Entanglement, Chaos and Quantum Computation

Arnold Sommerfeld Lectures (Munich 2023)

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Outline

The Classical Computer

- Classical Mechanics is the physical paradigm for classical computing.
- The classical computer uses bits to represent the values it is operating on.
- A bit can be either 0 (off) or 1 (on).
- The state of the classical computer at any given time is described by a collection of zeros and ones:

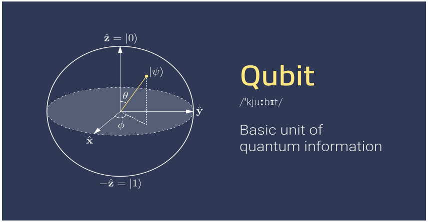
$$01100100000111111011101.... \tag{1}$$

- The set of gates (AND, NOT) is a complete set - every logical function can be written using these gates.

The Quantum Computer

- Quantum Mechanics is the physical paradigm for quantum computing.
- The quantum computer uses quantum bits called qubits to represent the values it is operating on.
- A qubit can represent the values 0 or 1, or a linear combination of both with complex coefficients (The principle of superposition):

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1 \tag{2}$$



Quantum Register

- A stack of n qubits is a quantum register.
- The state of the quantum register lives in the Hilbert space:

$$\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2, \qquad dim(\mathcal{H}) = 2^n \tag{3}$$

$$\begin{aligned} |\psi\rangle &= \sum c_{i_1 \dots i_n} |i_1\rangle \cdots |i_n\rangle, i_k = 0, 1 \\ \sum |c|^2 &= 1, \quad |\psi\rangle \sim e^{i\varphi}|\psi\rangle \implies CP^{2^n - 1} \end{aligned} \tag{4}$$

- Single qubit: CP^1

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle \tag{5}$$

Entanglement

- A generic state $|\psi\rangle \in \mathcal{H}$ is not a product state:

$$|\psi\rangle \neq |\psi\rangle_1 \otimes |\psi\rangle_2 \otimes \cdots \otimes |\psi\rangle_n \tag{6}$$

- Entanglement can be viewed as a measure of the deviation from being a product state, and is a measure for quantum correlation.
- Entanglement is necessary for quantum computation.
- Example:

$$\textit{Bell State}: \quad |\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{7}$$

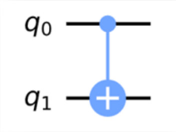
- It requires a delicate isolation of the quantum system from the classical environment (decoherence).

Entanglement

- The higher the entanglement is, the larger the information spread, such that we can read the information only after measuring a significant fraction of the qubits.
- As we will see, entanglement is responsible both for the success and failure of the quantum algorithms. The quantum algorithm will not be effective in the absence of entanglement, as well as with its excess.
- We will quantify an optimal region of information spreading and entanglement, which leads to the success of the optimization in the quantum/classical hybrid algorithms.

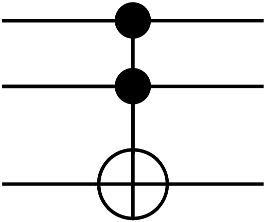
Quantum Gates

- Operations on the quantum register are performed by unitary transformations (Reversible).
- A universal set of one-qubit and two-qubit gates can approximate any unitary evolution.
- Single-qubit gates: $X|0\rangle = |1\rangle, X|1\rangle = |0\rangle$.
- Single-qubit rotations plus CNOT is a universal set of gates.
- Two-qubit gate (CNOT):



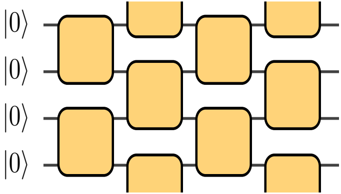
$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

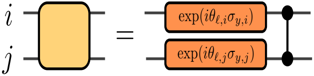
Toffoli Gate (Universal)



Circuit Architecture

- The architecture that we will be discussing (random quantum circuit) :





Quantum Circuit Architecture

- The qubits are arranged identically with period n, $i \simeq i + n$.
- At each time step, a chain of the two-qubit unitary gates act on the quantum register. The action is (alternating) on neighbouring qubit pairs.
- The two-qubit gate is made of independent Pauli-y rotations (with random parameter in $\mathcal{U}(0, 2\pi)$), acting on single qubits:

$$R(\theta) = \exp(i\sigma_y\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{8}$$

followed by the controlled-Z operation:

$$CZ = \mathrm{diag}\,(1, 1, 1, -1) \tag{9}$$

that generically creates a pairwise entanglement.

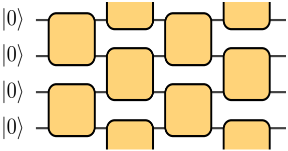
Noise

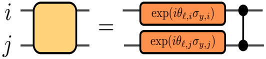
- Quantum system are extremely delicate: qubit phase errors, quantum gate errors, decoherence.
- There is a continuous manifold of errors.
- We cannot copy (no cloning).
- We cannot measure the data qubits to detect errors.
- Threshold theorem:

$$P_L \sim \left(\frac{p}{p_{th}}\right)^d \tag{10}$$

Quantum Algorithms

- Noisy Intermediate-Scale Quantum (NISQ) technology is being developed rapidly.
- It poses a great challenge to come up with efficient quantum algorithms that will run on the NISQ computers and perform better than classical ones.
- Many real-world use cases are associated with machine learning and optimization, for which random circuits offer an appropriate framework:





Hamiltonian Complexity

- The typical optimization tasks can be formulated as a search for the ground state of a κ-local Hamiltonian H, which is a sum of local operators that act on κ qubits:

$$H = \sum_i H_i \tag{11}$$

- H_i imposes a constraint analogous to κ-local clause in classical constraint satisfaction problems:

$$(x_i \vee x_j \vee x_k) \quad x_i, x_j, x_k \in \{0, 1\} \tag{12}$$

  implies $x_i x_j x_k \in \{001, 010, 011, 100, 101, 110, 111\}$.

