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Generating and Analyzing Models with Three-Body Hardcore Constraint

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SFB
Coherent Control of Quantum Systems

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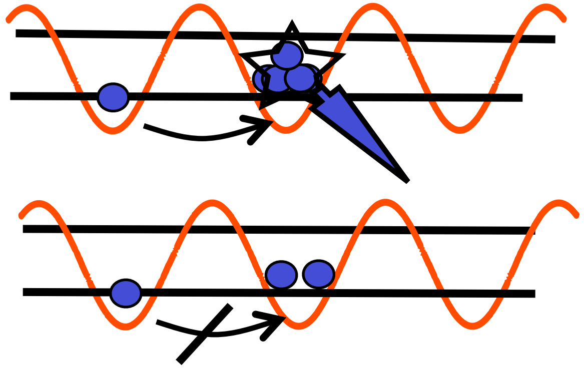
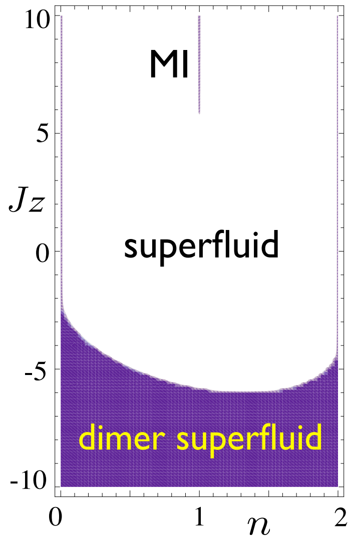
Outline

Part I: Dissipative generation of a three-body hardcore interaction



- Mechanism
- Experimental prospect
- Ground state preparation

Part II: Phase diagram for three-body hardcore Bosons



- First look: Dimer superfluid phase in Mean Field theory
- Construction of a Quantum Field Theory
- Beyond mean field results

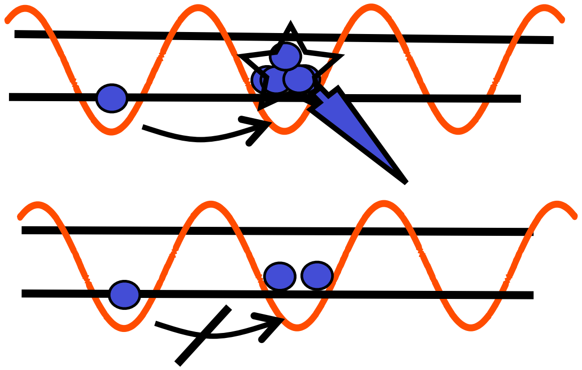
Part III: Atomic colour superfluid of three-component fermions

A. J. Daley, J. Taylor, SD, M. Baranov, P. Zoller, Phys. Rev. Lett. **102**, 040402 (2009)

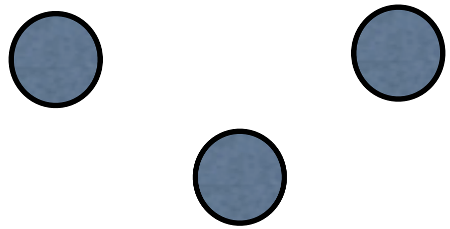
SD, M. Baranov, A. J. Daley, P. Zoller, arxiv:0910.1859 (2009); in preparation

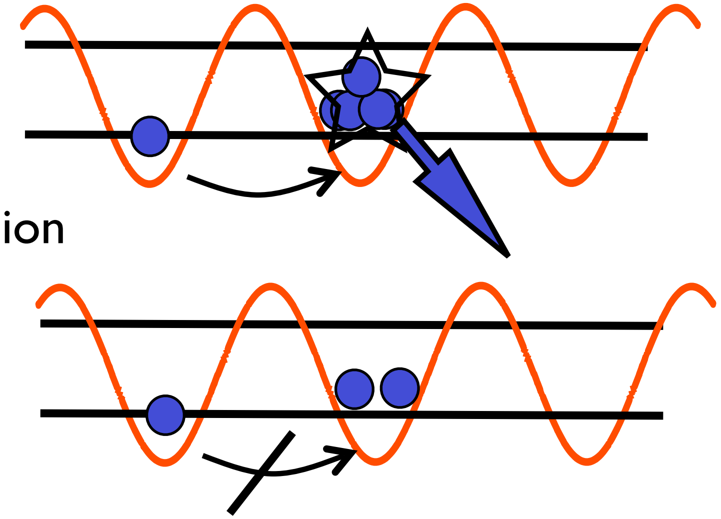
A. Kantian, M. Dalmonte, SD, W. Hofstetter, P. Zoller, A. J. Daley, arXiv:0908.3235 (2009)

Part I:
Generation of Three-Body Interactions from Three-Body Loss



Motivation





- 3-body loss processes (-)
  - ubiquitous, but typically undesirable inelastic 3 atom collision
  - inelastic 3 atom collision
  - molecule + atom ejected from lattice
- 3-body interactions (+)
  - Stabilize bosonic system with attractive interactions
  - Stabilize 3-component fermion system: atomic color superfluidity

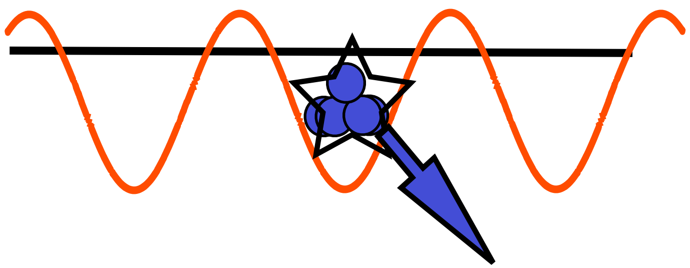
$$i\gamma_3 \longrightarrow \gamma_3$$

➡ We make use of strong 3-body loss to generate a 3-body hard-core constraint

Basic Mechanism: Interactions via Loss

- Model: Bosons on the optical lattice with three-body recombination
- Hamiltonian: $H = -J \sum_{\langle i,j \rangle} \hat{b}_i^{\dagger} \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1)$
- Three-body recombination: loss from lattice to continuum of unbound states
- Model on-site three-body loss: Master Equation

  in Lindblad form



couples density matrix sectors with n+3, n particles

$$\dot{\rho} = -i[H,\rho] + \frac{\gamma_3}{12} \sum_i 2\hat{b}_i^3 \rho \hat{b}_i^{\dagger} - \{\hat{b}_i^{\dagger\,3} \hat{b}_i^3, \rho\}$$

three-body loss rate

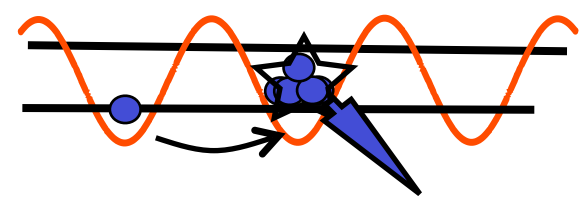
- zero temperature approximation: binding energy of deeply bound molecule much larger than lattice depth

Basic Mechanism: Interactions via Loss

- Rewrite the Master Equation as

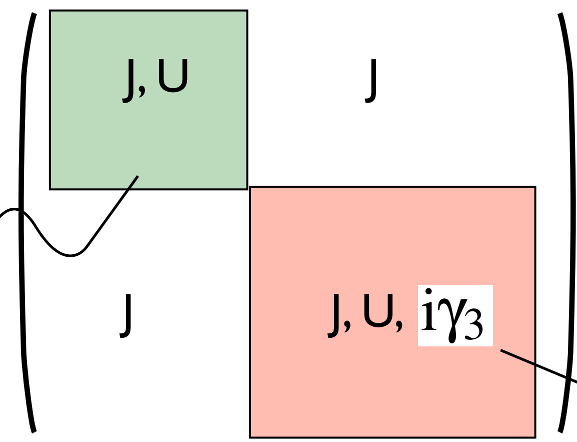
non-particle number conserving: couples sectors with n+3, n particles in the density matrix

$$\dot{\rho} = -i\left(H_{eff}\rho - \rho H_{eff}^{\dagger}\right) + \frac{\gamma_3}{12}\sum_i 2\hat{b}_i^3\rho(\hat{b}_i^{\dagger})^3$$



particle number conserving (but norm decays)

$$H_{eff} = H - i\frac{\gamma_3}{12}\sum_i(\hat{b}_i^{\dagger})^3\hat{b}_i^3 \quad =$$



up to double occupancy

triple and higher occupancy

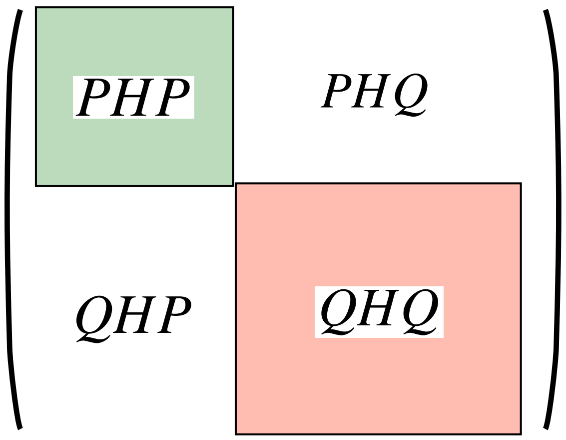
➡ Consider the limit $\gamma_3 \gg U, J$

