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The parameter space of locked inflation and corrections to modulated reheating

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Abstract

For a successfull inflation the curvatures of the potential fields should be lower than the Hubble parameter H. This is usually achieved by requiring the slow roll parameters to hold. Attempting to embed such a theory in a supergravity environment fails due to the mass of the moduli field of order H. Recently "New Old Inflation" was introduced to solve the problem. In such a model, moduli have masses of order H and a 50 e-folds inflationary period can be achieved. The main idea is that the inflationary field is trapped by an oscillatory field at the saddle point in the potential. In this way a De Sitter-like expansion is guaranteed by the false vacuum energy. In the present work two problems are inspected: first of all the parameter space of such a model is adressed in view of some consistency and phenomenological issues. Secondly, the mechanism of modulated reheating, necessary in such models to produce the desired density perturbations is inspected in depth. A general feature of such a mechanism is the production of non gaussianities of order one, which is relevant for current observations.

It is found that the parameter space of New Old Inflation is not ruled out and that the window of opportunity can be elevated into a prediction of the model. Regarding modulated reheating, a novel correction to the amount of

non gaussianities is obtained by considering the dynamical evolution of the modulating field. Such a correction is, however, model dependent.

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About this work

A period of accelerated expansion is globally accepted in the cosmological community as the canonical explaination to different questions such as the horizon and the flatness problems of the observed universe [1]. Treating the universe as a fluid, from Einstein's equation, it is possible to read off Friedmann's equations, through which a stage of accelerated expansion can be realised by the equation of state $p \approx -\rho$. Equality would generate a De Sitter universe, out of which a graceful exit would not be possible. In order to produce the desired equation of state the slow roll conditions are imposed which are nothing but conditions on the first and second field derivative on the potential proposed in each model. One of the main consequences of the slow roll conditions, is that the masses of the relevant fields for inflation should be lower than the Hubble parameter $H = \dot{a}/a$ in order not to spoil inflation. This was initially a problem for supersymmetric models of inflation where the various moduli have masses of order H. In [2] a new mechanism is introduced where the slow roll condition $\eta \sim m^2/H^2 \ll 1$ can be violated without affecting the inflationary dynamic. The main idea of the model is to use Guth's inflation [3] to nucleate a bubble (and transition from a false vacuum to a minimum) with the proper initial conditions for two weakly coupled scalar fields: ϕ and Φ. The first field nucleates at zero and has an higgs like potential through which it could, in principle, roll down. The second field nucleates at $\Phi \sim M_p$ with its minimum at $\Phi = 0$. If $M_\Phi \geq H_0$ with H_0 being sourced by the vacuum energy of the ϕ field at zero, then Φ will oscillate. If the oscillations are fast enough, the coupling term $\lambda\phi\Phi^2$ can be substituted by the average $\lambda\phi^2\langle\Phi^2\rangle$ generating an effective mass for ϕ which will not allow it to roll down the hill to its true minimum at $\phi = M_\star$. Hence the name locked inflation. In this configuration H_0 is the only contribution to the Energy Momentum Tensor of the fluid, and is constant, thus generating an exponential expansion. Φ's oscillatory amplitude will be redshifted and eventually the minimum in the effective ϕ's potential reappears letting it free to roll down, reheating the universe and inflation ends. The generated number of e-folds for the model turns out to be around fifty which is not problematic for the observed flatness because of the initial period of Guth's like inflation. In this model, reheating happens at a very low scale $O(Tev)$ and the usual method of density perturbations that serves as seeds for the today's observed inhomogeneities is disabled because $M \geq H$. A mechanism that can be used is modulated reheating [4]. The idea is to give a dynamical origin (using for example another modulus σ) to the coupling (that could be a Yukawa like coupling) dominating the decay of the inflaton in the reheating period. Doing so, fluctuations of the modulus during inflation are naturally imprinted in the reheating spectrum since reheating will happen at different time in different points of the universe. While the implementation is quite natural [5], the interesting imprint of such a mechanism is the production of non gaussianities with $f_{NL} \sim O(1)$ which is relevant for current experimental measurements of the parameter and is model indipendent. In this work, following the idea in [6], the dynamic of the modulus σ at reheating is taken into account finding an enhancement of f_{NL}. This correction is, however, model dependent.

Many constraints on locked inflation appear in the litterature such as [7] and [8] that seems to rule out the whole parameter space. The problems are mainly three: 1) rolling down to its true vacuum, ϕ might generate an extra period of inflation in the usual way, i.e. satisfying slow roll parameter, thus eliminating all the previous imprints, 2) loop corrections might move Φ's vacuum from zero, making the locking, and hence inflation, eternal, and 3) for certain choices of the parameter space, resonances happen within the first e-fold of inflation, backreacting on the background Φ and ending inflation immediately.

In the present work all these claims are analyzed taking a more general approach by relaxing the parameters of the model and constraining them in a bottom up fashion. In this context, a better condition for the averaging of Φ in the λ coupling (contributing to ϕ's mass) is discussed. It follows from it that $\alpha = M_\star/m_\phi \ll 1$. It is found that even though the previous articles restrict the parameters of the model, the parameter space is not fully ruled out. Moreover, already considering a complex scalar field, instead of a real scalar field for Φ, fully eliminates the problem parametric resonances of point 3). In this case, however, α cannot be assumed to be natural because of constraint 2) . Moreover it is found that, for a wide range of the parameter space, parametric resonances are unavoidable at the end of locked inflation. This, gives, anyway, a very small correction to the number of e-folds. Finally an estimate of the production of topological defects at ϕ's phase transition in an expanding universe is proposed. Due to the low temperature of reheating, Kibble mechanism cannot be applied [9]. Instead, in order to estimate the maximal correlator length of ϕ, a non adiabatic condition of the form $|d\omega/dt|_{t_c} \sim \omega(t_c)^2$ is imposed t_c being the moment when ϕ's minimum is restored. Due to the smallness of α no reasonable amount of topological defects is produced.

Organization

This work is organized as follows: the first two chapters are introductory. In this sense inflation is justified from a phenomenological point of view and the notation is introduced. In particular the second chapter is dedicated to the tools necessary to describe density perturbation. Chapter 3 is dedicated to the discussion of metric perturbation in a Friedmann's universe. In particular the well known δN formalism is derived from scratch. Such a formalism is then implemented in chapter 4 in order to understand the production of non gaussinities in modulated reheating. In this context a new formula is given for f_{NL} which specifies the amount of non gaussianities. A well known result is that, for modulated reheating, such f_{NL} is of order one indipendently of the considered model. In the derivation a new correction to this parameter is given by considering the dynamics of the modulating field during the imprint of density perturbation. Such a correction is indeed model dependent.

Chapter 5 is fully dedicated to explain locked inflation. In this context the decay rate for the nucleation of our universe is derived. The Hawking-Moss instanton responsible for such a process is understood in the context of stochastic inflation where the motion of the field undergoing the phase transition is described as a brownian-like motion. Hence the correspective Fokker-Planck equation is derived.

Chapter 6 focuses on the possible constraints restricting the parameter space of locked inflation in view of some consistency and phenomenological issues. It is shown, in particular, that the parameter space is not fully ruled out.

Chapter 1

Inflationary Universe

1.1 Shortcomings of the Big Bang Model

Modern cosmology is based on two observational facts: i) The universe is expanding and ii) On scales larger than 300 million light years the matter distribution is homogeneous and isotropic. The average spacetime is then described by the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{1.1}$$

where $k = 0, \pm 1$ for flat, positively curved and negatively curved spacelike 3-hypersurfaces and $c = 1$. On large scales, matter can be characterized by a perfect fluid with energy density ρ, pressure p and 4-velocity u^α. Its energy momentum tensor (EMT)

$$T^\alpha_\beta = (\rho + p)u^\alpha u_\beta - p\delta^\alpha_\beta \tag{1.2}$$

and $p = p(\rho) = w\rho$ is the equation of state.

The unknown functions $a(t)$ and $\rho(t)$ can be obtained through Einstein's equation $G^\alpha_\beta = R^\alpha_\beta - 1/2\delta^\alpha_\beta R = 8\pi G T^\alpha_\beta$ using the above metric and EMT. However a less rigorous derivation can be used in this context. According to the first principle of thermodynamics we have that in a volume $V \propto a^3$:

$$dE = -pdV \Rightarrow d\rho = -3(\rho + p)d\log a \Rightarrow \dot{\rho} = -3H(\rho + p) \qquad (1.3)$$

where $H = \frac{d\log a}{dt}$. The obtained relation is equivalent to the energy conservation $\nabla_\alpha T^{0\alpha} = 0$ [1]. As for the diagonal components of Einstein's equation one finds, using also (1.3)

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \qquad (1.4)$$

Equation (1.3) and (1.4) are known respectively as first and second Friedmann equations. Combining them together yields

$$\dot{H} = -4\pi G(\rho + p) \qquad (1.5)$$

Explicit solutions can be easily found for constant w [10]: $\rho \propto a^{-3(1+w)}$ and $a \propto t^{2/3(1+w)}$. Since for a radiation dominated universe ($w = 1/3$) and for a matter dominated universe ($w = 0$) the scales factor goes respectively as a^{-4} and a^{-3} it follows that at early times the universe was radiation dominated. As we rewind earlier and earlier in time we finally arrive to a singularity since for $t \to 0$ also $a \propto t^{1/2}$ goes to zero. Notice that it is not a coordinates singularity since, in this limit, also the scalar curvature becomes singular [11]. Hence the introduction of the Big Bang model. Alas, already at planckian time $t \sim 10^{-44}s$, non perturbative effects of gravity should be relevant spoiling the above approximation. However, even if we believe quantum gravity effects to be perturbative on subplanckian length, from observational data, we are lead to fine tuning problems on the initial conditions of our universe. Before moving to the next section notice that for a radiatio- like fluid the energy density $\rho \simeq T^4$ (where prefactors of $O(1)$ are ignored). It follows from Friedmann equation that $H^2 \propto a^{-4} \propto T^4$ i.e. $a \propto T^{-1}$.

1.1.1 Horizon and flatness problems

We briefly discuss and adress the fine tuning problem that arise without assuming an inflationary period. In principle one could argue that these are not problems. This is because the universe start expanding from scales of (or close to) the planckian scale. Hence a full theory of gravity would be required in order to adress the problem of initial conditions. However these initial conditions reflect a balancing condition between kinetic and potential energy. Through the inflationary scenario, the energetic distribution can be random. That means that the current state of our universe will be an attractor indipendent of whatever initial conditions hence making the theory of inflation not only predictive but also necessary.