- The Hamiltonian encodes an exact combinatorial problem, where each H_i is a constraint (QAOA, VQA, Hamiltonian complexity).

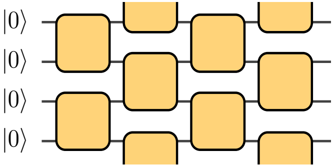
Hamiltonian Ground State

- The ground state is an eigenstate of the Hamiltonian with the lowest eigenvalue.
- The expectation value E of the Hamiltonian defines the cost function.
- The optimization task is the minimization of the cost function trying to reach the ground-level energy:

$$E(\theta) = \langle \psi_c(\theta)|H|\psi_c(\theta)\rangle, \quad |\psi_c(\theta)\rangle = U(\theta)|0\rangle^{\otimes n} \tag{13}$$

Random Circuit Model

- The random circuit model is a unitarily evolving closed $(1+1)$-dimensional chaotic system.
- The discrete space and time directions are spanned by the qubits and the circuit layers, respectively.
- The initial state $|0\rangle^{\otimes n}$ is pure, and so is the final state $|\psi_c(\theta)\rangle$ produced by the circuit.
- While the initial state is not entangled, the successive application of the circuit layers generates entanglement between the qubits in the final state:



Entanglement Entropies

- Divide the system of n qubits to two subsets A and B with n_A and $n_B = n - n_A$ qubits ($n_A = n_B = \frac{n}{2}$).
- The circuit's bipartite entanglement can be represented by the reduced density matrix:

$$\rho_A = \mathrm{Tr}_B\, \rho_c(\theta), \quad \rho_c(\theta) = |\psi_c(\theta)\rangle\langle\psi_c(\theta)| \tag{14}$$

- The k-th Renyi entropy reads:

$$\mathcal{R}_A^k = \frac{1}{1-k} \log \mathrm{Tr} \left(\rho_A^k \right) \tag{15}$$

- The limit $k \to 1$ corresponds to the von Neumann entanglement entropy:

$$S_{EE} = -\mathrm{Tr}\, \rho_A \log \rho_A \tag{16}$$

- We will use these entropies to diagnose the efficiency of the quantum algorithm.

A Simple Example

- The Bell State:

$$|\psi_{12}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) \tag{17}$$

- The density matrix reads:

$$\rho_{12} = |\psi_{12}\rangle\langle\psi_{12}| = \frac{1}{2}\left(|00\rangle + |11\rangle\right)\left(\langle 00| + \langle 11|\right) \tag{18}$$

- The reduced density matrix reads:

$$\rho_1 = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right) \equiv \begin{pmatrix} \frac{1}{2} & \\ 0 & \frac{1}{2} \end{pmatrix} \tag{19}$$

- The k-th Renyi entropy reads:

$$\mathcal{R}_A^k = 1 \tag{20}$$

Maximally Mixed State

- The density matrix of the maximally mixed state of N qubits is:

$$\rho_{mixed} = \frac{\mathbb{1}_{2^N}}{2^N} \tag{21}$$

- The Renyi entropies are:

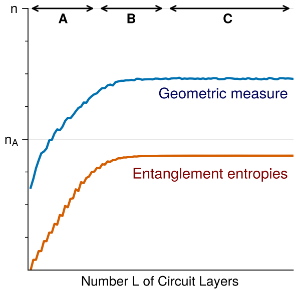
$$\mathcal{R}^k = N \tag{22}$$

- It is the limit of $\beta \to 0$ of the thermal density matrix:

$$\rho_{thermal} = \frac{e^{-\beta H}}{Tre^{-\beta H}} \tag{23}$$

Circuit Entanglement

- The general structure of the Renyi entropies:

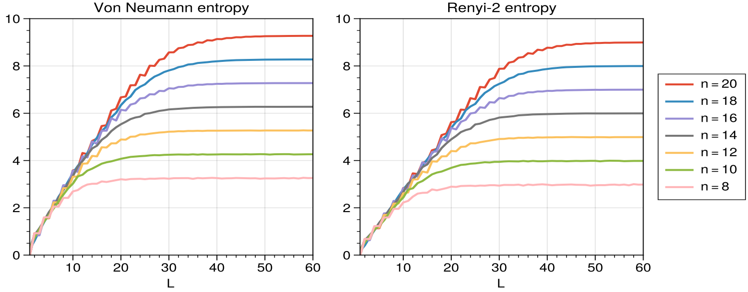


- The geometric measure of entanglement measures the distance of the state $|\psi\rangle$ from the closest product state:

$$\epsilon_g(|\psi\rangle) = -\log sup_{\alpha\in\mathcal{P}}|\langle\alpha|\psi\rangle|^2 \qquad (24)$$