Basic Mechanism II: Interactions via Loss

- Second order Perturbation Theory
  - Define projector P onto subspace with at most 2 atoms per site (Q=1-P)

$$H_{P,\,\mathrm{eff}} \approx PHP + \frac{2i}{\gamma_3} PHQHP = PHP - \frac{i\Gamma}{2}\sum_j P c_j^\dagger c_j P$$

$$c_j = b_j^2 \sum_{\langle k|j\rangle} b_k \,/\sqrt{2}$$



$$PHP = -J\sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) \quad \& \; b_i^{\dagger 3} \equiv 0$$

➡ Three-body hardcore constraint due to: dynamic suppression of triple onsite occupation

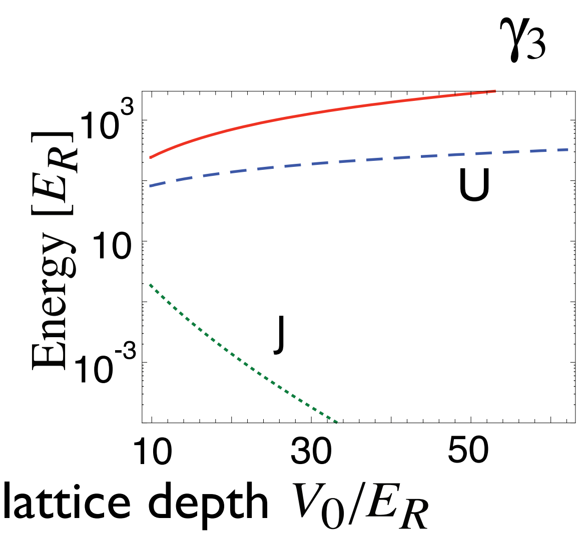
➡ Small decay constant in P subspace: $\Gamma = 12\dfrac{J^2}{\gamma_3}$

➡ Realization of a Hubbard-Hamiltonian with three-body hard-core constraint on time scales $\tau = 1/\Gamma$

Physical Realization in Cold Atomic Gases

- Estimate Loss rate: Integrate free space recombination rate over Wannier function
  - short length scale collisions not modified by lattice
- Cesium close to a zero crossing of the scattering length (e.g. Naegerl et al.)

parameter estimate



- Preparation of the ground state of PHP:
  - Nonequilibrium problem: role of residual heating effects
  - Approach: Exact numerical time evolution of full Master Equation in 1D; combine DMRG method with stochastic simulation of ME
  - Find optimal experimental sequence to avoid heating

Ground State Preparation

Quantum Trajectories: Stochastic Simulation

$$\dot{\rho} = -i[H,\rho] - \frac{\Gamma}{2}\sum_m \left[c_m^\dagger c_m \rho + \rho c_m^\dagger c_m - 2 c_m \rho c_m^\dagger\right]$$

- Evolve stochastic trajectories (states)

$$H_{\mathrm{eff}} = H - i\frac{\Gamma}{2}\sum_m c_m^\dagger c_m$$

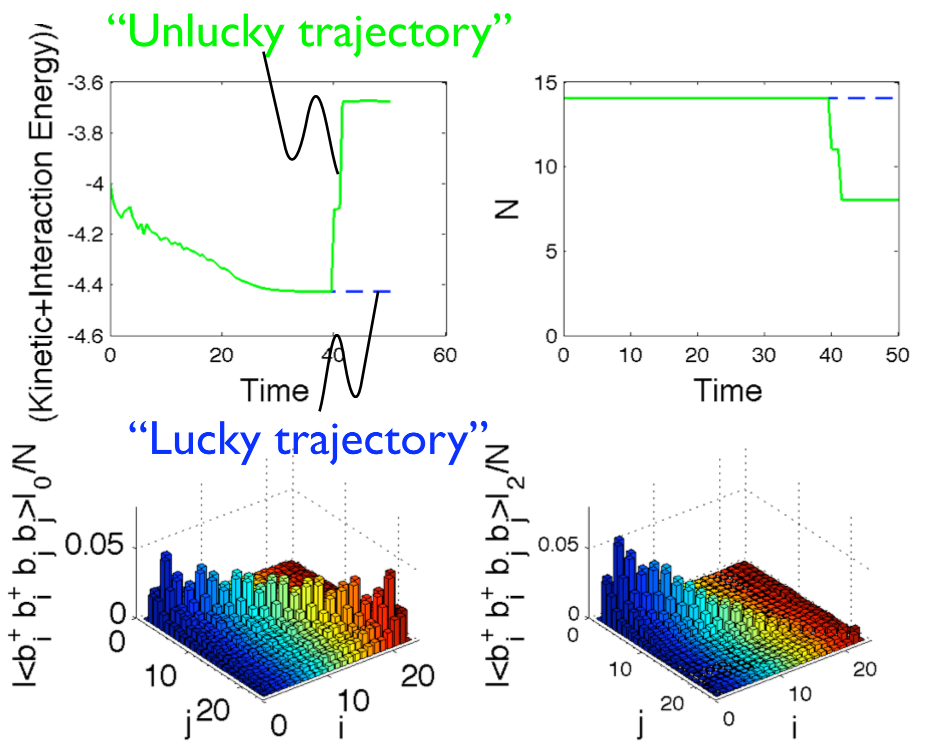
- Quantum Jumps

$$|\psi\rangle = \frac{c_m|\psi\rangle}{||c_m|\psi\rangle||}$$

  - Norm decays below random threshold
  - Jump operator chosen randomly

Features:

- Evolution of individual trajectories
- Expectation values by stochastic average

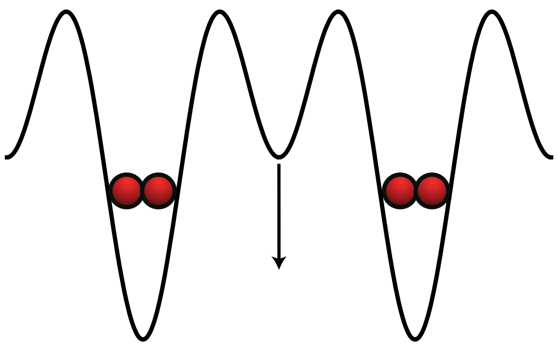


"Lucky trajectory": long range order

"Unlucky trajectory"

