Supergravity and Exceptional Field Theory

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Henning Samtleben

Université de Lyon, Laboratoire de Physique, ENS Lyon,
46 allée d'Italie, F-69364 Lyon CEDEX 07, France
henning.samtleben@ens-lyon.fr

Plan of the lectures

IV. Kaluza-Klein Supergravity

1. Dimensional reduction of pure gravity
2. Dimensional reduction of supergravity
3. Maximal $D = 4, \mathcal{N} = 8$ supergravity
4. Sphere compactifications

V. Gauged Supergravity

1. The embedding tensor
2. Deformed gauge algebra
3. Lagrangian
4. Examples

VI. Exceptional Field Theory

1. Generalized diffeomorphisms and section constraints
2. Gauge algebra
3. Exceptional form of supergravity

References

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D	spinors	$\mathcal{N}$	fields	n_B
11	M	1	$e_\mu{}^a$, ψ_μ, $C_{\mu\nu\rho}$	128
10	MW	(1,1)	$e_\mu{}^a$, $\psi_{+\mu}$, $\psi_{-\mu}$, $C_{\mu\nu\rho}$, $B_{\mu\nu}$, A_μ, λ_+, λ_-, ϕ	128
		(2,0)	$e_\mu{}^a$, $2\,\psi_{+\mu}$, $C^{(+)}_{\mu\nu\rho\sigma}$, $2\,B_{\mu\nu}$, $2\,\lambda_-$, $2\,\phi$	128
		(1,0)	$e_\mu{}^a$, $\psi_{+\mu}$, $B_{\mu\nu}$, λ_-, ϕ	64
			A_μ, λ_+	8
9	pM	2	$e_\mu{}^a$, $2\,\psi_\mu$, $C_{\mu\nu\rho}$, $2\,B_{\mu\nu}$, $3\,A_\mu$, $4\,\lambda$, $3\,\phi$	128
		1	$e_\mu{}^a$, ψ_μ, $B_{\mu\nu}$, A_μ, λ, ϕ	56
			A_μ, λ, ϕ	8
8	pM	2	$e_\mu{}^a$, $2\,\psi_\mu$, $C_{\mu\nu\rho}$, $3\,B_{\mu\nu}$, $6\,A_\mu$, $6\,\lambda$, $7\,\phi$	128
		1	$e_\mu{}^a$, ψ_μ, $B_{\mu\nu}$, $2\,A_\mu$, λ, ϕ	48
			A_μ, λ, $2\,\phi$	8
7	sM	4	$e_\mu{}^a$, $4\,\psi_\mu$, $5\,B_{\mu\nu}$, $10\,A_\mu$, $16\,\lambda$, $14\,\phi$	128
		2	$e_\mu{}^a$, $2\,\psi_\mu$, $B_{\mu\nu}$, $3\,A_\mu$, $2\,\lambda$, ϕ	40
			A_μ, λ, $3\,\phi$	8
6	sMW	(2,2)	$e_\mu{}^a$, $4\,\psi_{+\mu}$, $4\,\psi_{-\mu}$, $5\,B_{\mu\nu}$, $16\,A_\mu$, $20\,\lambda_+$, $20\,\lambda_-$, $25\,\phi$	128
		(2,1)	$e_\mu{}^a$, $4\,\psi_{+\mu}$, $2\,\psi_{-\mu}$, $5\,B^{(+)}_{\mu\nu}$, $B^{(-)}_{\mu\nu}$, $8\,A_\mu$, $10\,\lambda_+$, $4\,\lambda_-$, $5\,\phi$	64
		(1,1)	$e_\mu{}^a$, $2\,\psi_{+\mu}$, $2\,\psi_{-\mu}$, $B_{\mu\nu}$, $4\,A_\mu$, $2\,\lambda_+$, $2\,\lambda_-$, ϕ	32
			A_μ, $2\,\lambda_+$, $2\,\lambda_-$, $4\,\phi$	8
		(2,0)	$e_\mu{}^a$, $4\,\psi_{+\mu}$, $5\,B^{(+)}_{\mu\nu}$	24
			$B^{(-)}_{\mu\nu}$, $4\,\lambda_-$, $5\,\phi$	8
		(1,0)	$e_\mu{}^a$, $2\,\psi_{+\mu}$, $B^{(+)}_{\mu\nu}$	12
			A_μ, $2\,\lambda_+$	4
			$B^{(-)}_{\mu\nu}$, $2\,\lambda_-$, ϕ	4
			$2\,\lambda_-$, $4\,\phi$	4
5	spM	8	$e_\mu{}^a$, $8\,\psi_\mu$, $27\,A_\mu$, $48\,\lambda$, $42\,\phi$	128
		6	$e_\mu{}^a$, $6\,\psi_\mu$, $15\,A_\mu$, $20\,\lambda$, $14\,\phi$	64
		4	$e_\mu{}^a$, $4\,\psi_\mu$, $6\,A_\mu$, $4\,\lambda$, ϕ	24
			A_μ, $4\,\lambda$, $5\,\phi$	8
		2	$e_\mu{}^a$, $2\,\psi_\mu$, A_μ	8
			A_μ, $2\,\lambda$, ϕ	4
			$2\,\lambda$, $4\,\phi$	4
4	M	8	$e_\mu{}^a$, $8\,\psi_\mu$, $28\,A_\mu$, $56\,\lambda$, $70\,\phi$	128
		6	$e_\mu{}^a$, $6\,\psi_\mu$, $16\,A_\mu$, $26\,\lambda$, $30\,\phi$	64
		5	$e_\mu{}^a$, $5\,\psi_\mu$, $10\,A_\mu$, $11\,\lambda$, $10\,\phi$	32
		4	$e_\mu{}^a$, $4\,\psi_\mu$, $6\,A_\mu$, $4\,\lambda$, $2\,\phi$	16
			A_μ, $4\,\lambda$, $6\,\phi$	8
		3	$e_\mu{}^a$, $3\,\psi_\mu$, $3\,A_\mu$, λ	8
			A_μ, $4\,\lambda$, $6\,\phi$	8
		2	$e_\mu{}^a$, $2\,\psi_\mu$, A_μ	4
			A_μ, $2\,\lambda$, ϕ	4
			$2\,\lambda$, $4\,\phi$	4
		1	$e_\mu{}^a$, ψ_μ	2
			A_μ, λ	2
			λ, $2\,\phi$	2