Homogeneity problem

The present horizon scale is $l_0 \sim 10^{28} cm$. The original patch from which this originated is larger due to the ratio of the scale factor a_i/a_0 (where the subscript i stands for initial). Hence $l_i \sim l_0 a_i/a_0$. A causual region at the initial moment will be roughly $l_c \sim l_i$, giving

$$\frac{l_i}{l_c} \sim \frac{l_0}{l_i}\frac{a_i}{a_0} \tag{1.6}$$

taking as initial the moment where the universe had planckian size we can rewrite the ratio between the rescaled patch and the supposed casually connected patch as

$$\frac{l_i}{l_c} \sim \frac{10^{28}cm}{10^{-33}cm}\frac{T_0}{T_p} \sim 10^{28} \tag{1.7}$$

where $l_p \sim 10^{-33}cm$ and $T_p \sim 10^{32}K$ and $T_0 = 2.73K \sim 1K$ have been used. This means that in the early universe were present approximately 10^{84} casually disconnected regions. From cosmological observations, the fractional variation of energy distribution is $\delta\rho/\rho \sim 10^{-4}$ which seems unlikely for casually disconnected patches. Note that if we take $\dot{a} \sim a/t$ we get

$$\frac{l_i}{l_c} \sim \frac{\dot{a}_i}{\dot{a}_0} \tag{1.8}$$

Flatness Problem The cosmological parameter is defined as $\Omega(t) \doteq \frac{\rho(t)}{\rho^{cr}(t)}$ with $\rho^{cr} = \frac{3H^2}{8\pi G}$. This parameter tells us about the spatial flatness of our universe. It is bigger than one for a closed universe, smaller than one for an open universe and exactly one for a flat universe. Friedman equation can be rewritten as [1]

$$\Omega(t) - 1 = \frac{k}{(Ha)^2} \tag{1.9}$$

implying

$$\Omega_i - 1 = (\Omega_0 - 1)\left(\frac{\dot{a}_i}{\dot{a}_0}\right)^2 \leq 10^{-56} \tag{1.10}$$

where the estimate from (1.8) was used. Thus the cosmological parameter, corresponding to the splitting between kinetic and potential energy, must be extremely close to unity (up to $O(10^{-56})$). Hence the flatness problem.

1.2 Inflation as an elegant solution

In order to see how a period of accelerated expansion can solve these problems first one has to introduce the conformal time τ

$$\tau \doteq \int \frac{dt}{a} = \int \frac{da}{Ha^2} = \int_0^a d\log\tilde{a}\,\frac{1}{H\tilde{a}} \tag{1.11}$$

where $(aH)^{-1}$ is the comoving Hubble radius. The casuality interpretation of this formula is the following [12]: if particles are separated by distances greater than τ, they never could have communicated; if they are separated by distances grater than $(aH)^{-1}$ they cannot talk to each other now. Notice that during inflation, i.e. a period of accelerated exponential expansion, the initial comoving Hubble radius decreases as $H_I^{-1}e^{-N}$ where N is the number of e-foldings. Then what we would like to have is the present horizon H_0^{-1} to be reduced, through inflation, to a value smaller than H_I^{-1}

$$H_0^{-1}\frac{a_e}{a_0}e^{-N} = H_0^{-1}\frac{T_0}{T_e}e^{-N} \leq H_I^{-1} \tag{1.12}$$

where the subscript e characterizes the end of inflation. From this we can read off the following constraint for N

$$N \geq \log\left(\frac{T_0}{H_0}\right) - \log\left(\frac{T_e}{H_I}\right) \simeq 67 - \log\left(\frac{T_e}{H_I}\right) \tag{1.13}$$

from which we deduce we need at least 70 e-folds to solve the horizon problem. Notice that changing the temperature at the end of inflation, or the initial Hubble radius might lower the required number of e-folds. For this reason sometimes in the litterature simply a number of e-folds bigger than fifty is required [13].

Finally to solve the flatness problem we know that the initial moment of equation (1.10) is the end of inflation i.e. $\Omega_e - 1 \sim O(10^{-56})$ (necessary to observe an $\Omega_0 \sim 1$). Let us now compare Ω_e with Ω_i at the beginning of inflation:

$$\frac{\Omega_e - 1}{\Omega_i - 1} = \left(\frac{\dot{a}_i}{\dot{a}_e}\right)^2 = \left(\frac{a_i}{a_e}\right)^2 \simeq e^{-2N} \tag{1.14}$$

where in the last passage the fact that during inflation H is almost constant has been used.

Taking Ω_i of order unity (which corresponds to a random distribution $\delta\rho_i/\rho_i \sim O(1)$ or to an equal distribution between kinetic and potential energy [1]), fifty e-folds are enough to solve the flatness problem. The prediction $\Omega_0 = 1$ can legitimately be elevated into a prediction of inflation. Notice that the actual experimental value of the cosmological parameter is $\Omega_0^{exp} = 1.002 \pm .005$ expressing a preference for an open universe. However a "too long" period of inflation (e.g. $O(10^3)$ e-folds) might flatten this parameter too much with respect to the observed one.

1.2.1 Conditions for inflation

The first Friedman equation can be stated from (1.4) and (1.5) as

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p)a \tag{1.15}$$

implying that inflation is possible if $\rho + 3p < 0$ (violating strong energy condition). This implies that comoving hubble radius is shrinking i.e.

$$\frac{d}{dt}(aH)^{-1} = -\frac{\ddot{a}}{\dot{a}^2} < 0 \tag{1.16}$$

For later convenience (1.16) can be rewritten as

$$\frac{d}{dt}(aH)^{-1} = -\frac{\dot{a}H + a\dot{H}}{(aH)^2} = -\frac{1}{a}(1-\epsilon), \quad \epsilon \doteq -\frac{\dot{H}}{H^2} = -\frac{d\log H}{dN} > 0 \quad (1.17)$$

with $dN = d\log a = Hdt$ measuring the number of e-folds. In order for the acceleration to stay positive it is necessary to have $\epsilon < 1$. In order to solve the cosmological problems we also want this to be true for sufficiently long time (at least $N \sim 50$ e-folds). Thus we introduce a second parameter

$$\eta \doteq \frac{\dot{\epsilon}}{H\epsilon} = \frac{d\log\epsilon}{dN} \quad (1.18)$$

Requiring $\eta < 1$ will ensure the ϵ condition to last long enough. Notice that for the limiting case $H = const$ (or $\epsilon = 0$) one obtains pure De Sitter expansion $a \sim e^{\Lambda t}$. An inflationary universe, in this sense, can be thought as a quasi De Sitter spacetime with slowly varying Hubble parameter.

Next a discussion on the microscopical way to obtain such a condition is necessary.

1.2.2 False Vacuum Inflation

In 1981 Guth [3] suggested a description of the exponential expansion of the universe through supercooled vacuum state $\phi = 0$. His scenario was based on three propositions [14]:

- The universe initially expands in a state with superhigh temperature and restored symmetry, $\phi_{(T)} = 0$
- $V(\phi)$ still has a local minimum at $\phi = 0$ also at low temperature. In this way the field will be trapped in this local minimum for a long

period generating exponential expansion through the vacuum energy $\rho = V(0)$

- Inflation continues until a first order phase transition nucleates bubbles where $\phi = \phi_0$ is in the global minimum. The universe heats up due to bubble-wall collisions, and its subsequent evolution is described by the hot universe theory.

However, as noted by Guth himself [3], collisions of the walls of very large bubbles lead to an unacceptable level of inhomogeneities and anisotropies (no graceful exit). Cosmologists hence introduced the new inflationary scenario where the last two Guth's propositions are dropped. Even though this old inflationary model is ruled out, it can serve as a mean to produce the proper initial conditions for other inflationary model such as the proposed New Old Inflation [2] which will be discussed in detail later.

Decay of the metastable Vacuum

Suppose to have a potential with a local vacuum at $\phi = 0$ and a global minimum at $\phi = \phi_0$ separated by a barrier. Assume also the potential to be normalized in such a way that $V(0) = 0$. Denote $V(\phi_0) = -e$. The metastable vacuum decay happens via quantum tunnelling which can be described properly by an instanton. As already said, we will ignore temperature effects (e.g. decay via sphaleron). The action of the scalar field can be written as

$$S = \int \left(\frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\partial_i \phi)^2 + V(\phi) \right) d^3x dt \tag{1.19}$$

Notice that the space-measure has been kept differentiated by the time-measure on purpose. In order to have quantum tunnelling one has to reinterpret the potential as being a functional potential i.e. $\tilde{V}(\phi) \doteq \int d^3x V(\phi)$. This identification correspond to have a wave functional $\Psi(\phi(x,t))$ instead of the usual quantum mechanical wave function $\psi(x,t)$. Hence, the energy of the true vacuum is $\tilde{V}(\phi_0) = -e \times V$, V being the volume where the new phase is active. In this sense, an instanton can be understood as the classical solution with imaginary time describing the physical process of the field interpolating within the two vacua of the theory via quantum tunnelling (similarly to the quantum mechanical case). Thus, with euclidean metric we have

$$\ddot{\phi} + \Delta\phi = V,\phi \tag{1.20}$$

From quantum mechanics we know that for a tunnelling process the probability is [15]:

$$P_I \propto \exp\{-S_I\} \tag{1.21}$$

with S_I being the instantonic action. Analogously for our bubble we get a decay rate per unit time and unit volume:

$$\Gamma \simeq A \exp\{-S_4(\phi)\} \tag{1.22}$$

where the euclidean action is given by

$$S_4(\phi) = \int d^4x \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)\right] \tag{1.23}$$

and A is a complicated prefactor involving functional determinants calculated in [16]. On dimensional ground, the decay rate has dimension $[L]^{-4}$. Hence $A \sim O(R_I^{-4}, V_{,\phi\phi}^2, ...)$.

Solution under thin wall approximation

In principle, in order to find Γ, one should sum over all possible solutions of (1.20). Symmetry is a good indicator of which solutions minimize the Euclidean action. In particular, in this case, a possible symmetry is $O(4)$:

$$\phi = \phi(r = \sqrt{x^2 + t^2}) \tag{1.24}$$

In spherical coordinates the laplacian is [17]

$$\Delta f = \frac{\partial_i}{\sqrt{|g|}} \left(\sqrt{|g|} g^{ij} \partial_j f \right) \tag{1.25}$$

giving for dimension $d = 4$, $\Delta f = f'' + 3/rf' + \Delta_{angular} f$ with the prime denoting derivative with respect to r. Thus (1.20) becomes

$$\phi'' + \frac{3}{r}\phi' - V_{,\phi} = 0 \tag{1.26}$$

with boundary condition $\phi(\infty) = 0$ and, in order to avoid a singularity at the bubble center $\phi'(0) = 0$.

Multiplying (1.26) by ϕ' and integrating over the radial coordinates the first integral is:

$$\frac{1}{2}(\phi')^2 - V = \int_r^\infty \frac{3}{\rho}(\phi(\rho)')^2 d\rho \tag{1.27}$$

where $\phi(\infty) = 0$ has been used on the left side.