Ground State Preparation

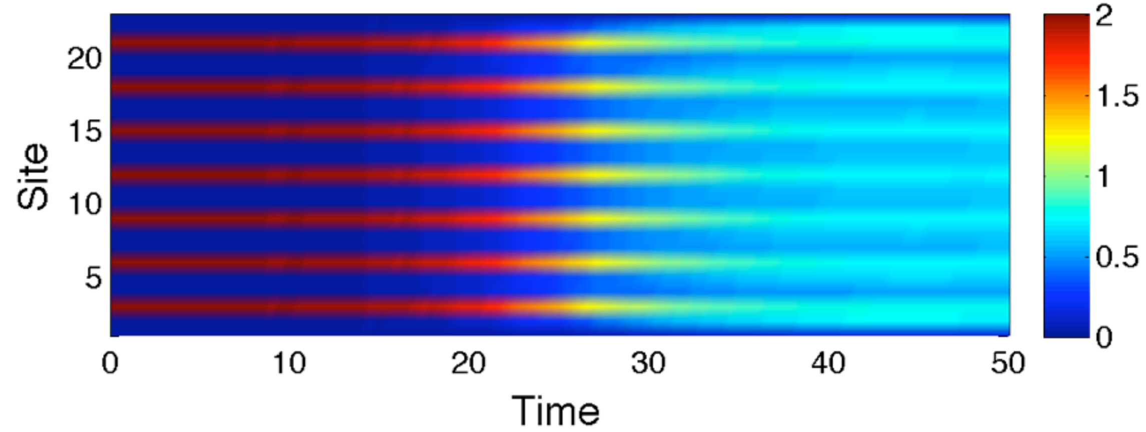
Ramping down a superlattice

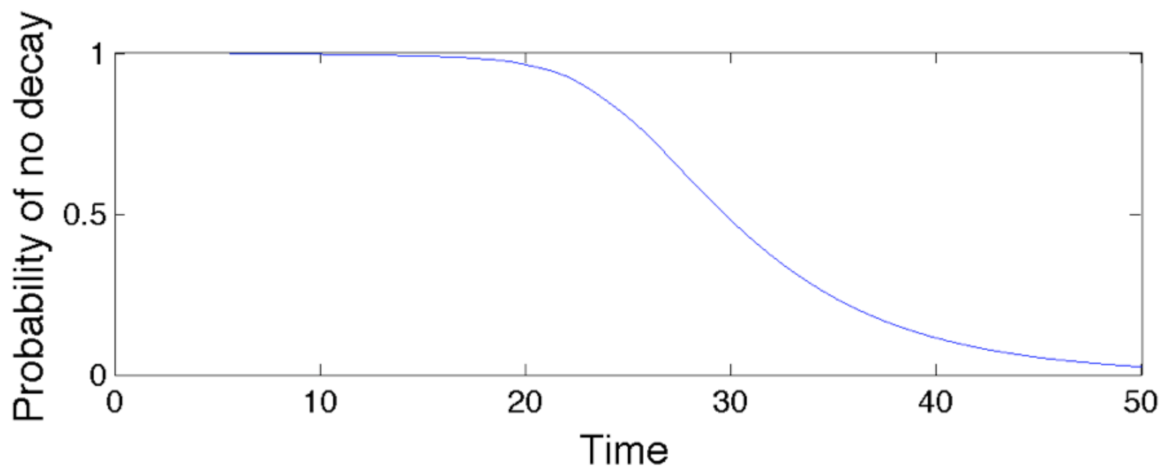


Ramp: Superlattice, V/J=30 to V/J=0, N=M=20; U/J =-8

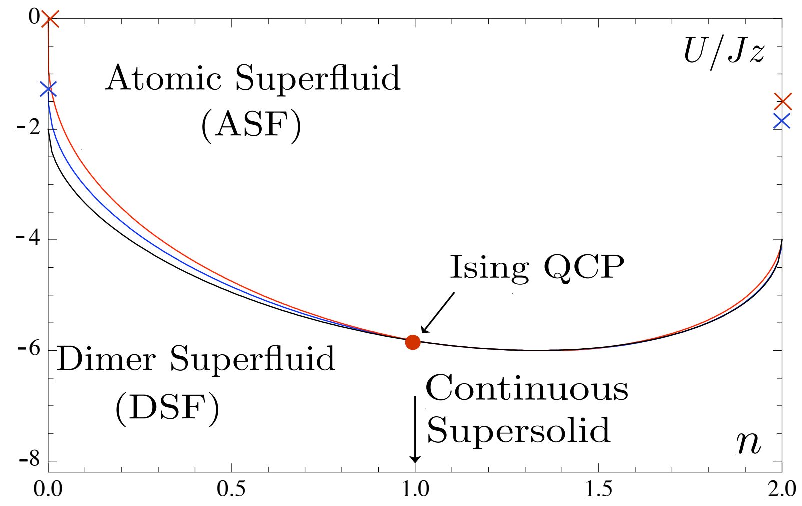
Buildup of long-range order in "lucky" case

$\Gamma = 250J$





Part II:
Phase Diagram for Three-Body Hardcore Bosons



microscopic

interactions

thermodynamic

interactions
condensation

long distance

interactions
condensation
spin waves

Physics of the projected Hamiltonian

- The constrained Bose-Hubbard Hamiltonian stabilizes attractive two-body interactions

$$PHP = -J\sum_{\langle i,j\rangle}\hat{b}_i^\dagger\hat{b}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) \;\; \& \; b_i^{\dagger 3} \equiv 0$$

$$U < 0$$

- Qualitative picture for ground state: Mean Field Theory
  - homogenous Gutzwiller Ansatz for projected on-site Hilbert space

$$|\Psi\rangle = \prod_i |\Psi\rangle_i \qquad |\Psi\rangle_i = f_0|0\rangle + f_1|1\rangle + f_2|2\rangle \qquad f_\alpha = r_\alpha e^{i\phi_\alpha}$$

  - Gutzwiller energy

$$\Phi = \phi_2 + \phi_0 - 2\phi_1$$

$$E(r_\alpha;\phi_\alpha) = U r_2^2 - JZ r_1^2\left(r_0^2 + 2\sqrt{2}\, r_2 r_0 \cos\Phi + 2 r_2^2\right)$$

Mean Field Phase Diagram

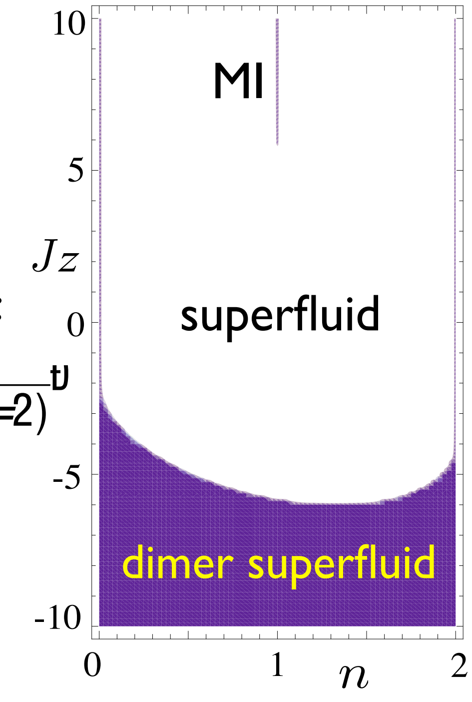
- Consider correlation functions:

  $\langle \hat{b} \rangle$ - Atomic SF order parameter

  $\langle \hat{b}^2 \rangle$ - Dimer SF order parameter

critical interaction strength:

$$\frac{U_c}{Jz} = -2\left(1 + n/2 + 2\sqrt{n(1-n/2)}\right)$$



- Symmetry breaking patterns:

  $\langle \hat{b} \rangle \neq 0; \quad \langle \hat{b}^2 \rangle \neq 0$ - Conventional SF

  $\langle \hat{b} \rangle \neq 0; \quad \langle \hat{b}^2 \rangle = 0$ - NO! phase locking in GW energy

  $\langle \hat{b} \rangle = 0; \quad \langle \hat{b}^2 \rangle \neq 0$

  - “Dimer SF”

$$E(\rho_\alpha; \phi_\alpha) = U\rho_2^2 - JZ\rho_1^2\left(\rho_0^2 + 2\sqrt{2}\rho_2\rho_0\cos\Phi + 2\rho_2^2\right)$$

- Phase transition reminiscent of Ising (cf Radzihovsky& ‘03; Stoof, Sachdev& ‘03):

$$\langle \hat{b} \rangle \sim \exp i\theta \qquad \langle \hat{b}^2 \rangle \sim \exp 2i\theta$$

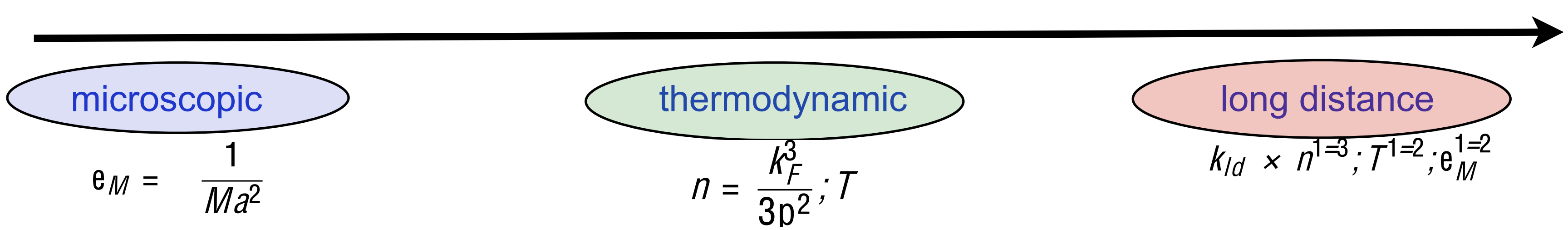
➡ Spontaneous breaking of Z_2 symmetry $\theta \to \theta + \pi$ of the DSF order parameter

➡ Second order within MFT

Quantum Field Theory

- Gutzwiller mean field theory: classical field theory for the amplitudes $\phi_{\alpha,i}(t), \quad \sum_\alpha \phi^*_{\alpha,i}\phi_{\alpha,i} = 1$
- Questions on various scales:
  - Vacuum problem: Dimer bound state formation expected for attractive interaction
  - Condensation/Thermodynamics: phase border, superfluid stiffness/ Goldstone Theorem, EFT in strongly interacting limit
  - Infrared limit: Nature of the Phase transition

➡ Quantized version of the Gutzwiller mean field description desirable



Implementation of the Hard-Core Constraint

- Introduce operators to parameterize on-site Hilbert space (Auerbach, Altman '98)

$$t^\dagger_{\alpha,i}|vac\rangle = |\alpha\rangle; \quad \alpha = 0,1,2$$

- They are not independent:

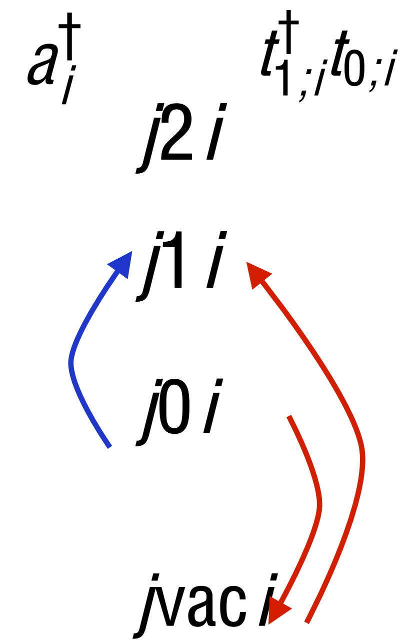
$$\sum_\alpha t^\dagger_{\alpha,i} t_{\alpha,i} = \mathbf{1}$$

