Introduction

Issues with "Unifying" QM and GR

- Discuss issues associated with
  BH's : special states
- Standard tool QFT in curved space ($N=\infty$, AdS/CFT)
- GR vs QM Equivalence principle vs. Unitarity

My proposal: BH's have to be treated as quantum states

Plan of lectures

1. The Hawking information paradox:
   Breakdown of Unitarity in gravitational collapse.
2. What if the evolution is unitary?
   - The Page model
   - The Firewall problem
     Breakdown of the equivalence principle in gravitational collapse
3. How to treat BH's as quantum states and what are the consequences

Detailed Plan of lectures

1. The Hawking information paradox
   - The collapsing shell model
   - The Hawking effect : particle production during gravitation collapse.
   - The density matrix of the produced particles
   - Conclusion : Unitarity is violated
   - The pair production picture
   - The near horizon (NH) state

2. What if the evolution is unitary ?
   - The Page model
   - Measures of purity, entanglement and information
   - The Page curve for entanglement & information
   - The firewall problem: The nature of the NH state
   - Equivalence principle vs. unitarity

3. BH's as quantum states
   - Semiclassical BH's, finite mass
   - The Hawking effect for semiclassical BH's
   - The modified Page curve
   - Fate of an infalling object: Nature of the NH state

4. Summary

The BH information paradox: Break down of Unitarity (predictability) in gravitational collapse; Hawking PRD '76

4D Schwarzschild BH geometry — Ford 9707062, B&D, Mukanov

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{1}{\left(1 - \frac{2M}{r}\right)} dr^2 + r^2 d\Omega^2$$

$\boxed{2GM = R_S}$

$v = t + r^*$, $u = t - r^*$ "tortoise coordinates"

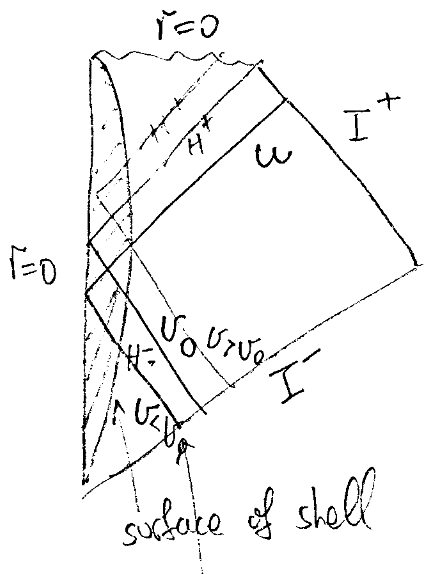
$$dr^* = \frac{dr}{\sqrt{\left(1 - \frac{2M}{r}\right)^2}} , \quad r^* = r + 2M \ln\left(\frac{r}{2M} - 1\right)$$

u - "retarded time"

v - "advanced time"

<u>A collapsing shell geometry</u>

<u>Penrose diagram</u> of the geometry of a collapsing matter shell



! Bigger diagram with colors.

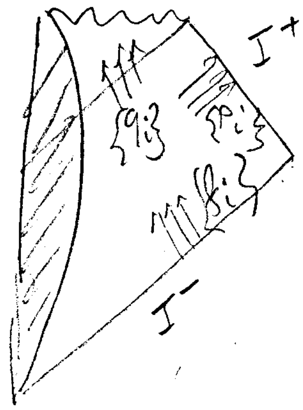
Problem 1: Draw a Penrose diagram of a collapsing star

$v < v_0$ trajectory of a lightray that passes through the shell and escapes to infinity

$v > v_0$ trajectory of a lightray that passes throug the shell and is trapped inside the BH.

Quantum (massless, scalar) fields in the classical space-time of the collapsing shell

$$\phi = \sum_i \left(f_i a_i + f_i^* a_i^\dagger \right)$$



BC - only positive frequencies ∂I^-, $f \sim e^{i\omega v}$

$a_i|0_-\rangle = 0$, $|0_-\rangle$ - "infalling vacuum"

"Almost" well defined (see below)

$$\phi = \sum_i \left(p_i b_i + p_i^* b_i^\dagger + q_i c_i + q_i^* c_i^\dagger \right) ,$$

$\{p_i\}$ - only positive frequencies $@ I^+$, zero Cauchy data $@ H^+$ } $p \sim e^{i\omega u}$ purely outgoing

$\{q_i\}$ - no outgoing component

However: cannot define positive frequency for $\{q_i\}$

$\Rightarrow$ Division into annihilation + creation operators ambiguous

Ambiguity in $\{q_i\}$ does not affect observables ∂I^+ !

