What is Dark Matter? and Classification of Effective operators

Hitoshi Murayama (Berkeley, Kavli IPMU, DESY)

Arnold Sommerfeld Seminar, July 25, 2019

東京大学 THE UNIVERSITY OF TOKYO

UTIAS 東京大学国際高等研究所 THE UNIVERSITY OF TOKYO INSTITUTES FOR ADVANCED STUDY

KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

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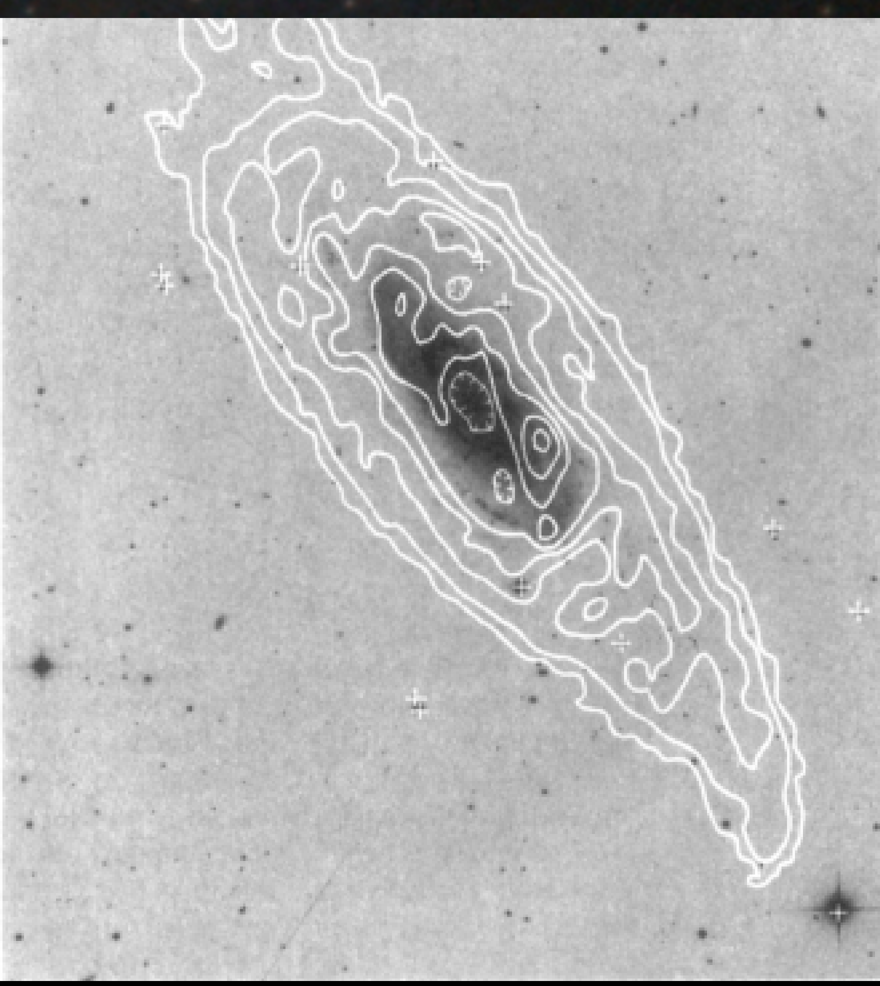
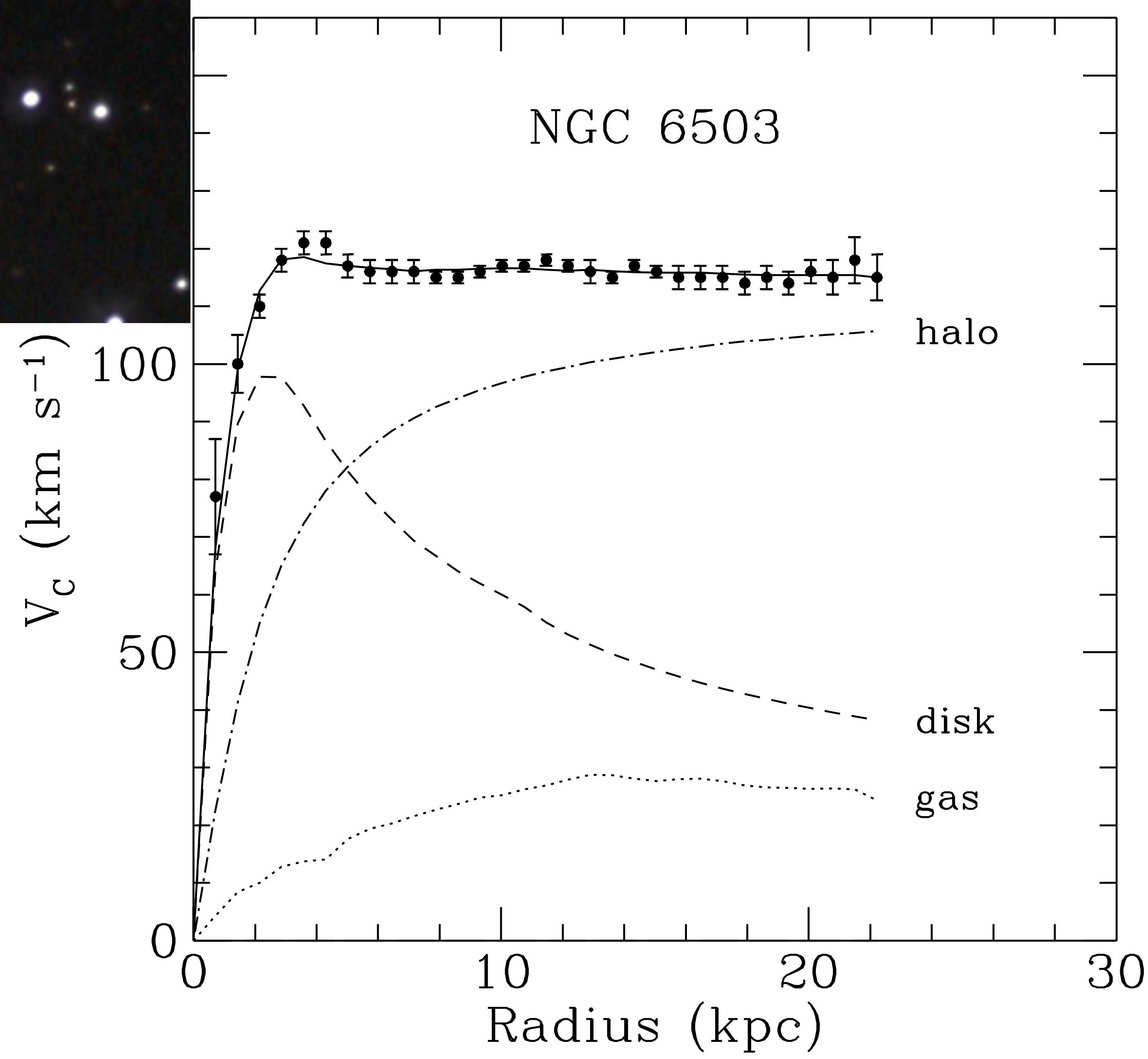
DESY.

BERKELEY CENTER FOR THEORETICAL PHYSICS

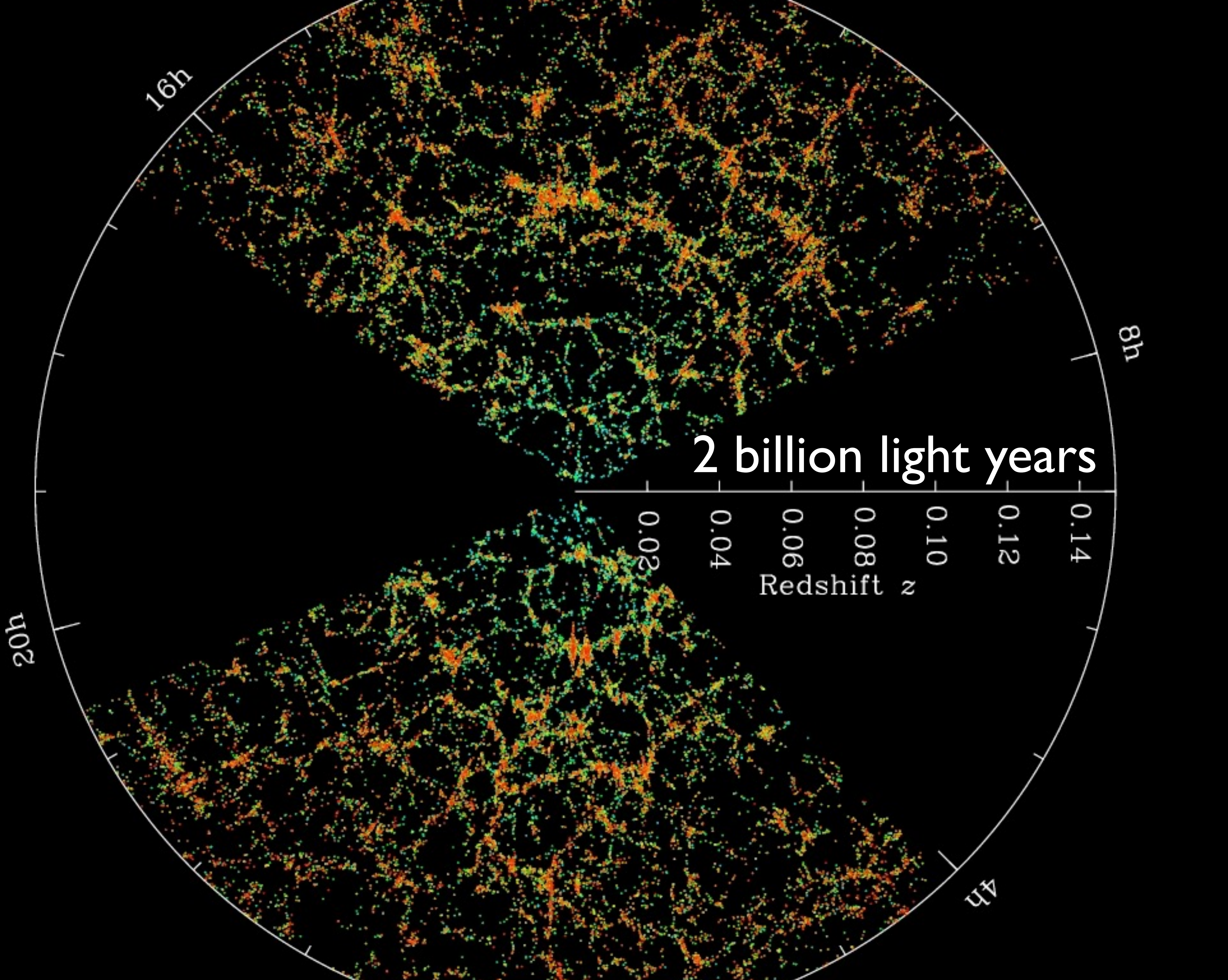
BERKELEY LAB

THE UNIVERSITY OF CALIFORNIA 1868 LET THERE BE LIGHT



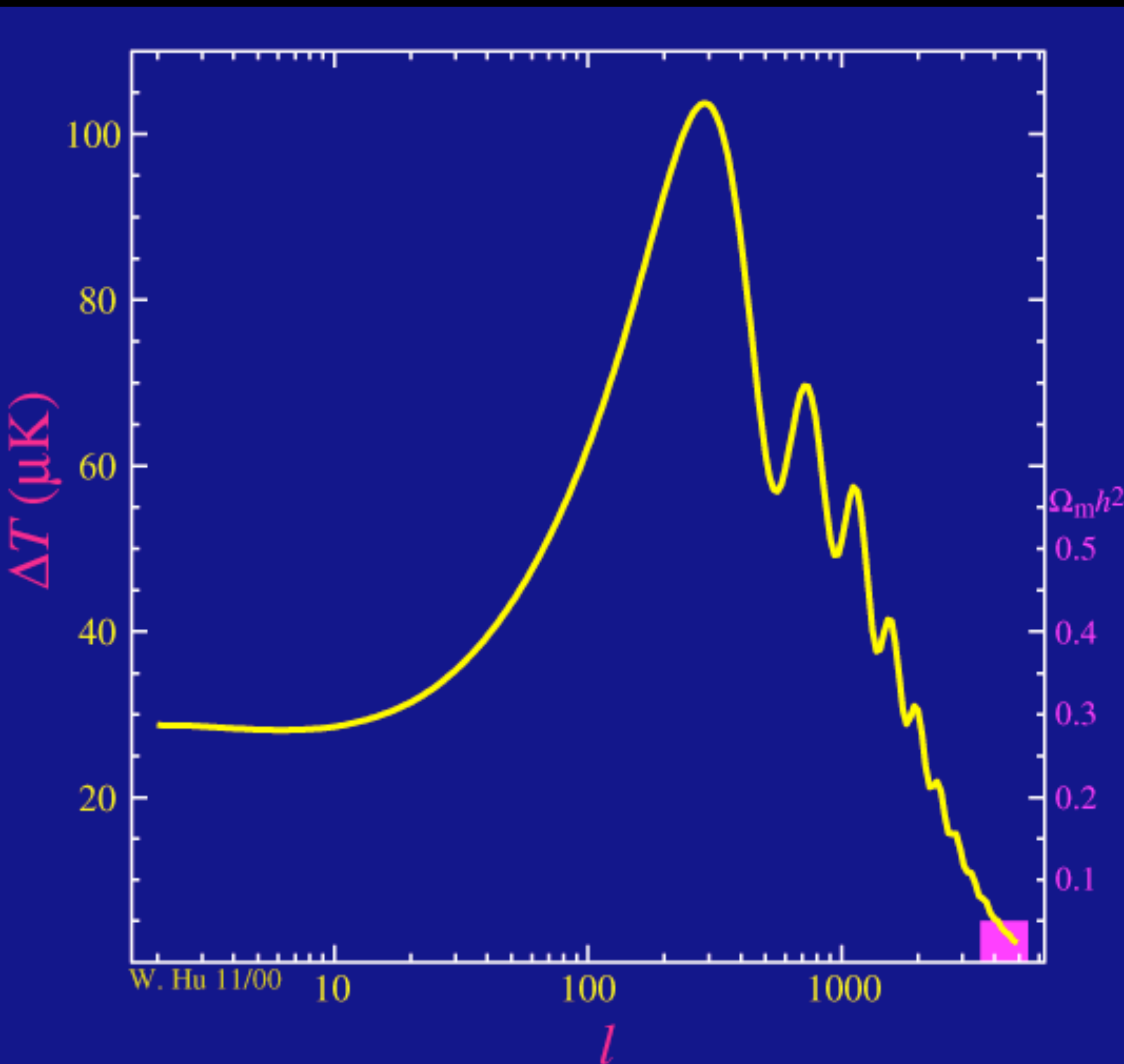


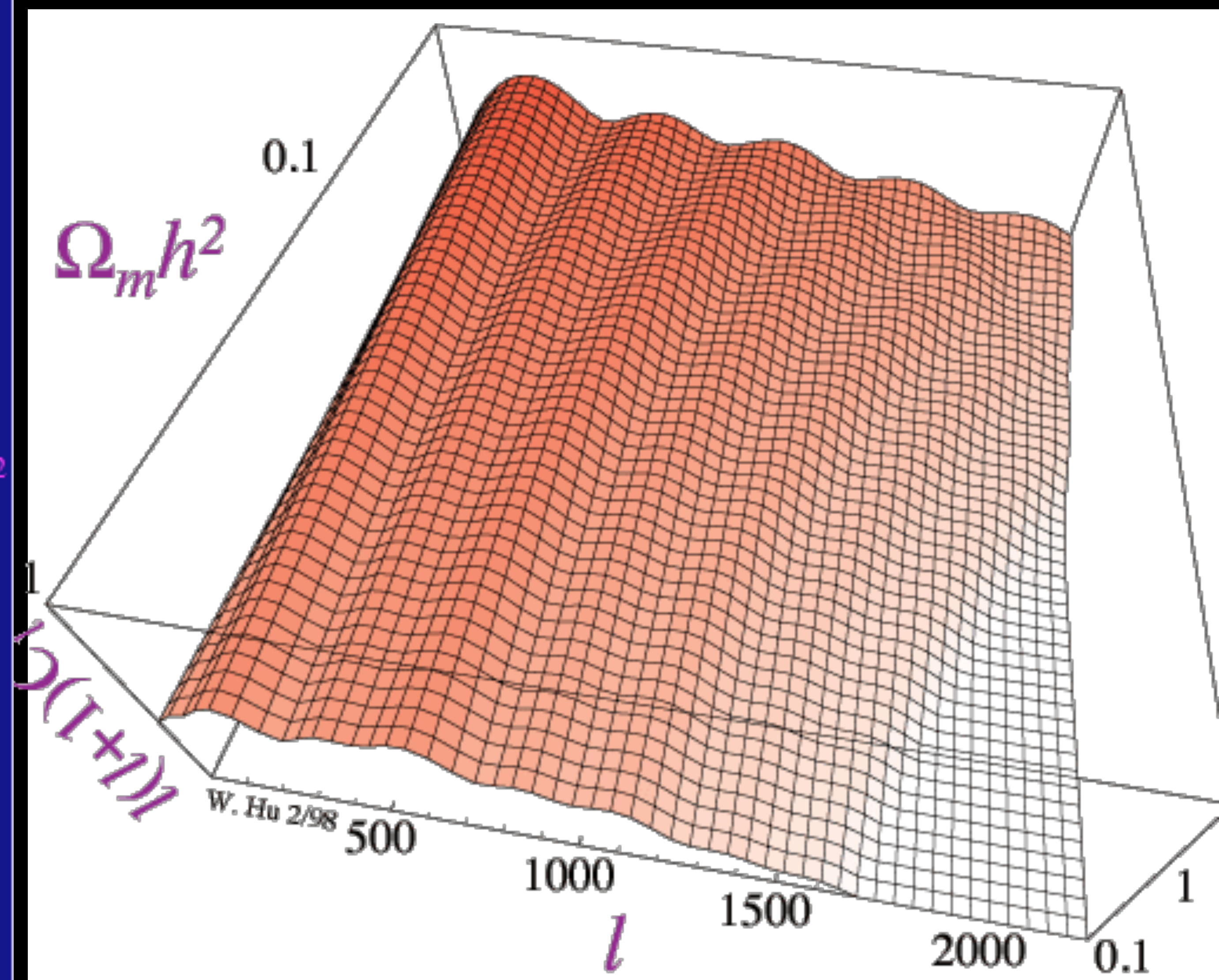
NGC 6503
halo
disk
gas
V_c (km s^{-1})
Radius (kpc)
0
50
100
0
10
20
30



2 billion light years
16h
8h
20h
4h
0.02
0.04
0.06
0.08
0.10
0.12
0.14
Redshift z

matter

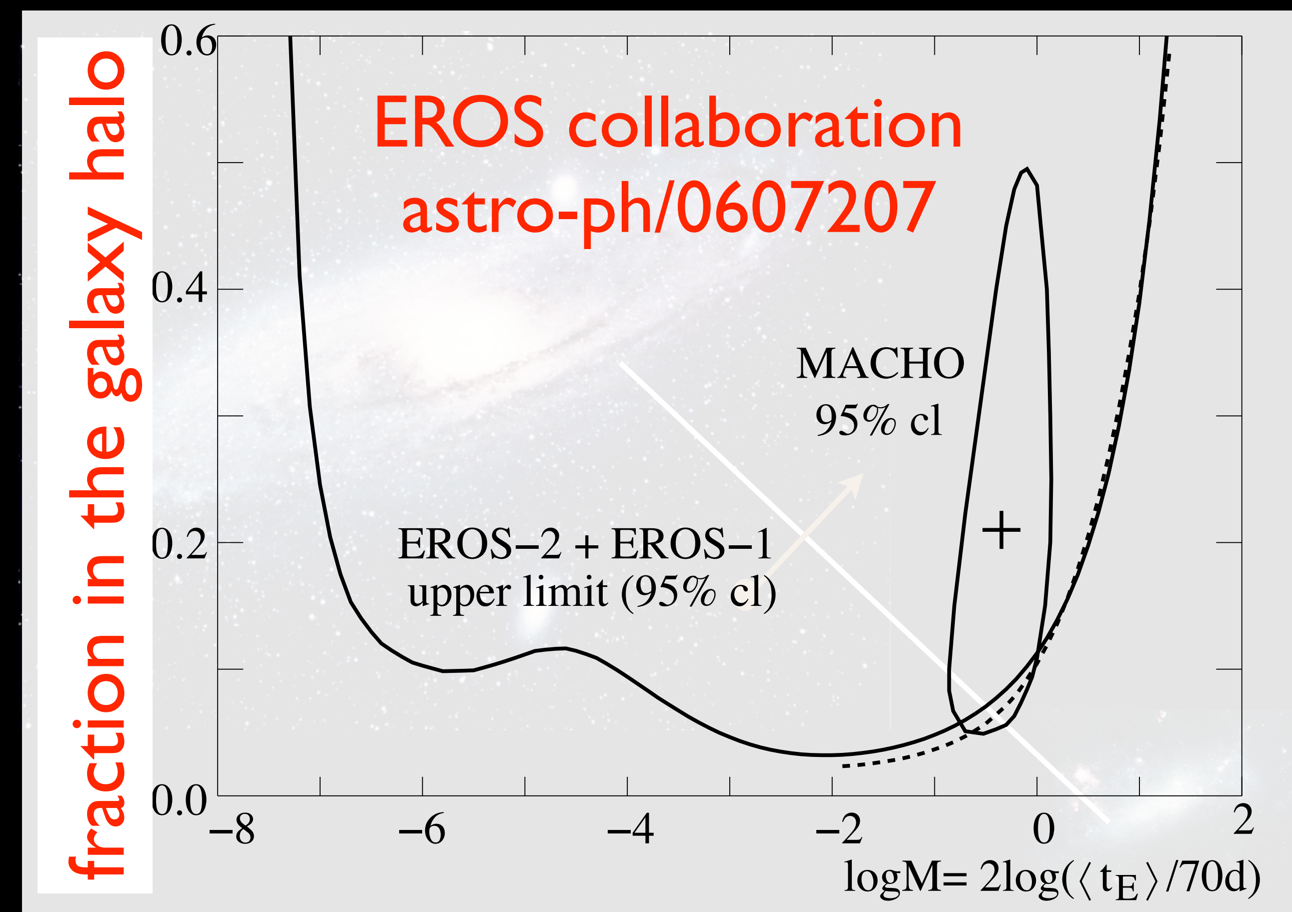


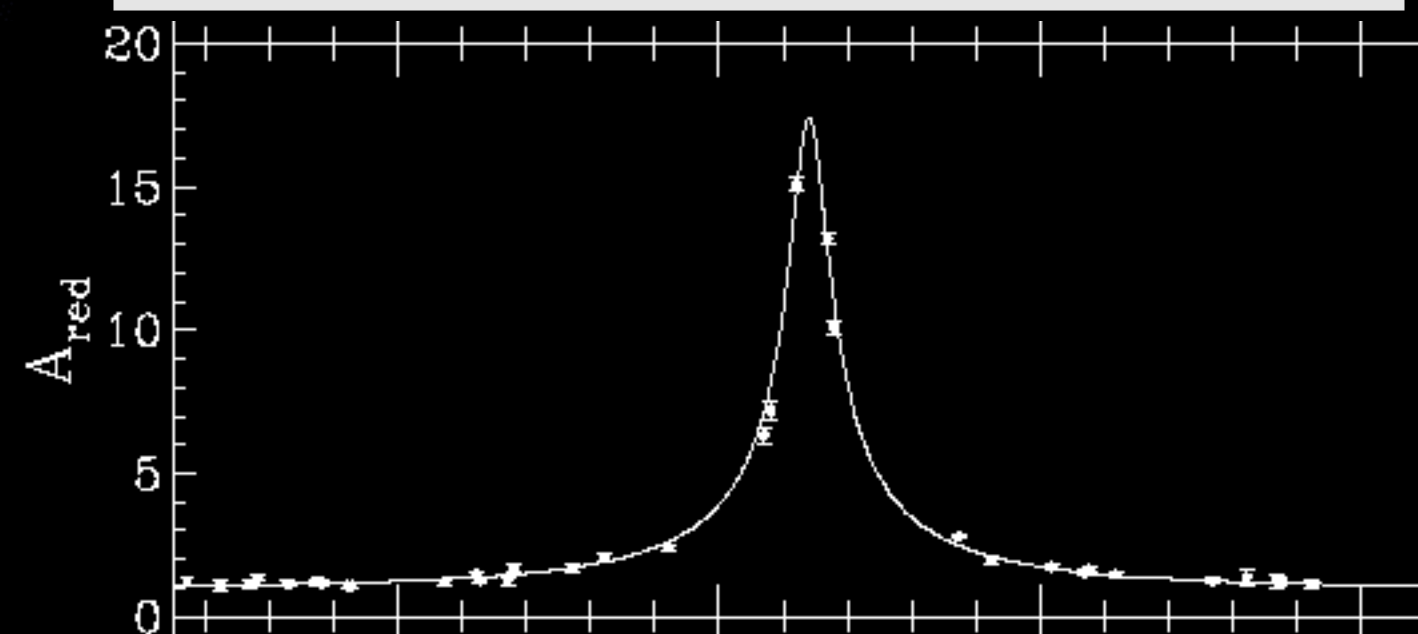


Ω_m changes the overall heights of the peaks

Dim Stars? Black Holes?