The other boundary condition, $\phi'(0) = 0$, gives:

$$-V(\phi(0)) = \int_0^\infty \frac{3}{\rho}(\phi(\rho)')^2 d\rho \tag{1.28}$$

Now, denote with R the radius of the bubble and with a the thickness of the wall. ϕ will be constant both inside and outside, while ϕ' will change only in a region around R of thickness a. This implies that the biggest contribution to the integral in (1.28) happens near the layer. Since $a \ll R$ the right handside of (1.27) is suppressed with respect to the left side yielding:

$$(\phi')^2 \simeq 2V \tag{1.29}$$

Plugging this approximation in (1.28) gives

$$-V(\phi(0)) \simeq \frac{3}{R}\int_0^{\infty} (\phi')^2 dr \simeq \frac{3}{R}\int_0^{\phi(0)} \sqrt{2V}d\phi \tag{1.30}$$

the integral is the surface tension of the bubble

$$S \doteq \int_0^{\infty} (\phi')^2 dr \tag{1.31}$$

We are now ready to evaluate the instanton action (1.23) which, in spherical coordinates, is

$$S_4(\phi) = 2\pi^2 \int \left(\frac{(\phi')^2}{2} + V\right) r^3 dr \tag{1.32}$$

The first term will just be equal to the surface tension. Since this varies in a small layer r^3 can be taken to be equal to R and constant. Most of the contribution to the second term will come from inside the bubble. Hence $V = V(\phi(0)) = const$. Hence one obtains:

$$S_4(\phi) \simeq 2\pi^2 SR^3 + \frac{\pi^2}{2} V(\phi = 0) R^4 \tag{1.33}$$

From (1.30) and the surface tension definition we have $-V(\phi = 0) \simeq 3S/R$. Substituting the radius of the bubble in (1.29) one gets

$$S_4 \simeq \frac{27\pi^2 S^4}{2|V(\phi(0))|^3} \tag{1.34}$$

in our case the action has the minimal value for $|V(\phi(0))| = e$. In this case inside the bubble the field has value ϕ_0 and

$$\Gamma \simeq A \exp\left(-\frac{27\pi^2 S^4}{2e^3}\right) \tag{1.35}$$

Observations

- After bubble nucleation, the found solution can be analytically continued back to the original time giving a solution of the form $\phi(x\sqrt{^2 - t^2})$ manifestly Lorentz invariant and describing the expansion of the bubble approaching the speed of light.

- The computation was done in Minkowski background. It is however clear that in Guth's inflation, the process should be described in a De Sitter background. If the curvature near the local minimum is small compared to H^2 gravitational effects cannot be ignored. A first Euclidean theory of tunnelling in de Sitter space was developed Coeleman and De Luccia [18]. The problem is that potentially both ϕ and $g_{\mu\nu}$ experience a quantum jump. In order to explore tunnelling in such a situation a full solution of the Schrödinger equation allowing for particle with complex momentum is necessary (and also complicated). In the original paper, however, they relied on the assumption $m^2 \gg H^2$ with m^2 being the curvature of the field sitting in the false vacuum meaning that gravity will only give a small correction to the Minkowski formulation.

A solution where the process is guided by gravitational processes, i.e. with $m^2 \ll H^2$, was proposed by Hawking and Moss [19] . Their solution for the transition probability was

$$P \sim A \exp\left(-3\frac{M_p^4}{8}\left(\frac{1}{V(0)} - \frac{1}{V(\phi_1)}\right)\right) \tag{1.36}$$

with ϕ_1 being the field at the top of the hill. The formula was not immediately fully understood by the community and generated much debate [20]. What will be interesting later in the present thesis, is not (1.36) but rather the fact that the field nucleated in the bubble starts with initial condition $\phi = \phi_1$ at the maximum of the potential. This was understood later thanks to the Starobinski's stocasthic approach to inflation described [20]. In order to understand this, a description of the fluctuations of a field in a De Sitter universe is necessary.

1.3 Slow-Roll Inflation

Consider the EMT of a scalar field ϕ with potential V [21] :

$$T^\alpha_\beta = \phi^{,\alpha}\phi_{,\beta} - \left(\frac{1}{2}\phi^{,\gamma}\phi_{,\gamma} - V(\phi)\right)\delta^\alpha_\beta \tag{1.37}$$

It is possible to simulate the EMT of a perfect fluid (1.2) through the identification

$$\rho \doteq \frac{1}{2}\phi^{,\gamma}\phi_{,\gamma} + V(\phi), \quad p \doteq \frac{1}{2}\phi^{,\gamma}\phi_{,\gamma} - V(\phi), \quad u^\alpha \doteq \phi^{,\alpha}/\sqrt{\phi^{,\gamma}\phi_{,\gamma}} \tag{1.38}$$

If the field is homogeneous (which is the case in a De Sitter spacetime because of the "no hair" theorem), i.e. $\phi_{,i} = 0$, then

$$\rho = \frac{(\dot{\phi})^2}{2} + V(\phi), \quad p = \frac{(\dot{\phi})^2}{2} - V(\phi) \tag{1.39}$$

Another way to justify the fact that the field is homogeneous is that the wavelength of each mode gets stretched during expansion, while the quadratic mass term does not (reflected in the fact that in the classical eom the laplacian term is divided by a^2 [22]).

The homogeneous field will then satisfy the Klein Gordon equation which is

$$\phi^{;\alpha}_{;\alpha} + V_{,\phi} = 0 \tag{1.40}$$

under FRW metric and using the fact that the contracted Christoffel symbol is $\Gamma^{\alpha}_{\mu\alpha} = \partial_\mu \log \sqrt{|g|}$ where g is the determinant of the metric, (1.40) can be rewritten as

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0 \tag{1.41}$$

The Friedman equation reads

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}(\dot{\phi})^2 + V(\phi)\right) \tag{1.42}$$

The continuity equation follows from (1.41) and (1.42) and is

$$\dot{H} = -\frac{8\pi G}{2}(\dot{\phi})^2 \tag{1.43}$$

Thus the slow-roll parameter becomes

$$\epsilon = -\frac{\dot{H}}{H^2} = 8\pi G\frac{\frac{1}{2}(\dot{\phi})^2}{H^2} = \frac{1}{2}\frac{(\dot{\phi})^2}{V} \tag{1.44}$$

requiring $\epsilon \ll 1$ implies $(\dot{\phi})^2 \ll V$. Friedmann equation thus reads

$$H^2 = \frac{8\pi G}{3} V(\phi) \tag{1.45}$$

Defining the quantity $\delta \doteq -\ddot{\phi}/(H\dot{\phi})$ one finds for the second slow roll parameter [12] $\eta = 2(\epsilon - \delta)$. Since $\eta \ll 1$ we have $\delta \ll 1$ implying that we can ignore the oscillatory term in the equation of motion of the ϕ field. Hence

$$3H\dot{\phi} \simeq -V_{,\phi} \tag{1.46}$$

(1.46) together with (1.45) gives

$$\epsilon = 8\pi G \frac{\frac{1}{2}(\dot{\phi})^2}{H^2} \simeq \frac{M_p^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \doteq \epsilon_V \tag{1.47}$$

where ϵ_V is the ϵ parameter under slow roll approximation and is known as the potential slow roll parameter and I set $8\pi G \doteq M_p^{-2}$. Similarly η_V can be defined, yielding [12]

$$|\eta_V| \doteq M_p^2 \frac{|V_{,\phi\phi}|}{V} \tag{1.48}$$

and the slow roll condition $|\eta_V| \ll 1$.
Clearly inflation will end when this parameters becomes of order 1. Thus the number of e-folds can be computed in terms of this parameters (see [12]).

1.3.1 On the meaning of a microscopic explaination of inflation

Recent observations have ruled out the simplest model of inflation: the massive scalar model [13]. This should be no surprise. Inflation is perfectly justified in terms of a perfect fluid with the approximate equation of state $p \simeq -\rho$. In this sense, the slow roll parameters are nothing but a mere tool to impose this on Friedmann's equations. The failure of a microscopic model is no surprise in the sense that the potential generating the slow roll conditions is based on a fundamental theory which is simply not yet full known. The failing of a model just means the failure of its parameters to parametrize our ignorance about this.

Since Guth's first model, many other models were proposed [13]. It is then very important to understand what are the main features of the various classes of inflationary models, and what their imprints might be on current and future observations in order to constrain them.

Hence the program of the following thesis. First of all a model with a potential where the potential slow-roll conditions are not satisfied will be introduced. After that the problem of density perturbations is introduced focusing on the possible imprints of the above model. To match observations, the implementation of a newly proposed mechanism known as modulated reheating [4] will be necessary.

Chapter 2

Density Perturbation

2.1 Preliminary Computation

Consider a massless scalar field living on a Minkowski background. After quantization, the field can be expressed in terms of modes as follow [21]

$$\int_k a_k^\dagger(t) e^{-ikx} + a_k(t)^\dagger e^{ikx} \tag{2.1}$$

with $\int_k$ being the three dimensional measure.

We want now to measure quantum fluctuations. In order to do that we introduce an apparatus of size l. Quantum fluctuations will be due to the two-point correlator

$$\langle 0|\phi(x)\phi(y)|0\rangle \tag{2.2}$$

where x and y have a space-distance of roughly the size of our apparatus. Then we have

$$\langle 0|\phi(x)\phi(y)|0\rangle = \int_k \int_{k'} \left\langle a_k a_{k'}^\dagger \right\rangle e^{ikx - ik'y} \tag{2.3}$$

the operator part yields a $\delta^3(k-k')$ function. After integration and passing to polar coordinates, up to numerical factors (assuming equal time):

$$\int d|k| \frac{k^2}{|k|} \frac{\sin |k| l}{|k| l} \tag{2.4}$$

where we can take the integral up to $|k| \sim O(1/l)$ i.e. the natural resolution of the apparatus. The contribution to the integral in the subset of integration where $|k|l \ll 1$ scales like

$$I \propto k^2 \ll \frac{1}{l^2} \tag{2.5}$$

because the last factor of (2.4) can be taken to be of order one. The maximum contribution to the integral comes, instead, from the subset where $|k| \sim l^{-1}$. In this subset, again, the last term in (2.4) can be taken to be simply a number. Hence also in this case the integral is proportional to $k^2 \sim l^{-2}$. The fluctuations then are of the order of the size of the box.

In the next paragraph the De Sitter case is analyzed and a rigorous method is presented.

