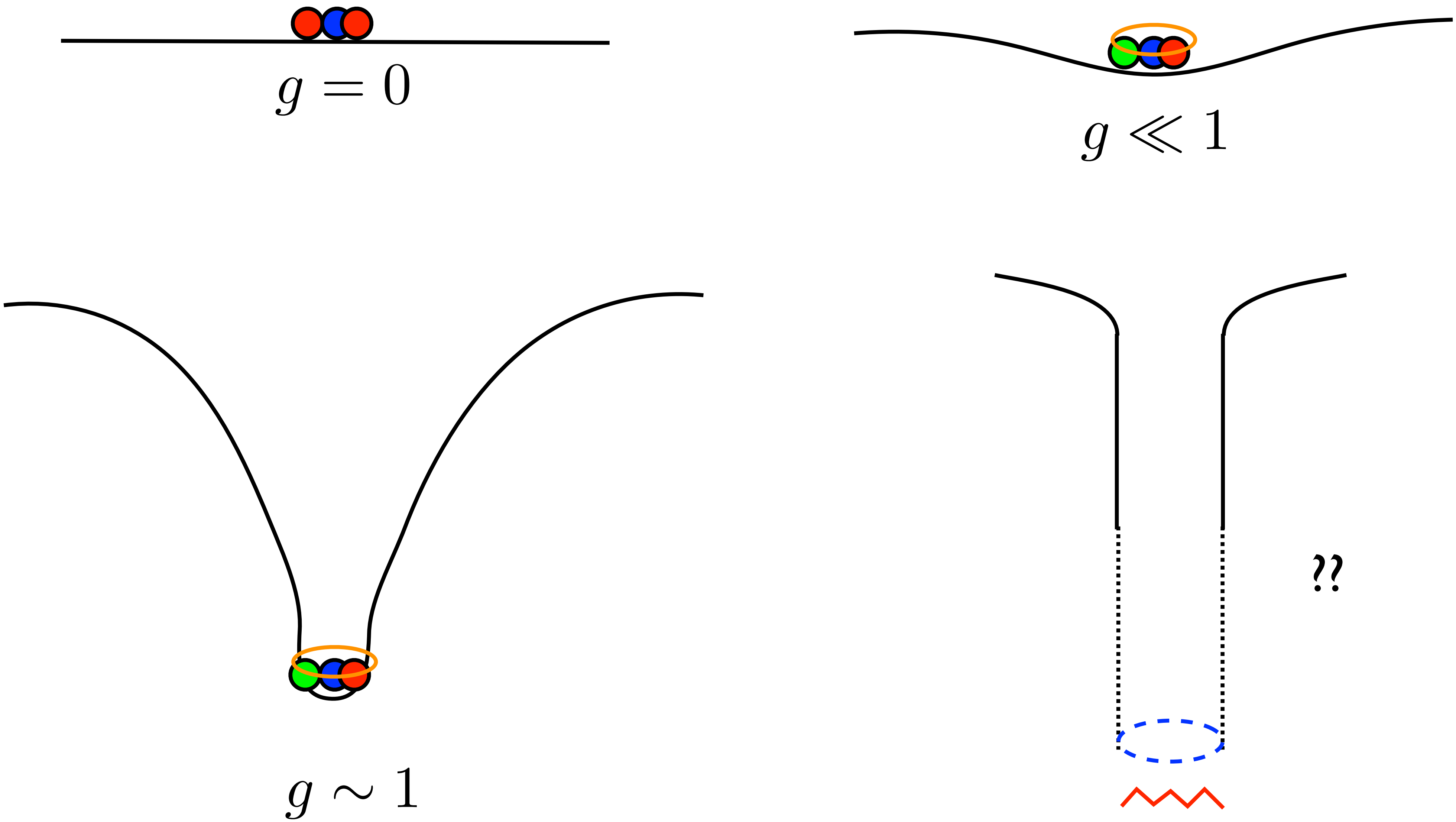
$$S_{bek} = \frac{A}{4G} \sim \frac{A}{l_p^2}$$



Since quantum gravity is difficult to study at the plank scale, in most theories it is hard to understand what is going on in a black hole

(A) In string theory, we have a coupling g which can be taken from small to large. Black hole states are at large effective coupling, but the weak coupling picture can be a useful guide.