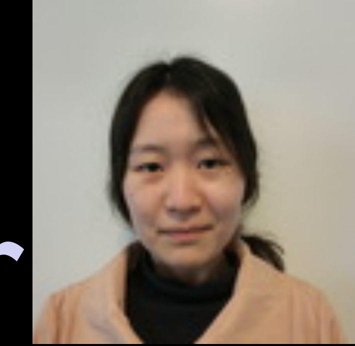
Search for MACHOs
(Massive Compact Halo Objects)



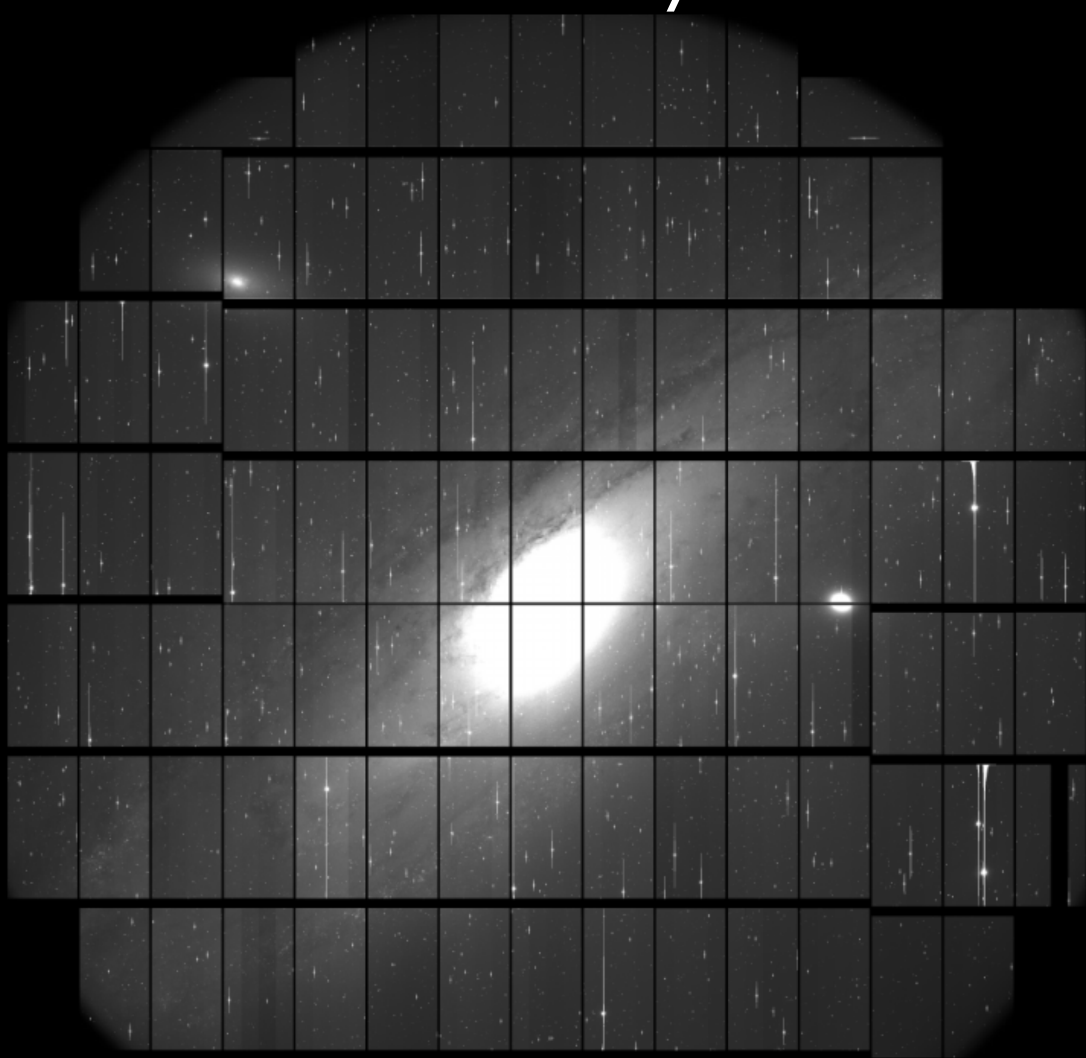


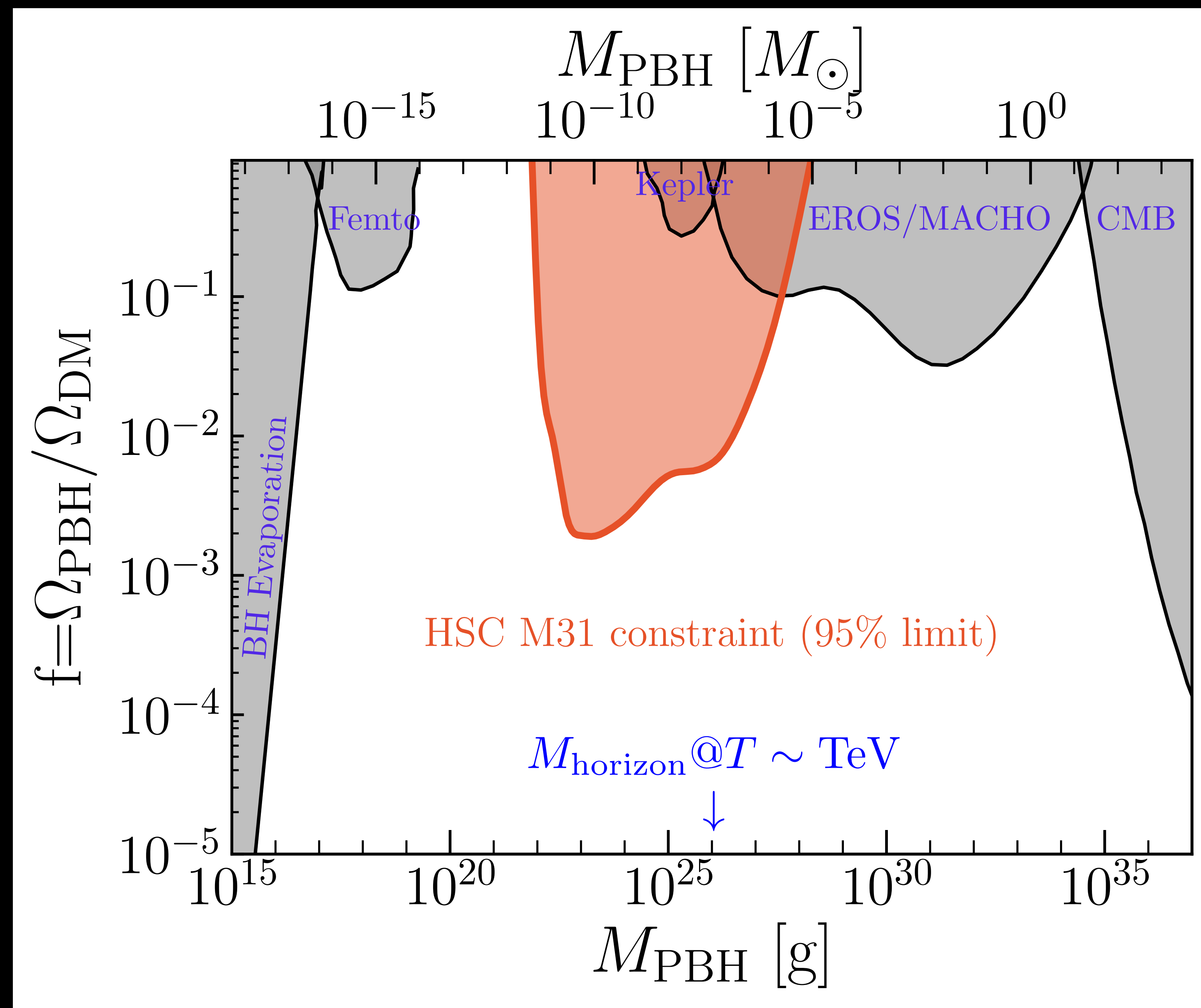
Not enough of them!

Best limit on Black Hole dark matter



observe Andromeda for one night
read out CCDs every 2 min



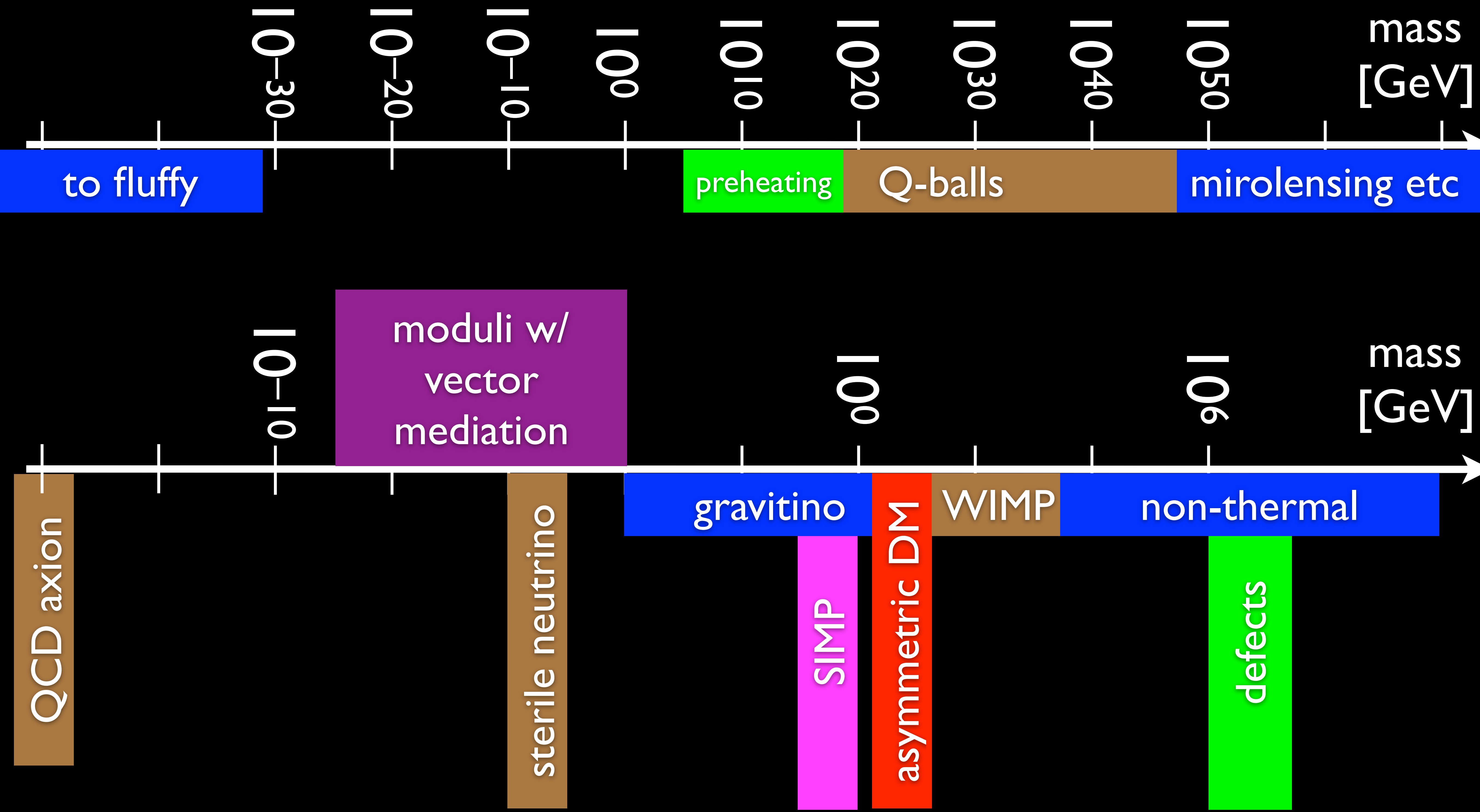


No detection ⇒ more stringent upper bound, than 2yr Kepler data (Griest et al.)

Niikura, Takada et al., Nature Astronomy

Mass Limits
"Uncertainty Principle"

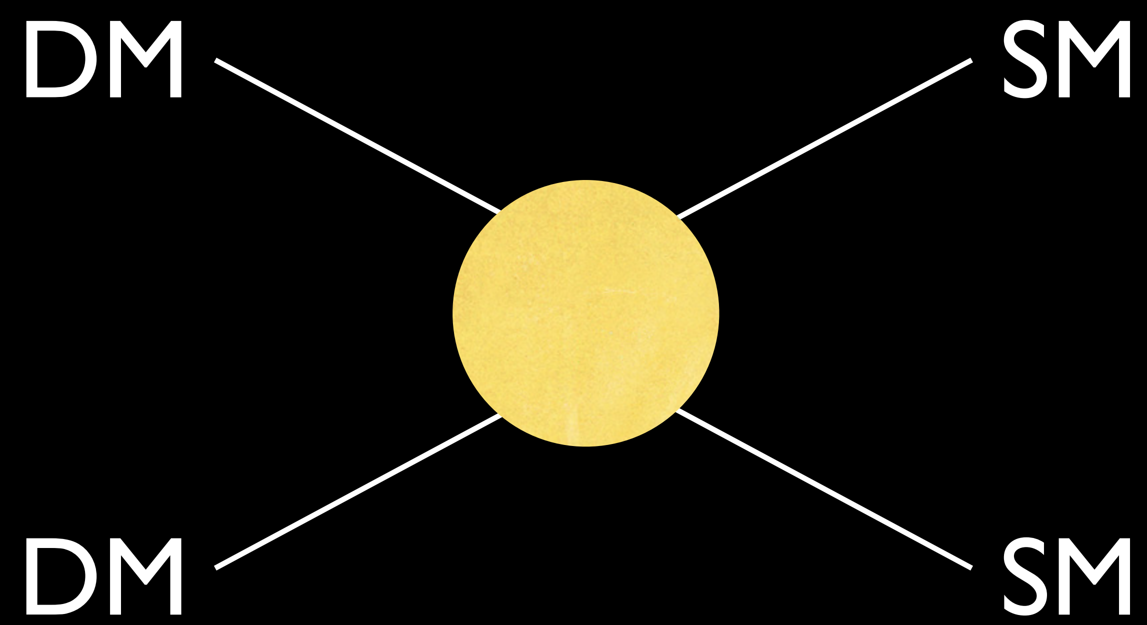
- Clumps to form structure
- imagine $V = G_N \frac{Mm}{r}$
- "Bohr radius": $r_B = \frac{\hbar^2}{G_N M m^2}$
- too small m ⇒ won't "fit" in a galaxy!
- m >10^{-22} eV "uncertainty principle" bound
  (modified from Hu, Barkana, Gruzinov, astro-ph/0003365)



SIMP: dark hadrons

m~0.3GeV, σ~10^{-24}cm^2

WIMP Miracle



$$\langle \sigma_{2\to 2} v \rangle \approx \frac{\alpha^2}{m^2}$$

$$\alpha \approx 10^{-2}$$

$$m \approx 300 \text{ GeV}$$

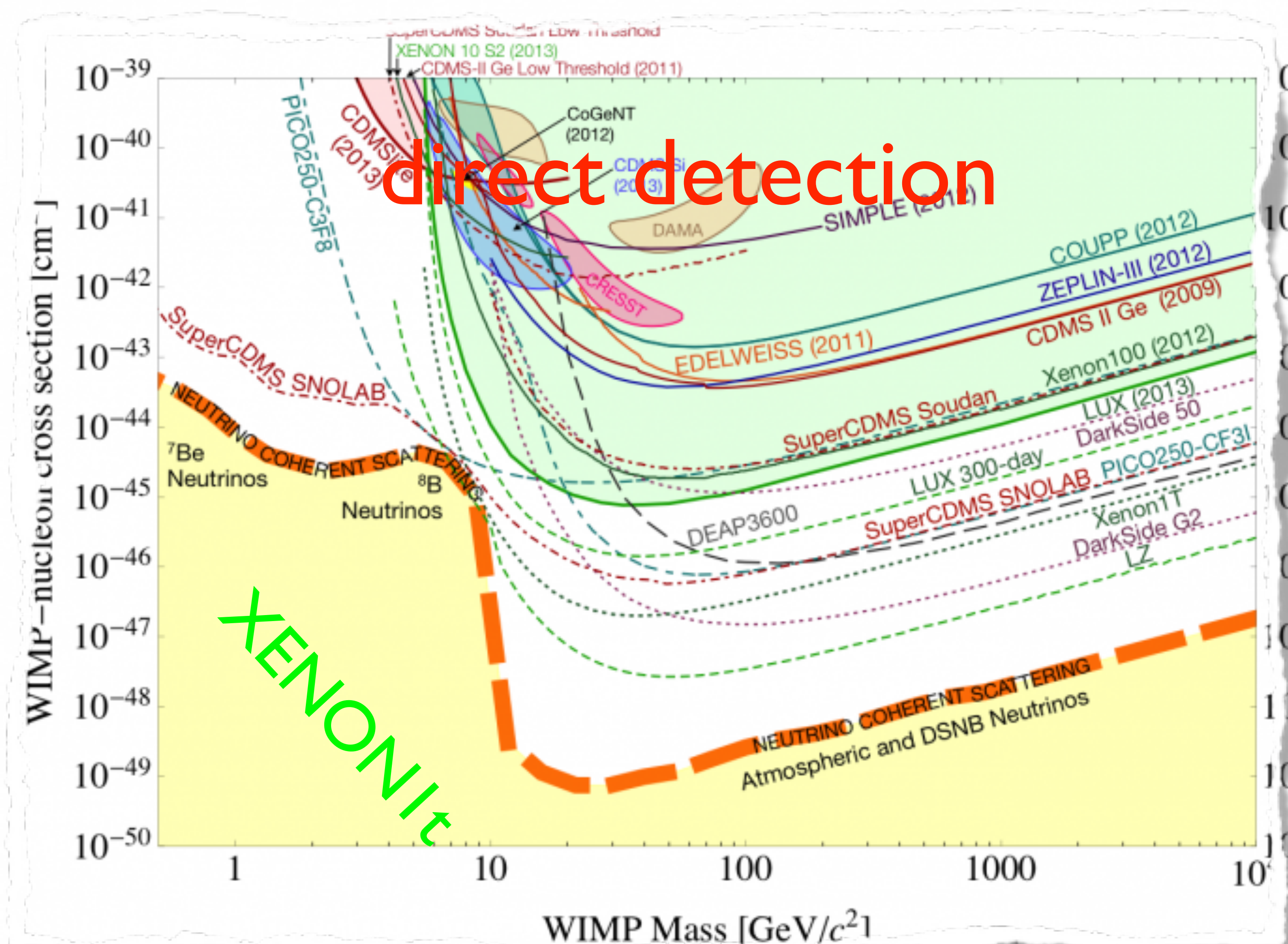
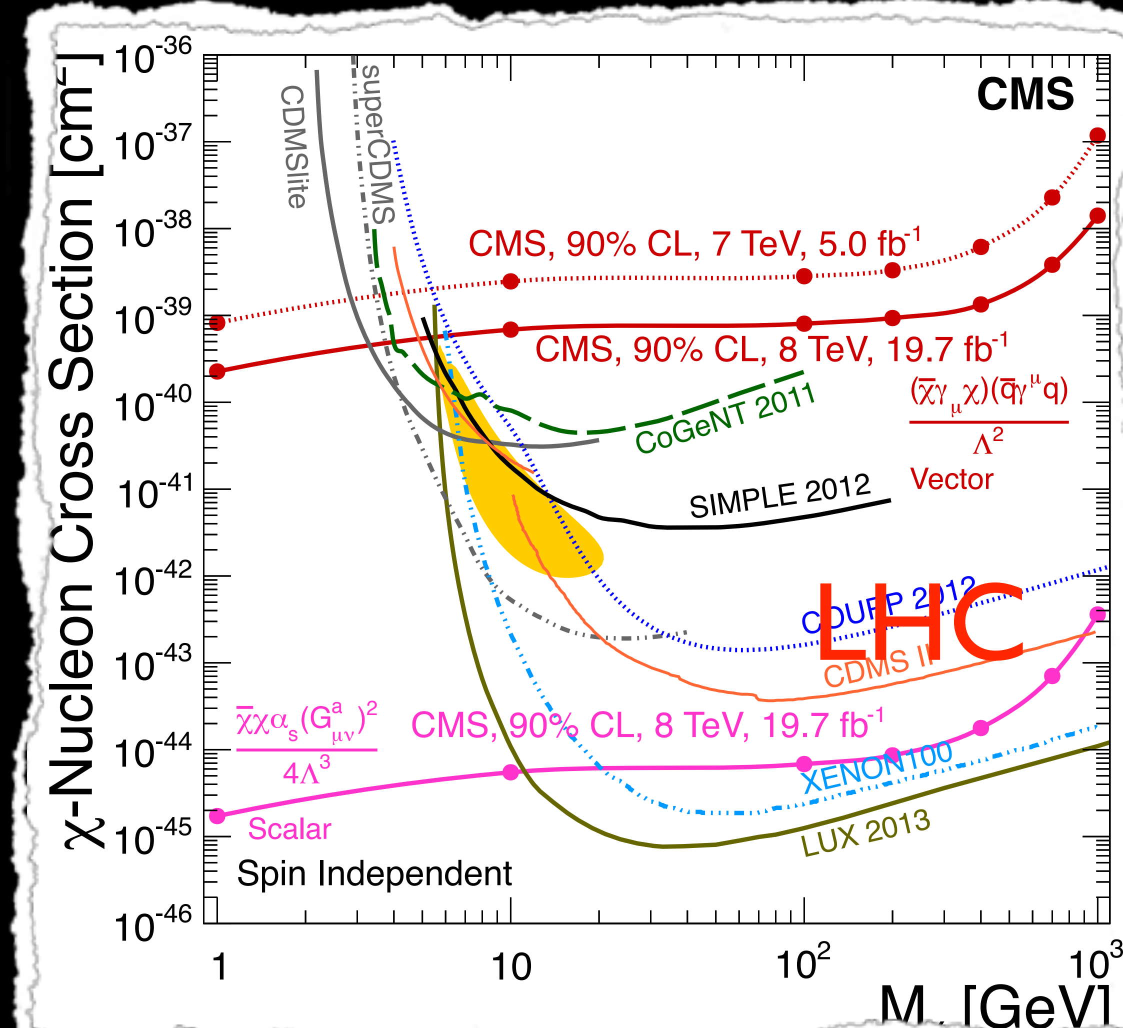
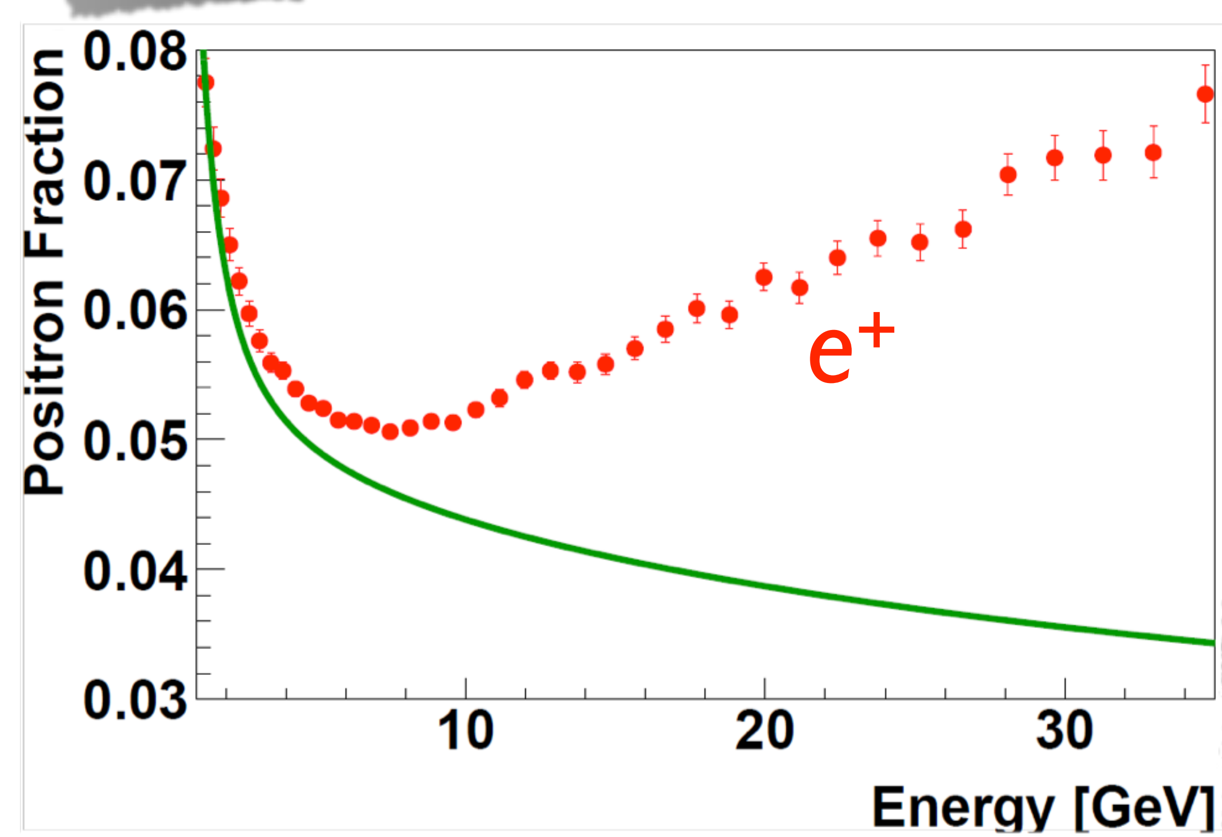
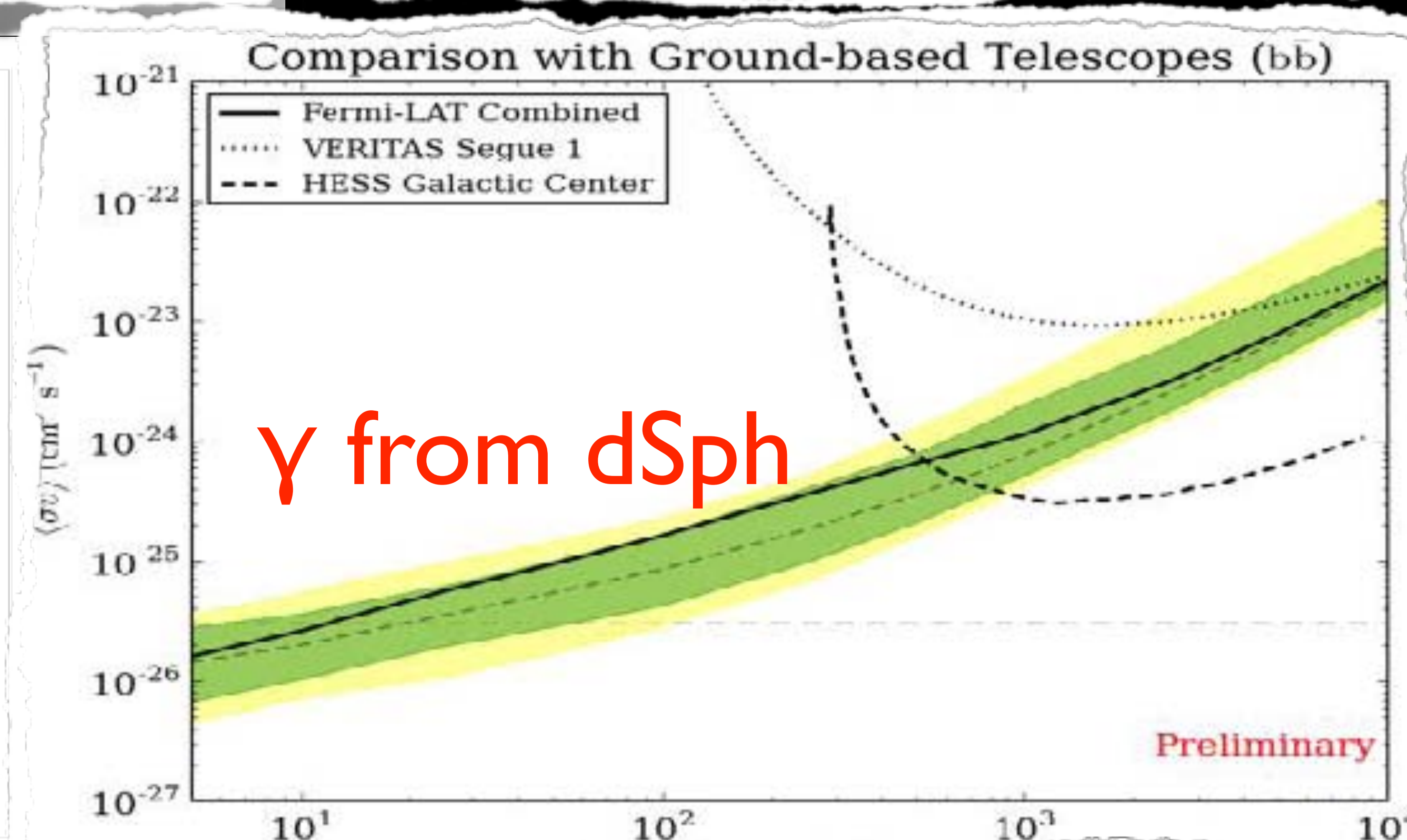
$$\frac{n_{\rm DM}}{s} = 4.4 \times 10^{-10} \frac{\text{GeV}}{m_{\rm DM}}$$