2.2 Fluctuations on De Sitter

First of all we have to rewrite our mode expansion, and manipulate it a bit in order to proceed:

$$\delta\phi(\vec{x}, t) = \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \delta\phi_k(t) \tag{2.6}$$

this mode has to satisfy the classical equation of motion (1.40). The extra $\vec{x}$ dependance is due to the fact that fluctuations spoil isotropy locally. Hence the explicit equation of motion will have an extra laplacian term suppressed by a^2 with respect to (1.41). This fact, in turn give, for the mode equation:

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \frac{k^2}{a^2}\delta\phi_k = 0 \tag{2.7}$$

Notice that $k \approx 1/\lambda$ is divided by a. Multiplication by this factor makes it physical $k/a \approx (\lambda a)^{-1} = \lambda_{phys}^{-1}$ since the scale factor a^2 is the metric element. Equation (2.7) admits two regimes (as it happens for Jeans theory): one when $\lambda_{phy} \gg H^{-1}$ and one when $\lambda_{phy} \ll H^{-1}$ respectively called super-Hubble and sub-Hubble wavelengths. In the first case, the last term of (2.7) can be ignored giving a constant solution. In the second case the gravitational term can be dropped, giving an oscillatory behaviour. Matching the two solutions at $\lambda_{phys} \simeq H^{-1}$ gives the final result. Before doing that it is necessary to redefine the field as $\delta\phi_k \doteq \delta\chi_k/a$ and to move to conformal time τ:

$$\tau \propto \int e^{-Ht}dt \tag{2.8}$$

implying $a = -1/(H\tau)$ with τ negative. Rearranging the equation of motion (2.7) becomes:

$$\delta\chi_k'' + (k^2 - \frac{a''}{a})\delta\chi_k = \delta\chi_k'' + (k^2 - \frac{2}{\tau^2})\delta\chi_k = 0 \tag{2.9}$$

on sub-hubble scales the τ term can be dropped and it follows:

$$\delta\chi_k = \frac{e^{-ik\tau}}{\sqrt{2k}} \tag{2.10}$$

at horizon crossing (i.e. $k = aH$), $|\delta\phi_k| = |\delta\chi_k|/a = 1/(\sqrt{2k}a) = H/\sqrt{2k^3}$ which remains constant outside the horizon. Now (2.3) will be proportional to

$$k^3|\delta\phi_k|^2 \propto H^2 \tag{2.11}$$

Proper menagement of the prefactors yields an extra $1/(2\pi)^2$ in the result [10]. Notice the analogy with the flat case. H^{-1} can be interpreted as the size l of our box. This has a nice physical interpretations. On distances bigger or equal to H^{-1}, the De-Sitter background becomes not neglegible making our notion of particles no longer reliable. This is because in a De Sitter universe the cosmological horizon also corresponds to the causal horizon, meaning that wavelength longer than that have no casual structure.

2.2.1 Massive Scalar Field in De Sitter

In the case of a massive field ϕ, H will not be the only size of the system and fluctuations will be shorter due to the fact that the mass term is screening them.

The EOM with a mass term are [22]:

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left(m^2 + \frac{k^2}{a^2}\right)\delta\phi_k = 0 \tag{2.12}$$

on the Super-Hubble scale the oscillatory term due to k^2 can be ignored. Hence, for $m \leq H$ the solution is given by:

$$\delta\phi_k \simeq Ae^{\mu t} \tag{2.13}$$

with μ being

$$\mu = -\frac{3H}{2} + \sqrt{\frac{9}{4}H^2 - m^2} \tag{2.14}$$

In (2.13) the other decaying mode can be dropped as it becomes negligible after few e-folds. A is a constant of integration that can be obtained by matching with the sub-Hubble solution at horizon crossing. The sub -horizon solution reads, as before, at horizon crossing $|\delta\phi_k| \simeq H/\sqrt{2k^3}$ which give us the initial value of A. To evaluate (2.13) remember that $t = -1/H \ln -\tau$ and notice that at horizon crossing $k\tau = 1$. Hence one obtains:

$$|\delta\phi_k| \simeq \frac{H}{\sqrt{2k^3}} \left(\frac{k}{aH}\right)^{\mu} \tag{2.15}$$

note that in the limit $m = 0$ the massless solution is recovered. Hence, in a De Sitter universe, the massive perturbations are no longer constant but present a slight dependance (a small tilt towards blue). For the slow parameter $\eta = m^2/3H^2 \ll 1$ we have $\mu = \eta$ by μ.

Interestingly enough this is opposite to what really happens during inflation. Indeed, the fact that H is no longer constant generates a tilt towards red (in the opposite direction). Before showing that, it is necessary to introduce some physical quantitites.

2.2.2 Power Spectrum and Spectral Index

The power spectrum for $\delta\phi_{\vec{k}}$ is defined as

$$\left\langle 0|\delta\phi_{\vec{k_1}}\delta\phi_{\vec{k_2}}|0\right\rangle \doteq (2\pi)^3\delta^3(\vec{k_1} + \vec{k_2})|\delta\phi_{\vec{k}}|^2 \tag{2.16}$$

in turn, the power spectrum in position space is then:

$$\left\langle 0|\delta\phi(\vec{x},t)^2|0\right\rangle = \int \frac{d^3\vec{k}}{(2\pi)^3}\delta\phi_{\vec{k}}\delta\phi_{-\vec{k}} = \int \frac{d^3\vec{k}}{(2\pi)^3}|\delta\phi_{\vec{k}}|^2 = \int \frac{dk}{k}P(k) \tag{2.17}$$

where the power spectrum of $\delta\phi(x,t)$ has been defined as

$$P(k) = \frac{k^3}{2\pi^2}|\delta\phi_{\vec{k}}|^2 \tag{2.18}$$

Finally the spectral index $n_{\delta\phi}$ is defined as

$$n_{\delta\phi} = \frac{d\ln P_{\delta\phi}}{d\ln k} \tag{2.19}$$

$n = 1$ for a scale invariant spectrum and smaller (bigger) than one for a red (blue) tilt.

2.2.3 Massive Case in Inflation

The fact that during inflation H is changing causes a small change in the effective mass of the field. Making the ansatz:

$$a(\tau) = -\frac{1}{H}\frac{1}{\tau^{1+\epsilon}} \tag{2.20}$$

with ϵ slow roll parameter, turns out to satisfy the condition $\dot{H} = -\epsilon H^2$. This change give, in the equation of motion (2.9), an extra term from a''/a which is

$$\frac{a''}{a} \simeq \frac{1}{\tau^2}(2 + 3\epsilon) \tag{2.21}$$

hence the exponential factor μ is modified to $\tilde{\mu} = \mu - \epsilon$. In the limit for small η parameter, $\tilde{\mu} \simeq \eta - \epsilon$. Hence the spectral index $n_{\delta\phi}$ is

$$n_{\delta\phi} - 1 = 2\eta - 2\epsilon \tag{2.22}$$

which can be negative.

The experimental result for the spectral index today is $n_s \simeq 0.96$. This tells us that in every phenomenological sensible inflationary model the combination of the slow roll-parameters $\eta - \epsilon \sim O(10^{-2})$. Moreover, notice that such a spectrum does not necessarily need to be prouced by the inflationary field, but it can be produced by any spectator field which during inflation is satisfying the slow roll conditions. This will be the case for locked inflation.

Chapter 3

Quantum fluctuation during inflation: density perturbations and δN formalism

3.1 Quantum Fluctuation

The theory of cosmological perturbations is a cornerstone of modern cosmology and is used to describe the observed structure formation in our universe. Fluctuations of the early field are stretched during inflation outside the Hubble patch. These perturbations, which have a quantum origin, enter back in the horizon during the later expansion of the universe, being the source of the gravitational field that lead to the formation of the galaxies.

During inflation, the inflaton field ϕ is dominating. Hence, a fluctuation in ϕ generates a fluctuation in the EMT:

$$\delta\phi \rightarrow \delta T_{\mu\nu} \rightarrow \delta_g \mu\nu \tag{3.1}$$

where the second arrow is due to Einstein's equation. A change in $\delta g_{\mu\nu}$ generates a backreaction in the evolution of the ϕ field through changes in the Klein Gordon (KG) equation, implying:

$$\delta\phi \leftrightarrow \delta g_{\mu\nu} \tag{3.2}$$

Consider the following field expansion $\phi(\vec{x},t) = \phi_0(t) + \delta\phi(\vec{x},t)$ where ϕ_0 is a solution to the KG equation. Then the equaiton for the perturbation $\delta\phi(\vec{x},t)$ is

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \frac{\nabla^2}{a^2}\delta\dot{\phi} + V_{,\phi\phi}\delta\phi = 0 \tag{3.3}$$

differentiating KG equation yields

$$\left(\dot{\phi}_0\right)^{\cdot\cdot} + 3H\left(\dot{\phi}_0\right)^{\cdot} + V_{,\phi\phi}\dot{\phi}_0 = 0 \tag{3.4}$$

where H is assumed to be approximately constant during inflation. Assuming $\frac{(\vec{k})^2}{a^2} \ll H$ i.e. dropping the gradient term in (3.3) implies that $\delta\phi$ and $\dot{\phi}_0$ are solution to the same equation of motion. This implies that the two solutions are related to each other by a constant depending on time i.e.

$$\delta_\phi = \dot{\phi}_0\delta t(\vec{x}) \tag{3.5}$$

implying that $\phi(\vec{x},t) = \phi_0(t + \delta t(\vec{x}), \vec{x})$. These fluctuations will, in turn, change the metric.

3.2 The problem of gauge invariance

Proper gauging is important in order to avoid the production of ficticious density perturbation. In order to understand this, consider a energy density distribution in a coordinate system of the form:

$$\rho(\vec{x}, t) = \rho(t) \tag{3.6}$$

Perform now a change of coordinates

$$t \to \tilde{t} = t + \delta t(\vec{x}, t) \tag{3.7}$$

under transformation (3.7) the energy density will transform as

$$\rho \to \tilde{\rho}(\tilde{t}, \vec{x}) = \rho(t(\tilde{t}, \vec{x})) \tag{3.8}$$

hence for infinitesimal time shift:

$$\rho(t) = \rho(\tilde{t} - \delta t(\vec{x}, t)) = \rho(\tilde{t}) - \frac{\partial \rho}{\partial t}\delta t = \rho(\tilde{t}) + \delta\rho(\vec{x}, t) \tag{3.9}$$

In the new coordinate system, it looks like there is a background energy depending only on time $\tilde{t}$ on top of which a linear perturbation is acting. Clearly its origin is ficticious and is only due to the coordinate choice.

3.2.1 Perturbation Classification

Consider the following perturbed metric

$$ds^2 = \left[{}^{(0)}g_{\mu\nu} + \delta g_{\mu\nu}\right] dx^\mu dx^\nu \tag{3.10}$$

where the unperturbed one is assumed to be the FRW metric:

$${}^{(0)}g_{\alpha\beta} dx^\alpha dx^\beta = a^2(\tau)\left(d\tau^2 - d\vec{x}^2\right) \tag{3.11}$$

We expect to have three propagating degrees of freedom: two from gravity (massless spin two theory) and one from the scalar field (inflaton). The metric has three possible type of perturbations: scalars, vectorial and tensorial.