"Outgoing vacuum" $b_i|0_+\rangle$, $c_i|0_+\rangle = 0$

$$p_i = \sum_j \alpha_{ij} f_j + \beta_{ij} f_j^* \ , \quad b_i = \sum \alpha_{ij}^* a_j - \beta_{ij}^* a_j^\dagger$$

Bogolubov coefficients.

The Hawking effect:

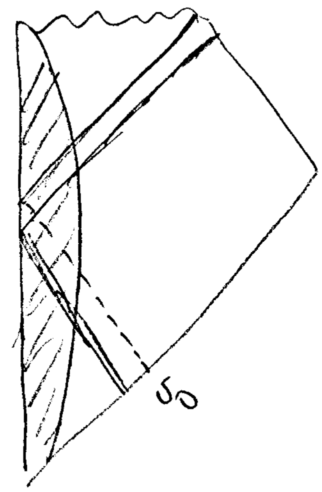
β's $\neq 0 \Rightarrow$ $|\text{outgoing}\rangle$ excited state w.r.t. to $|0_-\rangle$

$|\text{outgoing}\rangle$ is a thermal state

final state is a mixed state !

Calculation of the β coefficients (Ford '97, Birrel+Davies)

The main effect is the large phase shift of modes that pass through the shell and escape to infinity just before the horizon is formed



Use Fourier transform $f_\omega \sim \frac{1}{\sqrt{2\omega}} e^{i\omega v}$

$p_\omega \sim \frac{1}{\sqrt{2\omega}} e^{i\omega u}$

u, v very large near H^+

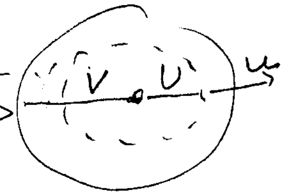
$\Rightarrow$ use geometric optics to evalute phase shift.

<u>Problem 2</u> : Calculate $\beta_{\omega\omega'}$ for a thin shell

Ford 9707062

Inside shell : Minkowski

Outside : Schwarzschild



a. Show that $u = -4M \ln\left(\frac{v_0 - v}{C}\right)$, C - const.

b. Show that $p_\omega = \begin{cases} e^{-4Mi\omega \ln\left(\frac{v_0 - v}{C}\right)} & v < v_0 \\ 0 & v > v_0 \end{cases}$

c. For $p_\omega = \int_0^\infty d\omega' \left(\alpha_{\omega'\omega} f_{\omega'} + \beta_{\omega'\omega} f^*_{\omega'}\right)$,

Show that $\beta_{\omega'\omega} = \frac{t_\omega}{2\pi} \frac{1}{\sqrt{\omega}} \Gamma(1 - i4M\omega)(\omega')^{-1/2 + i4M\omega} e^{-2\pi M\omega}$

$\times$ over all phase factor

t_ω, transmission coefficient (will be ignored)

Spectrum of emitted particles

$$\langle n_j \rangle = \langle 0_- | b_j^\dagger b_j | 0_- \rangle \simeq \int_0^\infty d\omega' \, \beta^*_{\omega'\omega} \beta_{\omega'\tilde{\omega}} =$$

$$= \frac{t^*_\omega t_{\tilde{\omega}}}{(2\pi)^2} \frac{1}{\sqrt{\omega\tilde{\omega}}} \Gamma(1 + i 4M\omega) \Gamma(1 - i 4M\tilde{\omega}) \times$$

$$\times \int_0^\infty d\omega' \, (\omega')^{-1 + i \cdot 4M(\tilde{\omega} - \omega)}$$

$$y = \ln \omega' \qquad \int_0^\infty d\omega' \, (\omega')^{-1 + i 4M(\tilde{\omega} - \omega)} = \int_{-\infty}^{\infty} dy \, e^{iy \cdot 4M(\omega - \tilde{\omega})}$$

$$= \frac{2\pi}{4M} \delta(\tilde{\omega} - \omega)$$

$$\langle n(\omega) \rangle = |t_\omega|^2 \frac{1}{e^{\frac{\hbar\omega}{T_H}} - 1}$$

Problem 3: Calculate explicitly $\langle n_\omega \rangle$ and show that $T_H = \frac{1}{8\pi M}$

$n(\omega, \tilde{\omega}) \propto \delta(\tilde{\omega} - \omega)$ indication that the density matrix of emitted radiation is diagonal $| \langle 0_- | b_i^\dagger b_j | 0_- \rangle \propto \delta_{ij}$

$n(\omega) \propto \frac{1}{e^{\frac{\hbar\omega}{T_H}} - 1}$ indication that the density matrix is thermal

$\Rightarrow$ density matrix is a mixed state

Initial state $\sim |0_-\rangle \otimes |\text{Shell}\rangle \rightarrow$ final state $= \hat{\rho}$

$\Rightarrow$ Evolution is <u>not</u> unitary!