- Representation of Hubbard operators:

$$a_i^\dagger = \sqrt{2}\, t^\dagger_{2,i} t_{1,i} + t^\dagger_{1,i} t_{0,i}$$

$$\hat{n}_i = 2 t^\dagger_{2,i} t_{2,i} + t^\dagger_{1,i} t_{1,i}$$

Action of operators



Implementation of the Hard-Core Constraint

- Hamiltonian:

$$H_{\mathrm{pot}} = -\mu \sum_i \left(2 t^\dagger_{2,i} t_{2,i} + t^\dagger_{1,i} t_{1,i} \right) + U \sum_i t^\dagger_{2,i} t_{2,i}$$

$$H_{\mathrm{kin}} = -J \sum_{\langle i,j \rangle} \Big[t^\dagger_{1,i} t_{0,i} t^\dagger_{0,j} t_{1,j} + \sqrt{2} \left(t^\dagger_{2,i} t_{1,i} t^\dagger_{0,j} t_{1,j} + t^\dagger_{1,i} t_{0,i} t^\dagger_{1,j} t_{2,j} \right) + 2 t^\dagger_{2,i} t^\dagger_{1,j} t_{1,i} t_{2,j} \Big]$$

- Properties:
  - Mean field: Gutzwiller energy (classical theory)
  - interaction: quadratic
  - hopping: higher order
    - } • Role of interaction and hopping reversed
    - } • Strong coupling approach
  - One phase is redundant: absorb via *local* gauge transformation

$$t_{1,i} = \exp i\varphi_{0,i} \, |t_{0,i}| \qquad t_{1,i} \to \exp\left(-i\varphi_{0,i}\right) t_{1,i}, \quad t_{2,i} \to \exp\left(-i\varphi_{0,i}\right) t_{2,i}$$

➡ e.g. t_0 can be chosen real

Implementation of the Hard-Core Constraint

- Resolve the relation between t-operators (zero density)

$$t^\dagger_{1,i} t_{0,i} = t^\dagger_{1,i}\sqrt{1 - t^\dagger_{1,i}t_{1,i} - t^\dagger_{2,i}t_{2,i}} \;\to\; t^\dagger_{1,i}(1 - t^\dagger_{1,i}t_{1,i} - t^\dagger_{2,i}t_{2,i})$$

  - justification: for projective operators one has from Taylor representation

$$X^2 = X \;\to\; f(X) = f(0)(1 - X) + X f(1) \qquad X = 1 - t^\dagger_{1,i}t_{1,i} - t^\dagger_{2,i}t_{2,i}$$

- Now we can interpret the remaining operators as standard bosons:
  - on-site bosonic space $\quad \mathcal{H}_i = \{|n\rangle^1_i |m\rangle^2_i\}, \quad n, m = 0, 1, 2, \ldots$
  - correct bosonic enhancement factors on physical subspace $\quad \forall\, n = 0, 1$
  - decompose into physical/unphysical space: $\quad \mathcal{H}_i = \mathcal{P}_i \oplus \mathcal{U}_i$

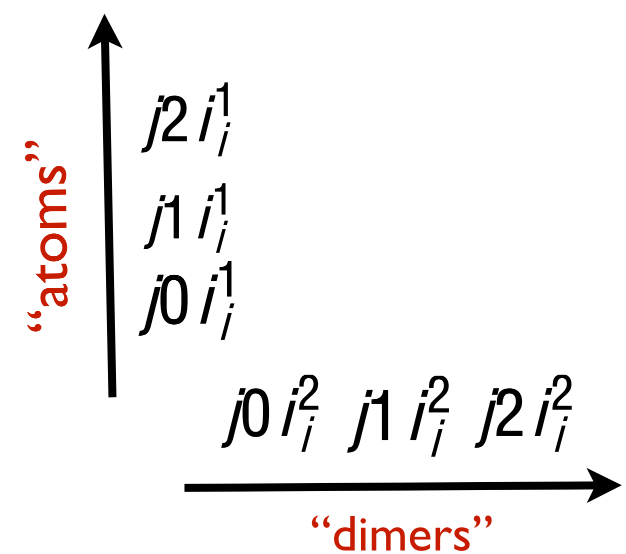
$$\mathcal{P}_i = \{|0\rangle^1_i|0\rangle^2_i, |1\rangle^1_i|0\rangle^2_i, |0\rangle^1_i|1\rangle^2_i\}$$

  - the Hamiltonian is an involution on P and U:

$$H = H_{PP} + H_{UU}$$

  - remaining degrees of freedom: "atoms" and "dimers"

➡ similarity to Hubbard-Stratonovich transformation



Implementation of the Hard-Core Constraint

- The partition sum does not mix U and P too:

$$Z = Tr\exp\ \ b H = Tr_{PP}\exp\ \ b H_{PP} + Tr_{UU}\exp\ \ b H_{UU}$$

- Need to discriminate contributions from U and P: Work with Effective Action

  - Legendre transform of the Free energy $W[J] = \log Z[J]$

$$G[c] = \ W[J] + \ J^T c; \quad c \quad \frac{dW[J]}{dJ}$$

Quantum Equation of Motion for J=0

  - Has functional integral representation:

$$\exp\ \ G[c] = \ D dc \exp\ \ S[c + dc] + \ J^T dc; \quad J = \frac{dG[c]}{c}$$

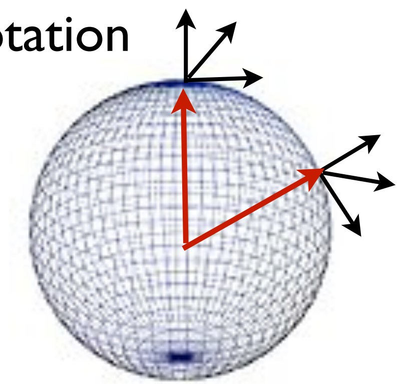
$$S[c = (t_1; t_2)] = \ dt\ \ \hat{A}_i\ t^{\dagger}_{1;i}\partial_t t_{1;i} + t^{\dagger}_{2;i}\partial_t t_{2;i} + H[t_1; t_2]$$

- Usually: Effective Action shares all symmetries of S
- Here: symmetry principles are supplemented with a constraint principle

Condensation and Thermodynamics

- Physical vacuum is continuously connected to the finite density case:

Introduce new, expectationless operators by (complex) Euler rotation



$$\vec{t} = (t_0; t_1; t_2)^T$$

$$\vec{b} = R_q \, R_j \, \vec{t}$$

- Hamiltonian in new coordinates takes form:

Quadratic part: Spin waves (Goldstone for n > 0)

$$H = E_{GW} + H_{SW} + H_{int}$$

Mean field: Gutzwiller Energy   higher order: interactions

microscopic	thermodynamic	long distance
interactions	interactions condensation	interactions condensation spin waves

Hard-Core Constraint: Summary

- Constrained Model can be mapped on coupled boson theory. This should be seen as a requantization of Gutzwiller mean field theory

- This theory automatically respects constraint: Decoupled physical and unphysical subspaces

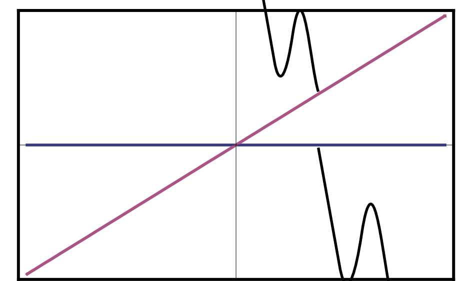
- Effective Action path integral quantization favorable: symmetry principles are supplemented with a constraint principle

Vacuum Problem (n=0)

- Hamiltonian to third order is of Yukawa/Feshbach type:
  - quadratic part:

$$H_{\text{pot}} = \sum_i (U - 2\mu) n_{2;i} - \mu n_{1;i}$$

detuning from atom level



  - leading interaction:

$$H_{\text{kin}} = -J \sum_{\langle i;j\rangle} \left[t^\dagger_{1;i} t_{1;j} + \sqrt{2}\,(t^\dagger_{2;i} t_{1;i} t_{1;j} + t^\dagger_{1;i} t^\dagger_{1;j} t_{2;j}) \right]$$