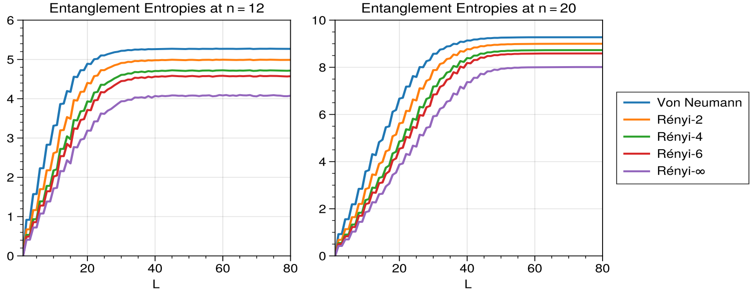
Numerical Simulations

- We see a linear growth of the entropies to a plateau with an approximate value $\frac{n}{2}$ (up to a numerical shift).
- The entanglement velocities are independent of the number of qubits but are different for the Von Neumann and Renyi-2 entropies.
- The Von Neumann entropy plateau is a little higher and the velocity is larger than that of Renyi-2.



Numerical Simulations

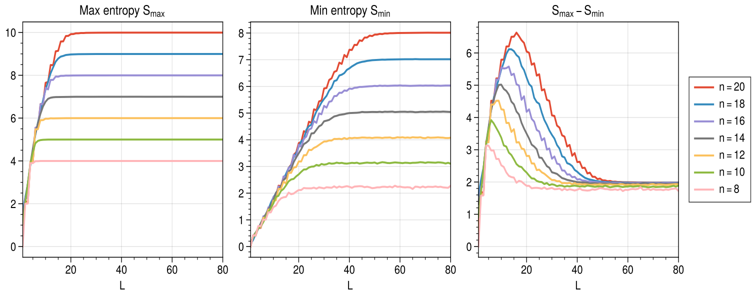
- Various k-th Renyi entropies for $n = 12$ and $n = 20$ qubits averaged over 50 samples as a function of the number of layers.
- We see the linear growth of the entropies to a plateau with an approximate value $\frac{n}{2}$, up to numerical shifts that depend on k.
- In the linear region, the growth rates are independent of the number of qubits but depend on k.



Renyi Entropies Monotonicity

- The Renyi-k entropies are non-increasing as a function of k:

$$\begin{aligned} \lim_{q\to\infty} \mathcal{R}^q(\rho) &\leq \cdots \leq \mathcal{R}^2(\rho) \leq S_{EE}(\rho) \leq \lim_{q\to 0} \mathcal{R}^q(\rho) \\ S_{max}(\rho) &= \lim_{k\to 0} \mathcal{R}^k(\rho) = \log(\text{rank } \rho) \\ S_{min}(\rho) &= \lim_{k\to\infty} \mathcal{R}^k(\rho) = -\log(\lambda_{max}(\rho)) \end{aligned} \tag{25}$$



Entanglement Scaling

- A ground state of κ-local Hamiltonians with a mass gap is typically a non-generic state whose entanglement entropy is expected to follow an area law scaling (proof in one dimension by Hastings).

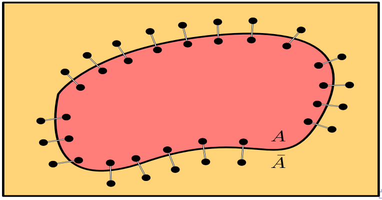
- In the linear growth region in the entanglement entropy curve:

$$\mathcal{R}_A^k(\rho_A) \simeq v_k L \cdot \mathrm{Area}(\partial A) = v_k L \tag{26}$$

  where v_k is the entanglement velocity that depends on k.

- In the plateau region:

$$\mathcal{R}_A^k(\rho_A) \simeq n/2 - c_k = \mathrm{Vol}(A) - c_k \tag{27}$$



Optimization Task

- Using the density matrix of the quantum circuit $\rho_c(\theta)$:

$$E(\theta) = \mathrm{Tr}\left(\rho_c(\theta) H\right) \tag{28}$$

- One performs multiple iterations of evaluating the density matrix $\rho_c(\theta)$ and updating the parameters (via the gradient descent) that will finally stop at $\theta = \theta_f$.
- We would like to reach the final parameter θ_f such that

$$\Delta E \equiv \mathrm{Tr}\left((\rho_c(\theta_f) - \rho_g) H\right) \simeq 0 \tag{29}$$

where ρ_g is the exact ground state of the Hamiltonian.

Trace Distance

- Cauchy-Schwarz inequality:

$$\mathrm{Tr}\left((\rho_c(\theta_f) - \rho_g)H\right) \leq \|\rho_c(\theta_f) - \rho_g\|_1 \cdot \|H\|_1, \tag{30}$$

  where the trace norm $\|O\|_1$ is the sum of singular values of an operator O, i.e., eigenvalues of $(O^\dagger O)^{1/2}$.
- A natural condition for efficient reduction of ΔE is arranging an initial circuit state $\rho_c(\theta_{in})$ to be in the proximity of the ground state with a small enough trace distance $\|\rho_c(\theta_{in}) - \rho_g\|_1$.
- We generally do not know the ground state, thus being unable to estimate the trace distance $\|\rho_c(\theta_{in}) - \rho_g\|_1$.
- The trace distance can be considerably large even when the energy difference between quantum states is minuscule.