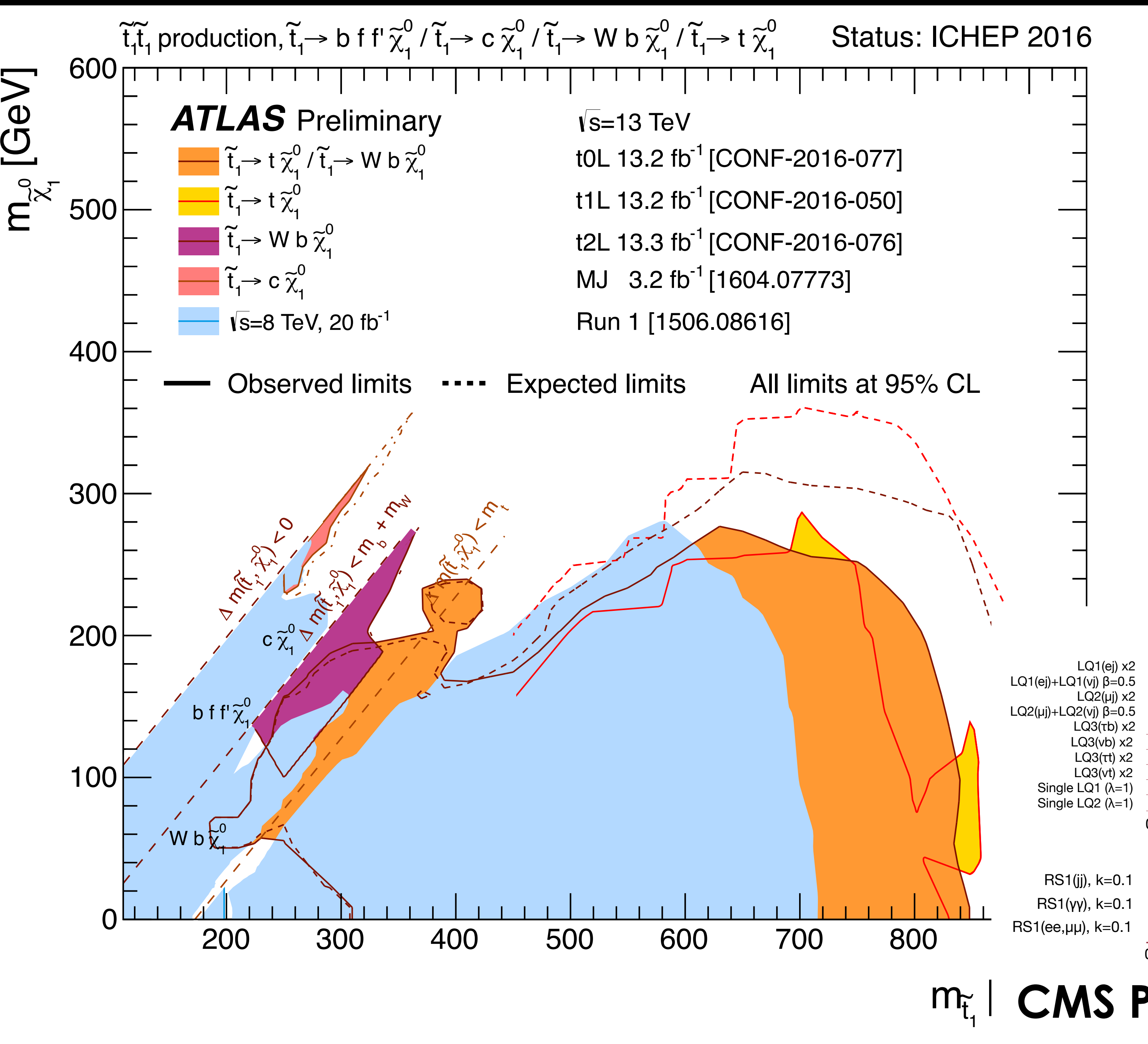
"weak" coupling
"weak" mass scale → correct abundance

Miracle^2

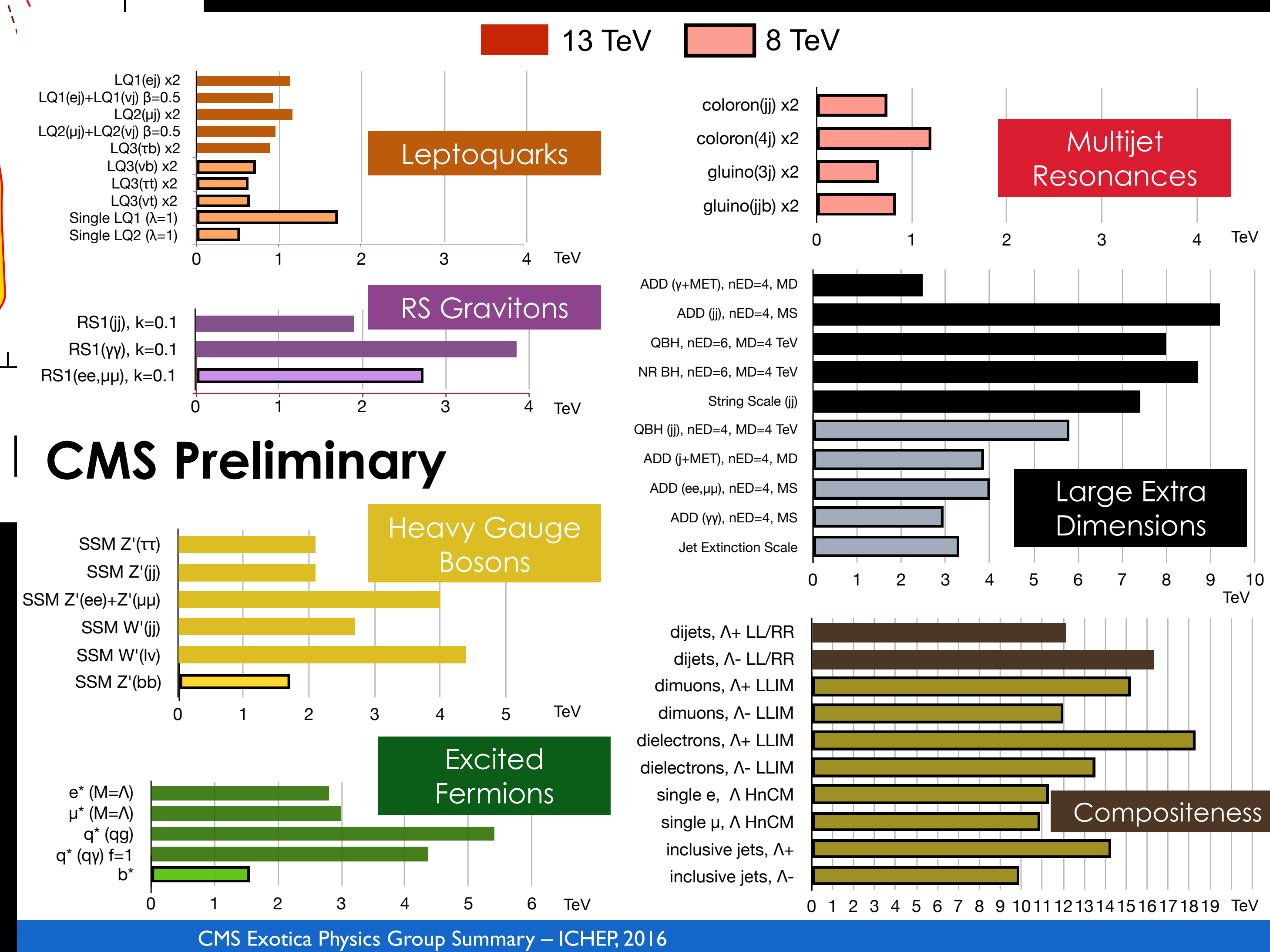
sociology

- We used to think
  - need to solve problems with the SM
  - hierarchy problem, strong CP, etc
  - it is great if a solution also gives dark matter candidate as an *option*
  - big ideas: supersymmetry, extra dim
  - probably because dark matter problem was not so established in 80's


direct detection
XENON1t
XENON 10 S2 (2013)
CDMS-II Ge Low Threshold (2011)
CoGeNT (2012)
DAMA
SIMPLE (2012)
COUPP (2012)
ZEPLIN-III (2012)
CDMS II Ge (2009)
EDELWEISS (2011)
Xenon100 (2012)
LUX (2013)
DarkSide 50
SuperCDMS Soudan
LUX 300-day
PICO250-CF3I
SuperCDMS SNOLAB
DEAP3600
Xenon1T
DarkSide G2
LZ
PICO250-C3F8
CRESST
SuperCDMS SNOLAB
NEUTRINO COHERENT SCATTERING
7Be Neutrinos
8B Neutrinos
Atmospheric and DSNB Neutrinos
WIMP–nucleon cross section [cm²]
WIMP Mass [GeV/c²]

LHC
CMS
CDMSlite
superCDMS
CMS, 90% CL, 7 TeV, 5.0 fb-1
CMS, 90% CL, 8 TeV, 19.7 fb-1
Vector
CoGeNT 2011
SIMPLE 2012
COUPP 2012
CDMS II
CMS, 90% CL, 8 TeV, 19.7 fb-1
XENON100
LUX 2013
Scalar
Spin Independent
χ-Nucleon Cross Section [cm²]
Mχ [GeV]

e+
Positron Fraction
Energy [GeV]

γ from dSph
Comparison with Ground-based Telescopes (bb)
Fermi-LAT Combined
VERITAS Segue 1
HESS Galactic Center
Preliminary

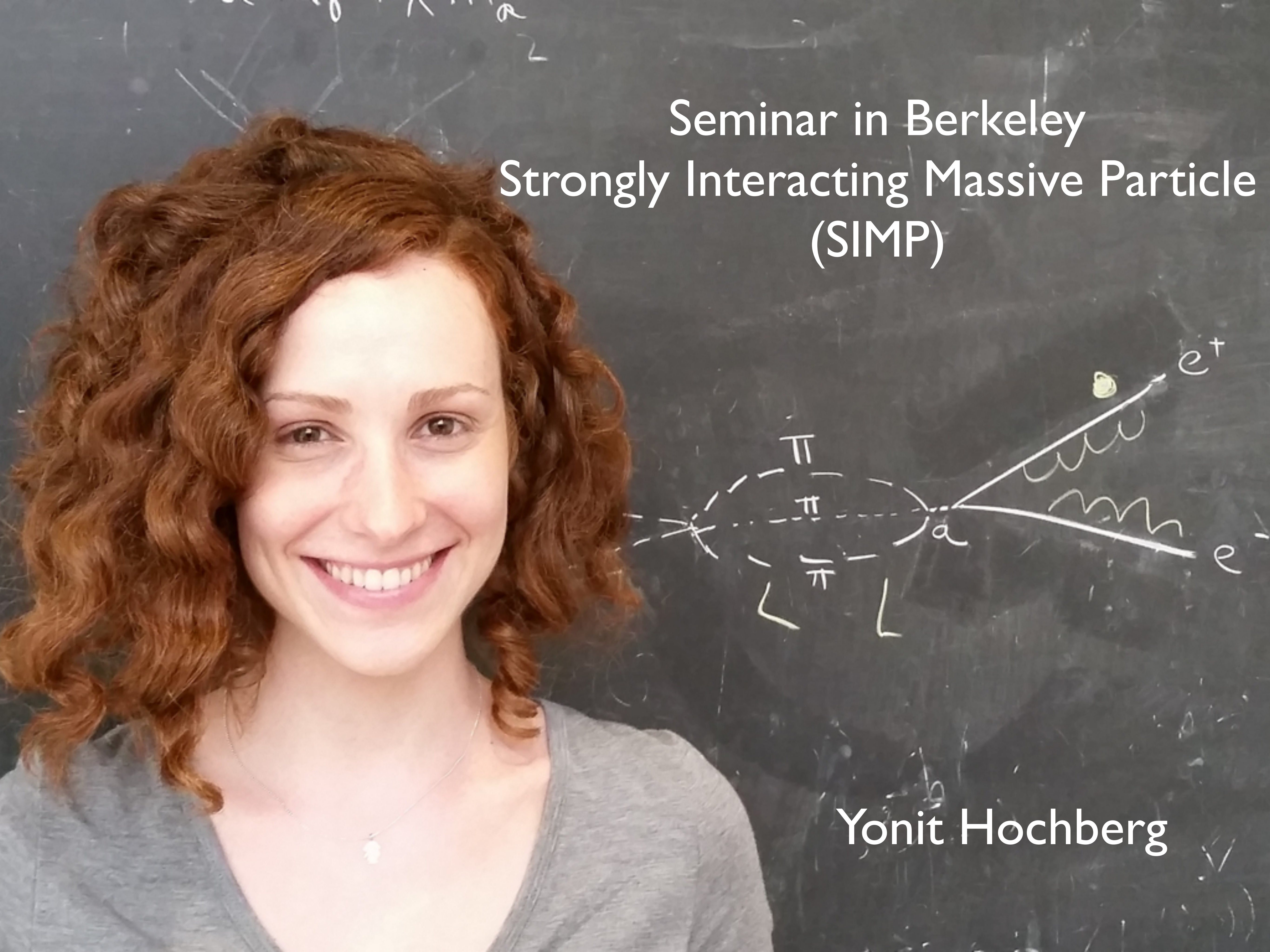


no sign of
new physics
that explains
naturalness!

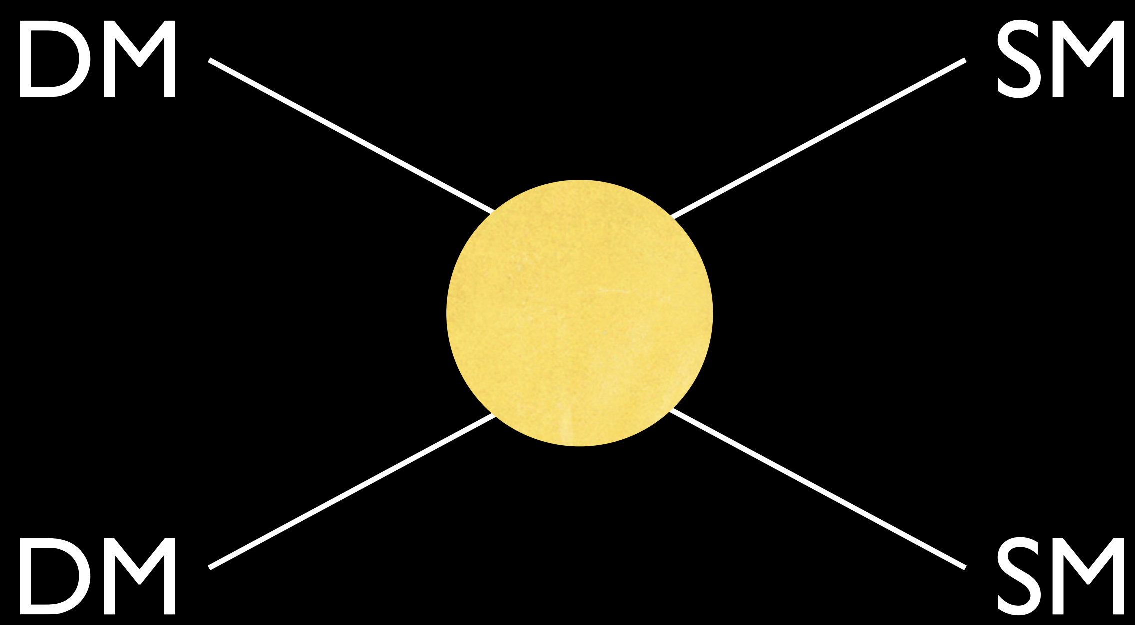


recent thinking

- dark matter definitely exists
  - naturalness problem may be optional?
- need to explain dark matter on its own
- perhaps we should decouple these two
- do we really need big ideas like SUSY?
- perhaps we can solve it with ideas more familiar to us?


Seminar in Berkeley
Strongly Interacting Massive Particle
(SIMP)
Yonit Hochberg

Miracles



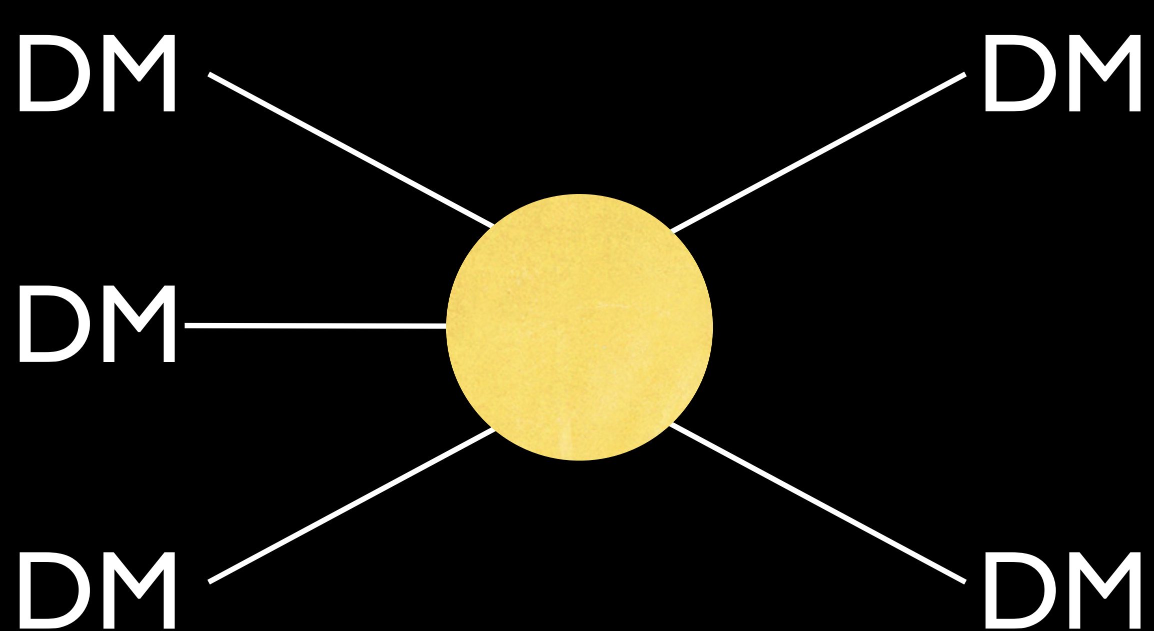
$$\langle \sigma_{2\to 2} v \rangle \approx \frac{\alpha^2}{m^2}$$

$$\alpha \approx 10^{-2}$$

$$m \approx 300\ \mathrm{GeV}$$

WIMP miracle!

$$\frac{n_{\mathrm{DM}}}{s} = 4.4 \times 10^{-10} \frac{\mathrm{GeV}}{m_{\mathrm{DM}}}$$



$$\langle \sigma_{3\to 2} v^2 \rangle \approx \frac{\alpha^3}{m^5}$$

$$\alpha \approx 4\pi$$

$$m \approx 300 \mathrm{MeV}$$

SIMP miracle!

Hochberg, Kuflik, Volansky, Wacker arXiv:1402.5143

~~LEE WEINBERG~~ 3→2 FREEZE-OUT

Back of the envelope calculation

$$\Gamma_{\rm ann} \simeq H|_{\rm freezeout}$$

$$\Gamma_{\rm ann} \simeq n_{\rm dm}^2 \langle \sigma v^2 \rangle_{3\to 2}$$

$$H \simeq \frac{T^2}{M_{\rm pl}}$$

$$m_{\rm dm} n_{\rm dm} \sim m_p n_b$$

$$\langle \sigma v^2 \rangle_{3\to 2} \simeq \frac{\alpha^3}{m_{\rm dm}^2}$$

$$n_b \sim \eta_b s$$

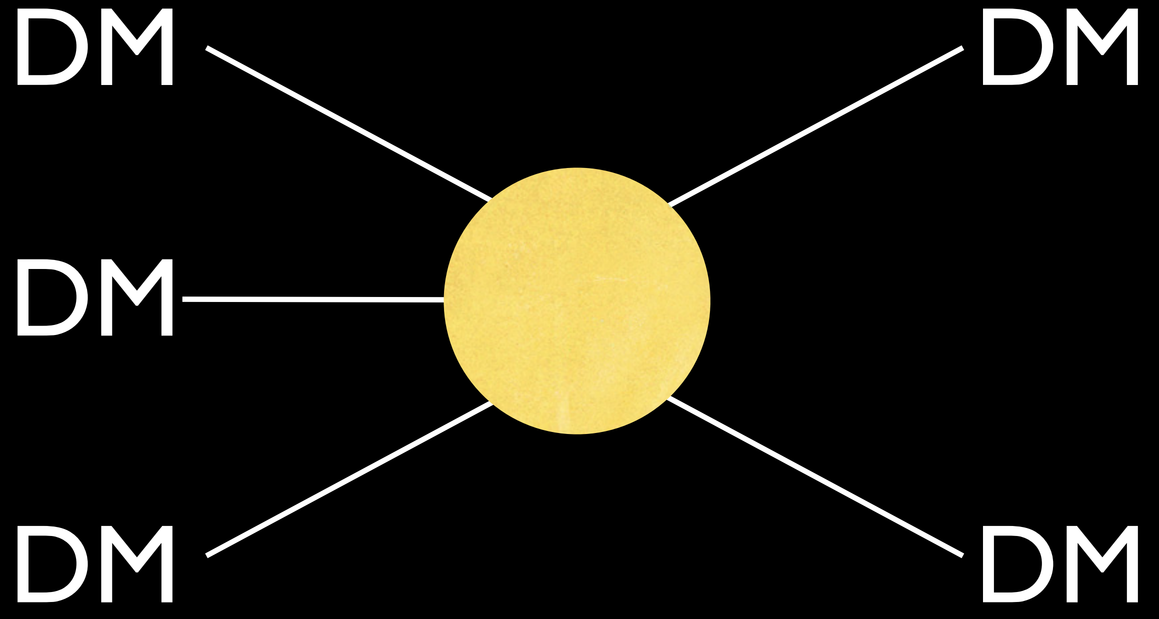
$$\eta_b \simeq T_{\rm eq}/m_p \qquad s \simeq T^3$$

$$\Gamma_{\rm ann} \simeq \frac{T_{\rm eq}^2 \alpha^3}{x_F^4 \times m_{\rm dm}} \quad = \quad H \simeq \frac{m_{\rm dm}^2}{M_{\rm pl} x_F^2}$$

Eric Kuflik

SIMPlest Miracle

- Not only the mass scale is similar to QCD
- dynamics itself can be QCD! Miracle[3]
- DM = pions
- e.g. SU(4)/Sp(4) = S^5



$$\mathcal{L}_{\rm chiral} = \frac{1}{16 f_\pi^2} {\rm Tr} \partial^\mu U^\dagger \partial_\mu U$$

Phys.Rev.Lett. 115 (2015) 021301

$$\mathcal{L}_{\rm WZW} = \frac{8N_c}{15\pi^2 f_\pi^5} \epsilon_{abcde} \epsilon^{\mu\nu\rho\sigma} \pi^a \partial_\mu \pi^b \partial_\nu \pi^c \partial_\rho \pi^d \partial_\sigma \pi^e + O(\pi^7)$$

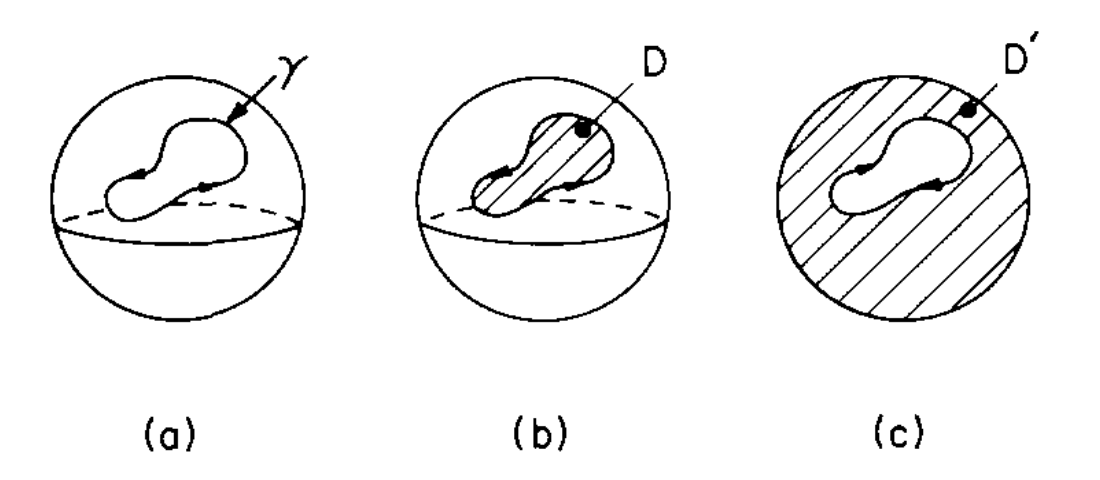
$$\pi_5(G/H) \neq 0$$

SIMPlest Miracle

- SU(2) gauge theory with four doublets
- SU(4)=SO(6) flavor symmetry
- $\langle q^i q^j \rangle \neq 0$ breaks it to Sp(2)=SO(5)
- coset space SO(6)/SO(5)=S^5
- $\pi_5(S^5)=\mathbb{Z} \Rightarrow$ Wess-Zumino term
- $\mathcal{L}_{WZ}=\varepsilon_{abcde}\, \varepsilon^{\mu\nu\rho\sigma}\, \pi^a \partial_\mu \pi^b \partial_\nu \pi^c \partial_\rho \pi^d \partial_\sigma \pi^e$

Wess-Zumino term

- SU(N_c) gauge theory
    - $\pi_5(SU(N_f))=\mathbb{Z}$ ($N_f \geq 3$)
- Sp(N_c) gauge theory
    - $\pi_5(SU(2N_f)/Sp(N_f))=\mathbb{Z}$ ($N_f \geq 2$)
- SO(N_c) gauge theory
    - $\pi_5(SU(N_f)/SO(N_f))=\mathbb{Z}$ ($N_f \geq 3$)



Witten

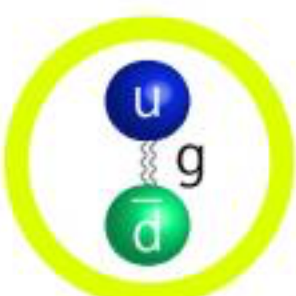
LAGRANGIANS

Quark theory

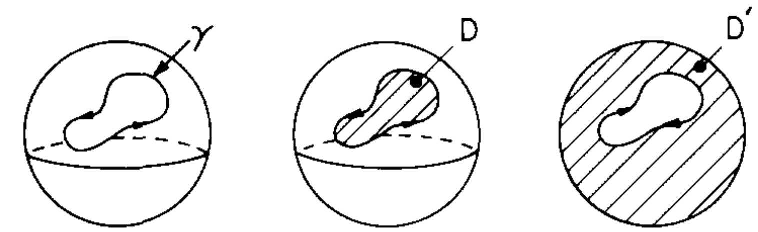
$$\mathcal{L}_{\rm quark} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \bar{q}_i i \not{D} q_i \; -\frac{1}{2}m_Q \; J^{ij} q_i q_j + h.c.$$

Sigma theory

$$\mathcal{L}_{\rm Sigma} = \frac{f_\pi^2}{16}{\rm Tr}\partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger - \frac{1}{2}m_Q\mu^3{\rm Tr}J\Sigma + h.c. - \frac{iN_c}{240\pi^2}\int {\rm Tr}(\Sigma^\dagger d\Sigma)^5$$



Pion theory

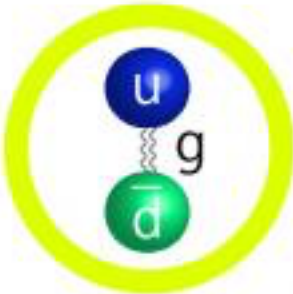


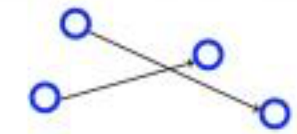
(a) (b) (c)