(bilocal) dimer splitting into atoms

- Compare to standard Hubbard-Stratonovich decoupling:

usually: decouple interaction U → detuning $\sim 1/U$

here: interaction in quadratic part: detuning $\sim U$

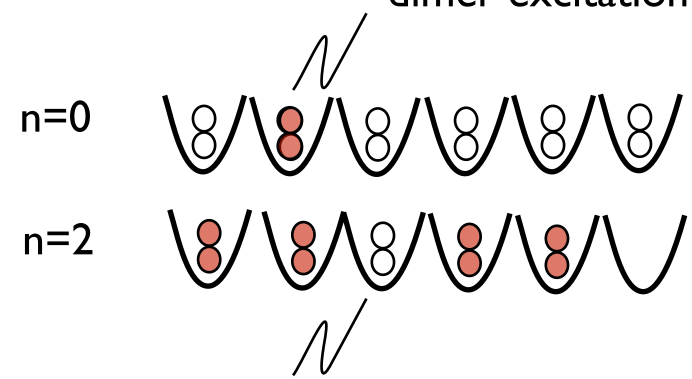
➡ realizes Feshbach model on the lattice

➡ we can expect resonant (strong coupling) phenomenology at weak coupling

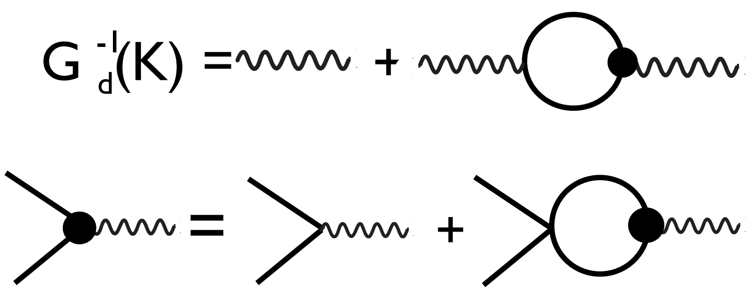
$$H_{\text{kin}} = -J \sum_{\langle i;j\rangle} \left[t^\dagger_{1;i}(1 - n_{1;i} - n_{2;i})(1 - n_{1;j} - n_{2;j}) t_{1;j} + \sqrt{2}\,(t^\dagger_{2;i} t_{1;i}(1 - n_{1;j} - n_{2;j}) t_{1;j} + t^\dagger_{1;i}(1 - n_{1;i} - n_{2;i}) t^\dagger_{1;j} t_{2;j}) + 2 t^\dagger_{2;i} t_{2;j} t^\dagger_{1;j} t_{1;i} \right]$$

Vacuum Problems

- The physics at n=0 and n=2 are closely connected:
  - no spontaneous symmetry breaking
  - low lying excitations:
    - n=0: dimers on the physical vacuum
    - n=2: di-holes on the fully packed lattice



- Two-body problems can be solved exactly
  - Bound state formation: $G_d^{-1}(\omega = \mathbf{q} = 0) = 0$



$$\frac{1}{a_n|\tilde{U}| + b_n} = \int \frac{d^d q}{(2\pi)^d} \frac{1}{-\tilde{E}_b + 2/d\sum_\lambda (1 - \cos \mathbf{q}\mathbf{e}_\lambda)}$$

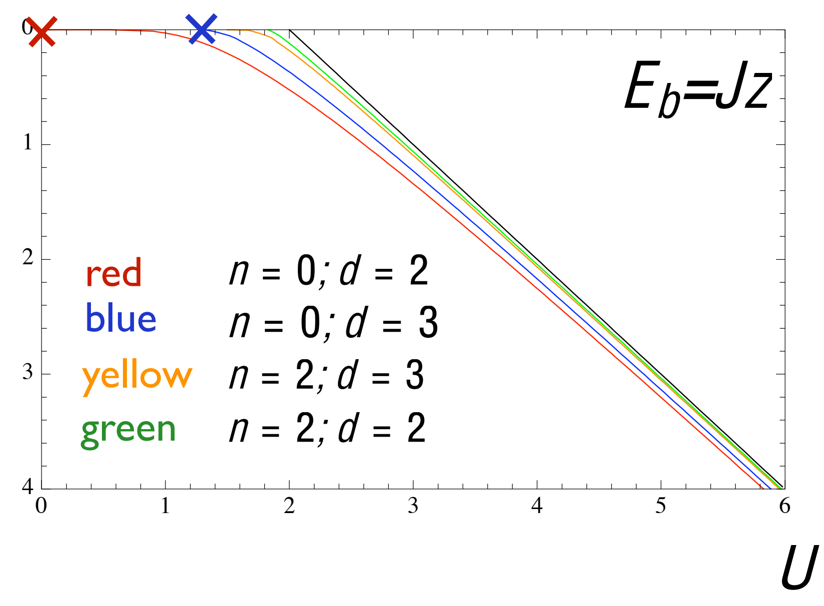
$$n = 0: \quad a_0 = 1, \; b_0 = 0$$

➡ reproduces Schrödinger Equation: benchmark

➡ Square root expansion of constraint fails

$$n = 2: \quad a_2 = 4, \; b_2 = -6 + 3\tilde{E}_b$$

➡ di-hole-bound state formation at finite U in 2D



ASF - DSF Phase Border

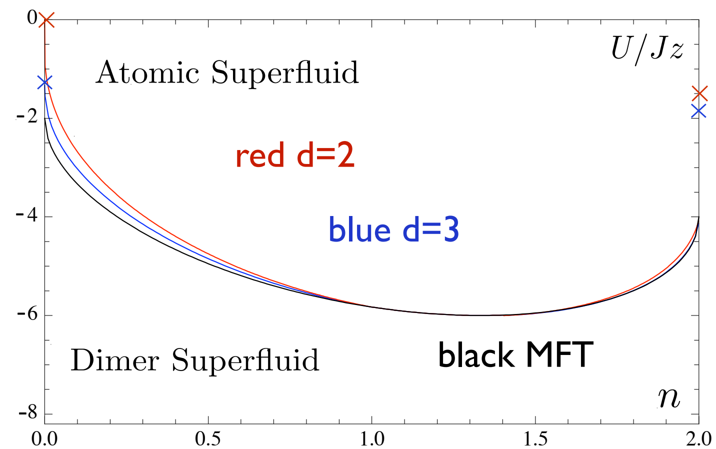
- Goal: Effects of quantum fluctuations on phase border
- Strategy:
  - Atomic mass matrix signals instability of ASF:

$$\det G_1^{\ 1}(w = \mathbf{k} = 0) = 0$$

  - ordering principle: small density expansion around

$$n \ ^3\ 0;\ n \ ^3\ 2$$

shifts of the phase border



- Results:
  - dominant fluctuations: associated to bound state formation
  - two scales: bound state formation (G_2) and atom criticality (G_1)

(i) low density: coincidence of scales

➡ strong shifts, nonanalytic nonuniversal behavior

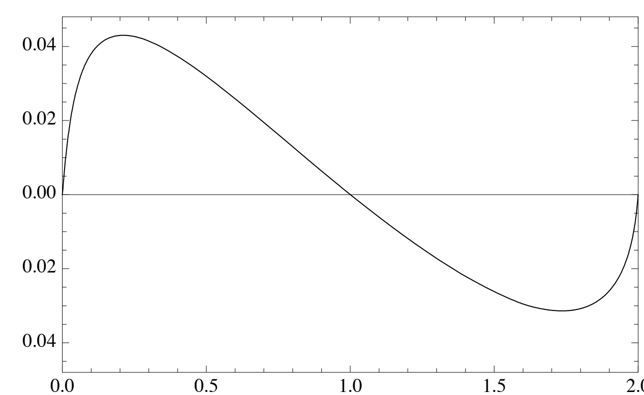
e.g. d=3: $\frac{U_c}{Jz} \ _3\ \frac{U_c(n=0)}{Jz} \ ' \ \sqrt{\frac{q|U_c(n=0)|}{2Jzs}};\quad s\ ^3\ 0.53$

condensate angle

(ii) maximum density: mismatch of scales, di-hole bound state forms prior to atom criticality

➡ mean field like behavior

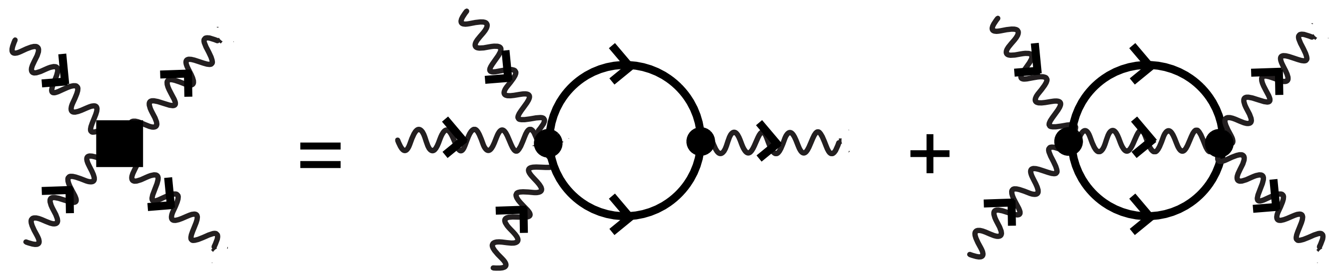
condensate depletion d=2



- Note: No particle-hole symmetry!