Since $g_{\mu\nu}$ is symmetric it has $4 \cdot 5/2 = 10$ components. A way to decompose them properly is to let us be guided by symmetries. The background is homogeneous and isotropic hence we have invariance under rotation and translation. Under these symmetries δg_{00} behaves as a scalar, δg_{0i} as a vector and δg_{ij} contains scalar, vectorial and tensorial parts. By Helmholtz decomposition theorem vectors can be decomposed into two components: a scalar and a transverse vector. Thus:

$$\delta g_{00} = 2a^2\phi, \quad \delta g_{0i} = a^2 \left(B_{,i} + S_i\right) \tag{3.12}$$

where $S^i_{,i} = 0$. The transversatility condition implies that the vector S has 2 indipendent components. The tensorial part decomposition is trickier. The trace part is invariant under rotations, and behaves as a scalar. Moreover there is a tensorial part responsible for gravitational waves. Finally in the most general term it is possible to add a vector term (which can be decomposed in a scalar and a transverse vector like term). Hence:

$$\delta g_{ij} = a^2 \left(2\psi\delta_{ij} + 2E_{,ij} + F_{i,j} + F_{j,i} + h_{ij}\right) \tag{3.13}$$

with $F^i_{,i} = 0$ and

$$h^i_i = h^i_{j,i} = 0 \tag{3.14}$$

If we get ten parameters in δg, then our decomposition is enough and correct in order to describe the metric fluctuations:

- Scalars: ϕ, B, ψ, E which give four components

- Vectors: S, F which are longitudinal, yielding other four components

- Tensors: h_{ij} which is a 3×3 symmetric tensor hence having $3\times4/2 = 6$ components. Other four components are killed by condition (3.14) giving in total $6-4=2$ components.

summing them all up together we get ten components making the decomposition correct.

3.2.2 Behaviour of the perturbations under gauge transformation

Gauge invariance can also be understood as invariance under local coordinates diffeomorphism. Consider the following coordinate transformation:

$$x^\alpha \to \tilde{x}^\alpha = x^\alpha + \xi^\alpha \tag{3.15}$$

with ξ being an infinitesimal local parameter. A generic quantity Q transforms under the coordinate transformation (3.15) as follow:

$$\delta\tilde{Q} = \delta Q + \mathfrak{L}_\xi Q_0 \tag{3.16}$$

Using (3.16) the transformation for both scalar, vector and tensor perturbation is easily found. However we want to derive them in a more physical way.

Scalar Field Consider the following scalar field

$$q(x^\rho) = {}^{(0)}q(x^\rho) + \delta q \tag{3.17}$$

under change of coordinates of the form ξ^α:

$$\tilde{q}(\tilde{x}^\rho) = {}^{(0)}\tilde{q}(\tilde{x}^\rho) + \delta\tilde{q} \tag{3.18}$$

A scalar field has the following property: $q(x^\rho) = \tilde{q}(\tilde{x}^\rho)$. It follows that:

$$\delta\tilde{q} = \tilde{q}(\tilde{x}^\rho) - {}^{(0)}\tilde{q}(\tilde{x}^\rho) = q(x^\rho) - {}^{(0)}\tilde{q}(x^\rho + \xi^\rho) \simeq q(x^\rho) - {}^{(0)}\tilde{q}(x^\rho) - \xi^\rho \partial_\rho {}^{(0)}\tilde{q}(x^\rho) \tag{3.19}$$

the unperturbed solution is not affected by the coordinate transformation i.e. ${}^{(0)}\tilde{q}(\tilde{x}^\rho) = {}^{(0)}q(\tilde{x}^\rho)$ implying for (3.19):

$$\delta\tilde{q} = q(x^\rho) - {}^{(0)}q(x^\rho) - \xi^\rho \partial_\rho {}^{(0)}q(x^\rho) = \delta q - \xi^\rho \partial_\rho {}^{(0)}q \tag{3.20}$$

Vector Field A vector field transforms as follow:

$$\tilde{u}_\alpha(\tilde{x}^\rho) = \frac{\partial x^\beta}{\partial \tilde{x}^\alpha} u_\beta(x^\rho) = u_\alpha(x^\rho) - \partial_\alpha \xi^\beta u_\beta(x^\rho) = {}^{(0)}u_\alpha(x^\rho) + \delta u - \partial_\alpha \xi^\beta u_\beta(x^\rho) \tag{3.21}$$

the left hand side of (3.21) is equal to ${}^{(0)}\tilde{u}_\alpha(\tilde{x}^\rho) + \delta\tilde{u}$. It follows that

$$\delta\tilde{u} = {}^{(0)}u_\alpha(x^\rho) + \delta u_\alpha - {}^{(0)}\tilde{u}_\alpha(\tilde{x}^\rho) - \partial_\alpha \xi^{\beta\,(0)} u_\beta(x^\rho) \tag{3.22}$$

where the last term is the background solution because we are working up to order $O(\xi, \delta)$. As in the scalar case it is possible to rewrite:

$${}^{(0)}\tilde{u}_\alpha(\tilde{x}\rho) = {}^{(0)}\tilde{u}_\alpha(x^\rho + \xi^\rho) \simeq {}^{(0)}\tilde{u}_\alpha(x^\rho) + \xi^\rho \partial_\rho {}^{(0)}\tilde{u}_\alpha(x^\rho) \tag{3.23}$$

since ${}^{(0)}\tilde{u}_\alpha(x^\rho) = {}^{(0)}u_\alpha(x^\rho)$ inserting (3.23) into (3.22) yields:

$$\delta\tilde{u}_\alpha = \delta u_\alpha - \partial_\alpha \xi^{\beta\,(0)} u_\beta - \xi^\rho \partial_\rho {}^{(0)}u_\alpha \tag{3.24}$$

Tensor Field Tensors transform as:

$$\tilde{g}_{\mu\nu}(\tilde{x}\rho) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}(x^\rho) = (\delta^\alpha_\beta - \partial_\mu \xi^\alpha)(\delta^\beta_\nu - \partial_\nu \xi^\beta) g_{\alpha\beta}(x) \tag{3.25}$$

working to first order in ξ, δ implies:

$$\tilde{g}_{\mu\nu}(\tilde{x}) = g_{\mu\nu}(x) - \partial_\mu \xi^{\alpha(0)} g_{\alpha\nu}(x) - \partial_\nu \xi^{\beta(0)} g_{\mu\nu}(x) \tag{3.26}$$

as before we have

$$\begin{aligned} \tilde{g}_{\mu\nu}(\tilde{x}) &= {}^{(0)}\tilde{g}_{\mu\nu}(\tilde{x}) + \partial \tilde{g}_{\mu\nu} = {}^{(0)}\tilde{g}_{\mu\nu}(x+\xi) + \delta \tilde{g}_{\mu\nu} = \\ &= {}^{(0)}g_{\mu\nu}(x) + \xi^\rho \partial_\rho {}^{(0)}g_{\mu\nu}(x) + \delta \tilde{g}_{\mu\nu} \end{aligned} \tag{3.27}$$

Inserting (3.27) into (3.26) gives:

$$\delta \tilde{g}_{\mu\nu} = \delta g_{\mu\nu} - \xi^\rho \partial_\rho {}^{(0)}g_{\mu\nu}(x) - \partial_\mu \xi^{\alpha(0)} g_{\alpha\nu}(x) - \partial_\nu \xi^{\beta(0)} g_{\mu\nu}(x) \tag{3.28}$$

3.2.3 Building Gauge Invariant Variables

From (3.20), (3.24) and(3.28) it is possible to read off the transformation behaviour of respectively δg_{00}, δg_{0i} and δg_{ij}. Decompose the infinitesimal vector $\xi^\alpha = (\xi^0, \xi_{,i})$ by Helmholtz theorem as

$$\xi^i = \xi^i_\perp + \varsigma^{,i} \tag{3.29}$$

with $\xi_\perp$ longitudinal. It follows:

$$\delta \tilde{g}_{00} = \delta g_{00} - \xi^0 \partial_0 a^2 - 2\partial_0 \xi^0 a^2 = \delta g_{00} - 2a(a\xi^0)' \tag{3.30}$$

for the vectorial component

$$\delta \tilde{g}_{0i} = \delta g_{0i} + \partial_0 \xi^i a^2 - \partial_i \xi^0 a^2 = \partial g_{0i} + a^2 \left[\xi'_{\perp i} + (\varsigma' - \xi^0)_{,i}\right] \tag{3.31}$$

and finally for the tensorial part:

$$\delta\tilde{g}_{ij} = \partial g_{ij} + a^2 \left[2\frac{a'}{a}\delta_{ij}\xi^0 + 2\varsigma_{,ij} + (\xi_{\perp i,j} + \xi_{\perp j,i} \right] \tag{3.32}$$

The metric including only scalar perturbations is:

$$ds^2 = a^2 \left[(1+2\phi)d\tau^2 + 2B_{,i}dx^i d\tau - ((1-2\psi)\delta_{ij} - 2E_{,ij})\, dx^i dx^j \right] \tag{3.33}$$

Comparing (3.30), (3.31) and (3.32) with (3.33) the transformation of the four scalar functions is obtained:

$$\phi \to \tilde{\phi} = \phi - \frac{1}{a}(a\xi^0)' \tag{3.34}$$

$$B \to \tilde{B} = B + \varsigma' - \xi^0 \tag{3.35}$$

$$\psi \to \tilde{\psi} = \psi + \frac{a'}{a}\xi^0 \tag{3.36}$$

$$E \to \tilde{E} = E + \varsigma \tag{3.37}$$

It follows that choosing ς and ξ^0 fixes the scalar part of the perturbations uniquely. This implies that out of the four scalars we had, we have only two degrees of freedom coupling to $\delta\phi$. Moreover it is possible to form gauge invariant observable by taking linear combination of the four scalars in an appropriate way.

Invariant energy density Suppose to have a background energy density $\rho_0(t)$. Since it is a scalar, under a coordinate transformation, it follows from (3.20) that

$$\delta\tilde{\rho} = \delta\rho - \rho_{0,\alpha}\xi^\alpha = \delta\rho - \rho_0'\xi^0 \tag{3.38}$$

the term ξ^0 can be obtained by the perturbation of $\delta(E' - B) = \xi^0$. It follows that I can define a gauge invariant perturbation as

$$\delta\rho_{GI} = \delta\rho - \rho_0'(B - E') \tag{3.39}$$

The previous computation tells us more. Using the freedom of the choice of ξ_0 it is always possible to take a coordinate transformation such that $\delta\rho = 0$. These surfaces are called uniform energy density surfaces and it follows that

$$\delta\rho \to \delta\rho_{unif} = \delta\rho - \rho_0'\xi^0 = 0 \tag{3.40}$$

implying that, in order to move to the uniform energy density slicing, one has to fix

$$\xi^0 = \frac{\delta\rho}{\rho'} \tag{3.41}$$

Bardeen's parameters As already mentioned it is possible to build gauge invariant variables. The most important are the Bardeen's parameters [23]:

$$\Phi = \phi - \frac{1}{a}\left[a(B - E')\right]' \quad \Psi = \psi + \frac{a'}{a}(B - E') \tag{3.42}$$

through these two variables it is possible to parametrize fluctuations. If both $\Phi = \Psi = 0$ then all the perturbations are ficticious. Choosing ξ^0 and ς' such that $E = B = 0$ yields $\phi = \Phi$ and $\Psi = \psi$ and it is known as longitudinal gauge. Another important parameter that will play an important role in building the δN formalism is the gauge invariant parameter ζ which is invariant under time translation

$$\zeta = -\psi + \frac{a'}{a}\xi_0 \tag{3.43}$$

In conformal time, the Friedmann energy conservation can be stated as [1]:

$$\rho' + 3\frac{a'}{a}(\rho + p) = 0 \tag{3.44}$$

inserting (3.44) in (3.43) gives

$$\zeta = -\psi - \frac{\delta\rho}{3(\rho + p)} \tag{3.45}$$

Notice that in the gauge choice where $\delta\rho = 0$ it follows $\zeta = -\psi$.