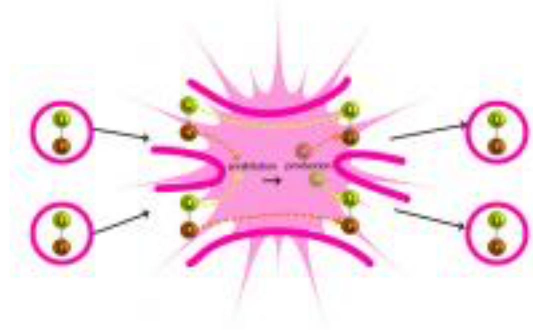
$$\mathcal{L}_{\rm pion} = \frac{1}{4}{\rm Tr}\partial_\mu\pi\partial^\mu\pi - \frac{m_\pi^2}{4}{\rm Tr}\pi^2 + \frac{m_\pi^2}{12 f_\pi^2}{\rm Tr}\pi^4 - \frac{1}{6f_\pi^2}{\rm Tr}\left(\pi^2\partial^\mu\pi\partial_\mu\pi - \pi\partial^\mu\pi\pi\partial_\mu\pi\right)$$

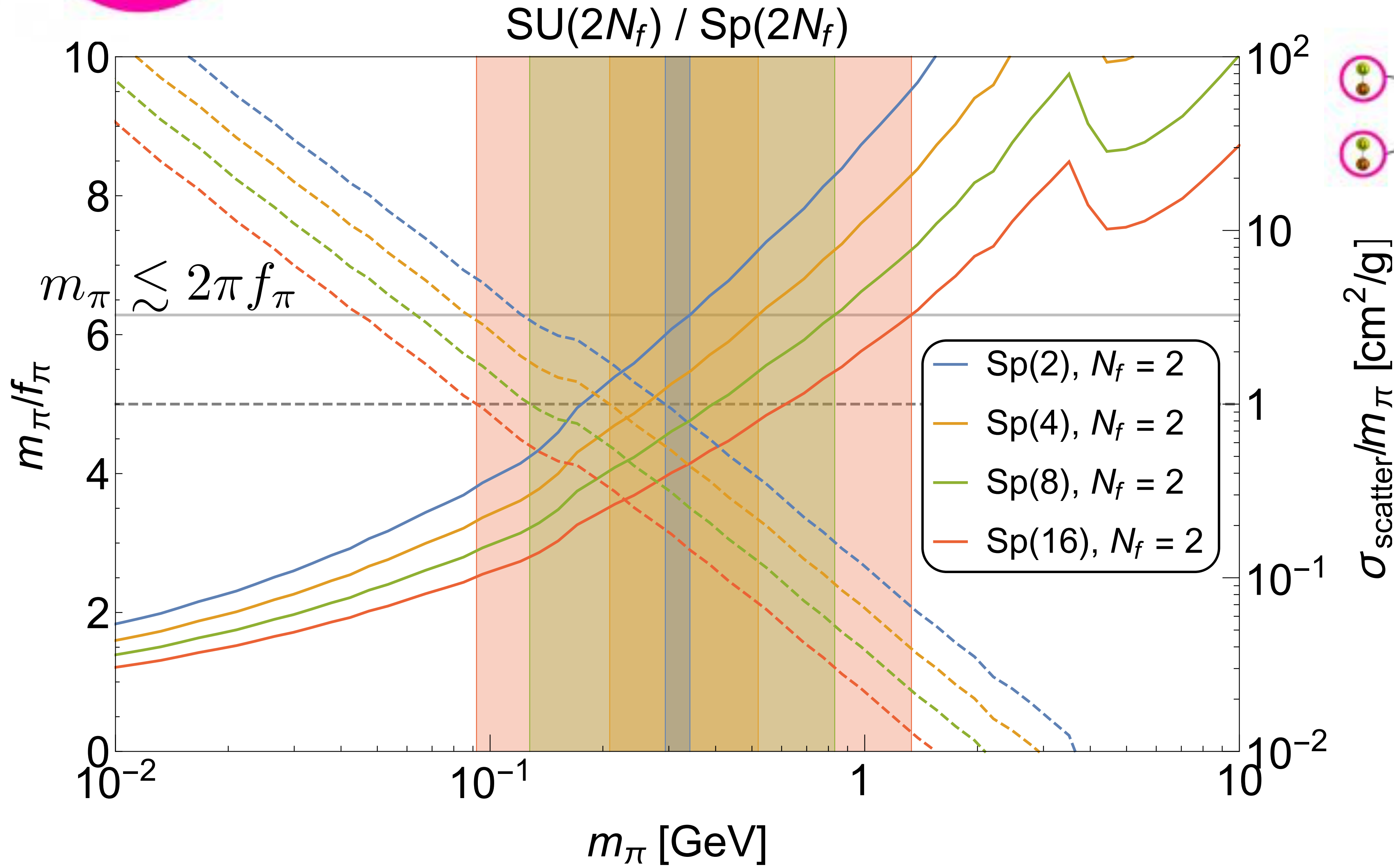
$$+ \frac{2N_c}{15\pi^2 f_\pi^5}\epsilon^{\mu\nu\rho\sigma}{\rm Tr}\left[\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi\right] + \mathcal{O}(\pi^6)$$

The Results









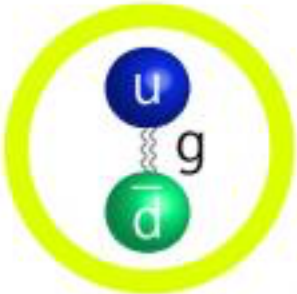
Solid curves: solution to Boltzmann eq.

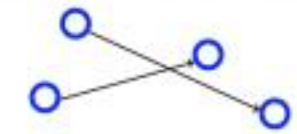
Dashed curves: along that solution

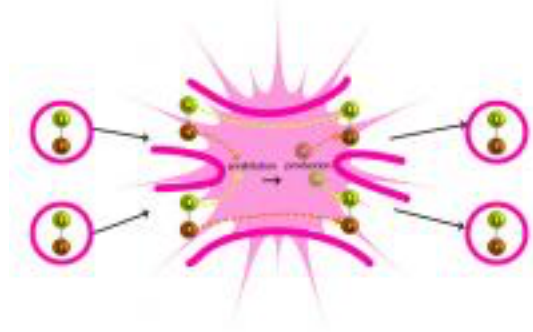
$$\frac{m_\pi}{f_\pi} \propto m_\pi^{3/10}$$

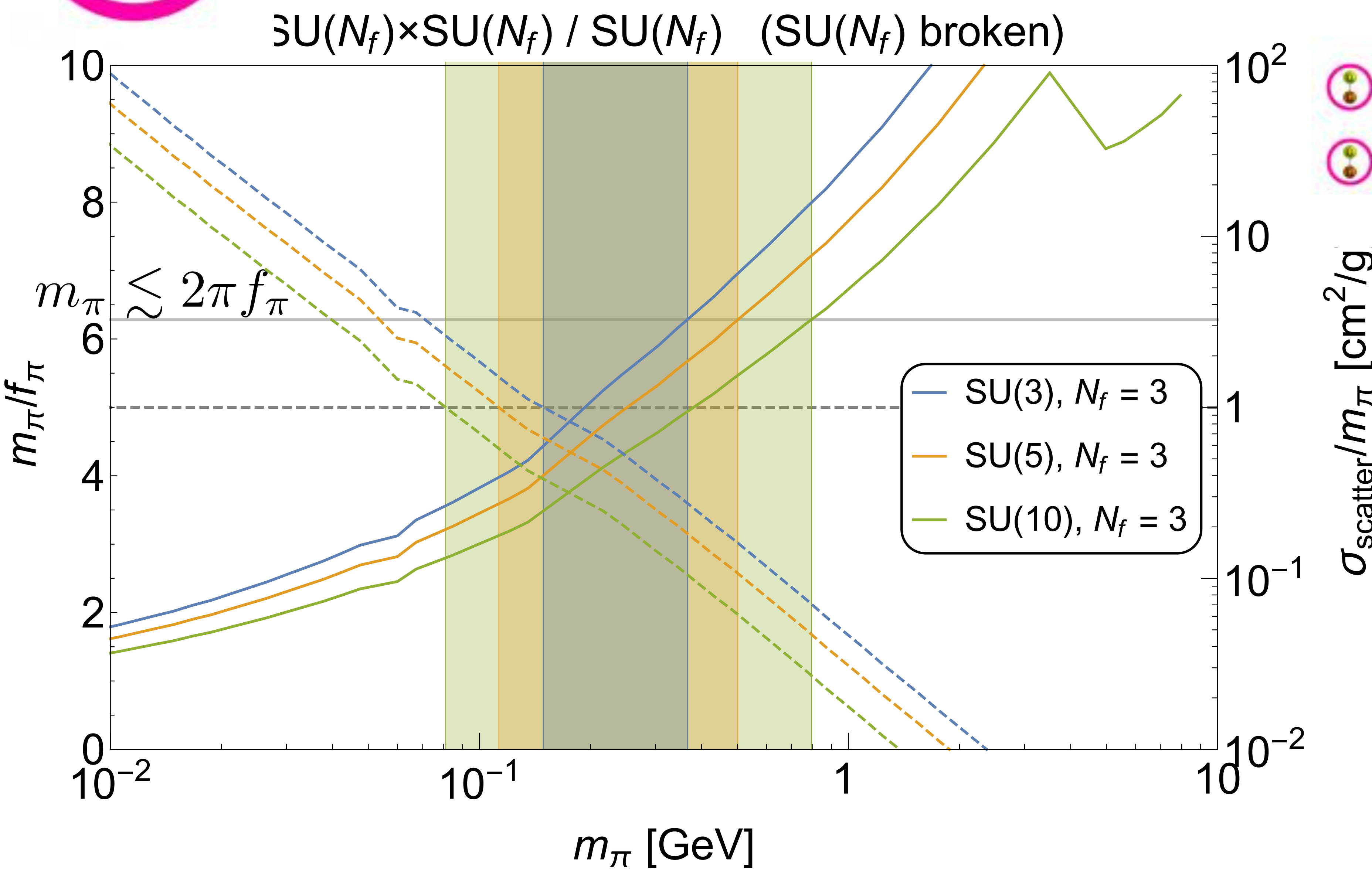
$$\frac{\sigma_{\text{scatter}}}{m_\pi} \propto m_\pi^{-9/5}$$

The Results









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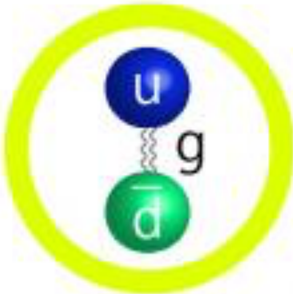

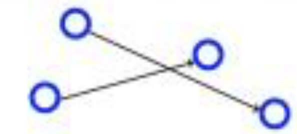

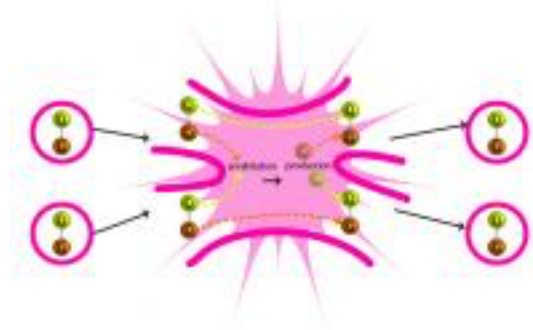

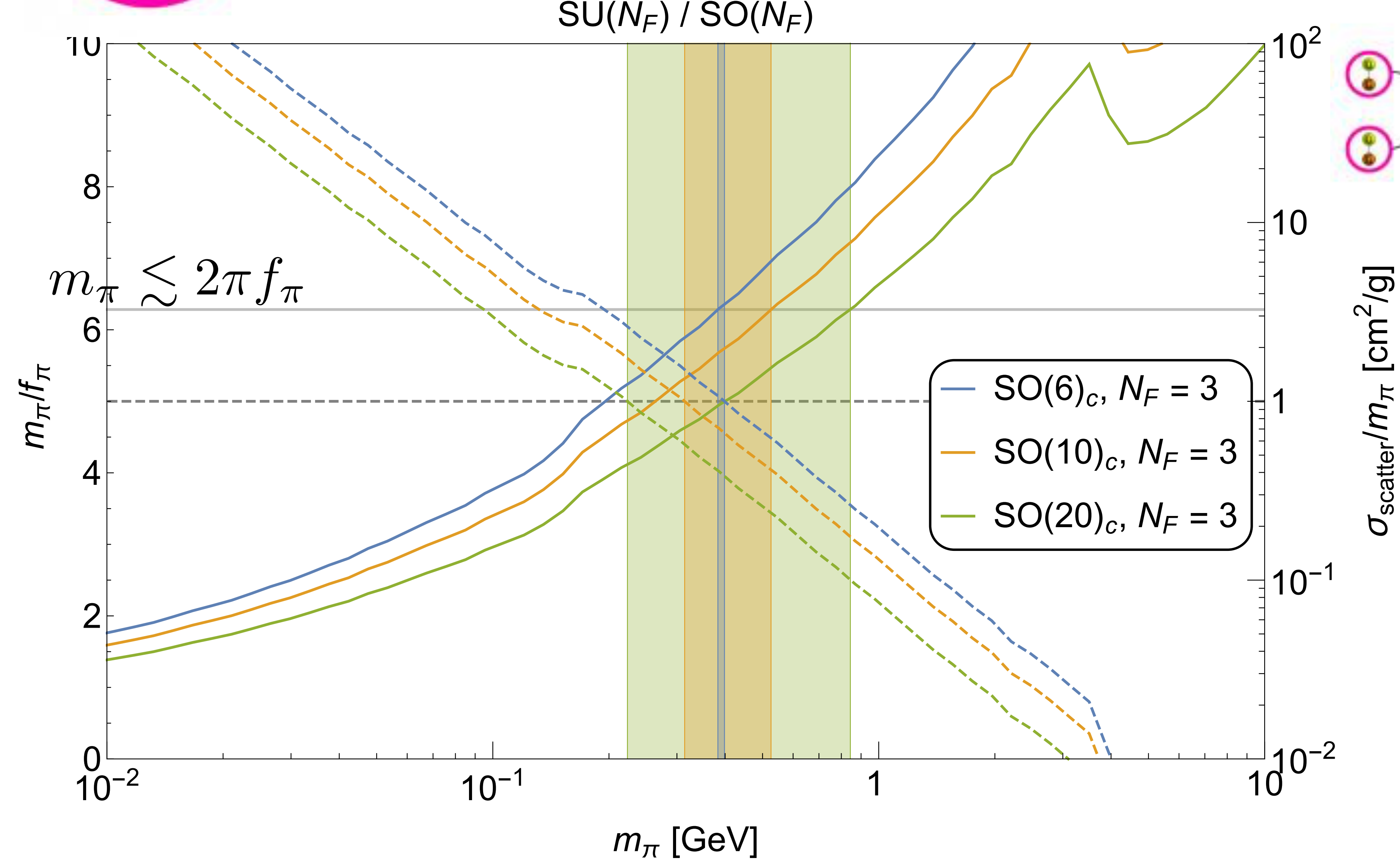

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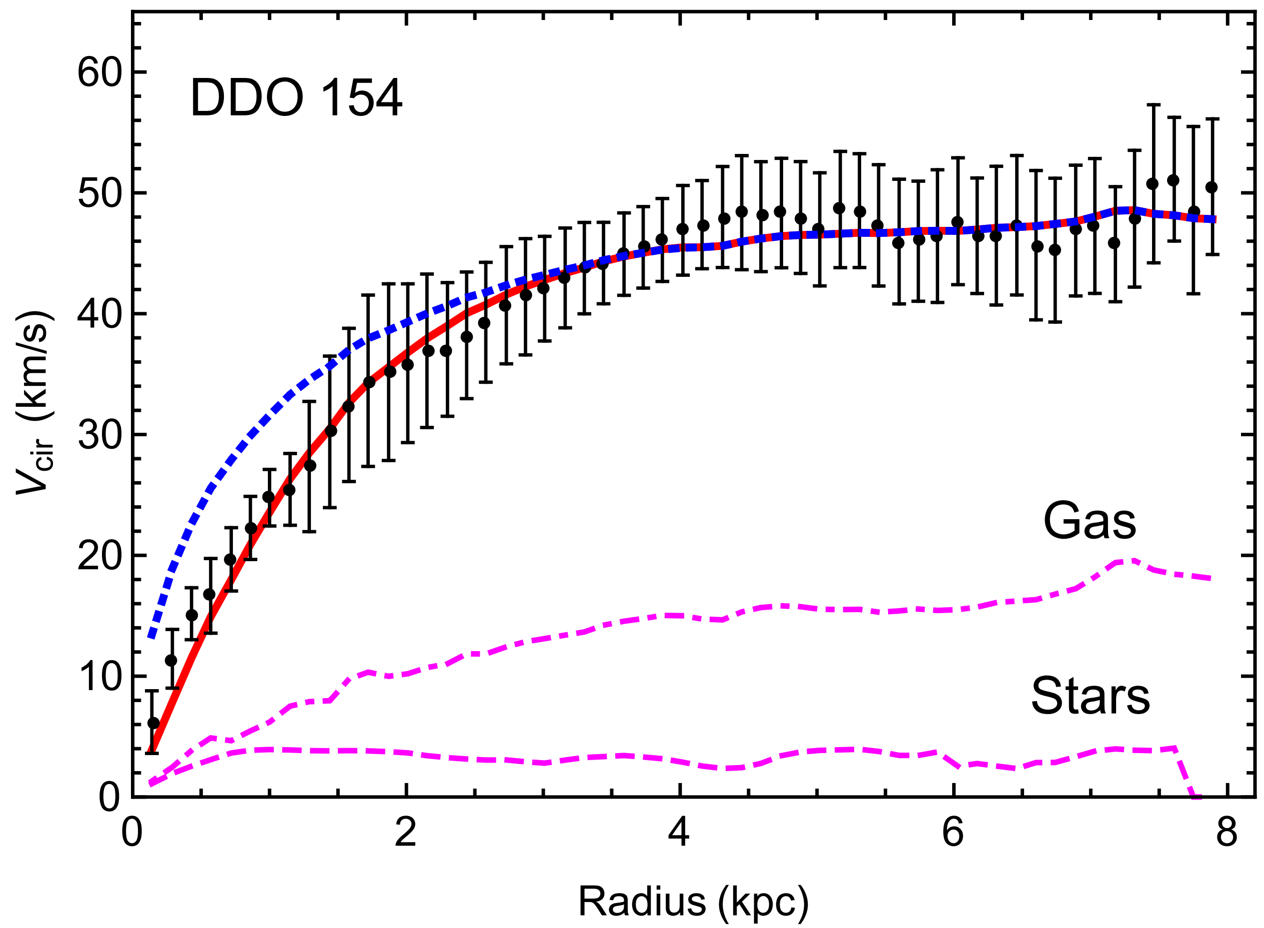
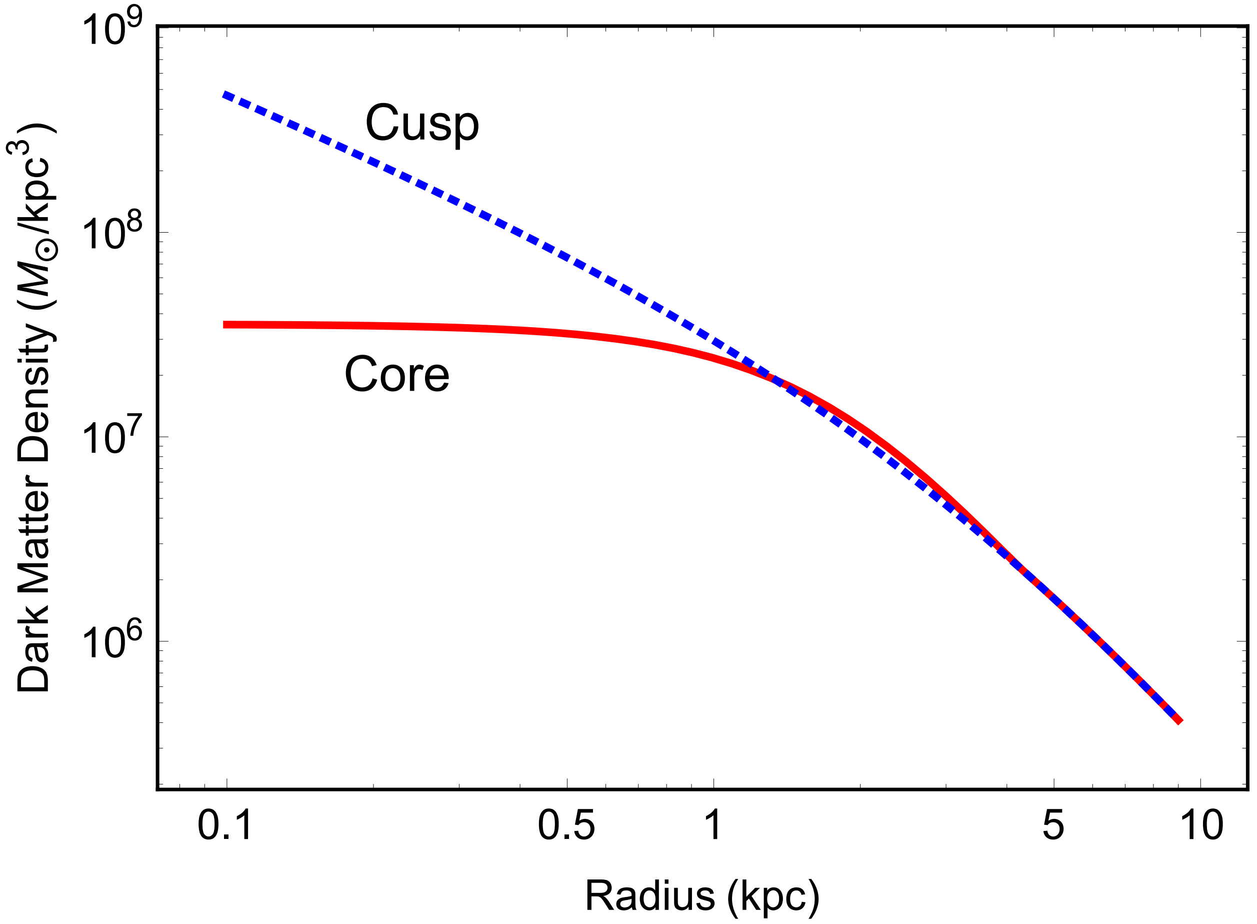
DDO 154 dwarf galaxy

DDO 154 dwarf galaxy



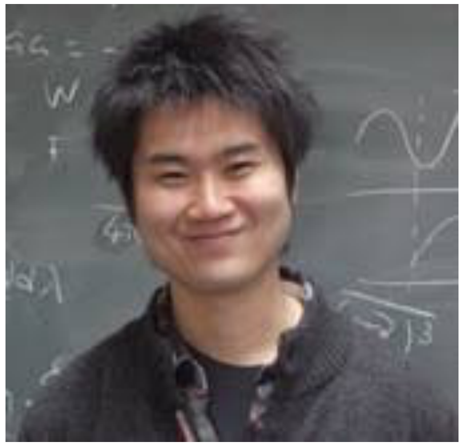


can be explained if dark matter scatters against itself

Need $\sigma/m \sim 1\text{b} / \text{GeV}$

only astrophysical information beyond gravity

Diversity in stellar distribution

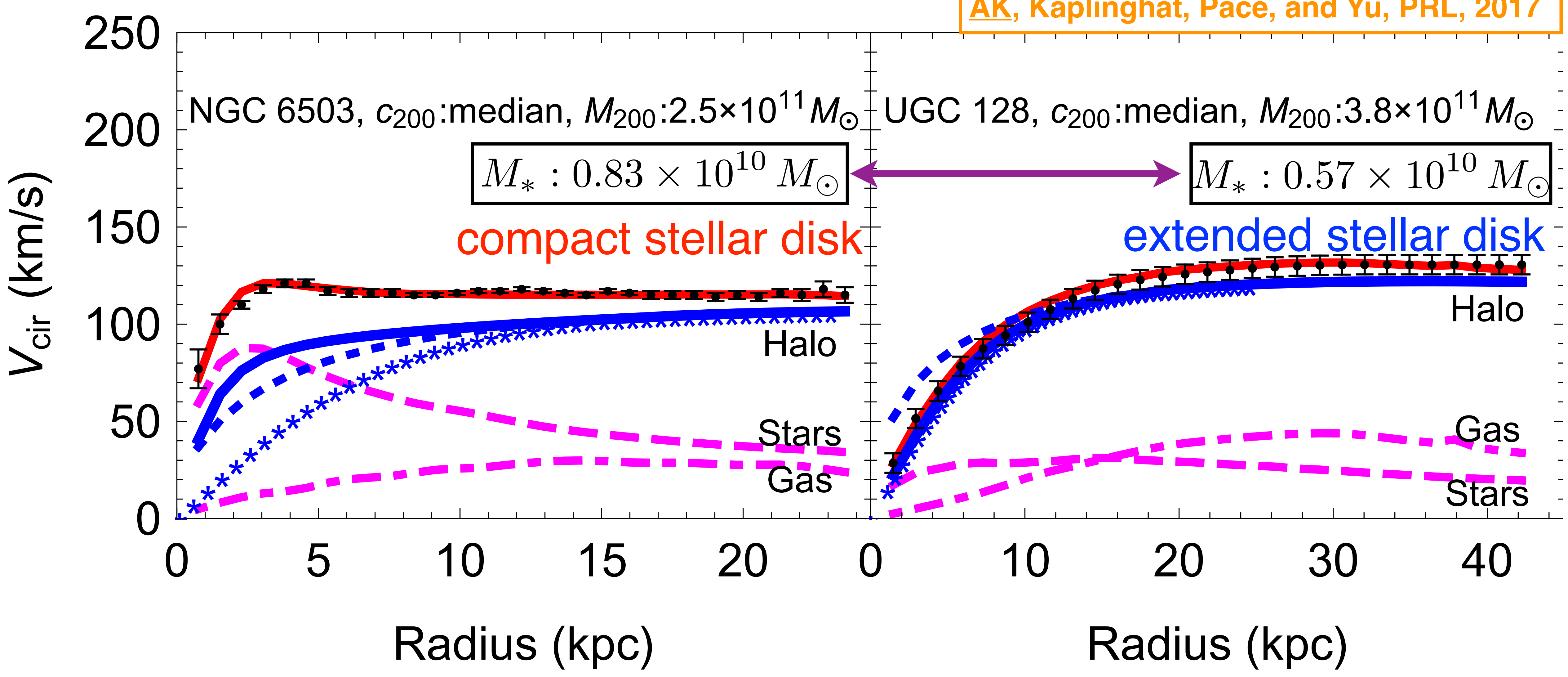


Ayuki Kamada

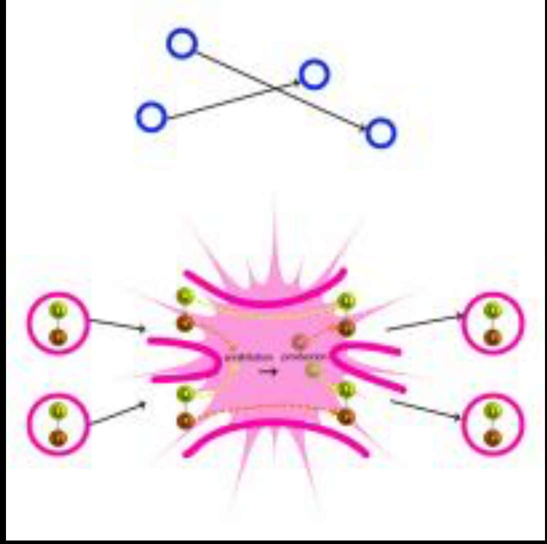
Similar outer circular velocity and stellar mass, but different stellar distribution

- compact → redistribute SIDM significantly
- extended → unchange SIDM distribution