Effective Field Theory in Strong Coupling

- Perturbative limit U >> J: expect dimer hardcore model
- Perturbation theory second order J for interaction coefficient:



  - Strong quantum mechanical fluctuations: one and two-loop graph contribute equally
  - Constraint vertices describe forbidden decay possibilities for dimers
  - Resulting Hamiltonian (use constraint principle)

$$H_{\mathrm{eff}} = \sum_{\langle i;j \rangle} t\, t^{\dagger}_{2;j}(1 - \hat{n}_{1;j} - \hat{n}_{2;j})(1 - \hat{n}_{1;i} - \hat{n}_{2;i})\, t_{2;i} + V \hat{n}_{2;i}\hat{n}_{2;j} + \mu_{\mathrm{eff}} \sum_i \hat{n}_{2;i}$$

constrained hopping — effective nn-repulsion

$$t = \frac{V}{2} = \frac{2J^2}{|U|}$$

Symmetry Enhancement

- Interpret EFT as a spin model in external field:

$$H_{\text{eff}} = -2t \sum_{\langle i,j \rangle} \left(s_i^x s_j^x + s_i^y s_j^y + \lambda s_i^z s_j^z \right)$$

- Leading (second) order perturbation theory:

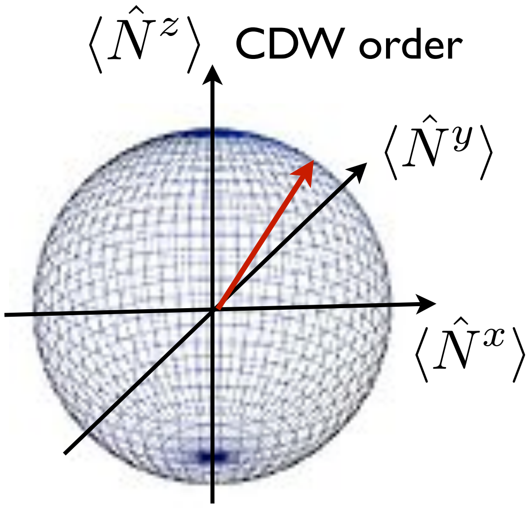
$$| = \frac{V}{2t} = 1$$

➡ Isotropic Heisenberg model (half filling n=1):
  - Emergent symmetry: SO(3) rotations vs. SO(2) sim U(1)
  - Bicritical point with Neel vector order parameter

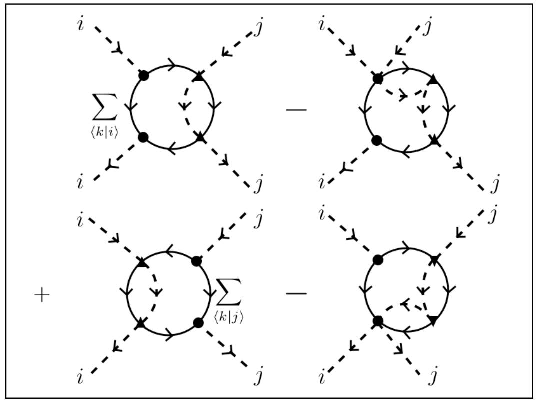
$$\hat{N}^\alpha = \sum_j (-)^j s_i^\alpha$$

  - charge density wave and superfluid exactly degenerate
    - CDW: Translation symmetry breaking
    - DSF: Phase symmetry breaking
  - physically distinct orders can be freely rotated into each other:

  "continuous supersolid"

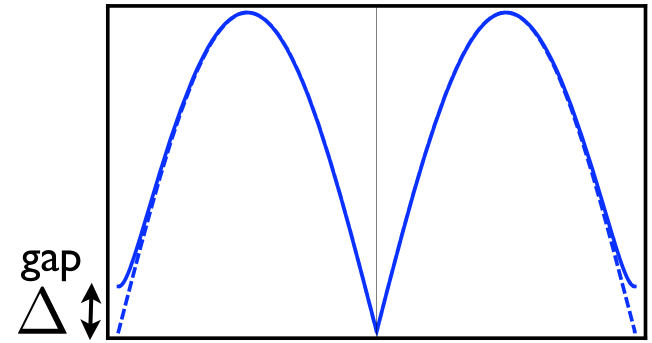


xy plane: superfluid order



- But: - Next (fourth) order PT slightly favors DSF: $\lambda = 1 - 8(z-1)(J/|U|)^2 < 1$
  - Deviations from half filling

Signatures of "continuous supersolid"

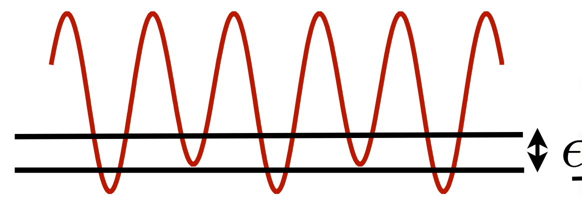


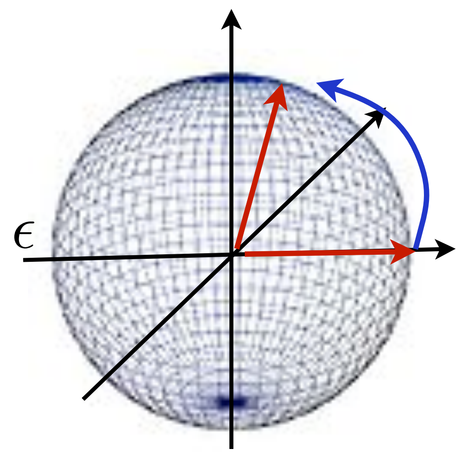
- Proximity to bicritical point governs the physics in strong coupling regime
  - Second collective (pseudo) Goldstone mode

$$\omega(\mathbf{q}) = tz\,\left[(\lambda\epsilon_{\mathbf{q}}+1)(1-\epsilon_{\mathbf{q}})\right]^{1/2}$$

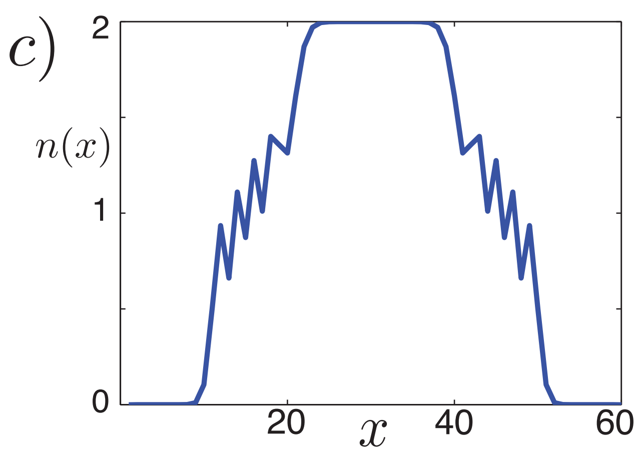
  - Use weak superlattice to rotate Neel order parameter

$$\epsilon/tz = \Delta/tz = 1-\lambda \approx 8(z-1)(J/U)^2$$

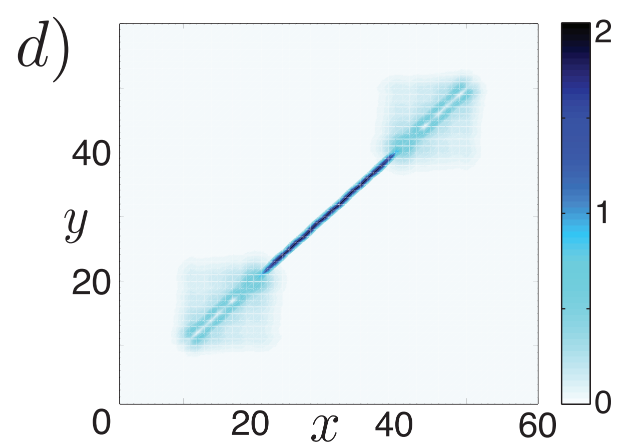




  - Simulation of 1D experiment in a trap (t-DMRG)