Entanglement Diagnostic

- Upper Bound:

$$\frac{1}{2}\left\|\rho_A - \frac{I_A}{2^{n_A}}\right\|_1^2 \leq n_A - S_{EE}(\rho_A) \tag{31}$$

- Lower bound ($n_A \gg 1$):

$$1 - \frac{S_{EE}(\rho_A)}{n_A} \lesssim \frac{1}{2}\left\|\rho_A - \frac{I_A}{2^{n_A}}\right\|_1 \tag{32}$$

Entanglement Diagnostic

- Our strategy will be to avoid having a maximally entangled state as an initial circuit state $\rho_c(\theta_{in})$ in order to locate it in the proximity of the ground state.
- "Expressible Circuits" that can express a volume-law entangled states have to be highly entangled.
- The entanglement diagnostic for circuit states is only a necessary condition to keep the initial and target states close.
- The gradient-based optimization indeed works efficiently for those variational circuits whose average entanglement entropy scales slower than the volume law.

Hamiltonians

- The one-dimensional Ising Hamiltonian:

$$H = \sum_{i=1}^{n} \sigma_{z,i}\sigma_{z,i+1} + g \sum_{i=1}^{n} \sigma_{x,i} \tag{33}$$

- The long-range Ising model:

$$H = \sum_{i<j} \frac{1}{d(i,j)^{\alpha}} \sigma_{z,i}\sigma_{z,j} + g \sum_{i=1}^{n} \sigma_{x,i} \tag{34}$$

  $d(i,j)$ is the shortest distance between the i'th and j'th spins.

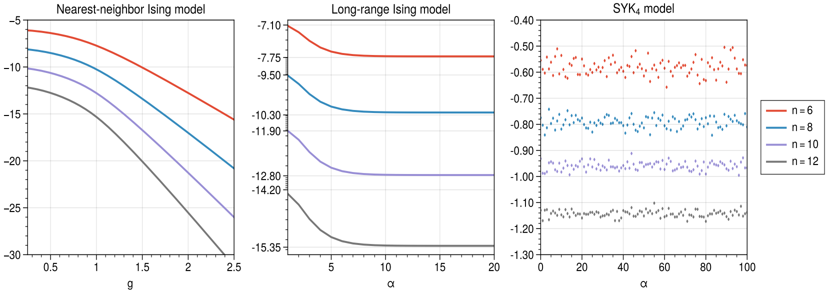
- The SYK model:

$$H = (i)^{q/2} \sum_{1\le i_1<\ldots<i_q\le 2n} J_{i_1\ldots i_q}\gamma_{i_1}\cdots\gamma_{i_q} \ , \tag{35}$$

  $\{\gamma_i\}_{1\le i\le 2n}$ are Majorana fermions and $J_{i_1\ldots i_q}$ are randomly drawn from the Gaussian distribution with mean 0 and variance $(q-1)!/(2n)^{q-1}$.

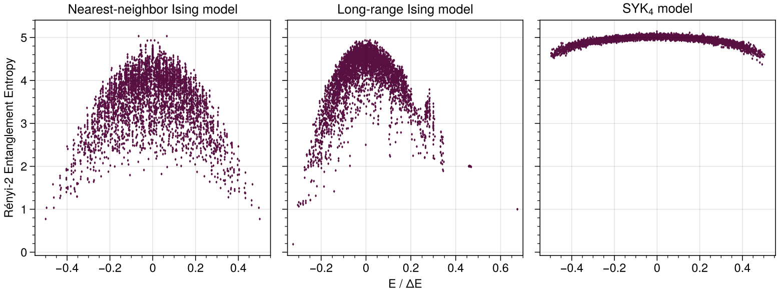
Ground State Energies

- The ground energies: (Left) Nearest-neighbor Ising Hamiltonian with different g. (Middle) Long-range Ising Hamiltonian at $g = 1$ with different α. (Right) SYK_4 Hamiltonian with 100 different instances of Gaussian random couplings:



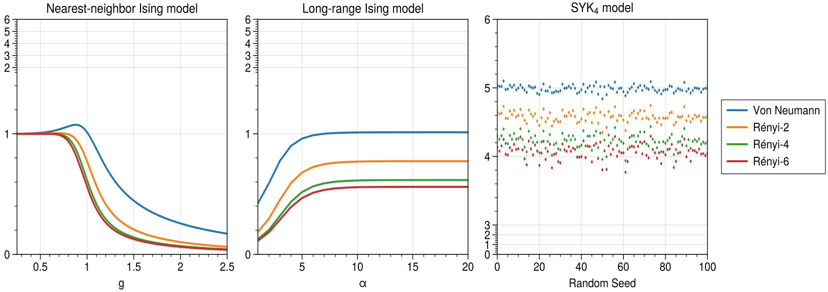
Renyi Entropies

- The scatter plot of energy and Rényi-2 entropy over all eigenstates of the $n = 12$ Hamiltonians: (Left) Nearest-neighbor Ising Hamiltonian at $g = 1$. (Middle) Long-range Ising Hamiltonian at $g = \alpha = 1$. (Right) SYK_4 Hamiltonian with a particular instance of Majorana fermion couplings sampled from Gaussian distribution:



Renyi Entropies

- The ground state entanglement entropies with 12 qubits: (Left) Nearest-neighbor Ising Hamiltonian with different g. (Middle) Long-range Ising Hamiltonian at $g = 1$ with different α. (Right) SYK_4 Hamiltonian with 100 different instances of Gaussian random couplings:



Optimization

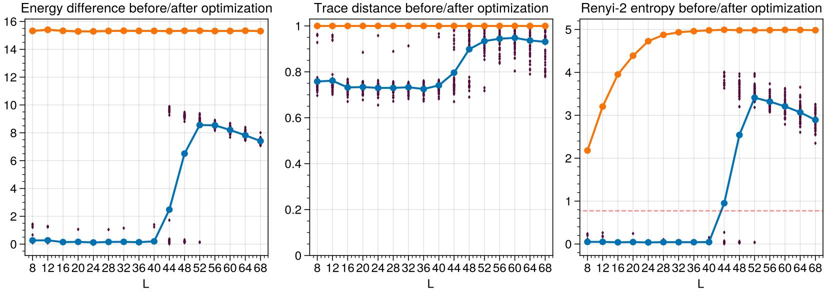
- The objective is to find a circuit parameter θ^* that can closely approximate the ground state energy, $E(\theta^*) \simeq E_g$.
- Iterative steps are proportional to the negative gradient of the energy function at each point:

$$\theta_{\tau+1} = \theta_\tau - \eta \nabla E(\theta_\tau) \tag{36}$$

- As we are interested in finding a general correlation between the entanglement diagnostics and the success of optimization.

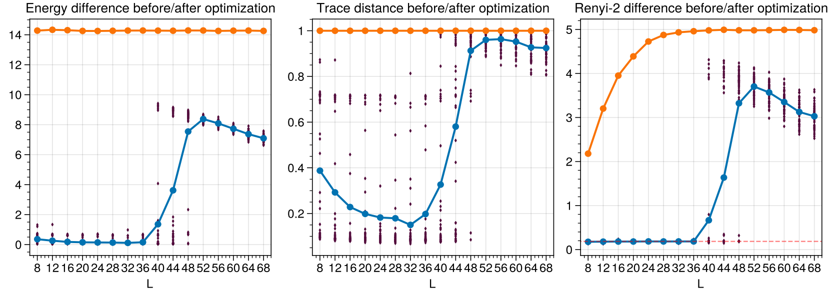
Optimization I

- The optimization to reach the nearest-neighbours transverse-field Ising model ground state at $g = 1$ as a function of the number of circuit layers:



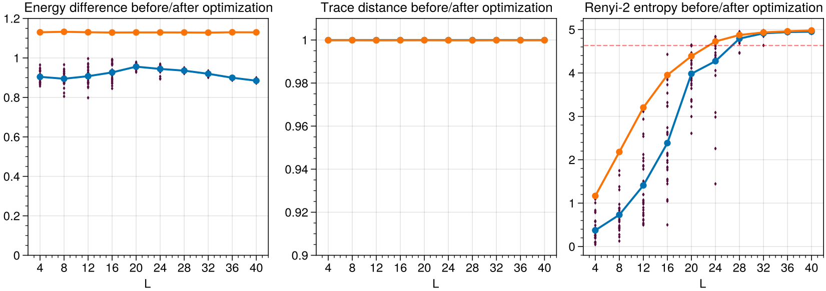
Optimization II

- The optimization to reach the non-local Ising Hamiltonian ground state at $\alpha = g = 1$ as a function of the number of circuit layers:



Optimization III

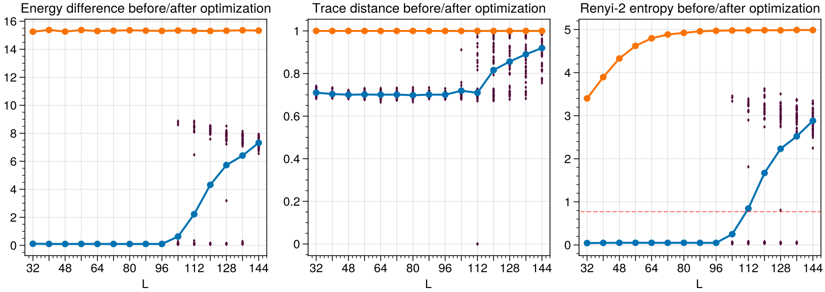
- Measurements averaged over 30 independent circuits, before/after the VQA optimization with a particular instance of the SYK_4 Hamiltonian at $n = 12$, as a function of the circuit depth L:



- One needs to overparametrize the circuit.

Random Graph Architecture

- Consider a simple stochastic variation of the circuit architecture that omits the CZ entangler inside the 2-qubit gate with a fixed probability $p = 1/2$:



Quantum Energy Landscape

- The energy function defines the quantum energy landscape:
$$E(\boldsymbol{\theta}) = \langle \psi(\boldsymbol{\theta}) | \mathcal{H} | \psi(\boldsymbol{\theta}) \rangle \quad (37)$$
- We argue that decreasing the entangling capability and increasing the number of circuit parameters have the same qualitative effect on the Hessian eigenspectrum.
- Both the low-entangling capability and the abundance of control parameters increase the curvature of non-flat directions, contributing to the efficient search of area-law entangled ground states as to the optimization accuracy and the convergence speed.