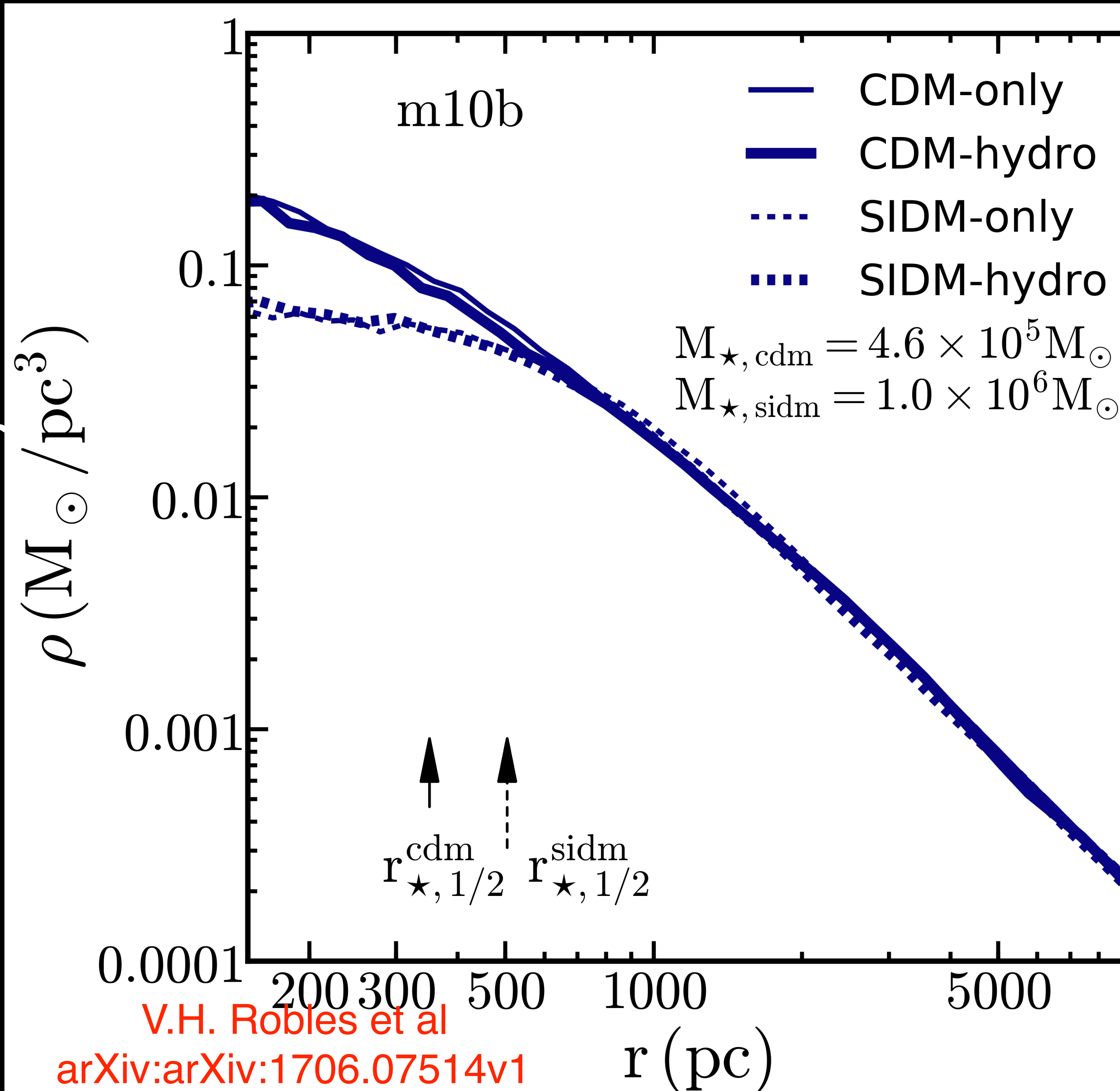
AK, Kaplinghat, Pace, and Yu, PRL, 2017



self interaction



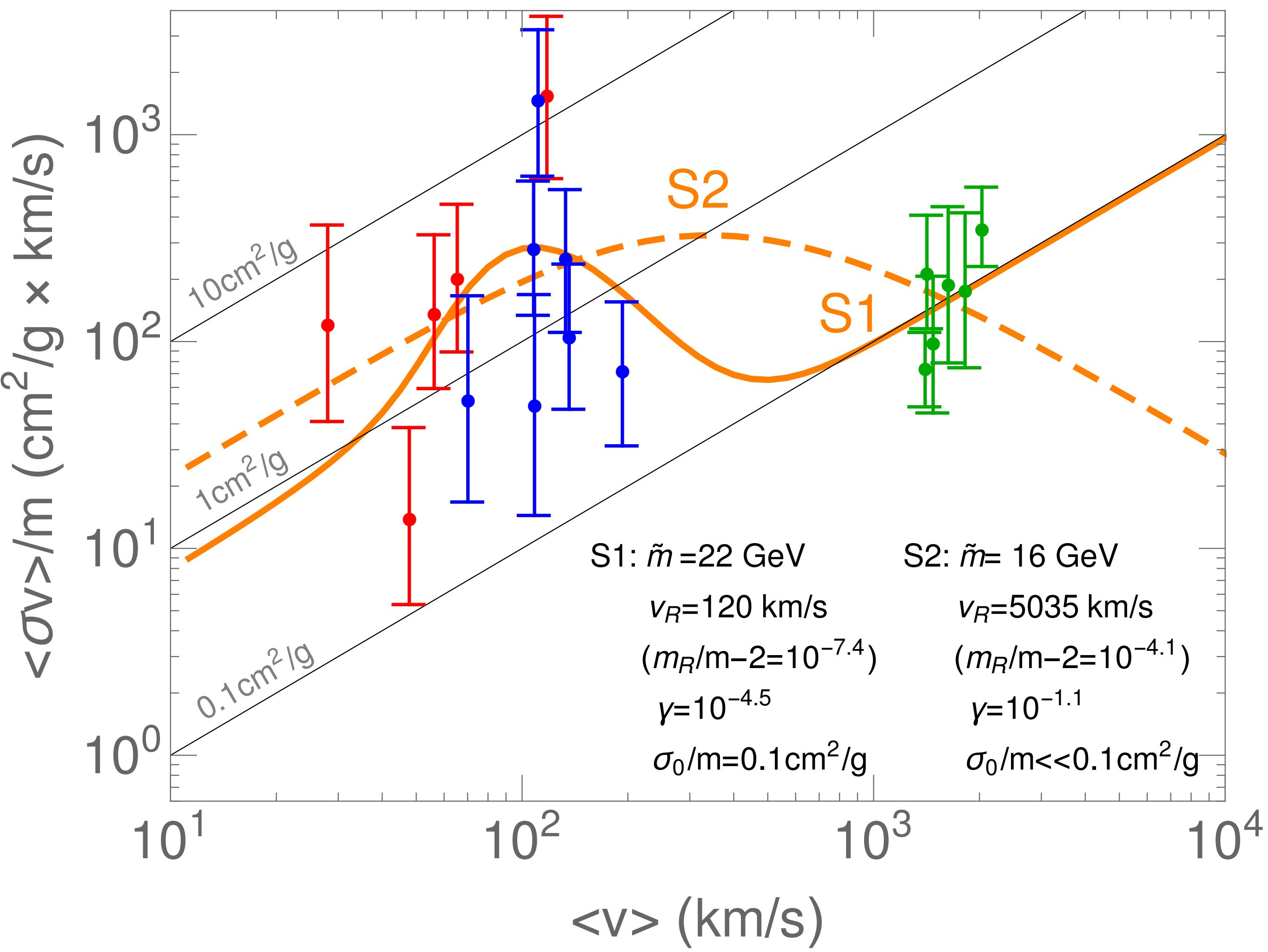
- σ/*m* ~ cm²/g
  ~10⁻²⁴cm² / 300MeV
- flattens the cusps in NFW profile
- suppresses substructur
- actually desirable for dwarf galaxies?

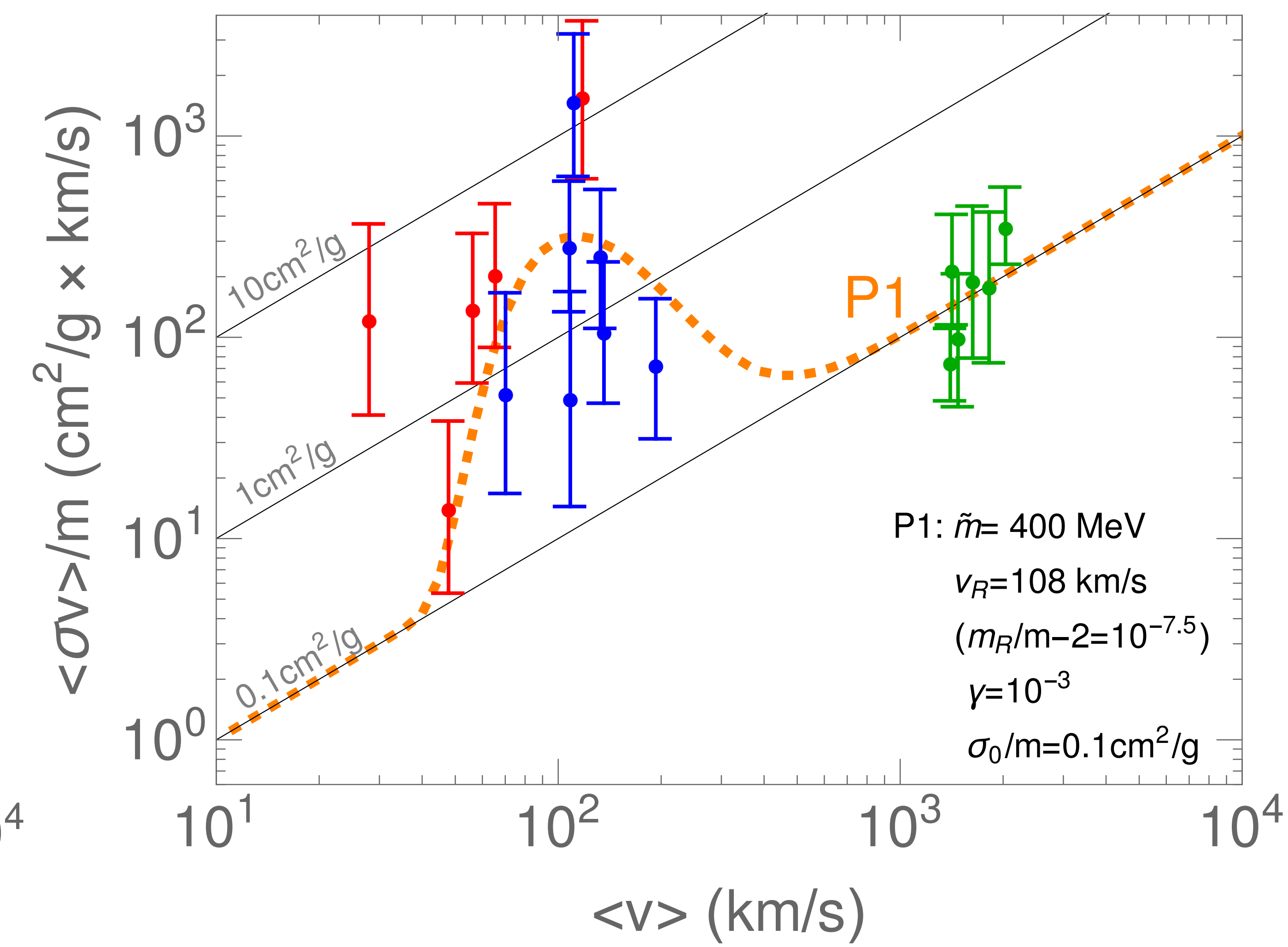


V.H. Robles et al
arXiv:arXiv:1706.07514v1

SIDM
Spergel & Steinhardt (2000)
now complete theory

Resonant scattering





Xiaoyong Chu, Camilo Garcia-Cely, HM,
Phys.Rev.Lett. 122 (2019) no.7, 071103

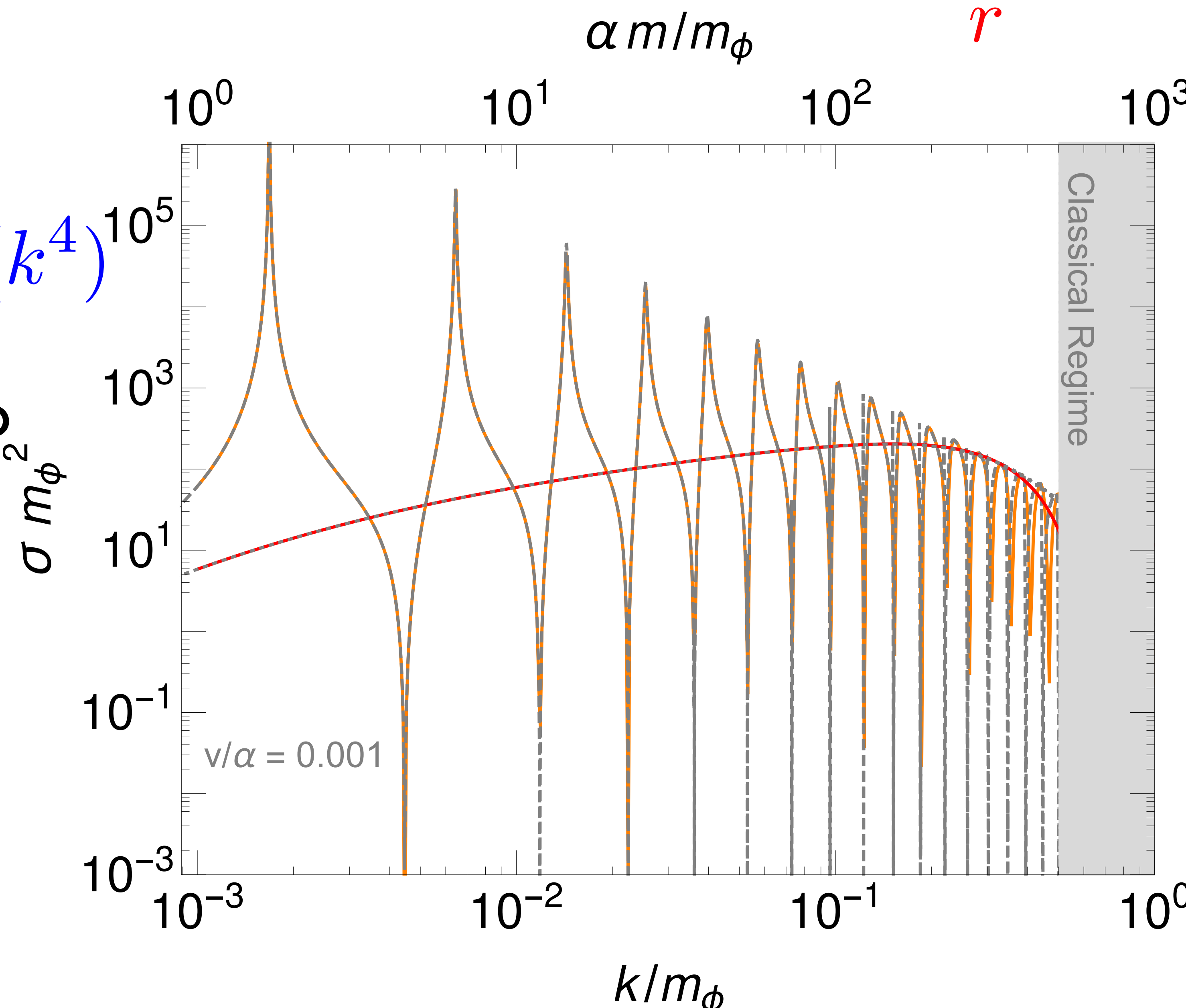
Unified description of SIDM

- Hans Bethe: effective range theory

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r_e k^2 + O(k^4)$$

- only two parameters to describe scattering at low velocities
- fully unitary and non-perturbative
- ideal for simulations!

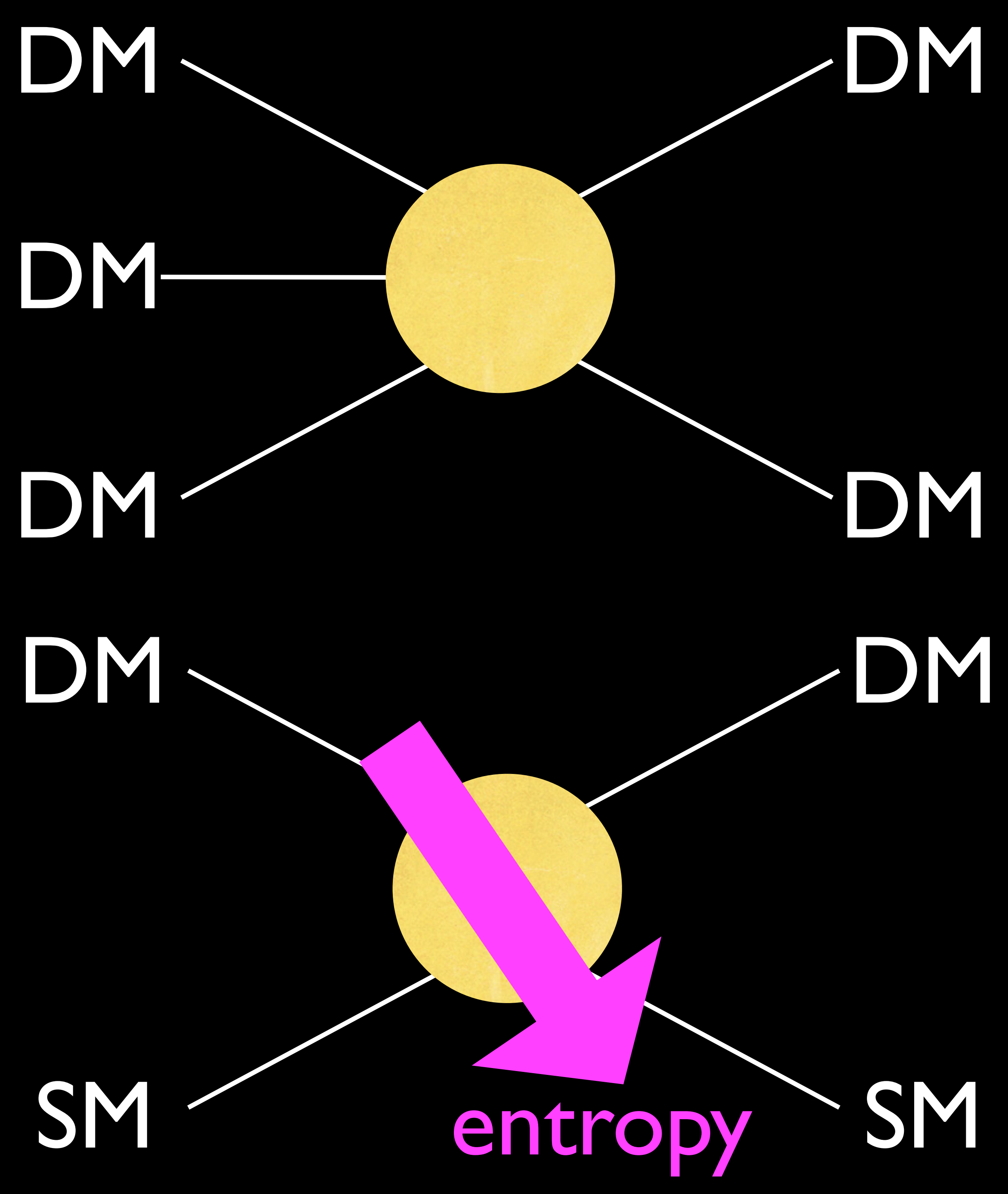
$$V = -\alpha \frac{e^{-m_\phi r}}{r}$$



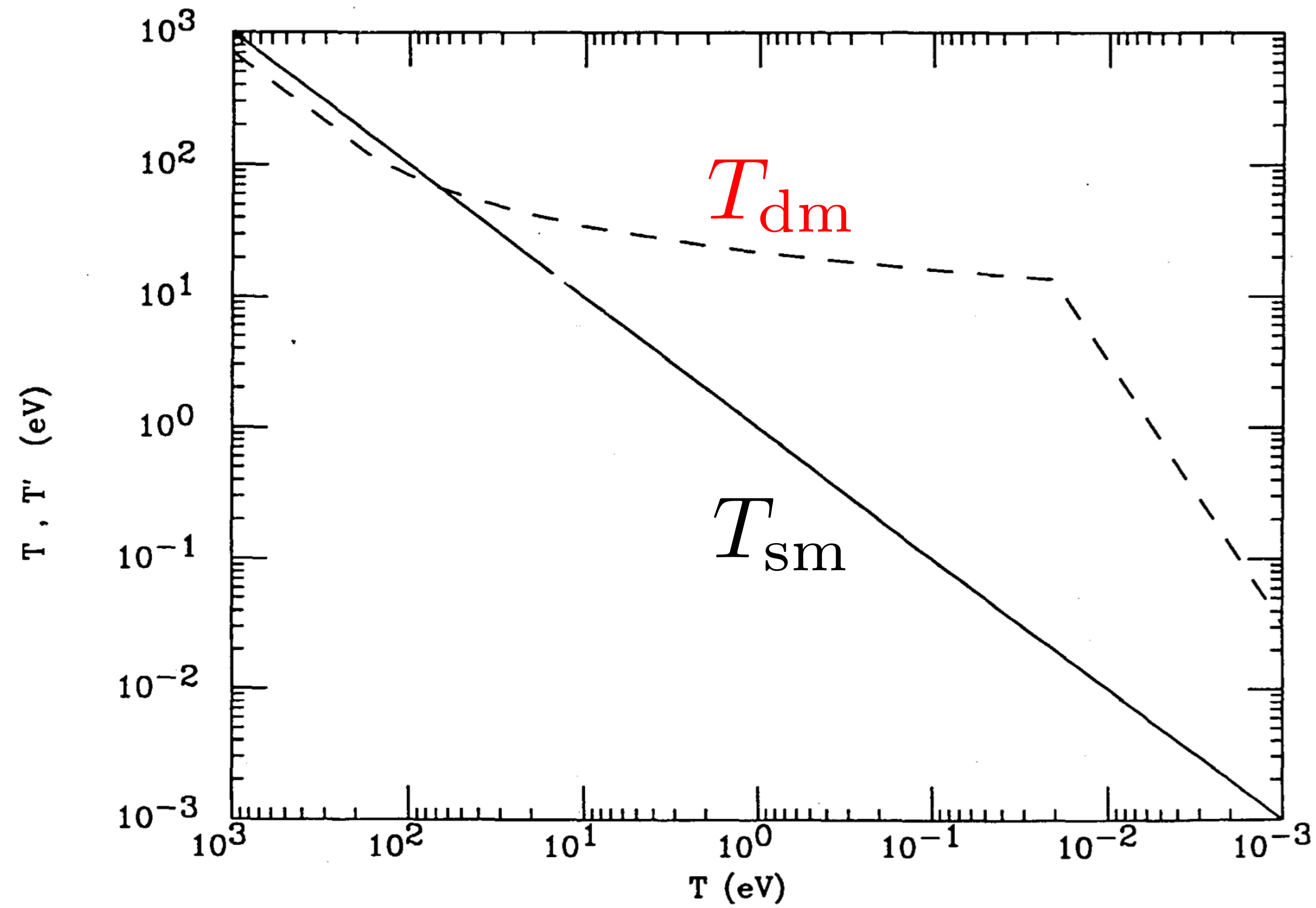
Xiaoyong Chu, Camilo Garcia-Cely, HM, in preparation

communication

- 3 to 2 annihilation
- excess entropy *must* be transferred to $e^{\pm}$, γ
- need communication at some level
- leads to experimental signal



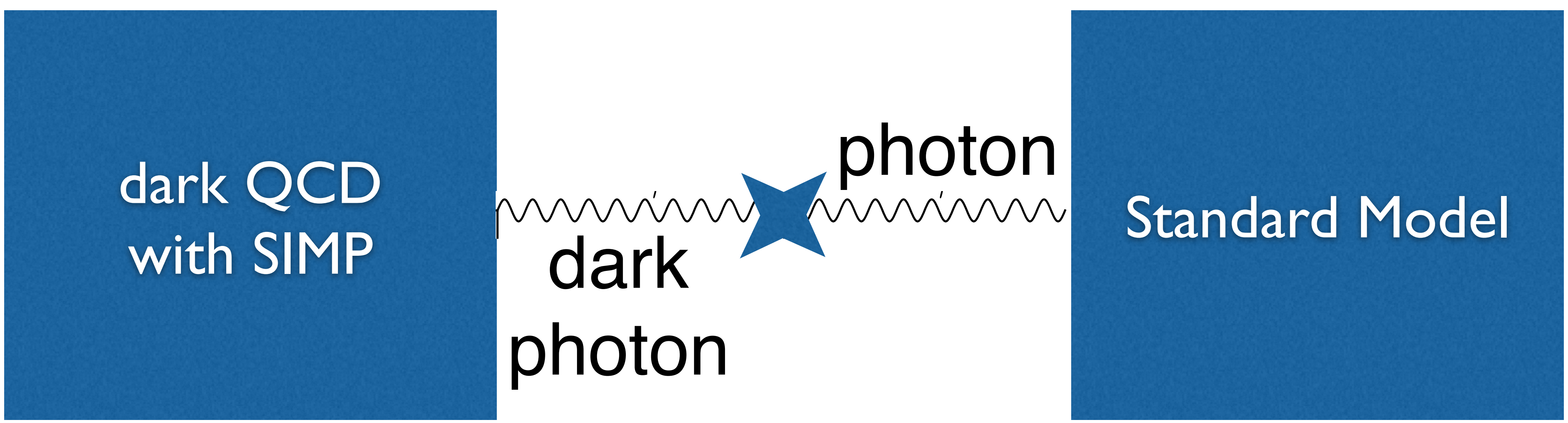
if totally decoupled



Carlson, Hall and Machacek, Astrophys. J. 398, 43 (1992)

- 3→2 annihilations without heat exchange is excluded by structure formation, [de Laix, Scherrer and Schaefer, Astrophys. J. 452, 495 (1995)]

vector portal



$$\frac{\epsilon_\gamma}{2c_W} B_{\mu\nu} F_D^{\mu\nu}$$

Kinetically mixed U(1)