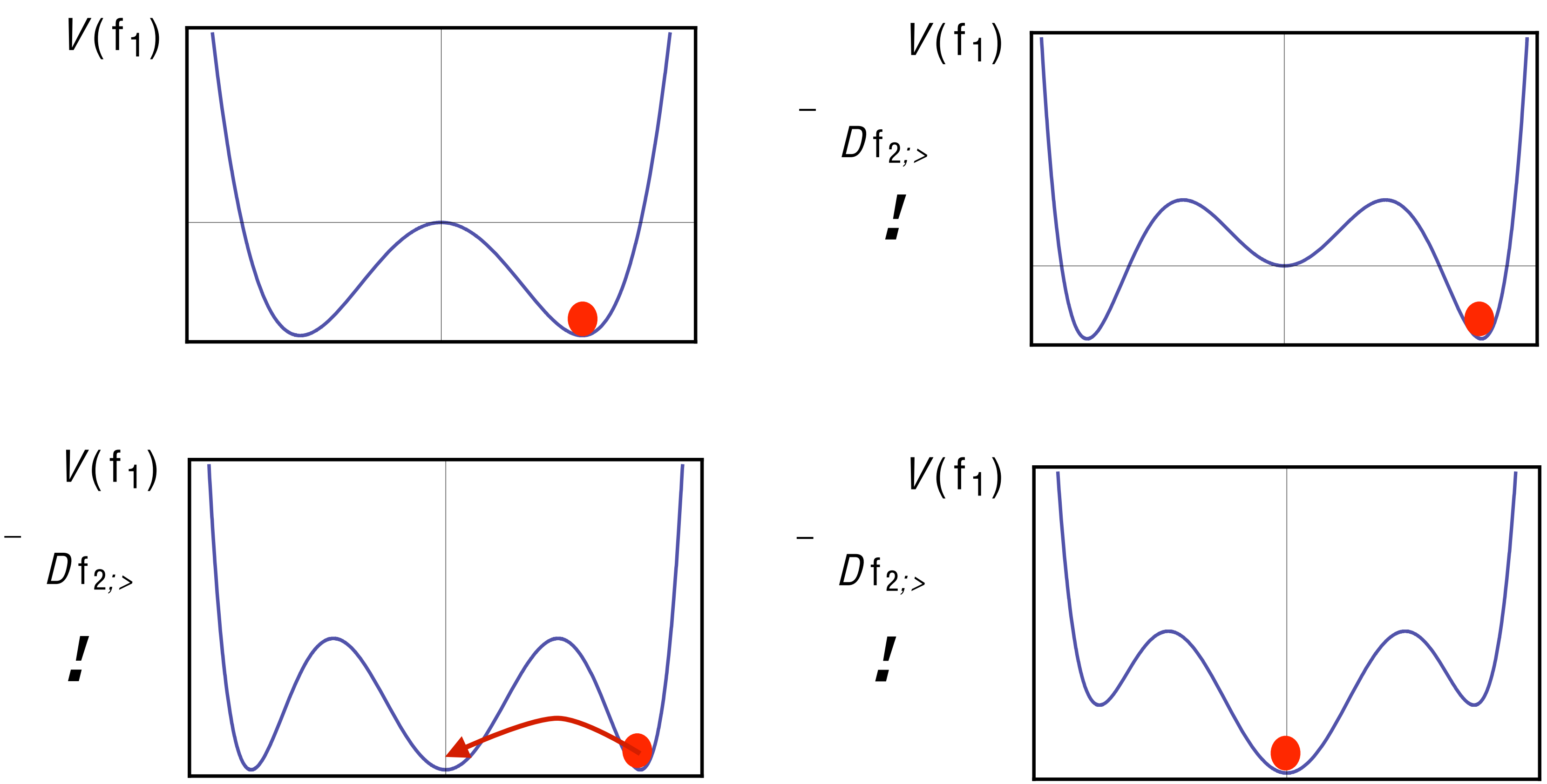
density profile: Onset of CDW



DSF order in textured regions

Infrared Limit: Nature of the Phase Transition

- Two near massless modes: Critical atomic field, dimer Goldstone mode
- Coleman-Weinberg phenomenon for coupled real fields: Radiatively induced first order PT



Infrared Limit: Nature of the Phase Transition

- Perform the continuum limit and integrate out massive modes:

pure Goldstone action

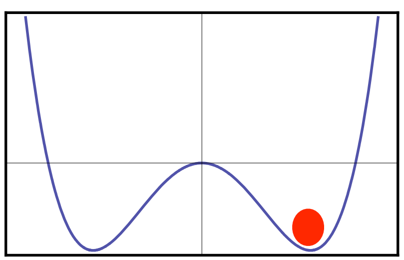
$$S[J;f] = S_I[f] + S_G[J] + S_{int}[J;f]$$

pure Ising action

coupling term

$$S_I[f] = \bar{\ } \ \partial_\mu f \partial^\mu f + m^2 f^2 + l f^4$$

Ising field: Real part of atomic field



Ising potential landscape:
Z_2 symmetry breaking

$$S_{int}[J;f] = ik \bar{\ } \ \partial_t J \ f^2$$

Frey, Balents; Radzihovsky&

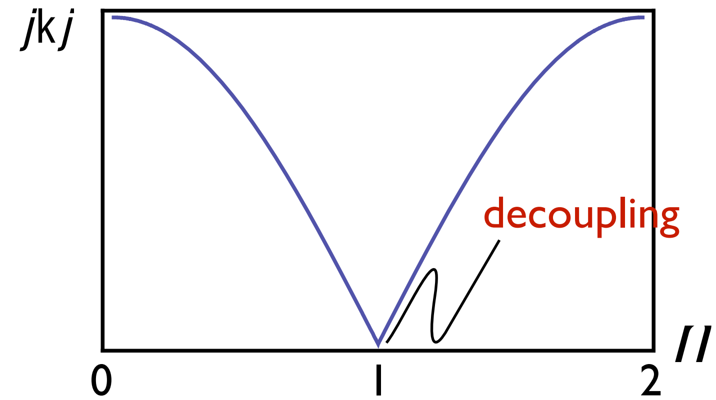
➡ Interactions persist to arbitrary long wavelength (cf. decoupling SW)

➡ $k \neq 0$: Phase transition is driven first order by coupling of Ising and Goldstone mode

d+1 Ising Quantum Critical Point at n=1

- Plot the Ising-Goldstone coupling:

$$S_{\text{int}}[J;f] = ik \quad \partial_t J \; f^2$$

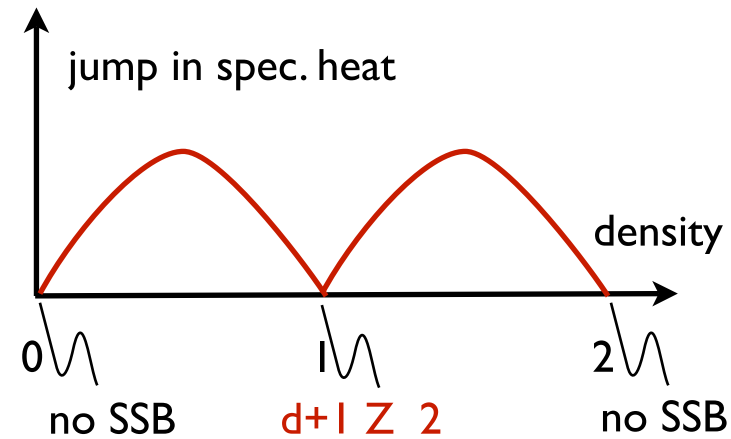


$$G \; 3 \quad b^{\dagger}_{2;i}(\quad g_2 \, \mu) \, b_{2;i}$$

- Symmetry argument:
  - dimer compressibility must have zero crossing
  - and is locked to other couplings by time-local gauge invariance and atom-dimer phase locking