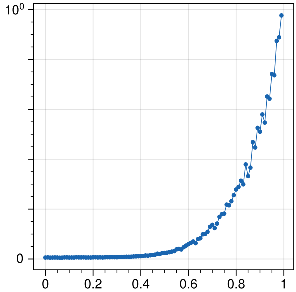
Quantum Energy Landscape

- The expectation values of the energy function and its derivatives (flat directions on average):

$$\mathbb{E}_{\boldsymbol{\theta}}[E(\boldsymbol{\theta})] = \mathrm{Tr}(H)/2^n \tag{38}$$

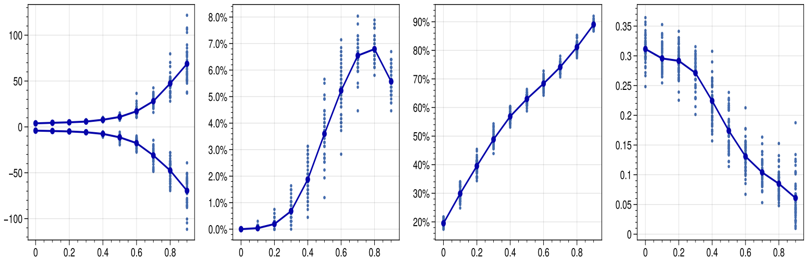
$$\mathbb{E}_{\boldsymbol{\theta}}[\partial_a E(\boldsymbol{\theta})] = \mathbb{E}_{\boldsymbol{\theta}}[\partial_a \partial_b E(\boldsymbol{\theta})] = \cdots = 0 \tag{39}$$

- The variance $Var_{\boldsymbol{\theta}}\left[\partial_a E(\boldsymbol{\theta})\right]$ of the energy function gradient with 56 circuit layers versus the probability p to remove the two-qubit entanglers:



The Hessian

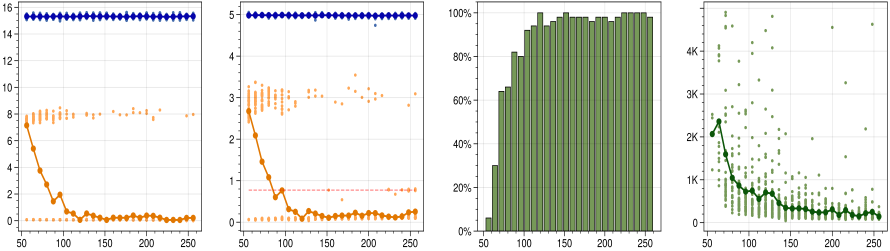
- The Hessian ($H_{ab} = \partial_a \partial_b E$) eigenspectrum with $n = 12$ qubits and $L = 56$ layers versus the probability p to omit the CZ-gates (at randomly initialized circuit parameters).
- (a) top/bottom eigenvalues, (b) % of large eigenvalues satisfying $|\lambda| > 5$, (c) % of small eigenvalues satisfying $|\lambda| < 0.2$, (d) gradient overlap with $\mathcal{P}_{\text{small}}$.



(a) (b) (c) (d)

Control Parameters

- Adding more control parameters (single-qubit rotations) for fixed entanglement ($L = 56$ layers) leads to better performance:

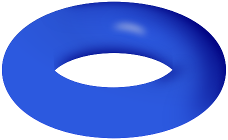


Energy Difference — Renyi-2 Entropy — Success Rate for $\Delta E < 0.1$ — Update Steps until $\Delta E < 0.1$

- Reminiscent of classical over-parametrized deep learning.

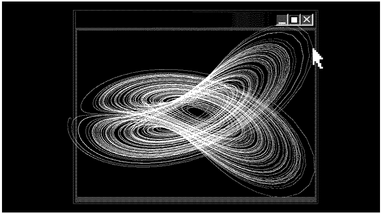
Classical Chaos

- A classical dynamical system with n degrees of freedom is integrable iff it has n conserved quantities in involution.



- A characteristic of chaotic system is the sensitivity to initial conditions (butterfly effect):

$$\delta|X(t)| \sim e^{\lambda t}\delta|X(0)| \tag{40}$$



Quantum Chaos

- Quantum Chaos is the quantum theory of classical chaotic dynamical systems.
- The variational circuit states are in general chaotic.
- We want to explore the connection between quantum chaos diagnostics of circuit states and the circuit performance.
- Result: the diagnostics that use the eigenvalue spectrum, e.g., operator spreading and entanglement entropy are more accurate measures of the optimization efficiency than those that use the level spacing distribution of the entanglement spectrum, such as r-statistics or spectral form factors.

Quantum Chaos

- Random matrix theory (RMT) describes statistical properties of the energy eigenvalues of a numerous chaotic systems (Wigner, Dyson, BGS).
- The level spacing distribution $p(s)$ is the probability to find two neighbouring (energy) eigenvalues separated by a distance s ($s_i = e_{i+1} - e_i$).
- For integrable models $p(s)$ follows the Poisson distribution:

$$p(s) = e^{-s} \tag{41}$$

- For RMT:

$$p_\beta(s) = \frac{s^\beta e^{-b_\beta s^2}}{\Gamma(\frac{1+\beta}{2})} \tag{42}$$

  GOE: $\beta = 1$, GUE: $\beta = 2$.
- In the limit $s \to 0$ $p(s) \to 0$ (level repulsion).