Hawking: The density matrix of outgoing radiation is diagonal (as presented valid only for free fields)

$$\hat{\rho}_H = \frac{1}{Z} e^{-\sum_i \frac{w_i}{T_H} b_i^\dagger b_i} \quad \text{- thermal state}$$

Quick and dirty "proof": the theory is free $\Rightarrow$ density matrix is Gaussian $\hat{\rho} = \frac{1}{Z} e^{-M^{ij} b_i^\dagger b_j}$

$$\langle 0_- | b_i^\dagger b_j | 0_- \rangle \propto \delta_{ij} \Rightarrow M_{ij} \propto \delta_{ij}$$

Best way to prove (valid also for interacting fields): Use PI methods, Gibbons-Hawking, Kabat-Strassler Brustein-Einhorn-Yarom

Weakness: Needs eternal BH geometry, infinite mass

Hawking's proof:

$$\phi = \sum_i \big(p_i b_i + p_i^* \underbrace{b_i^\dagger}_{\text{out}} + q_i c_i + q_i^* \underbrace{c_i^\dagger}_{\text{in}}\big)$$

$$|0_-\rangle = \sum_{A,B} \lambda_{AB} |A_{out}\rangle |B_{in}\rangle ,$$

$$|A_{out}\rangle = \prod_j \frac{1}{\sqrt{n_{ja}!}} (b_j)^{\dagger n_{ja}} |0_+\rangle , \quad |B_{in}\rangle = \prod_k \frac{1}{\sqrt{n_{kb}!}} (C_k^\dagger)^{n_{kb}} |0_{Hor}\rangle$$

n_{ja}, n_{kb} number of particles in j'th (k'th) mode

a, b configuration index.

$$\hat{\rho}_{out} = Tr_{in}\, \hat{\rho} = Tr_{in}\, |0_-\rangle\langle 0_-|$$

$$(\rho_{out})_{AC} = \sum_B \lambda_{AB} \lambda^*_{CB}$$

Plan: show that $\hat{\rho}_{out}$ is diagonal $(\rho_{out})_{\{n_{ja}\}\{n_{jb}\}} \propto \delta_{\{n_{ia}\}\{n_{jb}\}}$

ρ_{AC} can be completely determined by VEV's of the form $\langle 0_- | b_i^m (b_j^\dagger)^n | 0_- \rangle \Rightarrow$ Do not depend on the choice of $\{q_i\}$ of in-modes

For example $\langle 0_- | b_i | 0_- \rangle = 0$

$$= \mathrm{Tr}(\rho b_i) = \sum_N \rho_{N,N-1} C(N,i)$$

Similarly : $\langle 0_- | b_i^m (b_j^\dagger)^n | 0_- \rangle \propto \delta_{ij} \delta_{mn}$ (see problem 4)

and so on

$\Rightarrow$ Density matrix is diagonal $\rho_{\{n_{ia}\}\{n_{jb}\}} \propto \delta_{\{n_{ia}\}\{n_{jb}\}}$

by looking at $(b_i^\dagger b_i)^n$ it is possible to show that the state is thermal $\hat{\rho} = \frac{1}{Z} e^{-\sum \frac{\omega_i}{T_H} b_i^\dagger b_i}$

But important aspect : $\hat{\rho}$ is diagonal! $\Leftrightarrow$ mixed state.

$|\text{Initial}\rangle \sim |0_-\rangle \otimes |\text{shell}\rangle \longrightarrow |\text{Final}\rangle = \hat{\rho}$

$\Rightarrow$ Evolution is not unitary!

Result does not depend on assumptions about the ingoing modes q.

Hawking : Corrections exponential $e^{-R_S^2/l_P^2} \sim e^{-S_{BH}}$ - due to ambiguity in definition of positive frequency for f

turns out to be incorrect

corrections to $\langle 0_- | \hat{O} | 0_- \rangle \sim l_P^2 / R_S^2$

Problem 4 :

a. Show that $\langle 0_- | (b_j^{+})^{m} b_i^{n} | 0_- \rangle = 0 \quad i \neq j$

b. Show that $\hat{\rho}_{out}$ is completely determined by polynomials of $\{b_i\} \{b_i^{+}\}$

c. Shat that the density matrix is diagonal in total number of particles $N = \sum_i n_i$

d. Show that $\hat{\rho}_{out}$ is diagonal (As far as I know does not exist in lit.