Extremal states



Unless there is a phase transition, we cannot get the infinite throat of the traditional extremal hole

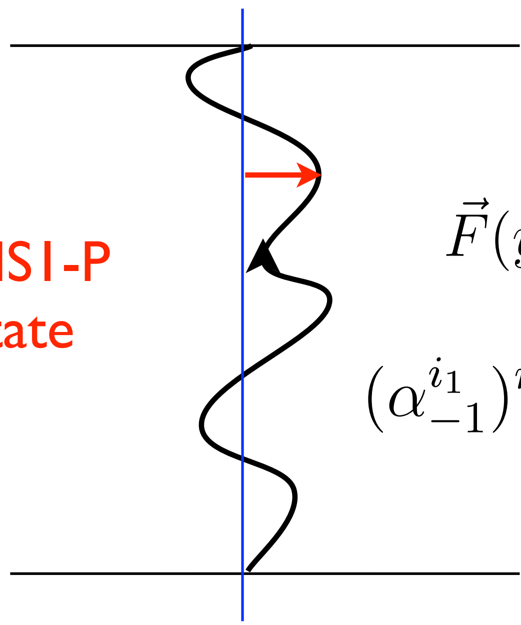
(B) The weak coupling brane picture gives an overall picture of all states

We then look for states ordered by SIMPLICITY

The simplest states are those where 'all excitations are in the same mode'
These generate coherent states, which can be studied by writing metrics

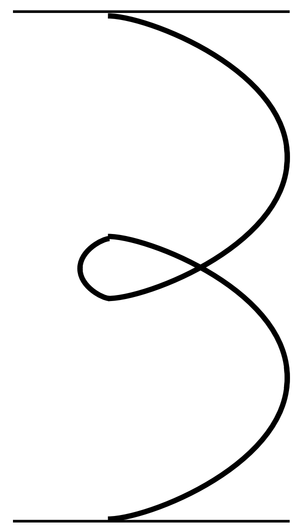
We then move to more complicated states etc.

NS1-P
state



$$\vec{F}(y-ct) =$$

$$(\alpha^{i_1}_{-1})^{n_1}(\alpha^{i_2}_{-2})^{n_2}\dots|0\rangle$$

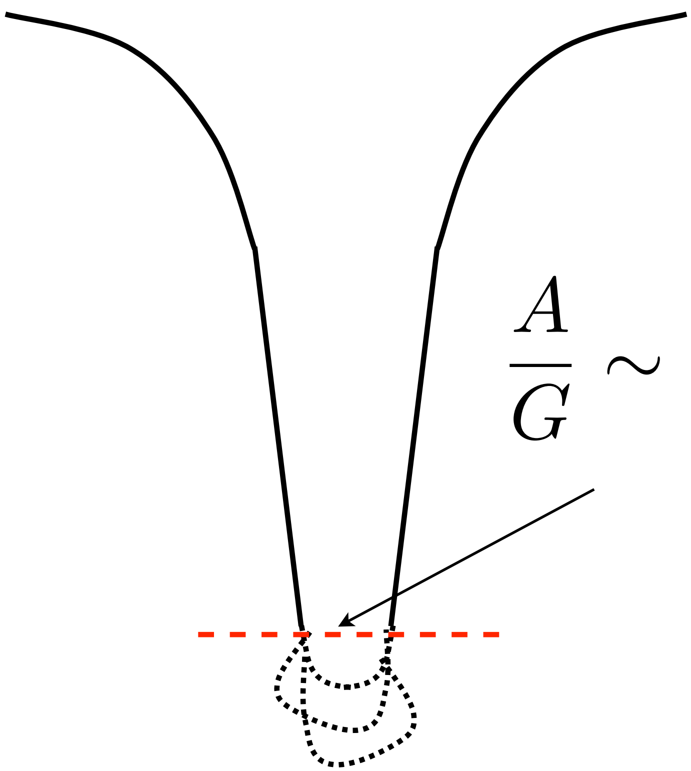


All momentum in
lowest harmonic

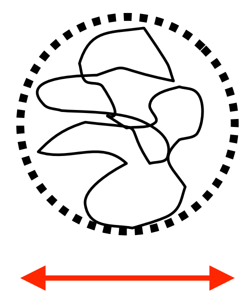
Then we move to more generic states: Use more and more higher harmonics. We do not find a horizon for any profile of the string.

Generic state has harmonic $k \sim \sqrt{n_1 n_p}$

Such a state will have structure at the string scale, but we can estimate the size of the region over which the metric deformations are nontrivial



$$\frac{A}{G} \sim \sqrt{n_1 n_p} \sim S_{micro}$$

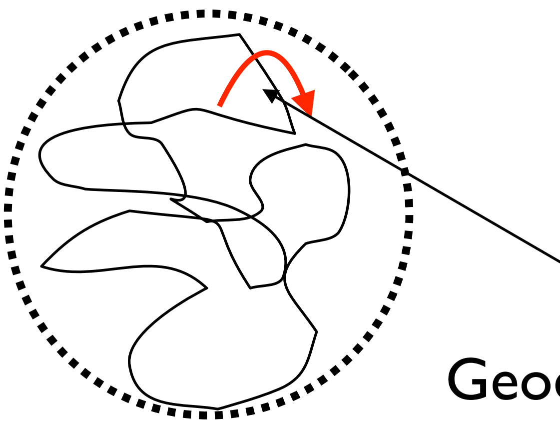


$$D \sim R_h$$

Evolution of wavemodes:

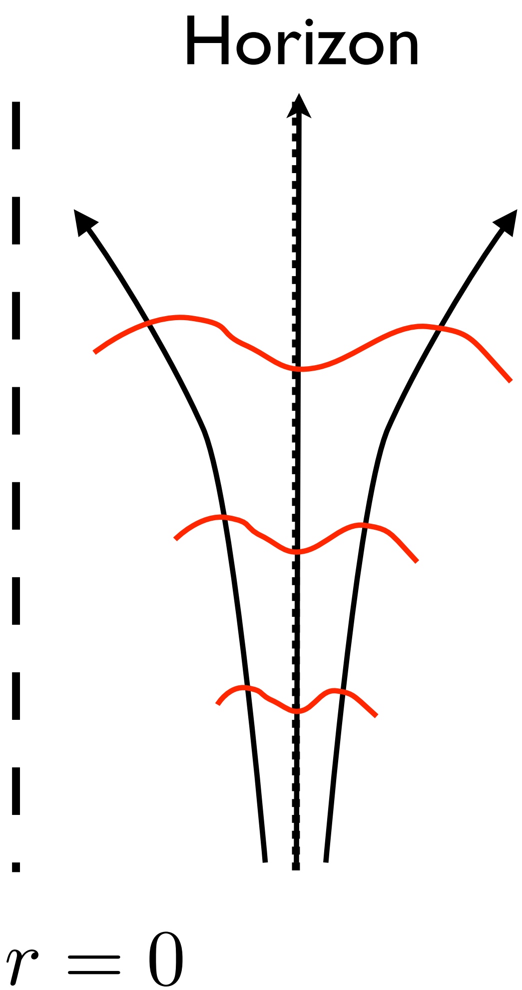
Hawking radiation arises from evolution of modes near the hole, on both sides of the horizon

We find that this evolution is altered by order unity, and the evolution of modes departs more and more from evolution in the vacuum as we move towards generic states

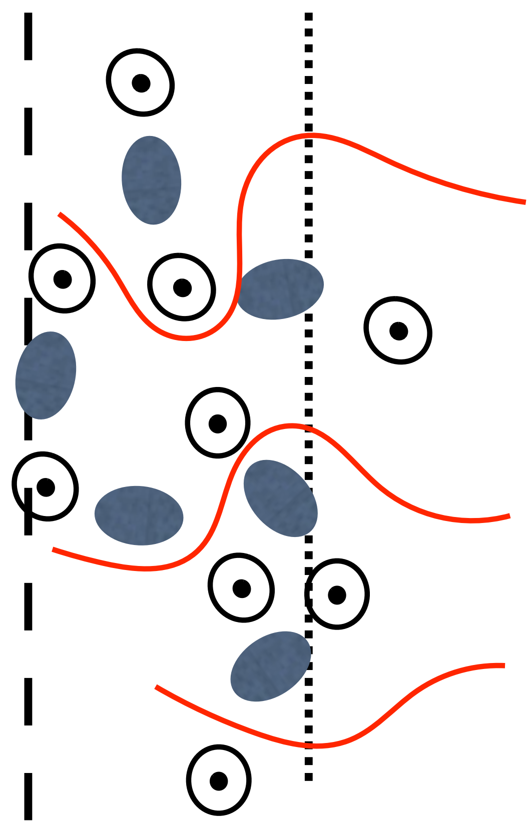


Geodesics bend by order unity
as they pass near curve

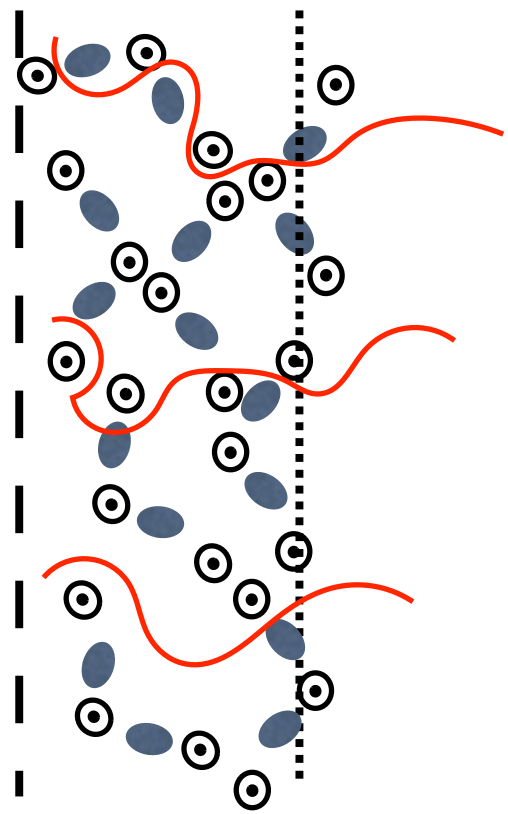
Evolution of modes in a fuzzball



Traditional picture



Simple fuzzball state



More complicated fuzzball state