3.2.4 δN formalism

The perturbed metric has the following form:

$$
\begin{aligned}
ds^2 =& a^2 \left[(1+2\phi)d\tau^2 + 2(B_{,i}+S_i)dx^i d\tau\right] + \\
&+ a^2 \left[((-1+2\psi)\delta_{ij} + 2E_{,ij} + F_{i,j} + F_{j,i} + h_{ij})dx^i dx^j\right]
\end{aligned} \tag{3.46}
$$

In the litterature the δN formalism is derived making use of the ADM (Arnowitt, Deser, Misner) metric [24]:

$$
ds^2 = -\hat{\alpha}^2 dt^2 + \hat{\gamma}_{ij}(dx^i + \hat{\beta}^i dt)(dx^j + \hat{\beta}^j dt) \tag{3.47}
$$

where $\hat{\gamma}_{ij} \doteq a^2(t)e^{2\hat{\psi}}(e^{\hat{h}})_{ij}$ with $\hat{h}_{ij} = \partial_i \hat{C}_j + \partial_j \hat{C}_i - 2/3\partial_k \hat{C}^k \delta_{ij} + \hat{h}^T_{ij}$. The perturbation parameters also contains ten parameters: one scalar due to $\hat{\alpha}$, three from the vector $\hat{\beta}_i$, and finally six from the tensor $\hat{\gamma}$. A mapping between the ADM metric and the old metric can be easily obtained by setting:

$$
\begin{aligned}
\hat{C}_{\perp i} = F_i, \quad \hat{M} = E \quad \hat{\beta}_i = a(B_{,i} + S_i), \\
\hat{\psi} = -\psi, \quad \phi = -\frac{1}{a^2}\frac{\hat{\alpha}^2 + \hat{\beta}_i\hat{\beta}^i}{2} - \frac{1}{2}
\end{aligned} \tag{3.48}
$$

Where $\hat{C}^i = \hat{C}^i_\perp + M^{,i}$ and all terms of order δ^2 are dropped (δ here being the order of the perturbative term).

As an excercise, to derive the relation between ψ and the number of e-folds, we will use metric (3.46). The derivation sugin the metric (3.47) is presented in [25]. Before proceeding it is usefull to rewrite the spatial part of the metric (3.46) as

$$
a^2 e^{-2\psi}(e^h)_{ij} \tag{3.49}
$$

with $(e^h)_{ij} = \delta_{ij} + 2E_{,ij} + F_{i,j} + F_{j,i} + h_{ij}$. Again it is meant that the order of perturbation is δ and perturbation of order δ^2 are dropped. Fix now part

of the gauge: $F_i = 0$, $E = 0$. In order to fix the gauge univocally another choice has to be made in the scalar sector. For $\psi = 0$ we obtain the flat gauge, for $\delta\rho = 0$ we obtain the uniform energy density gauge.
Consider a gauge transformation of the form

$$\tau \to T = \tau + \delta\tau, \quad x^i \to X^i = x^i + \xi^i \tag{3.50}$$

Note that the choice of the gauge sets $\xi^i = \xi^i_\perp + \varsigma^{,i} = 0$ since using (3.24) it folllows:

$$F_i \to \tilde{F}_i = F_i + \xi_{\perp i} \tag{3.51}$$

Hence setting $F_i = E = 0$ equals to set both $\xi_{\perp i}$ and ς to zero (see (3.37)).
Under (3.50):

$$\begin{aligned} g_{ij}(\tau, \vec{x}) =& \tilde{g}_{00}(T, \vec{X})\frac{\partial\delta T}{\partial x^i}\frac{\partial\delta T}{\partial x^j} + 2\tilde{g}_{0k}(T, \vec{X})\frac{\partial X^k}{\partial x^i}\frac{\partial\delta T}{\partial x^j} + \\ & \tilde{g}_{ij}(T, \vec{X})\frac{\partial X^k}{\partial x^i}\frac{\partial X^l}{\partial x^j} \end{aligned} \tag{3.52}$$

The first term is of order δT^2, the second term is of order $\delta \cdot \delta T$ and the last term is of order $O(\delta T^0, \delta^0)$ implying:

$$g_{ij}|_{E=F_i=0}(t, \vec{x}) = \tilde{g}_{ij}|_{E=F_i=0}(T, \vec{X}) + O(\delta T^2, \delta^2, \delta \cdot \delta T) \tag{3.53}$$

this implies in turn that

$$a^2 e^{-2\psi}(e^h)_{ij}(\tau, \vec{x}) \simeq a^2 e^{-2\tilde{\psi}}(e^{\tilde{h}})_{ij}(T, \vec{X}) \tag{3.54}$$

taking determinant on both side and also logarithm (noticing that $\det\{e^h\} = 1 + O(\delta)$) gives:

$$\ln a(\tau) - \psi(\tau, \vec{x})|_{E=F_i=0} = \ln a(T) - \tilde{\psi}(T, \vec{X})|_{E=F_i=0} \tag{3.55}$$

Finally taking $\psi(\tau)$ in the flat gauge, and $\psi(T)$ in the uniform density gauge gives:

$$\psi|_{\delta\rho=E=F_i=0}(T,\vec{X}) = -\ln\frac{a(T)}{a(\tau)} \tag{3.56}$$

finally we know the number of e-folds is:

$$N \doteq \int_{t^\star}^{t} H dt' = \int_{\tau^\star}^{\tau} \frac{a'}{a} d\tau' \tag{3.57}$$

suppose now that the change from flat gauge to uniform energy density gauge happens at $\hat{\tau} \in (\tau, T)$. It follows

$$-\psi|_{\delta\rho=E=F_i=0}(T,\vec{x}) = \ln\frac{a(\tau)}{a(\hat{\tau})} - \ln\frac{a(T)}{a(\hat{\tau})} \doteq N - \hat{N} \doteq \delta N \tag{3.58}$$

where $\hat{N}$ is the number of e-folds in the uniform density gauge. In this gauge $\psi = -\zeta$ implying $\zeta = \delta N$. Before moving on, a remark on the constancy of ζ in this gauge has to be made.

Let us work in the uniform energy density gauge ($\delta\rho = 0$). Then the scale factor is:

$$a(t)e^{\psi(t,\vec{x})} \tag{3.59}$$

the first thermodynamical principle implies

$$dE = d\rho V + \rho dV = -pdV \tag{3.60}$$

where $V \sim a^3 e^{3\psi}$. This implies that $dV = 3(a^2 e^{3\psi} da + a^3 e^{3\psi} d\psi)$. This implies for (3.60)

$$d\rho = -3(\rho + p)\frac{a^2 e^{3\psi} da + a^3 e^{3\psi} d\psi}{a^3 e^{3\psi}} = -3(\rho + p)\left(\frac{da}{a} + d\psi\right) \tag{3.61}$$

taking time derivative on both side, the perturbed energy density conservation becomes:

$$\dot{\rho} = -3(H + \dot{\psi})(\rho + p) \tag{3.62}$$

Now writing $\rho = \rho_0 + \delta\rho$ with ρ_0 satisfying the unperturbed equation (i.e. without the term $\dot{\psi}$), it follows at first order

$$\dot{\psi}(\rho_0 + p_0) = \delta\dot{\rho} = 0 \Rightarrow \dot{\psi} = 0 \tag{3.63}$$

where the $= 0$ equality is because of the gauge choice. This implies $\psi = -\zeta = const.$

Chapter 4

Modulated Reheating

Now that the δN formalism is developed, it will be possible to fully understand the non gaussianity production of order $O(1)$ due to modulated reheating. Such a result is already derived in ref [4]. After repeating the main step of such an article the result is derived using the machinery of the previous chapter.

4.1 Basics of reheating

At the end of inflation the inflaton field is still dominating the universe. However, since it is oscillating around its minimum, it behaves like matter. To understand why this is the case consider the classical equation of motion for the inflaton:

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = \ddot{\phi} + 3H\dot{\phi} + M^2\phi = 0 \tag{4.1}$$

where M is the mass of the field at its minimum. The equation follows from

$$\phi_{;\alpha}^{;\alpha} + V_{,\phi} = 0 \tag{4.2}$$

assuming ϕ only depends on time (the field is homogeneous at the classical level). Now multiplying (4.1) by ϕ and averaging over time implies:

$$\left\langle (\phi_{,t})^2 \right\rangle = M^2 \left\langle \phi^2 \right\rangle \tag{4.3}$$

where the term $\phi_{,tt}\phi$ has been rewritten as $(\phi_{,t}\phi)_{,t} - (\phi_{,t})^2$. The first term, as well as the friction term, can be dropped under time average.

For the energetic content:

$$\rho = \frac{1}{2}\phi_{,t}^2 + \frac{1}{2}M^2\phi^2 \tag{4.4}$$

Taking derivatives yields, using (4.1)

$$\rho_{,t} = \phi_{,t}(\phi_{,tt} + M^2\phi) = -3H\phi_{,t}^2 \tag{4.5}$$

From (4.3):

$$\langle \rho \rangle = M^2 \left\langle \phi^2 \right\rangle = \left\langle \phi_{,t}^2 \right\rangle \tag{4.6}$$

Inserting this in (4.5) yields

$$\frac{\langle \rho_{,t} \rangle}{\langle \rho \rangle} = -3H \tag{4.7}$$

upon integration, it is possible to see that the averaged energy density redshifts in a matter-like way:

$$\langle \rho \rangle = \frac{\rho_0}{a^3} \propto T^3 \tag{4.8}$$

Hence, while the inflaton is relaxing around its minimum, the universe is in a matter dominated era (assuming the inflaton energy density is still dominating in the universe). An oscillating homogeneus scalar field can be interpreted as a condensate of heavy particles of mass M at rest. The idea of reheating is that this oscillatory energy, instead of being fully redshifted, is converted into some other degrees of freedom or, equivalently, the inflaton decays. Then, depending on the temperature of reheating, the newly formed particles might be relativistic or not, generating a period of either radiation domination or matter like domination.

A full comprehensive theory of reheating is still not present. The reason is that a full theory of inflation is not known, as well as the degrees of freedom taking part in the process.

For practical purpose we can say, for example, that the inflaton decays in some degrees of freedom (denote them as ψ) through the renormalizable coupling

$$\lambda\bar{\psi}\phi\psi \tag{4.9}$$

In principle the terms appearing in the lagrangian, and describing the decay, might be non renormalizable, depending on which degrees of freedom have been integrated out at the scale of reheating.