Quantum Chaos

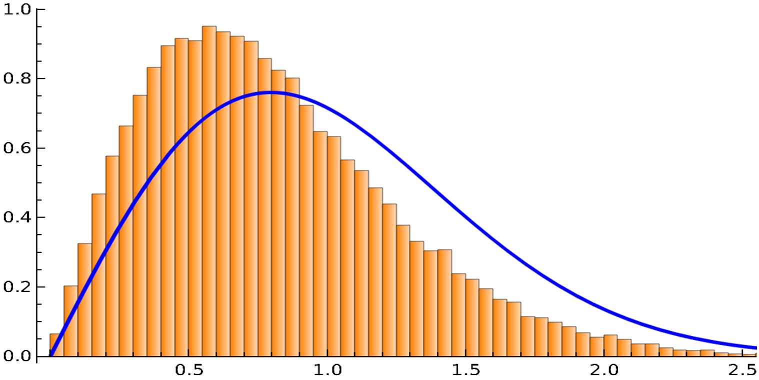
- Denote the density matrix of the circuit state by ρ_c and divide n qubits of the quantum register into two subsets A and B of the equal size, $n_A = n_B = \frac{n}{2}$.
- The modular Hamiltonian $H(\rho_A)$ of the reduced density matrix $\rho_A = \mathrm{Tr}_B \rho_c$ is:

$$\rho_A \equiv \frac{e^{-H(\rho_A)}}{Z_A} \tag{43}$$

  $Z_A = \mathrm{Tr}_A e^{-H(\rho_A)}$ is the partition function of the modular Hamiltonian.
- Chaotic properties of Hamiltonian systems reveal themselves also in the level spacing distribution of the modular Hamiltonian spectrum.

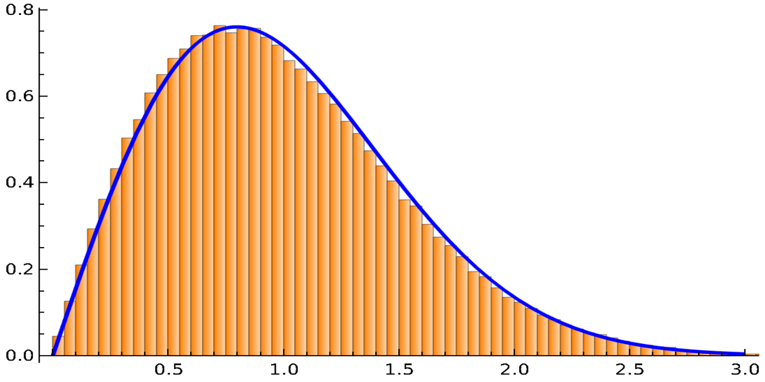
Circuit Distribution

- $H_A(\rho)$ eigenvalue-difference distribution at 30 layers compared to GOE:



Circuit Distribution

- $H_A(\rho)$ eigenvalue-difference distribution at 50 layers compared to GOE:



Operator Spreading

- Operator spreading is a diagnostic of the chaotic dynamics and information scrambling.
- A local operator $\mathcal{O}$ that acts on a small number of qubits at time step $t = 0$ evolves at time step t to:

$$\mathcal{O}(t) = U(t)^{\dagger}\mathcal{O}(0)U(t) \tag{44}$$

that acts on a large number of qubits.
- This spread can be quantified and its growth is ballistic with a characteristic velocity called the butterfly velocity.

Operator Spreading

- Any Hermitian operator $\mathcal{O}(t)$ acting on n qubit systems can be written in the Pauli string basis:

$$\mathcal{O}(t) = \frac{1}{2^{n/2}} \sum_{j_1,\cdots,j_n} h_{j_1,\cdots,j_n}(t)\, \sigma_{j_1}^{(1)} \otimes \cdots \otimes \sigma_{j_n}^{(n)} \tag{45}$$

where

$$h_{j_1,\cdots,j_n}(t) \equiv \frac{1}{2^{n/2}} \mathrm{Tr}(\sigma_{j_1}^{(1)} \otimes \cdots \otimes \sigma_{j_n}^{(n)} \cdot \mathcal{O}(t)) \tag{46}$$

- Under the unitary time evolution (44)

$$\mathrm{Tr}(\mathcal{O}(t)^\dagger \mathcal{O}(t)) = \mathrm{Tr}(\mathcal{O}(0)^\dagger \mathcal{O}(0)) = \frac{1}{2^n} \sum |h_{j_1,\cdots,j_n}(t)|^2 = \text{constant} \tag{47}$$