Table 1: Supermultiplets in different space-time dimensions. The red background indicates the supergravity multiplets. The last column gives the number of bosonic degrees of freedom.

	maximal		half-maximal	
$D=11$	1			
	↓	IIB		
$D=10$	GL(1)	$\frac{\mathrm{SL}(2)}{\mathrm{SO}(2)}$	GL(1)	
	↓	↙	↓	
$D=9$	$\frac{\mathrm{GL}(1)\times\mathrm{SL}(2)}{\mathrm{SO}(2)}$		$\frac{\mathrm{GL}(1)\times\mathrm{SO}(1,1+n)}{\mathrm{SO}(1+n)}$	
	↓		↓	
$D=8$	$\frac{\mathrm{SL}(2)\times\mathrm{SL}(3)}{\mathrm{SO}(2)\times SO(3)}$		$\frac{\mathrm{GL}(1)\times\mathrm{SO}(2,2+n)}{\mathrm{SO}(2)\times\mathrm{SO}(2+n)}$	
	↓		↓	
$D=7$	$\frac{\mathrm{SL}(5)}{\mathrm{SO}(5)}$		$\frac{\mathrm{GL}(1)\times\mathrm{SO}(3,3+n)}{\mathrm{SO}(3)\times\mathrm{SO}(3+n)}$	
	↓		↓	(2,0)
$D=6$	$\frac{\mathrm{SO}(5,5)}{\mathrm{SO}(5)\times\mathrm{SO}(5)}$		$\frac{\mathrm{GL}(1)\times\mathrm{SO}(4,4+n)}{\mathrm{SO}(4)\times\mathrm{SO}(4+n)}$	$\frac{\mathrm{SO}(5,5+n)}{\mathrm{SO}(5)\times\mathrm{SO}(5+n)}$
	↓		↓	↙
$D=5$	$\frac{\mathrm{E}_{6(6)}}{\mathrm{USp}(8)}$		$\frac{\mathrm{GL}(1)\times\mathrm{SO}(5,5+n)}{\mathrm{SO}(5)\times\mathrm{SO}(5+n)}$	
	↓		↓	
$D=4$	$\frac{\mathrm{E}_{7(7)}}{\mathrm{SU}(8)}$		$\frac{\mathrm{SL}(2)\times\mathrm{SO}(6,6+n)}{\mathrm{SO}(2)\times\mathrm{SO}(6)\times\mathrm{SO}(6+n)}$	
	↓		↓	
$D=3$	$\frac{\mathrm{E}_{8(8)}}{\mathrm{SO}(16)}$		$\frac{\mathrm{SO}(8,8+n)}{\mathrm{SO}(8)\times\mathrm{SO}(8+n)}$	

Table 2: Symmetric spaces in maximal and half-maximal supergravity and their relation by dimensional reduction. In the half-maximal case, the integer n denotes the number of vector multiplets in 10 dimensions.