The decay rate, from purely dimensional analysis, has the form (up to numerical prefactors of order one):

$$\Gamma(\phi \to \psi\psi) = \lambda^2 M \tag{4.10}$$

Such a channel should be effective only after the end of inflation. It follows immediately a consistency bound during inflation

$$\Gamma \leq H \tag{4.11}$$

The reason of such a bound is casual. If the process takes distances (or times) longer than the Hubble patch, then the process is exponentially suppressed.

Parametrize $\phi(t) \simeq \Phi(t)\cos(Mt)$ where $\Phi(t)$ is a slowly decaying amplitude. The number density is:

$$n_\phi \simeq \frac{\langle\rho\rangle}{M} = \frac{1}{2}M\Phi^2 \tag{4.12}$$

These equations will be usefull for later considerations. Notice also that the average life of a particle ϕ is $\sim \Gamma^{-1} \sim M^{-1}$. Another problem affecting reheating are parametric resonances which will be tackled when dealing with locked inflation.

4.2 Basic mechanism of modulated reheating

The key idea of modulated reheating is to give to the coupling λ in (4.9) a dynamical origin i.e. $\lambda = \lambda(S)$, S being a scalar. Suppose, also, that at the end of inflation, the deacying channel becomes enabled i.e.

$$\Gamma \geq H \tag{4.13}$$

Another assumption is that during the inflationary period $m_S \ll H$ so that the scalar field S fluctuates as in a quasi De Sitter universe without spoiling inflation which is driven by another field. Assuming that the decay channel (4.9) is the only responsible for the reheating of the universe, the only source of temperature will be Γ. Under this transition, let us assume that the universe goes from a matter dominated universe, to a radiation dominated universe.

In a radiation dominated universe, the energy density ρ scales as T^4. Friedmann's equation in this case reads $1/t^2 \sim T^4$ where $M_p = 1$ and other constants of order one have been dropped. The time length dominating during reheating is Γ^{-1}. It follows that:

$$T_R \sim \sqrt{\Gamma M_p} \sim \lambda\sqrt{M M_p} \tag{4.14}$$

where M_p has been reintroduced using dimensional analysis. For a precise derivation see [12]. In (4.14) few factors of order one are missing, as well as the number of degrees of freedom at that temperature. Since these quantities will not affect our future treatment we set them to one. Notice that in principle, however, it might be of order 10^2 if all the degrees of freedom of the standard model are available.

From (4.14) it follows that a fluctuation in the coupling λ might generate a fluctuation in the temperature:

$$\frac{\delta T_R}{T_R} \sim \frac{\delta\Gamma}{\Gamma} \sim \frac{\delta\lambda}{\lambda} \tag{4.15}$$

To proceed further a two separate universe argument is given. Suppose to have two separated universes where the coupling λ has fluctuated differently. Wlog assume that $\lambda_1 > \lambda_2$ i.e.

$$\frac{1}{\Gamma_1} < \frac{1}{\Gamma_2} \tag{4.16}$$

What will happen is that the two universes will undergo the same evolution, at different values of the scale factor a generating density perturbation. To better understand this now we will compute how much they differ.

Assume that the decay happens instantaneously at the moment (and the radiation domination immediately begins thereafter):

$$\Gamma = H \sim \frac{\sqrt{\rho}}{M_p} \tag{4.17}$$

Call the $t_\star$ the moment when $\Gamma_1 = H(t_\star)$.

It follows that

$$\rho_1(t_\star) = \rho_2(t_\star) = \rho_0 \tag{4.18}$$

from this moment onward, the evolution of the two universes will be different. In the first , ρ_1 will evolve as radiation, while in the second ρ_2 will evolve as matter i.e.

$$\rho_1(t) = \frac{\rho_0}{a^4}, \qquad \rho_2(t) = \frac{\rho_0}{a^3} \tag{4.19}$$

The second universe will evolve until the final moment t_f when

$$\Gamma_2 = H_{t_f} \tag{4.20}$$

from (4.19) it follows that:

$$a(t_f) = \sqrt[3]{\frac{\rho_0}{\rho_2(t_f)}} \tag{4.21}$$

The energy density $\rho_1(t_f)$ can be rewritten as

$$\rho_1(t_f) = \frac{\rho_0}{\rho_0^{4/3}}\rho_2(t_f)^{4/3} = \frac{\rho_2(t_f)^{4/3}}{\rho_0^{1/3}} \tag{4.22}$$

Since, by previous considerations, $\rho_0 = (\lambda_1^2 m)^2 M_p^2$, and $\rho_2(t_f) = (\lambda_2^2 m)^2 M_p^2$ with m being the mass of the inflaton at its minimum, (4.22) becomes

$$\rho_1(t_f) = \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{4}{3}} \lambda_2^4 m^2 M_p^2 = \left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{4}{3}} \rho_2(t_f) \tag{4.23}$$

Then (4.15) implies:

$$\frac{\delta\rho}{\rho} \sim \frac{\delta\lambda}{\lambda} \sim \frac{\delta\Gamma}{\Gamma} \tag{4.24}$$

this completes our computation since we showed that a fluctuation of the field S during inflation, will be imprinted in the energy spectrum at later time during reheating.

Two remarks are in order

- in the above computation the ususal contribution to density perturbations due to the inflaton field has been ignored: this is the case if one considers low scale inflationary models unable to produce enough density perturbation wrt observations
- the whole computation has been realized assuming thermal equilibrium. This implies that this mechanism is generating adiabatic fluctuations.

4.2.1 On the origin of fluctuations

The dependance of λ on a scalar S can be parametrized in a general way as follows:

$$\lambda(S) = \lambda_0 \left(1 + \frac{S}{M} + ...\right) \tag{4.25}$$

with M being the scale at which the degree of freedom was integrated out. It can either be the planckian or a lower scale. Some terms in (4.25) might be missing due to symmetries.

Since by assumption $m_s \ll H$ it follows that the fluctuations $\delta S \sim H$ during inflation (massless De Sitter case). We have that

$$\frac{\delta\Gamma}{\Gamma} = \frac{\delta\lambda}{\lambda} = \frac{\delta S}{M} = \frac{\delta S}{\langle S\rangle}\frac{\langle S\rangle}{M} \doteq f\frac{\delta S}{S} \tag{4.26}$$

with $\langle \cdots \rangle$ denoting vacuum expectation value (which the field assumes during inflation) and f can be interpreted as the fraction of fluctuations controlled by the coupling.

Since observationally

$$\frac{\delta\Gamma}{\Gamma} \sim 10^{-5} \tag{4.27}$$

There are two possibilities: either $\langle S\rangle \gg H \sim \delta S$ or $f \ll 1$. In the first case perturbation will be gaussian like, while in the second case non gaussian.

Equation (4.25) implies that there are mainly two channels contributing to the decay. A direct channel dominated by the constant coupling λ_0 and a modulating channel (call it s-channel) where $\lambda = \langle S \rangle / M$. Contributions to the energy density come from both channels. However the contribution to fluctuations comes only from the s-channel implying:

$$\frac{\delta\rho}{\rho} = \frac{\delta\rho_s}{\rho_s + \rho_{direct}} \tag{4.28}$$

where $\delta\rho_s$ estimates as follows:

$$\rho_s = \left(\frac{\langle S \rangle}{M}\right)^4 m^2 M_p^2 \quad \rightarrow \quad \delta\rho_s = \left(\frac{\langle S \rangle}{M}\right)^3 \frac{\delta S}{M} m^2 M_p^2 \tag{4.29}$$

let us now consider the two limits for (4.28) where either the s-channel or the d-channel dominates.

If the s-channel dominates, (4.28) reduces to

$$\frac{\delta\rho}{\rho} \sim \frac{\delta S}{\langle S \rangle} \tag{4.30}$$

and since $\delta S \sim H$ it follows that

$$\langle S \rangle \sim 10^5 H \tag{4.31}$$

implying gaussian like perturbations.

In the case the direct channel dominates:

$$\frac{\delta\rho}{\rho} \sim \delta S \frac{\langle S \rangle^3}{M^4 \lambda_0^4} = \frac{\delta S}{M \lambda_0^4} \sim \frac{H}{M \lambda_0^4} \tag{4.32}$$

where the minimum of S has been assumed at M (take S to be, for example, a particle with an Higgs like potential). In this case $\delta S \gg \langle S \rangle$ implying the production of non gaussianities. Notice that up to now, the balance between the two channels has been considered arbitrary. When such a mechanism is embedded in a model, it will tells us what are the weights of each different channel.

Mass domination Before moving to a model indipendent estimate on the production of non gaussianities, it is worth mentioning another modulated like mechanism described in [26]. Suppose the universe undergoes a period of mass domination starting at the moment t_0

$$\rho_0 = M^4 \tag{4.33}$$

During this period the universe number one will evolve in a matter like way until the mass decays in other degrees of freedom when $\rho_f \sim \Gamma^2$. After that, universe number one will start again evolving in a radiation-like way. We will compare this evolution with the one of universe number two, where the mass domination never happens, and simply evolves like radiation. For universe 1 we have

$$\frac{\rho_1(t_0)}{\rho_1(t_f)} = \left(\frac{a_f}{a_0}\right)^3 = \frac{M^4}{\Gamma^2} \tag{4.34}$$

In universe 2:

$$\frac{\rho_2(t_0)}{\rho_2(t_f)} = \frac{\rho_1(t_0)}{\rho_2(t_f)} = \left(\frac{a_f}{a_0}\right)^4 = \left(\left(\frac{a_f}{a_0}\right)^3\right)^{4/3} = \frac{M^{16/3}}{\Gamma^{8/3}} \tag{4.35}$$

From (4.34), using (4.35) we have

$$\rho_1(t_f) = \frac{\Gamma^2}{M^4}\rho_1(t_0) = \frac{M^{4/3}}{\Gamma^{2/3}}\rho_2(t_f) \tag{4.36}$$

finally implying:

$$\frac{\delta\rho}{\rho} = \frac{4}{3}\frac{\delta M}{M} - \frac{2}{3}\frac{\delta\Gamma}{\Gamma} \tag{4.37}$$

in the case $\delta M = 0$ we recover the case of previous section with the opposite sign. An example of a model where this type of domination is studied is presented in [26].

4.2.2 Non Gaussianity

Physical observables in quantum field theory are fully encoded in correlator functions. Taking their amplitude square correspond to evaluating the probability of such a process. A free field theory without interactions is Gaussian. In fact the probability distribution is given by the Feynmann path integral

$$\int \mathfrak{D}\phi e^{iS(\phi)} \tag{4.38}$$

and observable can be obtained by computing

$$\langle F \rangle = \int \mathfrak{D}\phi F e^{iS(\phi)} \tag{4.39}$$

up to a normalization factor. It can be easily proven that, for odd number of fields, the correlator function gives zero (Wick's theorem). This is no longer the case when interactions are considered. However, since the canonical approach to the problem of interaction is through perturbation theory, it means that our distribution can be considered still Gaussian, with small deviations. These deviations make in turn correlators different from zero. Hence the best way to measure non gaussianities is, for example, to measure the three point correlator. This term will be smaller compared to the expectation value of the two point correlator because it will at least be multiplied by the order parameter of the perturbative expansion (to some power depending on the theory).