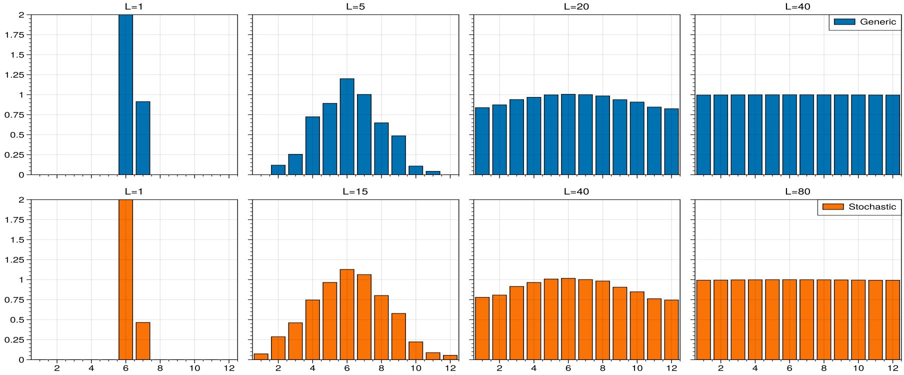
Operator Spreading

- The size of the operator $\mathcal{O}(t)$ is measured by the size of the region where $\mathcal{O}(t)$ does not commute with a typical local operator $\sigma_a^{(x)}$ at position x.
- This can be quantified as (OTOC):

$$
\begin{aligned}
\mathcal{C}(x,t) &= \frac{1}{2}\mathrm{Tr}(\rho_\infty[\mathcal{O}(t),\sigma_a^{(x)}]^\dagger[\mathcal{O}(t),\sigma_a^{(x)}]) \\
&= \frac{1}{2}\mathrm{Tr}([\mathcal{O}(t),\sigma_a^{(x)}]^\dagger[\mathcal{O}(t),\sigma_a^{(x)}]) \\
&= \sum_{\substack{j_1,\cdots,j_n \\ j_x\neq 0,a}} 2|h_{j_1,\cdots,j_n}(t)|^2 \qquad (48)
\end{aligned}
$$

Operator Spreading

- We numerically measure (48) for 12 qubits, where the operator $\mathcal{O}(0)$ is the Hamiltonian whose expectation value we are minimizing:



r-Statistics

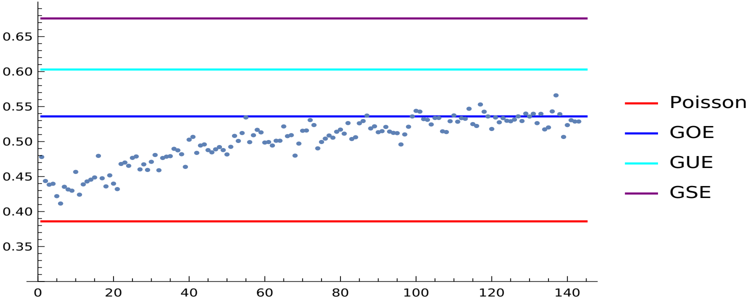
- r-statistics is a short-range diagnostics of quantum chaos.

$$r_i = \frac{\mathrm{Min}(s_i, s_{i+1})}{\mathrm{Max}(s_i, s_{i+1})}, \quad s_i = e_{i+1} - e_i \tag{49}$$

- $r_i \approx 0.53590$, $r_i \approx 0.60266$ and $r_i \approx 0.67617$ for GOE, GUE and GSE ensembles, respectively.
- For an integrable system the values of r_i are typically smaller, approaching the value of $r_i \approx 0.38629$ for a Poisson process.

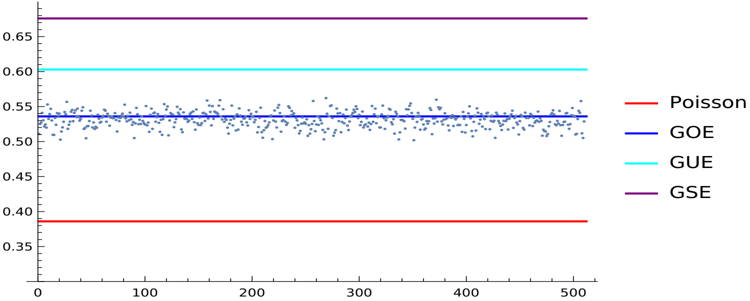
Circuit r-Statistics

- With 12 layers:



Circuit r-Statistics

- With 50 layers:

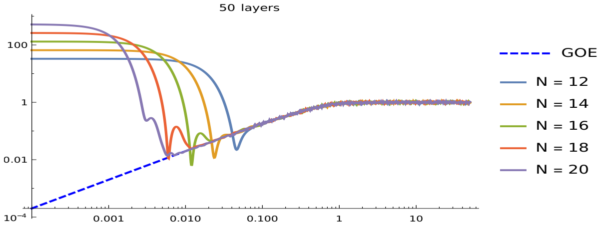


Spectral Form Factor

- Define:

$$Z(\tau) = Tre^{-i\tau H_A(\rho)} = \sum_{i,j} e^{-i\tau(e_i - e_j)} \quad (50)$$

- The spectral form factor $\frac{|Z(\tau)|^2}{Z(0)^2}$ is another diagnostic for quantum chaos:



- The universal structure begins at the "ramp time" t_{ramp}.
- The linear ramp is a diagnostic for quantum chaos at "large separation".

Challenges

- Develop analytical tools to study quantum circuits (Tensor networks, condensed matter applications, quantum gravity)
- Understand which computations can a quantum computer do better than the classical one (quantum complexity, NISQ applications)
- Construct quantum algorithms for real world use cases.
- Study the relationship between classical and quantum, e.g. classical and quantum deep learning.