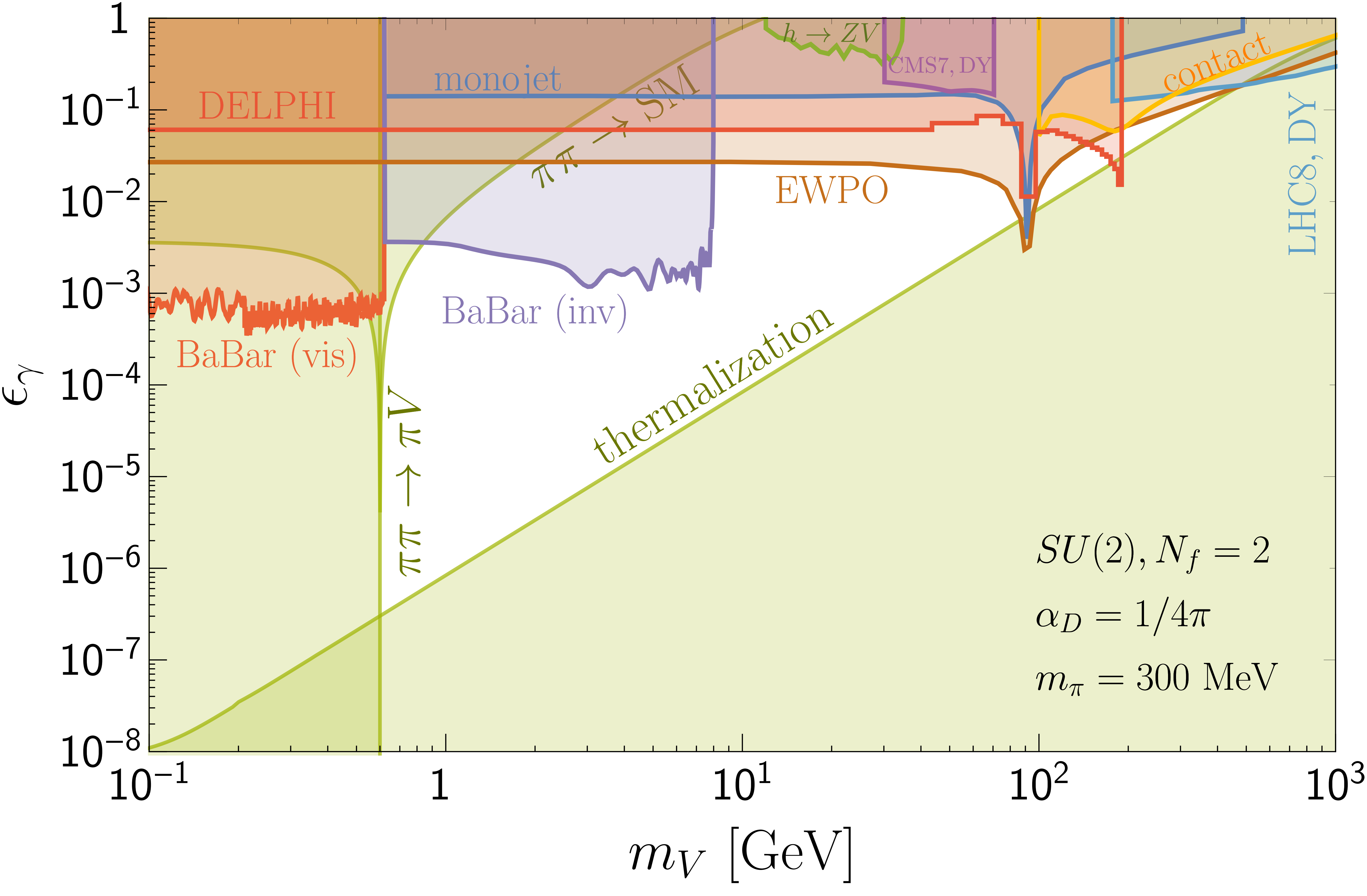
- e.g., the SIMPlest model SU(2) gauge group with N_f=2 (4 doublets)
- gauge U(1)=SO(2) ⊂ SO(2) ×SO(3) ⊂ SO(5)=Sp(4)
- maintains degeneracy of quarks
- near degeneracy of pions for co-annihilation

$$SU(4)/Sp(4) = S^5$$

$$(q^+, q^+, q^-, q^-)$$

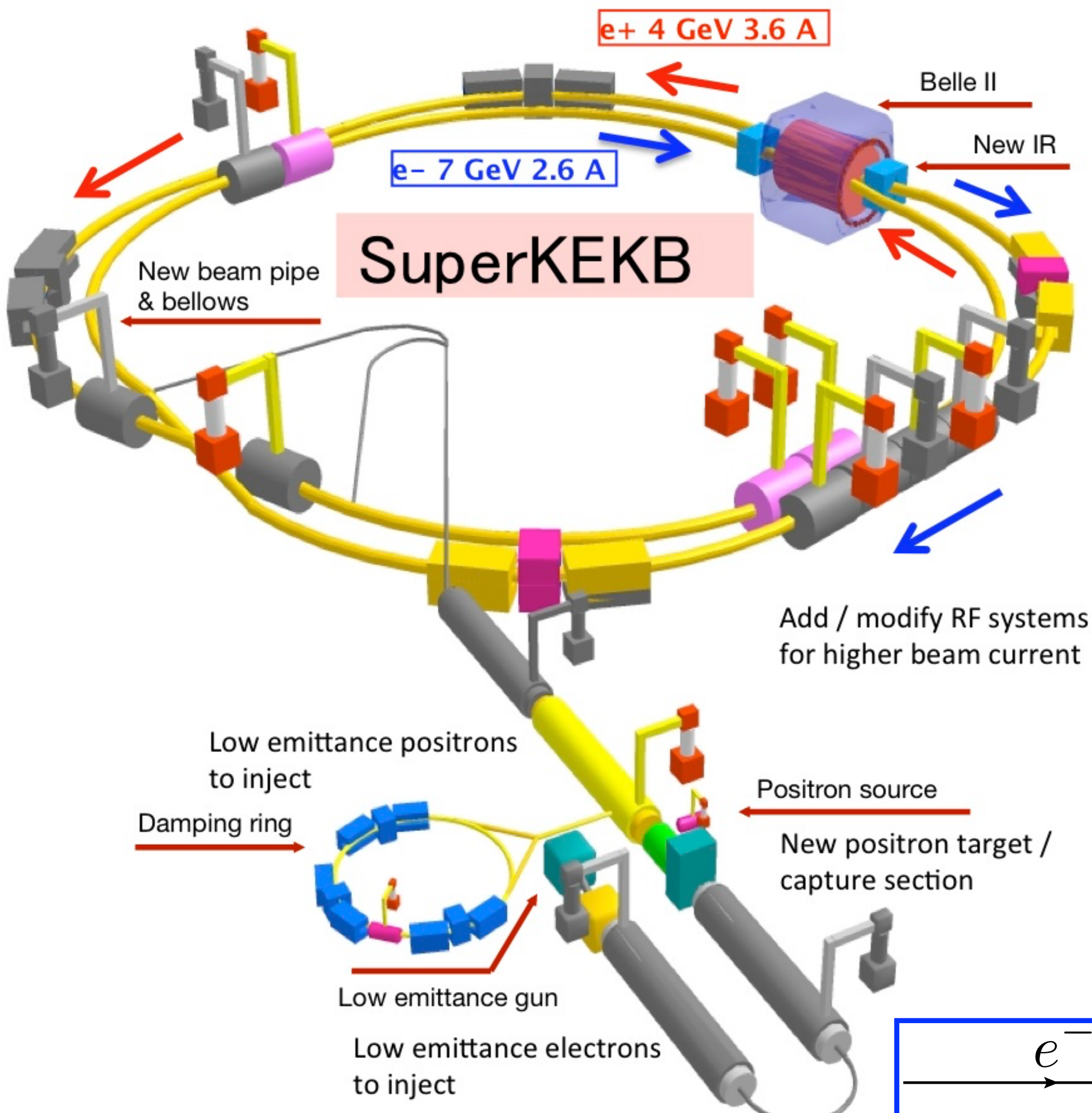
$$(\pi^{++}, \pi^{--}, \pi^0_x, \pi^0_y, \pi^0_z)$$

$$\frac{\epsilon_\gamma}{2c_W} B_{\mu\nu} F_D^{\mu\nu}$$

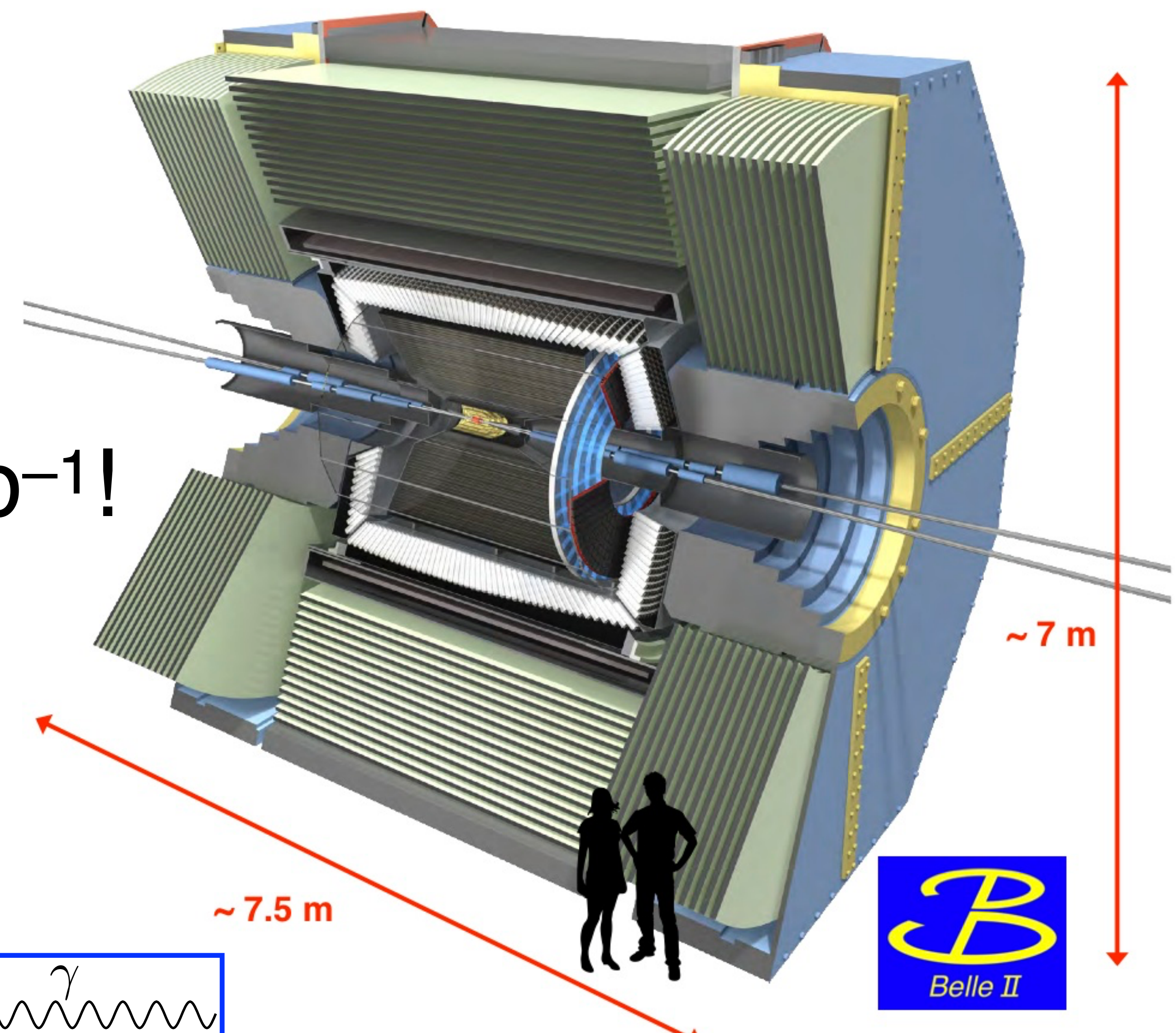


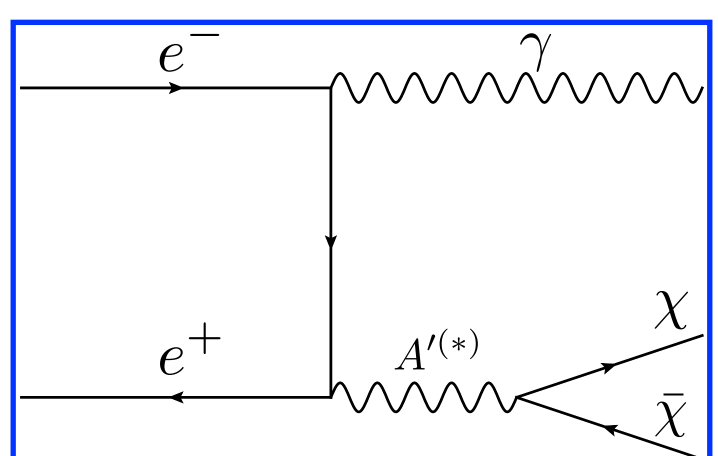
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
ϵ_γ
10^{-1}
1
10^{1}
10^{2}
10^{3}
m_V [GeV]
DELPHI
monojet
$\pi\pi \to SM$
$h \to ZV$
CMS7, DY
contact
LHC8, DY
EWPO
BaBar (inv)
BaBar (vis)
$\pi\pi \to \pi V$
thermalization
$SU(2), N_f = 2$
$\alpha_D = 1/4\pi$
$m_\pi = 300$ MeV

Super KEK B & Belle II

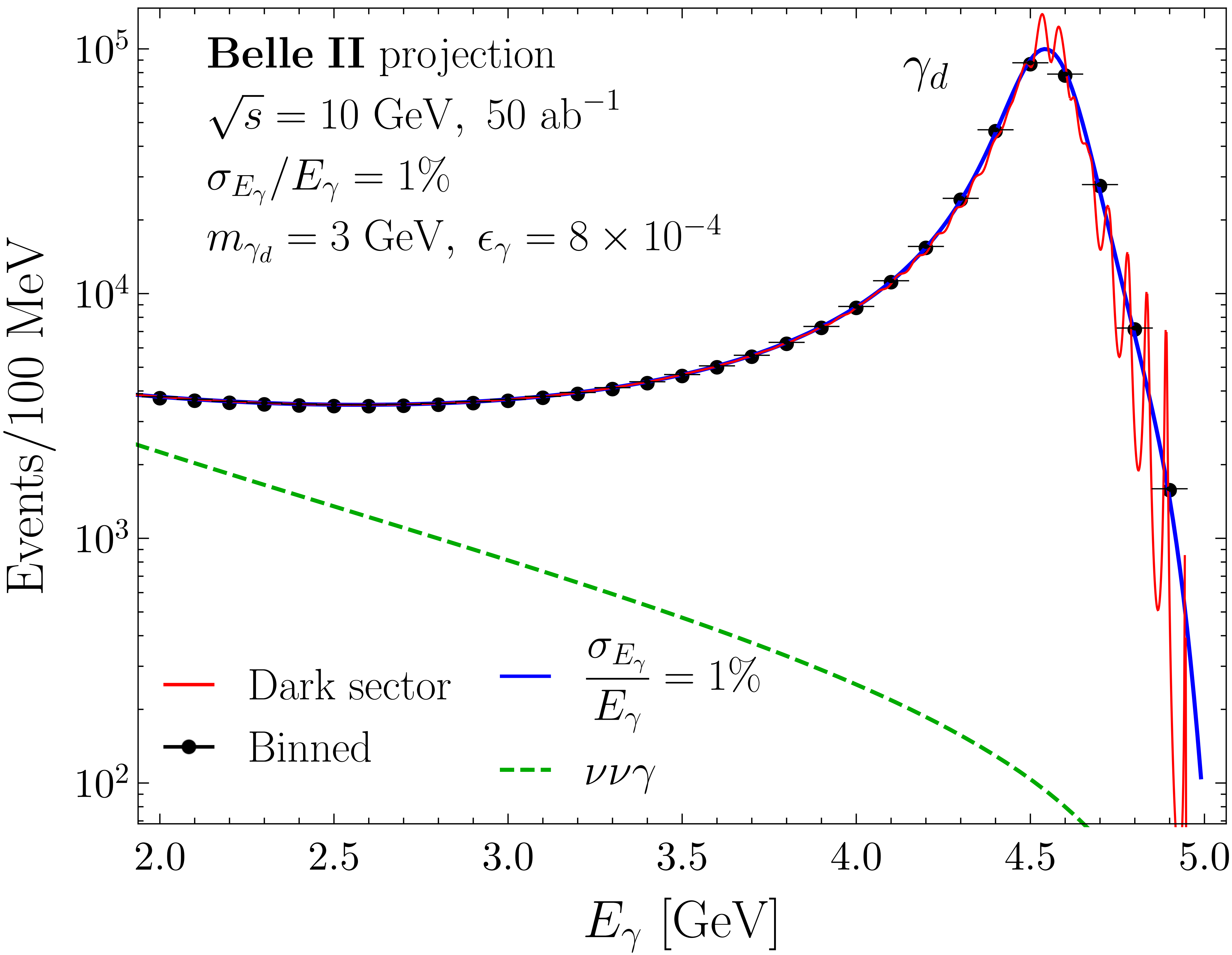


50 ab^{-1}!

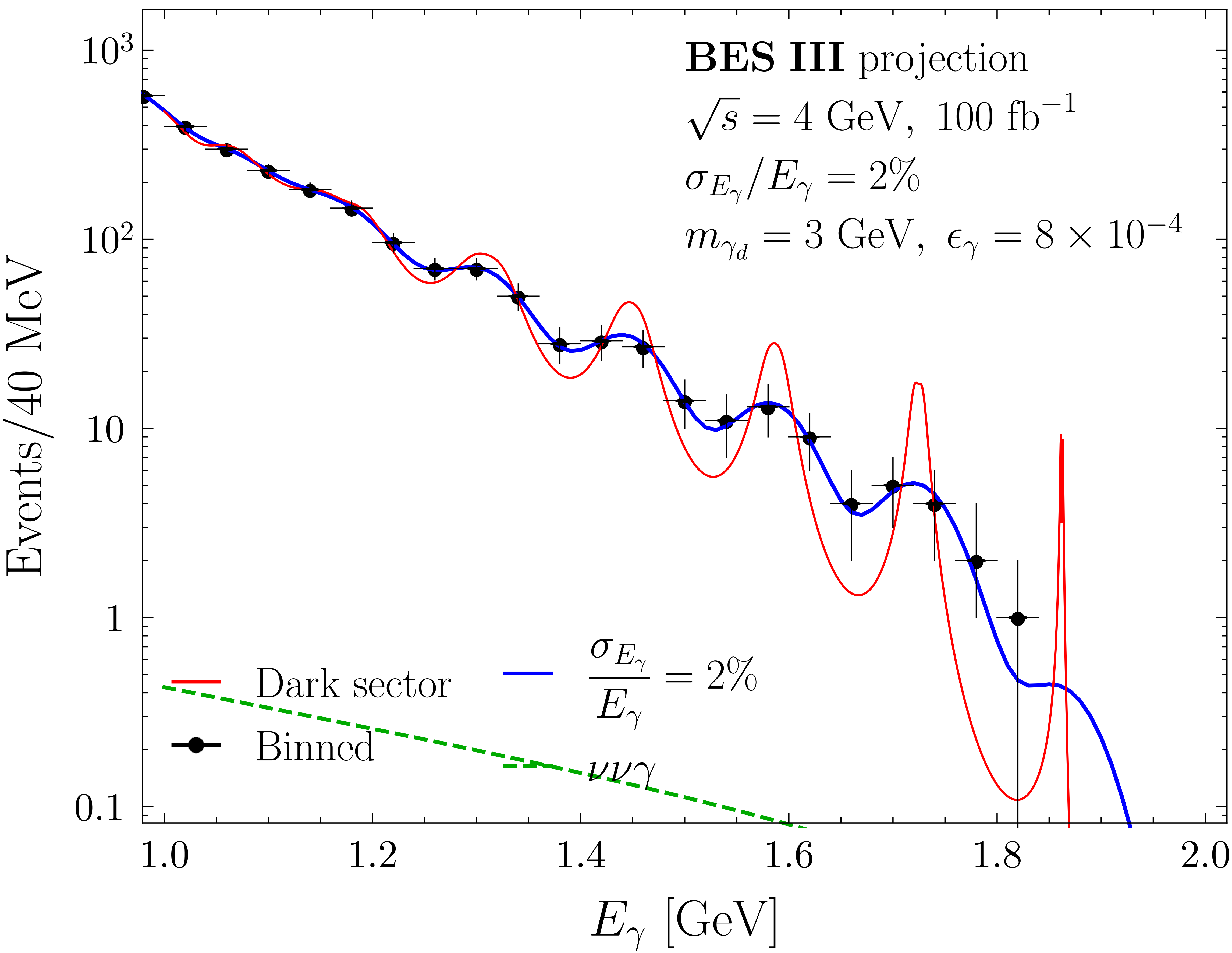




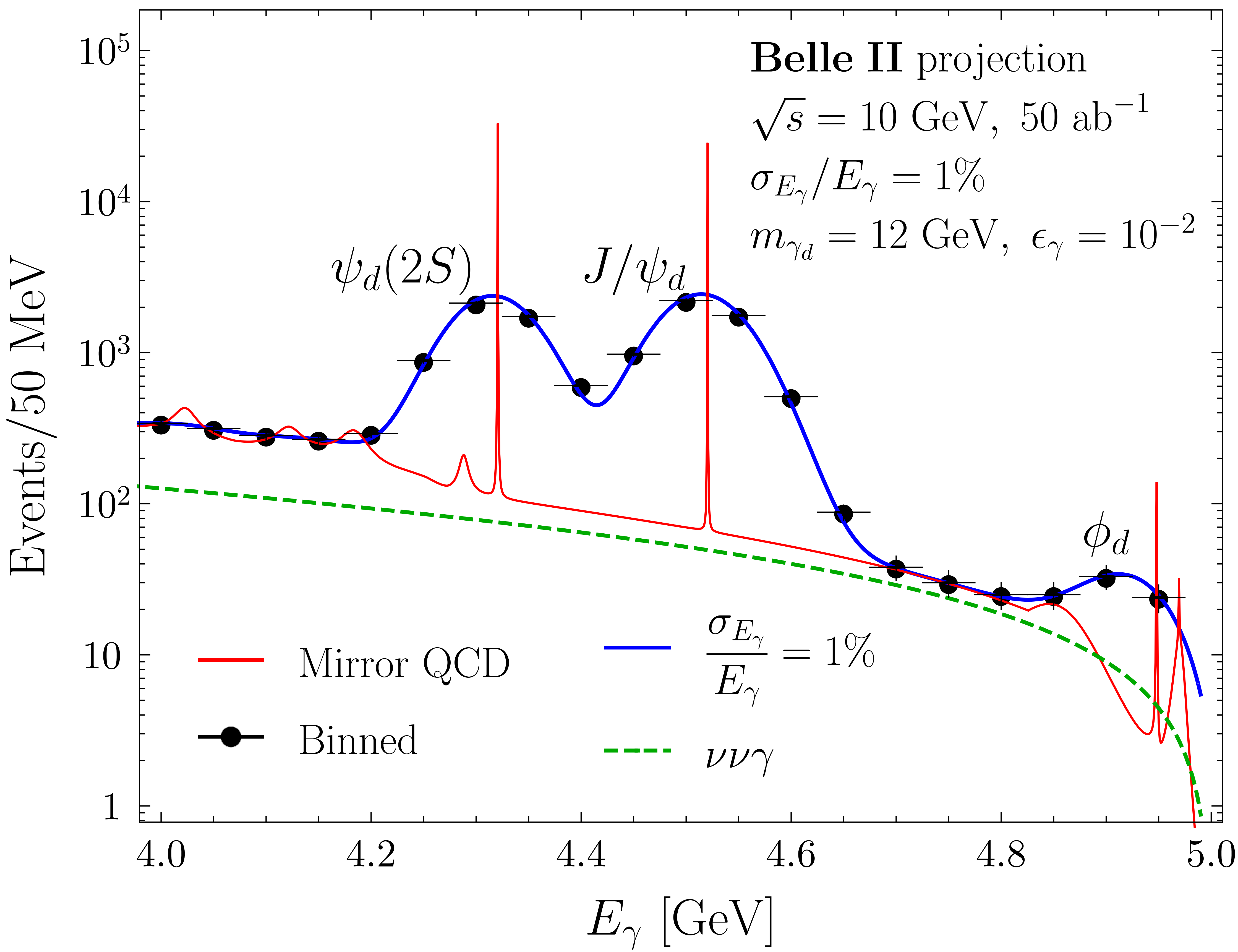
$$E_\gamma = \frac{\sqrt{s}}{2}\left(1 - \frac{M_{\rm inv}^2}{s}\right)$$



Belle II projection
$\sqrt{s} = 10$ GeV, 50 ab^{-1}
$\sigma_{E_\gamma}/E_\gamma = 1\%$
$m_{\gamma_d} = 3$ GeV, $\epsilon_\gamma = 8 \times 10^{-4}$
γ_d
Dark sector
Binned
$\frac{\sigma_{E_\gamma}}{E_\gamma} = 1\%$
$\nu\nu\gamma$
Events/100 MeV
E_γ [GeV]
10^5
10^4
10^3
10^2
2.0
2.5
3.0
3.5
4.0
4.5
5.0

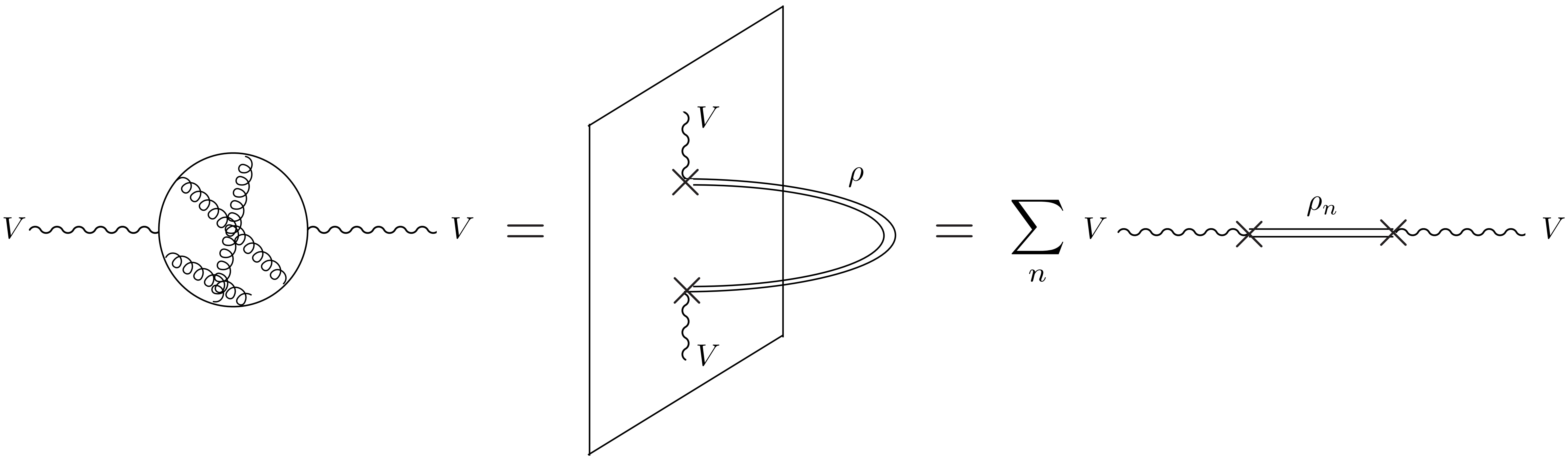


BES III projection
$\sqrt{s} = 4$ GeV, 100 fb^{-1}
$\sigma_{E_\gamma}/E_\gamma = 2\%$
$m_{\gamma_d} = 3$ GeV, $\epsilon_\gamma = 8 \times 10^{-4}$
Dark sector
Binned
$\frac{\sigma_{E_\gamma}}{E_\gamma} = 2\%$
$\nu\nu\gamma$
Events/40 MeV
E_γ [GeV]
10^3
10^2
10
1
0.1
1.0
1.2
1.4
1.6
1.8
2.0