  ➡ emergent relativistic symmetry: isotropic d+1 dimensional model

  ➡ K must have zero crossing: true quantum critical Ising transition

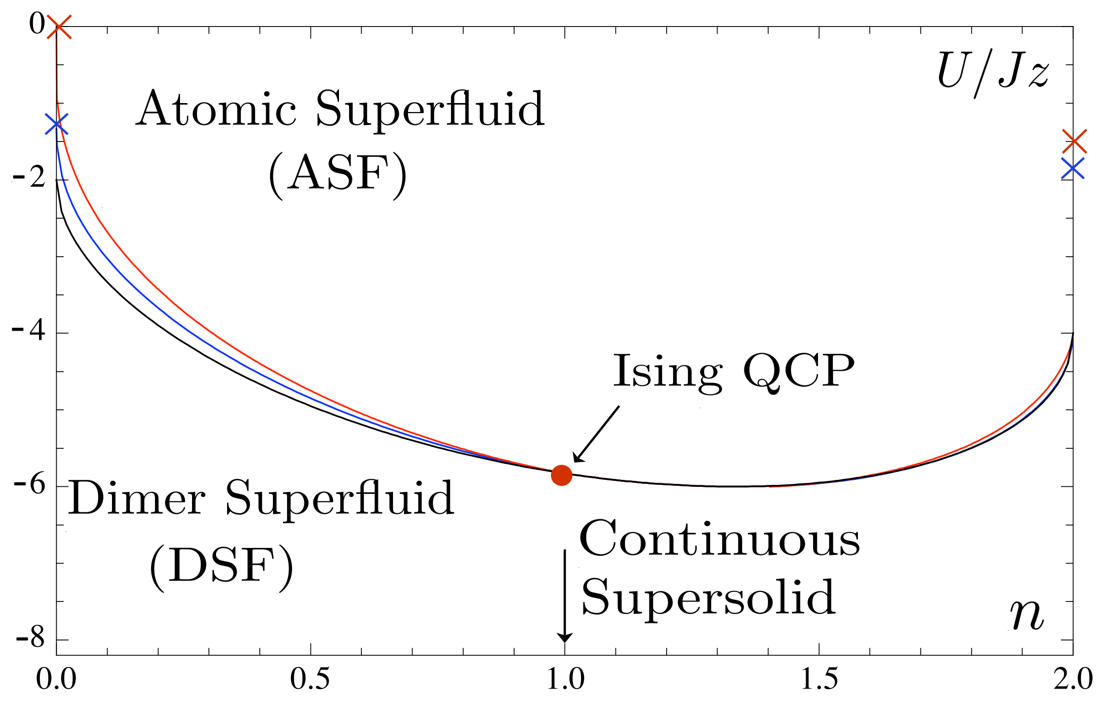


- Estimate correlation length: $\xi/a \sim \kappa^{-6} \sim |1-n|^{-6}$

  ➡ weakly first order, broad near critial domain

➡ Second order quantum critical behavior is a lattice+constraint effect

Summary: Beyond mean field phase diagram

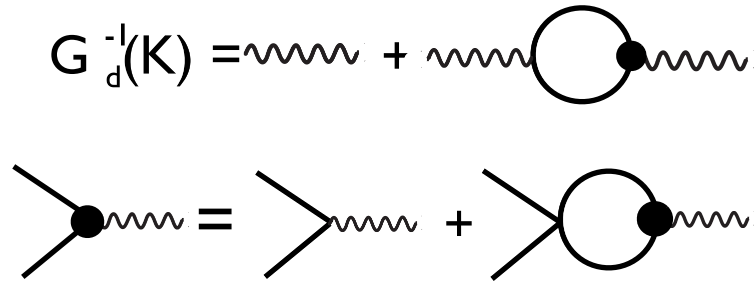


- ➡ Quantum fluctuations shift the phase border for low densities
- ➡ Radiatively induced first order ASF-DSF transition terminates into Ising QCP
- ➡ Symmetry enhancement in strong coupling leads to "continuous supersolid"

✓ Uniquely tied to interactions

✓ Uniquely tied to three-body constraint

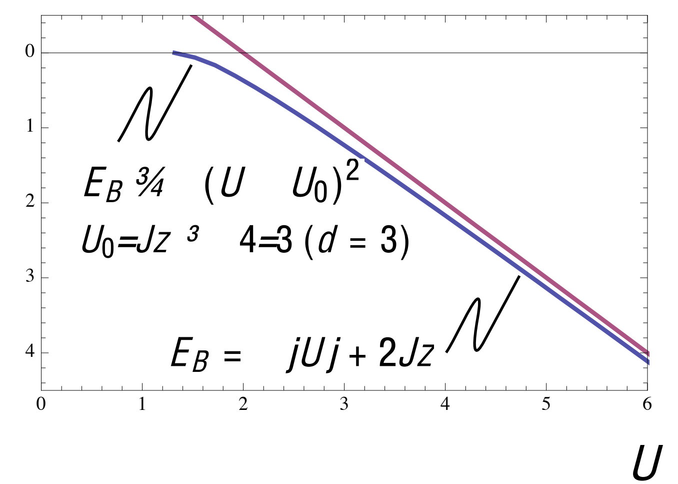
Vacuum Problem (n=0)



- Exact solution of non-local two-body problem possible:

  Recover nonperturbative Lippmann-Schwinger Eq.

$$G_d^{-1}(\omega;\mathbf{k}) = -\left[U^{-1} + \int \frac{d^3q}{(2\pi)^3} \frac{1}{2(\epsilon_{\mathbf{k}} + \epsilon_{\mathbf{q}-\mathbf{k}}) - 2\mu - \omega} \right]^{-1}$$



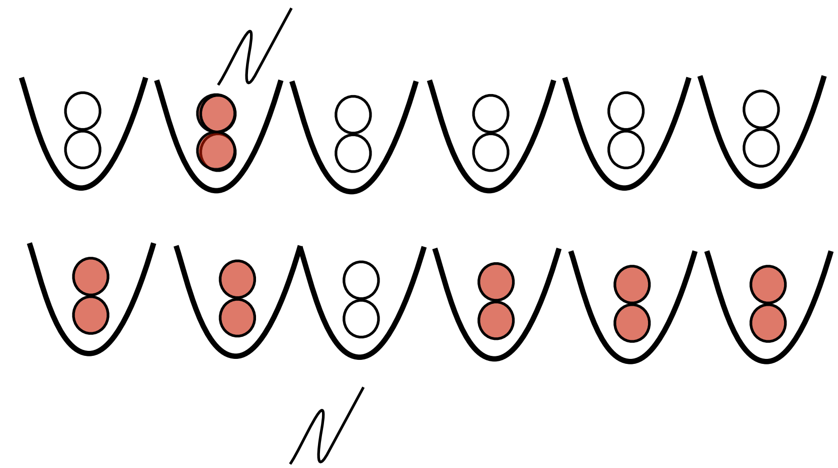
➡ Fluctuation induced formation of dynamic dimer bound state

➡ Benchmark with otherwise available results passed

➡ Square root expansion of constraint fails

Dimer - Di-hole Mapping

- The physics at n=0 and n=2 are closely connected:
  - no spontaneous symmetry breaking
  - low lying excitations:
    - n=0: dimers on the physical vacuum
    - n=2: di-holes on the fully packed lattice



- There exists a 1-1 mapping:

$$t_2 \to t_0; \quad \mu \to \mu + U; \quad U \to U; \quad J_{hop} \quad J \to 2J; \quad g_{split} \quad \sqrt{2}J \to \sqrt{2}J; \quad g_{exchange} \quad 2J \to J$$

dimer-di-hole trafo   adding di-hole = removing dimer

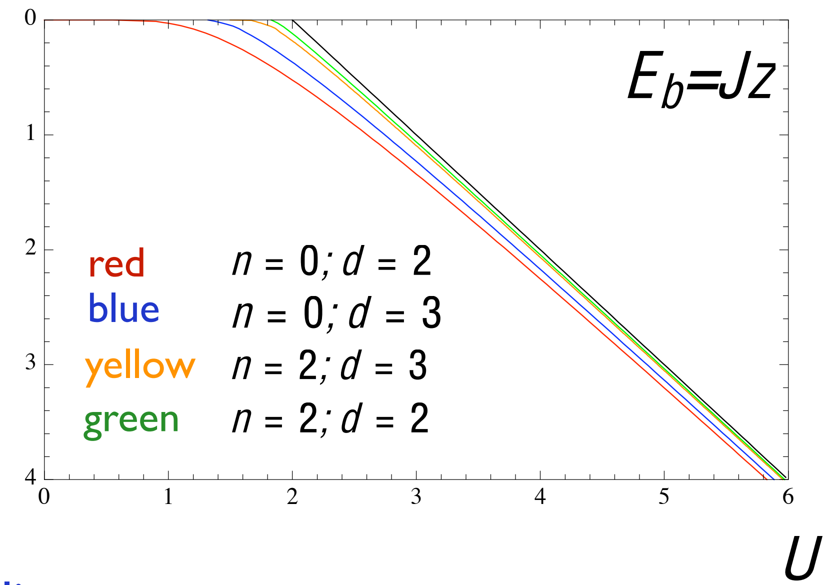
➡ Modified Lippman-Schwinger equation for di-hole allows to determine exactly the spectrum on top of the n=2 state

Bound state formation at n=0,2

- Bound state formation: critical coupling $\frac{U_c}{Jz}; \quad z = 2d$

n\d	2	3
0	0	-4/3
2	-3/2	-11/6

MFT: 2



- Onset behavior:
  - 3d: quadratic
  - 2d: exponential due to log-divergence
  - Finite onset in 2d n=2: generalized LSE!

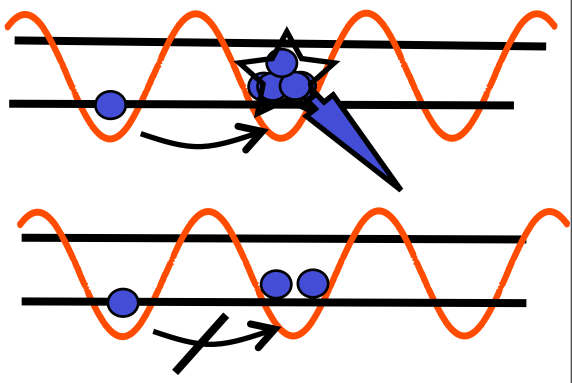
MFT: linear

- Di-hole spectrum: gapless and quadratic:

  not a Mott Insulator for attractive interaction

MFT: none!

Conclusions and Outlook



- Strong 3-particle dissipation can create 3-body hard-core interactions
  - New equilibrium phases: dimer superfluid
  - Preparation of these phases feasible, but of non-equilibrium nature
- Field theoretical framework for the treatment of certain constrained lattice models
  - Constrained system can be mapped to coupled boson theory
  - Beyond mean field physics on various scales
    - Bound state formation: dimers and di-holes
    - Phase Border: nonuniversal shifts of the critical point
    - Effective spin model with enhanced symmetries in strong coupling regime
    - Nature of phase transition: d+1 dimensional Ising QCP at n=1
  - Further applications:
    - Nonperturbative effects on the MI-SF phase boundary
    - Spin models?