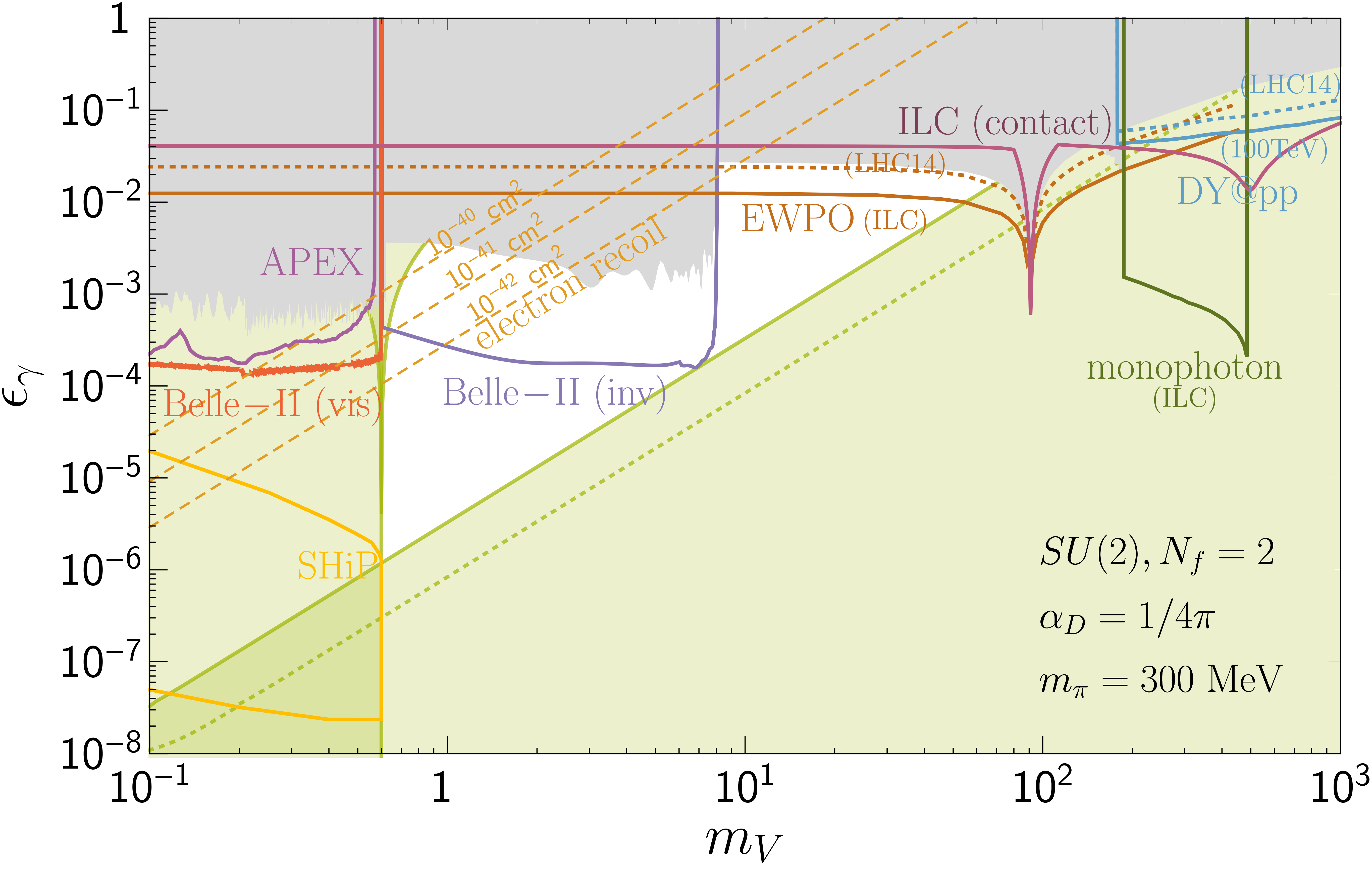


Belle II projection
$\sqrt{s} = 10$ GeV, 50 ab^{-1}
$\sigma_{E_\gamma}/E_\gamma = 1\%$
$m_{\gamma_d} = 12$ GeV, $\epsilon_\gamma = 10^{-2}$
$\psi_d(2S)$
J/ψ_d
ϕ_d
Mirror QCD
Binned
$\frac{\sigma_{E_\gamma}}{E_\gamma} = 1\%$
$\nu\nu\gamma$
Events/50 MeV
E_γ [GeV]
10^5
10^4
10^3
10^2
10
1
4.0
4.2
4.4
4.6
4.8
5.0

Holographic QCD



inspired by AdS/CFT from string theory



1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
ϵ_γ
10^{-1}
1
10^{1}
10^{2}
10^{3}
m_V
ILC (contact)
(LHC14)
(LHC14)
(100TeV)
DY@pp
EWPO (ILC)
10^{-40} cm^2
10^{-41} cm^2
10^{-42} cm^2
electron recoil
APEX
Belle−II (vis)
Belle−II (inv)
monophoton
(ILC)
SHiP
$SU(2), N_f = 2$
$\alpha_D = 1/4\pi$
$m_\pi = 300$ MeV

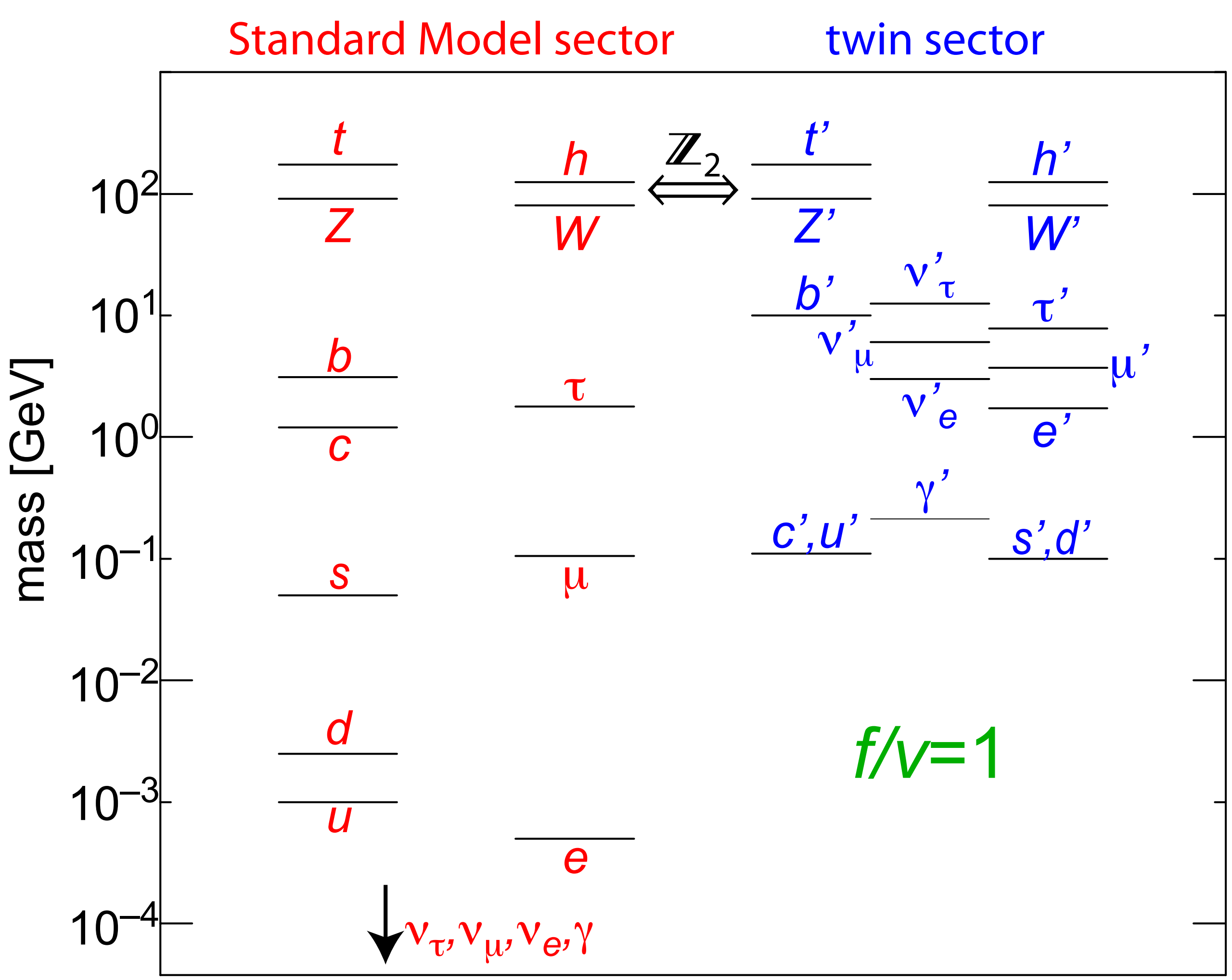


FIG. 1. A sample spectrum of twin particles. Here we use $f/v = 1$ to demonstrate the $\mathbb{Z}_2$ invariance between the visible and twin sectors for t, h, Z, W; lighter particles are subject to $\mathbb{Z}_2$-breaking effects without spoiling the solution to the hierarchy problem. In practice, twin sector masses are of course raised by a factor of $f/v \gtrsim 3$.

They are stable since they are the lightest particle with a conserved $SU(2)_f$ quantum number. (Here and below, we denote particles in the twin sector with a prime on the

meson M	particle content	$m_M^2 \propto$	m_M
$\theta^0(\mathbf{3},\mathbf{1})$	$u'\bar{c}', c'\bar{u}', \frac{1}{\sqrt{2}}(u'\bar{u}' - c'\bar{c}')$	$2m_{u'}$	$m_\pi(1+\Delta)$
$D^+(\mathbf{2},\mathbf{2})$	$u'\bar{d}', c'\bar{d}', u'\bar{s}', c'\bar{s}'$	$m_{u'} + m_{d'}$	$m_\pi(1+\frac{\Delta}{2})$
$D^-(\mathbf{2},\mathbf{2})$	$d'\bar{u}', s'\bar{u}', d'\bar{c}', s'\bar{c}'$	$m_{u'} + m_{d'}$	$m_\pi(1+\frac{\Delta}{2})$
$\eta^0(\mathbf{1},\mathbf{1})$	$\frac{1}{2}(d'\bar{d}' + s'\bar{s}' - u'\bar{u}' - c'\bar{c}')$	$m_{u'} + m_{d'}$	$m_\pi(1+\frac{\Delta}{2})$
$\pi^0(\mathbf{1},\mathbf{3})$	$d'\bar{s}', s'\bar{d}', \frac{1}{\sqrt{2}}(d'\bar{d}' - s'\bar{s}')$	$2m_{d'}$	m_π

TABLE I. Decomposition of the meson $SU(4)_f$ 15-plet under $SU(2)_U \times SU(2)_D \times U(1)_{\rm EM}$. The third column shows the linear combination of quark masses that determines the meson masses-squared. From top to bottom, the meson masses go from heaviest to lightest, assuming $m_{d'} = m_{s'} < m_{u'} = m_{c'} = m_{d',s'}(1+\Delta)$.

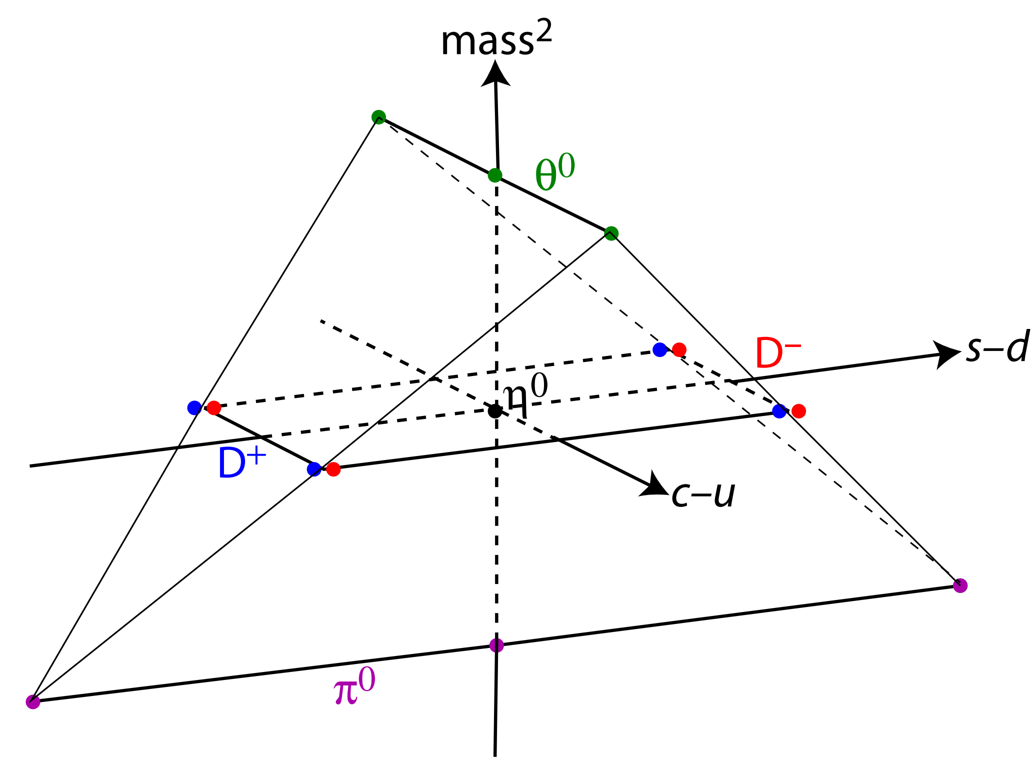


FIG. 2. A visual representation of the meson spectrum.

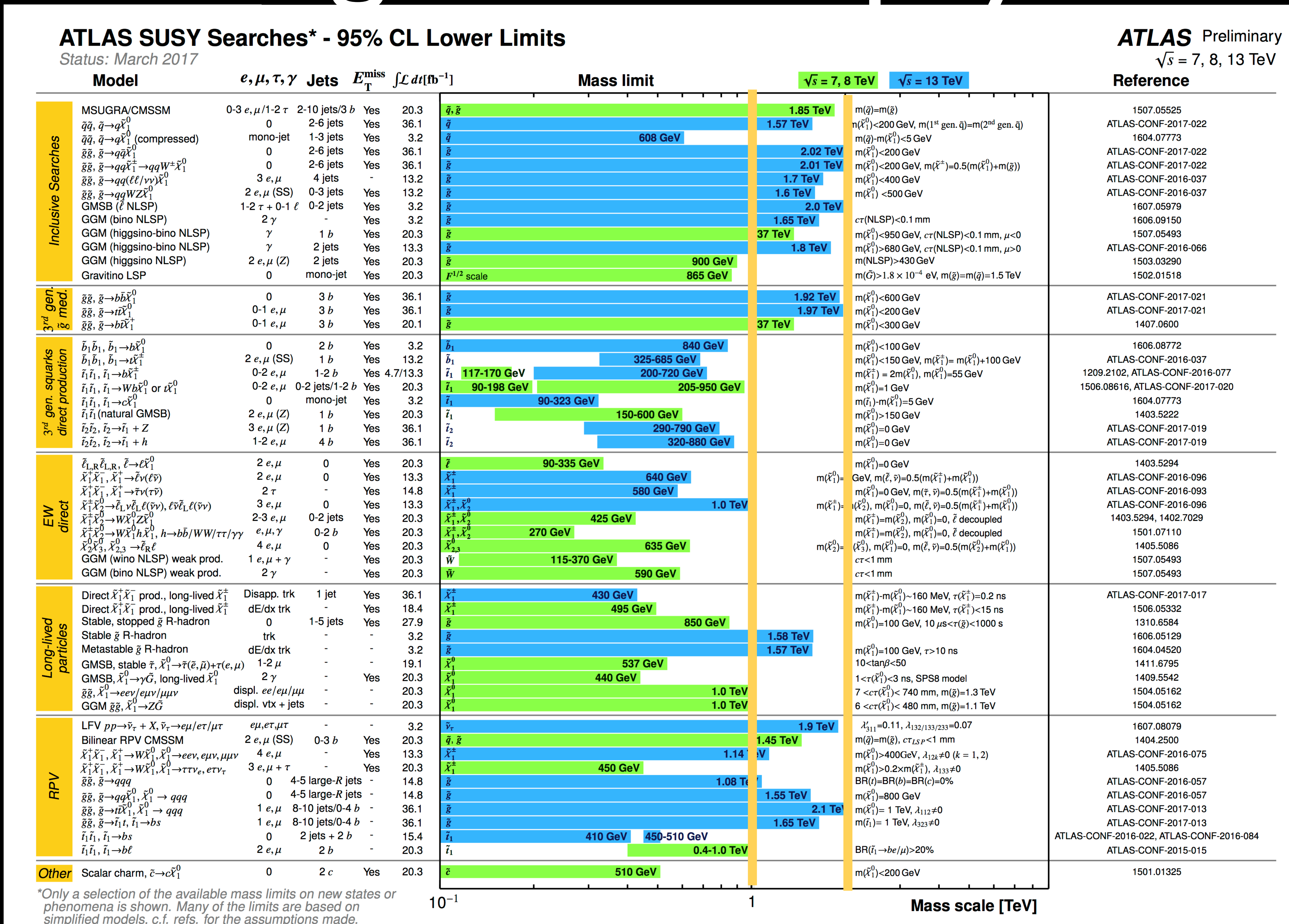
Conclusion

- surprisingly an *old* theory for dark matter
- SIMP Miracle^3
  - mass ~ QCD
  - coupling ~ QCD
  - theory ~ QCD
- can solve problem with DM profile
- very rich phenomenology
- can also be spin 1, axion mediation
- can be a part of twin Higgs
- Exciting *dark spectroscopy*!

Effective operators

Hitoshi Murayama (Berkeley, Kavli IPMU, DESY)
with Brian Henning, Xiaochuan Lu, Thomas Melia

no sign of new physics



"The 2 TeV line has been reached for some scenarios"

ENGINEERING
Machines That Change Shape

MEDICINE
An Off Switch for Cancer

NEUROSCIENCE
How to Reach "Vegetative" Patients

SCIENTIFIC AMERICAN

ScientificAmerican.com

A

IF SUPERSYMMETRY

CRISIS

DOESN'T PAN OUT,

IN

SCIENTISTS NEED A NEW WAY

PHYSICS

TO EXPLAIN THE UNIVERSE

?

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why effective operators

- No signal of BSM @ LHC so far
- use effective operators to parametrize physics at higher energies
  - precision electroweak
  - precision Higgs
  - precision flavor
  - *B*, *L* violation
  - coupling to the dark sector
- once deviation $\Longrightarrow$ BSM theory
- similar to four-fermion operators in weak interactions $\Rightarrow$ Standard Model

Effective Operators

- Surprisingly difficult question
- In the case of the Standard Model
  - Weinberg (1980) on D=6 $\not{B}$, D=5 $\not{L}$
  - Buchmüller-Wyler (1986) on D=6 ops
    - 80 operators for N_f=1, B, L conserving
  - Grzadkowski et al (2010) removed redundancies and discovered one missed
    - 59 operators for N_f=1, B, L conserving
  - Mahonar et al (2013) general N_f
  - Lehman-Martin (2014,15) D=7 for general N_f, D=8 for N_f=1 (incomplete)

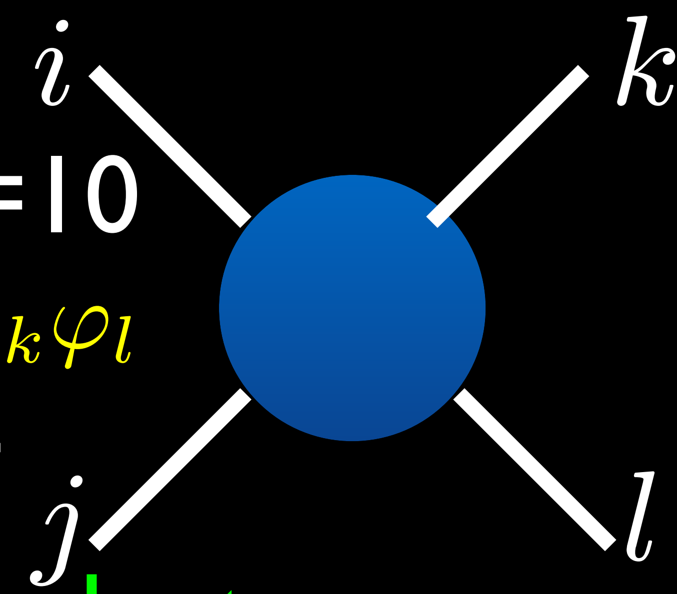
$$
\begin{aligned}
\widehat{H}_6 \; &= H^3 H^{\dagger\,3} + u^\dagger Q^\dagger H H^{\dagger\,2} + 2Q^2 Q^{\dagger\,2} + Q^{\dagger\,3} L^\dagger + Q^3 L + 2QQ^\dagger LL^\dagger + L^2 L^{\dagger\,2} + uQH^2 H^\dagger \\
&+ 2uu^\dagger QQ^\dagger + uu^\dagger LL^\dagger + u^2 u^{\dagger\,2} + e^\dagger u^\dagger Q^2 + e^\dagger L^\dagger H^2 H^\dagger + 2e^\dagger u^\dagger Q^\dagger L^\dagger + eLHH^{\dagger\,2} + euQ^{\dagger\,2} \\
&+ 2euQL + ee^\dagger QQ^\dagger + ee^\dagger LL^\dagger + ee^\dagger uu^\dagger + e^2 e^{\dagger\,2} + d^\dagger Q^\dagger H^2 H^\dagger + 2d^\dagger u^\dagger Q^{\dagger\,2} + d^\dagger u^\dagger QL \\
&+ d^\dagger e^\dagger u^{\dagger\,2} + d^\dagger e Q^\dagger L + dQHH^{\dagger\,2} + 2duQ^2 + duQ^\dagger L^\dagger + de^\dagger QL^\dagger + deu^2 + 2dd^\dagger QQ^\dagger + dd^\dagger LL^\dagger \\
&+ 2dd^\dagger uu^\dagger + dd^\dagger ee^\dagger + d^2 d^{\dagger\,2} + u^\dagger Q^\dagger H^\dagger G_R + d^\dagger Q^\dagger H G_R + HH^\dagger G_R^2 + G_R^3 + uQHG_L \\
&+ dQH^\dagger G_L + HH^\dagger G_L^2 + G_L^3 + u^\dagger Q^\dagger H^\dagger W_R + e^\dagger L^\dagger H W_R + d^\dagger Q^\dagger H W_R + HH^\dagger W_R^2 + W_R^3 \\
&+ uQHW_L + eLH^\dagger W_L + dQH^\dagger W_L + HH^\dagger W_L^2 + W_L^3 + u^\dagger Q^\dagger H^\dagger B_R + e^\dagger L^\dagger H B_R \\
&+ d^\dagger Q^\dagger H B_R + HH^\dagger B_R W_R + HH^\dagger B_R^2 + uQHB_L + eLH^\dagger B_L + dQH^\dagger B_L + HH^\dagger B_L W_L \\
&+ HH^\dagger B_L^2 + 2QQ^\dagger HH^\dagger \mathcal{D} + 2LL^\dagger HH^\dagger \mathcal{D} + uu^\dagger HH^\dagger \mathcal{D} + ee^\dagger HH^\dagger \mathcal{D} + d^\dagger u H^2 \mathcal{D} + du^\dagger H^{\dagger\,2} \mathcal{D} \\
&+ dd^\dagger HH^\dagger \mathcal{D} + 2H^2 H^{\dagger\,2} \mathcal{D}^2 \,. \qquad (3.16)
\end{aligned}
$$

$\mathcal{D}$: space time derivative

redundancies

- effective operators are invariants under the gauge group, Lorentz group, etc
- their classifications go back to Hilbert, Weyl
- applied to superpotentials, Standard Model
- but so far no general discussions on operators with derivatives
- two sources of redundancies
  - equation of motion (EOM)
  - integration by parts (IBP)

Simplest Example



- scalars four-point at $O(p^2)$: 4(4+1)/2=10

$$(\partial_\mu \partial_\mu \varphi_i)\varphi_j \varphi_k \varphi_l \qquad (\partial_\mu \varphi_i)(\partial_\mu \varphi_j)\varphi_k \varphi_l$$

- $\partial^2 \varphi_i = m_i^2 \varphi_i$ removes the first class: 4
- We know only 2 out of 6 are independent
  - s, t, u, $s+t+u=m_1^2+m_2^2+m_3^2+m_4^2$

$$(\partial_\mu \varphi_i)(\partial_\mu \varphi_j)\varphi_k \varphi_l - \varphi_i \varphi_j (\partial_\mu \varphi_k)(\partial_\mu \varphi_l) = \frac{1}{2}\partial^2(\varphi_i \varphi_j)(\varphi_k \varphi_l) - \frac{1}{2}(\varphi_i \varphi_j)\partial^2(\varphi_k \varphi_l) \approx 0$$

$$\partial_\mu \varphi_i \partial_\mu \varphi_j \varphi_k \varphi_l + \partial_\mu \varphi_i \varphi_j \partial_\mu \varphi_k \varphi_l + \partial_\mu \varphi_i \varphi_j \varphi_k \partial_\mu \varphi_l = \partial_\mu \varphi_i \partial_\mu (\varphi_j \varphi_k \varphi_l) \approx 0$$

- In addition, there are only *d* linearly independent momenta in *d*-dimensions for higher-point functions

Main idea

- Take kinetic terms as the zeroth order Lagrangian $(\partial\phi)^2,\ \bar{\psi}i\partial\!\!\!/\psi,\ (F_{\mu\nu})^2$
- Classically, it is conformally invariant under SO(4,2)≃SO(6,$\mathbb{C}$)
- Operator-State correspondence in CFT tells us that operators fall into representations of the conformal group
  - equation of motion: short multiplets
  - remove total derivatives: primary states

Master formula

- Define a multi-variate Hilbert series

$$H(p,\phi_1,\cdots,\phi_n) = \int d\mu_{\rm conformal} d\mu_{\rm gauge} \sum_{n=1}^{\infty} p^n \chi^*_{[n;0]} \prod_i PE[\phi_i \chi_i(q,\alpha,\beta)]$$

- integration over the gauge groups pick up gauge invariants
- integration over the conformal group picks only the primary states and Lorentz scalars
- expand it in power series in ϕ_i and p to find operators at given order in them
- Possible for any Lorentz-inv "free" QFT

*There are corrections for operators d≤4 due to lack of orthonormality among characters for short multiplets

Standard Model

```
χH[t_, α_, β_, x_, y_, z1_, z2_] := χscal[t, α, β] * u1[3, x] * su2f[y];
χHd[t_, α_, β_, x_, y_, z1_, z2_] := χscal[t, α, β] * u1[-3, x] * su2fb[y];
χQ[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[1, x] * su2f[y] * su3f[z1, z2];
χQd[t_, α_, β_, x_, y_, z1_, z2_] :=
  χfermR[t, α, β] * u1[-1, x] * su2fb[y] * su3fb[z1, z2];
χu[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[-4, x] * su3fb[z1, z2];
χud[t_, α_, β_, x_, y_, z1_, z2_] := χfermR[t, α, β] * u1[4, x] * su3f[z1, z2];
χd[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[2, x] * su3fb[z1, z2];
χdd[t_, α_, β_, x_, y_, z1_, z2_] := χfermR[t, α, β] * u1[-2, x] * su3f[z1, z2];
χL[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[-3, x] * su2f[y];
χLd[t_, α_, β_, x_, y_, z1_, z2_] := χfermR[t, α, β] * u1[3, x] * su2fb[y];
χe[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[6, x];
χed[t_, α_, β_, x_, y_, z1_, z2_] := χfermR[t, α, β] * u1[-6, x];
χBl[t_, α_, β_, x_, y_, z1_, z2_] := χfsL[t, α, β];
χBr[t_, α_, β_, x_, y_, z1_, z2_] := χfsR[t, α, β];
χWl[t_, α_, β_, x_, y_, z1_, z2_] := χfsL[t, α, β] * su2ad[y];
χWr[t_, α_, β_, x_, y_, z1_, z2_] := χfsR[t, α, β] * su2ad[y];
χGl[t_, α_, β_, x_, y_, z1_, z2_] := χfsL[t, α, β] * su3ad[z1, z2];
χGr[t_, α_, β_, x_, y_, z1_, z2_] := χfsR[t, α, β] * su3ad[z1, z2];
```

$$H(p,\phi_1,\cdots,\phi_n)=\int d\mu_{\text{conformal}}d\mu_{\text{gauge}}\sum_{n=1}^{\infty}p^n\chi^*_{[n;0]}\prod_i PE[\phi_i\chi_i(q,\alpha,\beta)]$$

Terminal Shell Edit View Window Help 100% Wed Jul 5 1:03:08 AM Hitoshi Murayama

Terminal — tcsh — ttys001

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Standard Model with
a heavy electroweak

Remove all

Terminal Shell Edit View Window Help 100% Wed Jul 5 1:06:42 AM Hitoshi Murayama

Terminal — tcsh — ttys001

```
Hitoshi-no-MacBook-Pro.local 49: form hssm8.frm
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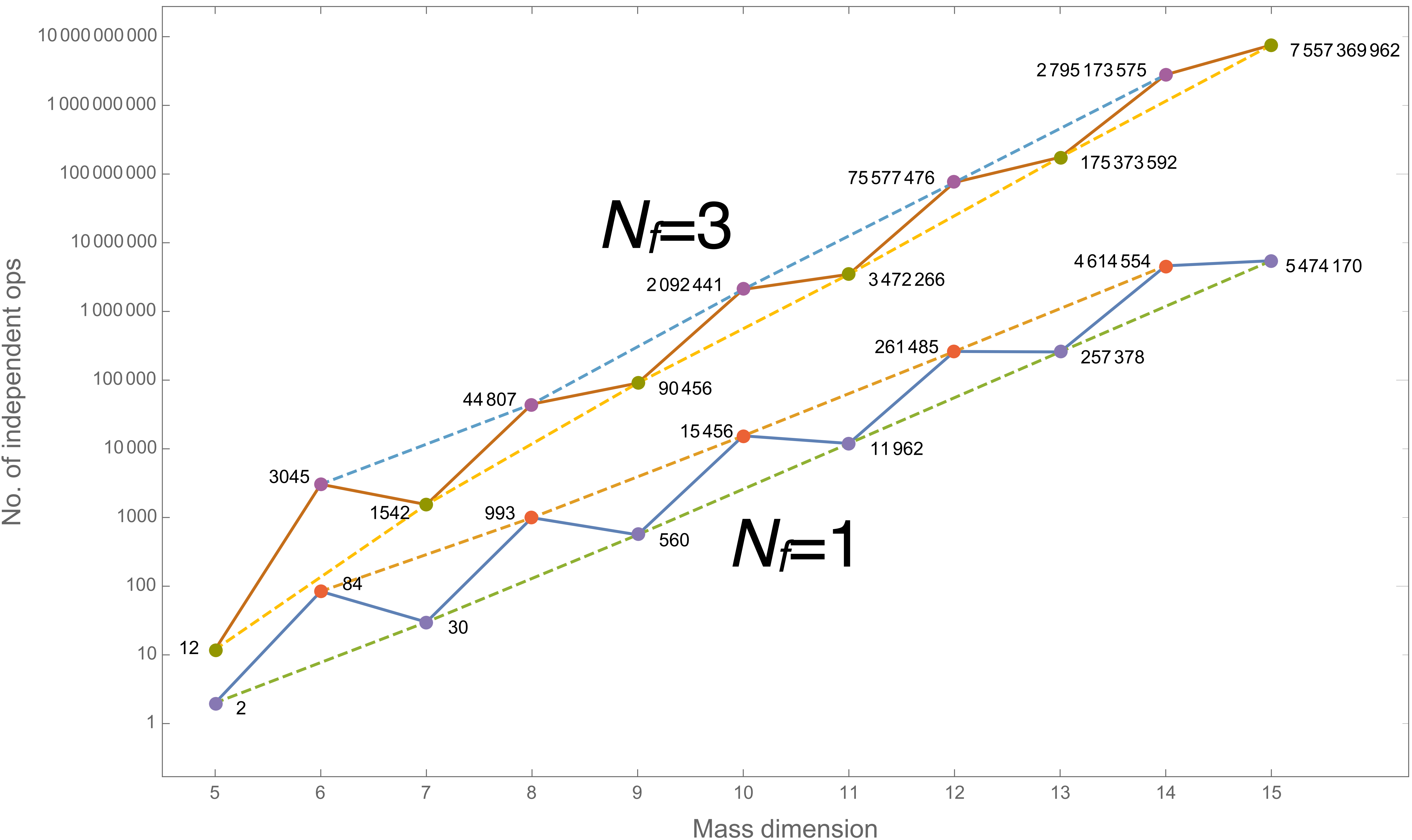
Standard Model with
a heavy electroweak

Remove all

D=8 operators

```
f =
   2*L^2*Ld^2*t^2 + 2*ee*ed*L*Ld*t^2 + ee^2*ed^2*t^2 + 2*d*dd*L*Ld*t^2 + 2*
   d*dd*ee*ed*t^2 + 2*d^2*dd^2*t^2 + ud^2*dd*ed*t^2 + 2*u*ud*L*Ld*t^2 + 2*u
   *ud*ee*ed*t^2 + 4*u*ud*d*dd*t^2 + u^2*d*ee*t^2 + 2*u^2*ud^2*t^2 + 2*Qd*
   dd*ee*L*t^2 + 3*Qd*ud*ed*Ld*t^2 + 2*Qd*u*d*Ld*t^2 + 3*Qd^2*ud*dd*t^2 +
   Qd^2*u*ee*t^2 + Qd^3*Ld*t^2 + 2*Q*d*ed*Ld*t^2 + 2*Q*ud*dd*L*t^2 + 3*Q*u*
   ee*L*t^2 + 4*Q*Qd*L*Ld*t^2 + 2*Q*Qd*ee*ed*t^2 + 4*Q*Qd*d*dd*t^2 + 4*Q*Qd
   *u*ud*t^2 + Q^2*ud*ed*t^2 + 3*Q^2*u*d*t^2 + 4*Q^2*Qd^2*t^2 + Q^3*L*t^2
    + Wr*L^2*Ld^2 + Wr*ee*ed*L*Ld + Wr*d*dd*L*Ld + Wr*u*ud*L*Ld + Wr*Qd*dd*
   ee*L + 3*Wr*Qd*ud*ed*Ld + Wr*Qd*u*d*Ld + 3*Wr*Qd^2*ud*dd + Wr*Qd^2*u*ee
    + 2*Wr*Qd^3*Ld + Wr*Q*d*ed*Ld + Wr*Q*ud*dd*L + 3*Wr*Q*Qd*L*Ld + Wr*Q*Qd
   *ee*ed + 2*Wr*Q*Qd*d*dd + 2*Wr*Q*Qd*u*ud + 2*Wr*Q^2*Qd^2 + Wr^2*L*Ld*t
    + Wr^2*Q*Qd*t + 2*Wr^4 + Wl*L^2*Ld^2 + Wl*ee*ed*L*Ld + Wl*d*dd*L*Ld +
   Wl*u*ud*L*Ld + Wl*Qd*dd*ee*L + Wl*Qd*u*d*Ld + Wl*Q*d*ed*Ld + Wl*Q*ud*dd*
   L + 3*Wl*Q*u*ee*L + 3*Wl*Q*Qd*L*Ld + Wl*Q*Qd*ee*ed + 2*Wl*Q*Qd*d*dd + 2*
   Wl*Q*Qd*u*ud + Wl*Q^2*ud*ed + 3*Wl*Q^2*u*d + 2*Wl*Q^2*Qd^2 + 2*Wl*Q^3*L
    + 2*Wl*Wr*L*Ld*t + Wl*Wr*ee*ed*t + Wl*Wr*d*dd*t + Wl*Wr*u*ud*t + 2*Wl*
   Wr*Q*Qd*t + Wl^2*L*Ld*t + Wl^2*Q*Qd*t + 2*Wl^2*Wr^2 + 2*Wl^4 + Gr*d*dd*L
   *Ld + Gr*d*dd*ee*ed + Gr*d^2*dd^2 + 3*Gr*ud^2*dd*ed + Gr*u*ud*L*Ld + Gr*
   u*ud*ee*ed + 4*Gr*u*ud*d*dd + Gr*u^2*ud^2 + Gr*Qd*dd*ee*L + 3*Gr*Qd*ud*
   ed*Ld + 2*Gr*Qd*u*d*Ld + 6*Gr*Qd^2*ud*dd + Gr*Qd^2*u*ee + 2*Gr*Qd^3*Ld
    + Gr*Q*d*ed*Ld + 2*Gr*Q*ud*dd*L + 2*Gr*Q*Qd*L*Ld + Gr*Q*Qd*ee*ed + 4*Gr
   *Q*Qd*d*dd + 4*Gr*Q*Qd*u*ud + Gr*Q^2*ud*ed + 2*Gr*Q^2*Qd^2 + Gr*Wr*Q*Qd*
   t + Gr*Wl*Q*Qd*t + Gr^2*d*dd*t + Gr^2*u*ud*t + Gr^2*Q*Qd*t + 2*Gr^2*Wr^2
    + Gr^2*Wl^2 + 3*Gr^4 + Gl*d*dd*L*Ld + Gl*d*dd*ee*ed + Gl*d^2*dd^2 + Gl*
   u*ud*L*Ld + Gl*u*ud*ee*ed + 4*Gl*u*ud*d*dd + 3*Gl*u^2*d*ee + Gl*u^2*ud^2
    + Gl*Qd*dd*ee*L + 2*Gl*Qd*u*d*Ld + Gl*Qd^2*u*ee + Gl*Q*d*ed*Ld + 2*Gl*Q
   *ud*dd*L + 3*Gl*Q*u*ee*L + 2*Gl*Q*Qd*L*Ld + Gl*Q*Qd*ee*ed + 4*Gl*Q*Qd*d*
   dd + 4*Gl*Q*Qd*u*ud + Gl*Q^2*ud*ed + 6*Gl*Q^2*u*d + 2*Gl*Q^2*Qd^2 + 2*Gl
   *Q^3*L + Gl*Wr*Q*Qd*t + Gl*Wl*Q*Qd*t + Gl*Gr*L*Ld*t + Gl*Gr*ee*ed*t + 3*
   Gl*Gr*d*dd*t + 3*Gl*Gr*u*ud*t + 3*Gl*Gr*Q*Qd*t + Gl*Gr*Wl*Wr + Gl^2*d*dd
   *t + Gl^2*u*ud*t + Gl^2*Q*Qd*t + Gl^2*Wr^2 + 2*Gl^2*Wl^2 + 3*Gl^2*Gr^2
    + 3*Gl^4 + Br*ee*ed*L*Ld + Br*d*dd*L*Ld + Br*d*dd*ee*ed + 2*Br*ud^2*dd*
   ed + Br*u*ud*L*Ld + Br*u*ud*ee*ed + 2*Br*u*ud*d*dd + Br*Qd*dd*ee*L + 3*
   Br*Qd*ud*ed*Ld + Br*Qd*u*d*Ld + 3*Br*Qd^2*ud*dd + Br*Qd^3*Ld + Br*Q*d*ed
   *Ld + Br*Q*ud*dd*L + 2*Br*Q*Qd*L*Ld + Br*Q*Qd*ee*ed + 2*Br*Q*Qd*d*dd + 2
   *Br*Q*Qd*u*ud + Br*Q^2*ud*ed + Br*Wr*L*Ld*t + Br*Wr*Q*Qd*t + Br*Wl*L*Ld*
   t + Br*Wl*Q*Qd*t + Br*Gr*d*dd*t + Br*Gr*u*ud*t + Br*Gr*Q*Qd*t + Br*Gr^3
    + Br*Gl*d*dd*t + Br*Gl*u*ud*t + Br*Gl*Q*Qd*t + Br*Gl^2*Gr + 2*Br^2*Wr^2
    + Br^2*Wl^2 + 2*Br^2*Gr^2 + Br^2*Gl^2 + Br^4 + Bl*ee*ed*L*Ld + Bl*d*dd*
   L*Ld + Bl*d*dd*ee*ed + Bl*u*ud*L*Ld + Bl*u*ud*ee*ed + 2*Bl*u*ud*d*dd + 2
   *Bl*u^2*d*ee + Bl*Qd*dd*ee*L + Bl*Qd*u*d*Ld + Bl*Qd^2*u*ee + Bl*Q*d*ed*
   Ld + Bl*Q*ud*dd*L + 3*Bl*Q*u*ee*L + 2*Bl*Q*Qd*L*Ld + Bl*Q*Qd*ee*ed + 2*
   Bl*Q*Qd*d*dd + 2*Bl*Q*Qd*u*ud + 3*Bl*Q^2*u*d + Bl*Q^3*L + Bl*Wr*L*Ld*t
    + Bl*Wr*Q*Qd*t + Bl*Wl*L*Ld*t + Bl*Wl*Q*Qd*t + Bl*Gr*d*dd*t + Bl*Gr*u*
   ud*t + Bl*Gr*Q*Qd*t + Bl*Gl*d*dd*t + Bl*Gl*u*ud*t + Bl*Gl*Q*Qd*t + Bl*Gl
   *Gr^2 + Bl*Gl^3 + Bl*Br*L*Ld*t + Bl*Br*ee*ed*t + Bl*Br*d*dd*t + Bl*Br*u*
   ud*t + Bl*Br*Q*Qd*t + Bl*Br*Wl*Wr + Bl*Br*Gl*Gr + Bl^2*Wr^2 + 2*Bl^2*
   Wl^2 + Bl^2*Gr^2 + 2*Bl^2*Gl^2 + Bl^2*Br^2 + Bl^4 + 3*Hd*ee*L^2*Ld*t +
   Hd*ee^2*ed*L*t + 3*Hd*d*dd*ee*L*t + 3*Hd*ud*d*ed*Ld*t + 2*Hd*ud^2*dd*L*t
    + 2*Hd*u*d^2*Ld*t + 3*Hd*u*ud*ee*L*t + 6*Hd*Qd*ud*L*Ld*t + 3*Hd*Qd*ud*
   ee*ed*t + 6*Hd*Qd*ud*d*dd*t + 3*Hd*Qd*u*d*ee*t + 3*Hd*Qd*u*ud^2*t + 3*Hd
   *Qd^2*d*Ld*t + Hd*Qd^3*ee*t + 6*Hd*Q*d*L*Ld*t + 3*Hd*Q*d*ee*ed*t + 3*Hd*
   Q*d^2*dd*t + 2*Hd*Q*ud^2*ed*t + 6*Hd*Q*u*ud*d*t + 6*Hd*Q*Qd*ee*L*t + 6*
   Hd*Q*Qd^2*ud*t + 3*Hd*Q^2*ud*L*t + 6*Hd*Q^2*Qd*d*t + Hd*Wr*ee*L*t^2 + 2*
   Hd*Wr*Qd*ud*t^2 + Hd*Wr*Q*d*t^2 + Hd*Wr^2*ee*L + 2*Hd*Wr^2*Qd*ud + Hd*
   Wr^2*Q*d + 2*Hd*Wl*ee*L*t^2 + Hd*Wl*Qd*ud*t^2 + 2*Hd*Wl*Q*d*t^2 + 2*Hd*
   Wl^2*ee*L + Hd*Wl^2*Qd*ud + 2*Hd*Wl^2*Q*d + 2*Hd*Gr*Qd*ud*t^2 + Hd*Gr*Q*
   d*t^2 + 2*Hd*Gr*Wr*Qd*ud + Hd*Gr*Wr*Q*d + Hd*Gr^2*ee*L + 3*Hd*Gr^2*Qd*ud
    + 2*Hd*Gr^2*Q*d + Hd*Gl*Qd*ud*t^2 + 2*Hd*Gl*Q*d*t^2 + Hd*Gl*Wl*Qd*ud +
   2*Hd*Gl*Wl*Q*d + Hd*Gl^2*ee*L + 2*Hd*Gl^2*Qd*ud + 3*Hd*Gl^2*Q*d + Hd*Br*
   ee*L*t^2 + 2*Hd*Br*Qd*ud*t^2 + Hd*Br*Q*d*t^2 + Hd*Br*Wr*ee*L + 2*Hd*Br*
   Wr*Qd*ud + Hd*Br*Wr*Q*d + 2*Hd*Br*Gr*Qd*ud + Hd*Br*Gr*Q*d + Hd*Br^2*ee*L
    + Hd*Br^2*Qd*ud + Hd*Br^2*Q*d + 2*Hd*Bl*ee*L*t^2 + Hd*Bl*Qd*ud*t^2 + 2*
   Hd*Bl*Q*d*t^2 + 2*Hd*Bl*Wl*ee*L + Hd*Bl*Wl*Qd*ud + 2*Hd*Bl*Wl*Q*d + Hd*
   Bl*Gl*Qd*ud + 2*Hd*Bl*Gl*Q*d + Hd*Bl^2*ee*L + Hd*Bl^2*Qd*ud + Hd*Bl^2*Q*
   d + Hd^2*ee^2*L^2 + Hd^2*ud*d*t^3 + Hd^2*ud*d*L*Ld + Hd^2*Qd*ud*ee*L + 2
   *Hd^2*Qd^2*ud^2 + 2*Hd^2*Q*d*ee*L + 2*Hd^2*Q*Qd*ud*d + 2*Hd^2*Q^2*d^2 +
   Hd^2*Wr*ud*d*t + Hd^2*Wl*ud*d*t + Hd^2*Gr*ud*d*t + Hd^2*Gl*ud*d*t + Hd^2
   *Br*ud*d*t + Hd^2*Bl*ud*d*t + 3*H*ed*L*Ld^2*t + H*ee*ed^2*Ld*t + 3*H*d*
   dd*ed*Ld*t + 2*H*ud*dd^2*L*t + 3*H*u*dd*ee*L*t + 3*H*u*ud*ed*Ld*t + 2*H*
   u^2*d*Ld*t + 6*H*Qd*dd*L*Ld*t + 3*H*Qd*dd*ee*ed*t + 3*H*Qd*d*dd^2*t + 6*
   H*Qd*u*ud*dd*t + 2*H*Qd*u^2*ee*t + 3*H*Qd^2*u*Ld*t + 3*H*Q*ud*dd*ed*t +
   6*H*Q*u*L*Ld*t + 3*H*Q*u*ee*ed*t + 6*H*Q*u*d*dd*t + 3*H*Q*u^2*ud*t + 6*H
   *Q*Qd*ed*Ld*t + 6*H*Q*Qd^2*dd*t + 3*H*Q^2*dd*L*t + 6*H*Q^2*Qd*u*t + H*
   Q^3*ed*t + 2*H*Wr*ed*Ld*t^2 + 2*H*Wr*Qd*dd*t^2 + H*Wr*Q*u*t^2 + 2*H*Wr^2
   *ed*Ld + 2*H*Wr^2*Qd*dd + H*Wr^2*Q*u + H*Wl*ed*Ld*t^2 + H*Wl*Qd*dd*t^2
    + 2*H*Wl*Q*u*t^2 + H*Wl^2*ed*Ld + H*Wl^2*Qd*dd + 2*H*Wl^2*Q*u + 2*H*Gr*
   Qd*dd*t^2 + H*Gr*Q*u*t^2 + 2*H*Gr*Wr*Qd*dd + H*Gr*Wr*Q*u + H*Gr^2*ed*Ld
    + 3*H*Gr^2*Qd*dd + 2*H*Gr^2*Q*u + H*Gl*Qd*dd*t^2 + 2*H*Gl*Q*u*t^2 + H*
   Gl*Wl*Qd*dd + 2*H*Gl*Wl*Q*u + H*Gl^2*ed*Ld + 2*H*Gl^2*Qd*dd + 3*H*Gl^2*Q
   *u + 2*H*Br*ed*Ld*t^2 + 2*H*Br*Qd*dd*t^2 + H*Br*Q*u*t^2 + 2*H*Br*Wr*ed*
   Ld + 2*H*Br*Wr*Qd*dd + H*Br*Wr*Q*u + 2*H*Br*Gr*Qd*dd + H*Br*Gr*Q*u + H*
   Br^2*ed*Ld + H*Br^2*Qd*dd + H*Br^2*Q*u + H*Bl*ed*Ld*t^2 + H*Bl*Qd*dd*t^2
    + 2*H*Bl*Q*u*t^2 + H*Bl*Wl*ed*Ld + H*Bl*Wl*Qd*dd + 2*H*Bl*Wl*Q*u + H*Bl
   *Gl*Qd*dd + 2*H*Bl*Gl*Q*u + H*Bl^2*ed*Ld + H*Bl^2*Qd*dd + H*Bl^2*Q*u + 4
   *H*Hd*L*Ld*t^3 + 2*H*Hd*L^2*Ld^2 + 2*H*Hd*ee*ed*t^3 + 2*H*Hd*ee*ed*L*Ld
    + H*Hd*ee^2*ed^2 + 2*H*Hd*d*dd*t^3 + 2*H*Hd*d*dd*L*Ld + H*Hd*d*dd*ee*ed
    + H*Hd*d^2*dd^2 + H*Hd*ud^2*dd*ed + 2*H*Hd*u*ud*t^3 + 2*H*Hd*u*ud*L*Ld
    + H*Hd*u*ud*ee*ed + 2*H*Hd*u*ud*d*dd + H*Hd*u^2*d*ee + H*Hd*u^2*ud^2 +
   2*H*Hd*Qd*dd*ee*L + 4*H*Hd*Qd*ud*ed*Ld + 2*H*Hd*Qd*u*d*Ld + 4*H*Hd*Qd^2*
   ud*dd + H*Hd*Qd^2*u*ee + 2*H*Hd*Qd^3*Ld + 2*H*Hd*Q*d*ed*Ld + 2*H*Hd*Q*ud
   *dd*L + 4*H*Hd*Q*u*ee*L + 4*H*Hd*Q*Qd*t^3 + 5*H*Hd*Q*Qd*L*Ld + 2*H*Hd*Q*
   Qd*ee*ed + 4*H*Hd*Q*Qd*d*dd + 4*H*Hd*Q*Qd*u*ud + H*Hd*Q^2*ud*ed + 4*H*Hd
   *Q^2*u*d + 3*H*Hd*Q^2*Qd^2 + 2*H*Hd*Q^3*L + 6*H*Hd*Wr*L*Ld*t + 2*H*Hd*Wr
   *ee*ed*t + 2*H*Hd*Wr*d*dd*t + 2*H*Hd*Wr*u*ud*t + 6*H*Hd*Wr*Q*Qd*t + 2*H*
   Hd*Wr^2*t^2 + H*Hd*Wr^3 + 6*H*Hd*Wl*L*Ld*t + 2*H*Hd*Wl*ee*ed*t + 2*H*Hd*
   Wl*d*dd*t + 2*H*Hd*Wl*u*ud*t + 6*H*Hd*Wl*Q*Qd*t + 2*H*Hd*Wl*Wr*t^2 + 2*H
   *Hd*Wl^2*t^2 + H*Hd*Wl^3 + 2*H*Hd*Gr*d*dd*t + 2*H*Hd*Gr*u*ud*t + 4*H*Hd*
   Gr*Q*Qd*t + H*Hd*Gr^2*t^2 + H*Hd*Gr^3 + 2*H*Hd*Gl*d*dd*t + 2*H*Hd*Gl*u*
   ud*t + 4*H*Hd*Gl*Q*Qd*t + H*Hd*Gl*Gr*t^2 + H*Hd*Gl^2*t^2 + H*Hd*Gl^3 + 4
   *H*Hd*Br*L*Ld*t + 2*H*Hd*Br*ee*ed*t + 2*H*Hd*Br*d*dd*t + 2*H*Hd*Br*u*ud*
   t + 4*H*Hd*Br*Q*Qd*t + 2*H*Hd*Br*Wr*t^2 + H*Hd*Br*Wr^2 + H*Hd*Br*Wl*t^2
    + H*Hd*Br^2*t^2 + 4*H*Hd*Bl*L*Ld*t + 2*H*Hd*Bl*ee*ed*t + 2*H*Hd*Bl*d*dd
   *t + 2*H*Hd*Bl*u*ud*t + 4*H*Hd*Bl*Q*Qd*t + H*Hd*Bl*Wr*t^2 + 2*H*Hd*Bl*Wl
   *t^2 + H*Hd*Bl*Wl^2 + H*Hd*Bl*Br*t^2 + H*Hd*Bl^2*t^2 + 6*H*Hd^2*ee*L*t^2
    + 6*H*Hd^2*Qd*ud*t^2 + 6*H*Hd^2*Q*d*t^2 + 2*H*Hd^2*Wr*Qd*ud + 2*H*Hd^2*
   Wl*ee*L + 2*H*Hd^2*Wl*Q*d + H*Hd^2*Gr*Qd*ud + H*Hd^2*Gl*Q*d + H*Hd^2*Br*
   Qd*ud + H*Hd^2*Bl*ee*L + H*Hd^2*Bl*Q*d + H*Hd^3*ud*d*t + H^2*ed^2*Ld^2
    + H^2*u*dd*t^3 + H^2*u*dd*L*Ld + 2*H^2*Qd*dd*ed*Ld + 2*H^2*Qd^2*dd^2 +
   H^2*Q*u*ed*Ld + 2*H^2*Q*Qd*u*dd + 2*H^2*Q^2*u^2 + H^2*Wr*u*dd*t + H^2*Wl
   *u*dd*t + H^2*Gr*u*dd*t + H^2*Gl*u*dd*t + H^2*Br*u*dd*t + H^2*Bl*u*dd*t
    + 6*H^2*Hd*ed*Ld*t^2 + 6*H^2*Hd*Qd*dd*t^2 + 6*H^2*Hd*Q*u*t^2 + 2*H^2*Hd
   *Wr*ed*Ld + 2*H^2*Hd*Wr*Qd*dd + 2*H^2*Hd*Wl*Q*u + H^2*Hd*Gr*Qd*dd + H^2*
   Hd*Gl*Q*u + H^2*Hd*Br*ed*Ld + H^2*Hd*Br*Qd*dd + H^2*Hd*Bl*Q*u + 3*H^2*
   Hd^2*t^4 + 4*H^2*Hd^2*L*Ld*t + H^2*Hd^2*ee*ed*t + H^2*Hd^2*d*dd*t + H^2*
   Hd^2*u*ud*t + 4*H^2*Hd^2*Q*Qd*t + 2*H^2*Hd^2*Wr*t^2 + 2*H^2*Hd^2*Wr^2 +
   2*H^2*Hd^2*Wl*t^2 + 2*H^2*Hd^2*Wl^2 + H^2*Hd^2*Gr^2 + H^2*Hd^2*Gl^2 +
   H^2*Hd^2*Br*t^2 + H^2*Hd^2*Br*Wr + H^2*Hd^2*Br^2 + H^2*Hd^2*Bl*t^2 + H^2
   *Hd^2*Bl*Wl + H^2*Hd^2*Bl^2 + H^2*Hd^3*ee*L + H^2*Hd^3*Qd*ud + H^2*Hd^3*
   Q*d + H^3*Hd*u*dd*t + H^3*Hd^2*ed*Ld + H^3*Hd^2*Qd*dd + H^3*Hd^2*Q*u + 2
   *H^3*Hd^3*t^2 + H^4*Hd^4;
```

993 of them for N_f=1



No. of independent ops
Mass dimension
10 000 000 000
1 000 000 000
100 000 000
10 000 000
1 000 000
100 000
10 000
1000
100
10
1
5
6
7
8
9
10
11
12
13
14
15
N_f=3
N_f=1
12
3045
1542
44 807
90 456
2 092 441
3 472 266
75 577 476
175 373 592
2 795 173 575
7 557 369 962
2
84
30
993
560
15 456
11 962
261 485
257 378
4 614 554
5 474 170

Conclusions

- Nailed the question of classifying effective operators in any given Lorentz-inv theory
- Also for chiral Lagrangians
- useful techniques for matching
- careful mapping to observables
- hope for deviations from Standard Model
- inverse problem to identify BSM physics