Non Gaussianity in inflation

Proper measurement of the three point correlator function is a good way to measure non gaussianities. Simple models of inflation satisfying the following conditions:

- single field
- canonical normalization
- slow-roll condition satisfied
- vacuum initial state

all predict a tiny amount of non gaussianities. Too tiny to actually be detected by today measurements. Their order of magnitude is $10^{-2}P$ where P is the order of magnitude of the two-point correlator function. It follows that measuring primordial non gaussianities today, might rule out all the simple one field inflationary models requiring, then, a richer physics of inflation.
A natural question is which field should one use to evaluate perturbations since a wrong choice might lead to ficticious results. The action of a scalar field minimally coupled to gravity reads:

$$S = \frac{1}{2}\int d^4x\sqrt{g}\left(R - (\nabla\phi)^2 - 2V(\phi)\right) \tag{4.40}$$

where g is the metric determinant, R the scalar curvature and $M_p = 1$. When perturbed, we have an ADM metric (3.47). Under that, the action (4.40) can be rewritten as:

$$\begin{aligned} S = \frac{1}{2}\int \sqrt{\hat{\gamma}}(\hat{\alpha}R^{(3)} - 2\hat{\alpha}V + \hat{\alpha}^{-1}(E_{ij}E^{ij} - E^2)+ \\ \hat{\alpha}^{-1}(\dot{\phi} - \hat{\beta}^i\partial_i\phi)^2 - \hat{\gamma}^{ij}\partial_i\phi\partial_j\phi) \end{aligned} \tag{4.41}$$

with

$$E_{ij} = \frac{1}{2}\left(\frac{d}{dt}\hat{\gamma}_{ij} - \nabla_i\hat{\beta}_j - \nabla_j\hat{\beta}_i\right) \tag{4.42}$$

A proper choice of the gauge could be the uniform energy density gauge (i.e. $\psi = -\zeta$) and $\delta\phi = 0$. This condition can be imposed by gauge fixing also the vectorial part. Notice that the scalar field ϕ, dominating the energy density of the universe during inflation, under coordinate transformation behaves like:

$$\delta\phi \to \delta\phi - \phi_0 \frac{\delta\rho}{\dot{\rho}} + \phi_{,i}\xi^i \tag{4.43}$$

the second term is already zero because of the uniform energy density slicing, while $\delta\phi + \phi_{,i}\xi^i$ can be made to vanish by chosing ξ^i appropriately.
Under this gauge the equation of motions for $\hat{\alpha}$ and $\hat{\beta}^i$ corresponds respectively to the hamiltonian and to the momentum constraints. Solving these constraints and inserting them back in the action, one finds, at second order [27]:

$$S = \frac{1}{2}\int d^4x \frac{\dot{\phi}^2}{\dot{\rho}^2}\left[e^{3\rho}\dot{\zeta}^2 - e^{\rho}(\partial_i\zeta)^2\right] \tag{4.44}$$

where $e^{\rho} \doteq a(t)$. Since (4.44) describes a free field we can expand them in terms of harmonic oscillators:

$$\zeta(x) = \int \frac{d^3k}{(2\pi)^3}\zeta_k(t)e^{i\vec{k}\cdot\vec{x}} \tag{4.45}$$

$\zeta_k(t)$ is a harmonic oscillator with time dependent mass and spring constant. To quantize the field take two classical solutions of the EOM

$$\frac{\partial L}{\partial \zeta} = 0 \tag{4.46}$$

and write

$$\zeta_{\vec{k}}(t) = \zeta_k^{cl}(t)a_{\vec{k}}^{\dagger} + \zeta_k^{cl*}(t)a_{-\vec{k}} \tag{4.47}$$

imposing canonical a, $a^\dagger$ commutation relation we obtain its renormalization. Hence we can compute non gaussianities evaluating the correlator:

$$\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\right\rangle \doteq (2\pi)^3\delta^3(\vec{k_1}+\vec{k_2}+\vec{k_3})B_\zeta(k_1,k_2,k_3) \tag{4.48}$$

to proceed further let us use δN formalism (notice that the gauge choice is the same as the one used there). To compute it we can use the so-called "gradient expansion method" [25] for which we systematically ignore terms involving spatial derivatives. This approximation is valid as long that the perturbations that we are dealing with are longer than the Hubble length i.e. $\lambda_{phys} \gg H^{-1}$ (which is the case for separate universes). Notice that at the zeroth order this corresponds to ignore all inhomogeneities implying that the equation for the perturbed quantitites is simply the Friedmann's equation for the background

$$H^2 = \frac{1}{M_p^2}\rho \tag{4.49}$$

This corresponds to say that when the wavelenght of ζ is larger than the hubble horizon, each horizon patch evolves as if they were separated universes. Under this approximation, to obtain δN it is sufficient to solve the background evolution for different initial conditions, and to calculate perturbations as the differences between initial conditions.

Following [28], we write an expansion for the field ζ in terms of slow-rolling multi fields. Since these fields are slow-rolling, it follows from Klein Gordon equation, that ζ is only a function of the field ϕ^I and not of its derivative $\dot{\phi}^I$. It follows from (3.58)

$$\zeta = \delta N \simeq \frac{\partial N}{\partial\phi^I}\delta\phi^I + \frac{1}{2}\frac{\partial^2 N}{\partial\phi^I\partial\phi^J}\delta\phi^I\delta\phi^J + ... - \hat{N} \tag{4.50}$$

where the last term is the number of e-folds in the uniform energy density gauge. The label runs $I = 1, ..., N$, N being the number of fields contributing to ζ. Now it is clear that the perturbation term $\delta\phi^I$ can be expanded in modes, and hence (4.48) can be calculated. Another useful quantity is the two-point function

$$\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2} \right\rangle = (2\pi)^3 P_\zeta \delta^{(3)}(\vec{k}_1 + \vec{k}_2) \tag{4.51}$$

To proceed further define

$$\left\langle \delta\phi^I_{\vec{k}1}\delta\phi^I_{\vec{k}_2} \right\rangle = (2\pi)^3 P^{IJ}(k_1)\delta^{(3)}(\vec{k}_1 + \vec{k}_2) \tag{4.52}$$

with $P^{IJ} \doteq P\delta^{IJ}$. Also rewrite $N_I \doteq \partial N/\partial\phi^I$.

From the previous definitions it follows:

$$\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2} \right\rangle = N_I N_J \left\langle \delta\phi^I_{\vec{k}_1}\delta\phi^J_{\vec{k}_2} \right\rangle = N_I N_J P^{IJ}(2\pi)^3\delta^{(3)}(\vec{k}_1 + \vec{k}_2) \tag{4.53}$$

from which one deduces, for (4.51)

$$P_\zeta(k_1) = N_I N_J \delta^{IJ} P \tag{4.54}$$

Similarly for the three point function:

$$\begin{aligned} \left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3} \right\rangle =& N_A N_B N_C \left\langle \delta\phi^A\delta\phi^B\delta\phi^C \right\rangle + \\ & \left(N_A N_B N_{CD} \left\langle \delta\phi^A_{\vec{k}_1}\delta\phi^B_{\vec{k}_2}\delta\phi^C_{\vec{k}_3}\delta\phi^D_{\vec{k}_3} \right\rangle + \text{perms} \right) + ... \end{aligned} \tag{4.55}$$

we assume the fields ϕ to be weakly interacting, and their modes $\delta\phi$ to be fully Gaussian. It follows that the first term in (4.55) vanishes, while the second can be contracted in the following way

$$\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3} \right\rangle = \left(N_A N_B N_{CD} \left\langle \delta\phi^A_{\vec{k}_1}\delta\phi^C_{\vec{k}_3} \right\rangle \left\langle \delta\phi^C_{\vec{k}_2}\delta\phi^D_{\vec{k}_3} \right\rangle + \text{perms} \right) \tag{4.56}$$

using (4.52)

$$\left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\right\rangle = (2\pi)^3 N_A N_B N_{CD}\left(P(k_1)\delta^{AC}P(k_2)\delta^{CD} + \text{perms}\right) \quad (4.57)$$

The f_{NL} parameter is defined in the following way [28]:

$$B_\zeta(k_1,k_2,k_3) = \frac{6}{5}f_{NL}\left(P_\zeta(k_1)P_\zeta(k_2) + 2\text{perms}\right) \quad (4.58)$$

comparing (4.58) together with (4.57) and (4.48):

$$f_{NL} = \frac{5}{6}\frac{B_\zeta(k_1,k_2,k_3)}{(P_\zeta(k_1)P_\zeta(k_2) + 2\text{perms})} = \frac{5}{6}\frac{N_A N_B N^{AB}}{(N_K N^K)^2} \quad (4.59)$$

A non analytic computation of f_{NL} for modulated reheating in the case of more than one field, is given in [28]. Here we would like to specialize the computation for one single field, computing analytically f_{NL}.

As we said in the discussion of modulated reheating the coupling depends only on one scalar field, namely S. Hence the expansion for ζ is:

$$\zeta \simeq \frac{\partial N}{\partial S}\delta S + \frac{\partial^2 N}{\partial S^2}\delta S^2 + ... \quad (4.60)$$

it immediately follows for f_{NL}:

$$f_{NL} = \frac{5}{6}\frac{\partial^2 N}{\partial S^2}\left(\frac{\partial N}{\partial S}\right)^{-2} \quad (4.61)$$

now we simply have to compute N.

In the case of a slow rolling single field inflation the number of e-folds can be written as [10]

$$\frac{\partial N}{\partial \phi} \sim \frac{H}{\dot{\phi}} \quad (4.62)$$

it follows immediately from the definition of f_{NL} that

$$f_{NL} \simeq \frac{5}{6}\frac{d}{d\phi}\left(\frac{H}{\dot{\phi}}\right)\frac{\dot{\phi}^2}{H^2} \quad (4.63)$$

now rewriting $d/d\phi$ as $\phi^{-1}d/dt$ gives:

$$f_{NL} = \frac{5}{6}\dot{\phi}^{-1}\frac{d}{dt}\left(\frac{H}{\dot{\phi}}\right)\frac{\dot{\phi}^2}{H^2} = \frac{5}{6}\left(\frac{\dot{H}}{H^2} - \frac{\ddot{\phi}}{H\dot{\phi}}\right) = \frac{5}{6}(\eta - 2\epsilon) \sim O(10^{-2}) \quad (4.64)$$

for a slow rolling field. As already said at the beginning, is too small to be detected by current measurements.

In the case of modulated reheating the number of e-folds is computed differently. As we said, in the gradient expansion, we are looking at $k \leq aH$ i.e. modes outside the horizon. Define $\hat{t}$ the moment at which $k = aH$ i.e. horizon exit. After the end of inflation the inflaton starts oscilating around its minimum. When decaying into different channels, the universe gets reheated. Hence we have the following transition: inflation, matter dominated universe and finally radiation dominated universe. Let t_m be that time in the radiation dominated universe where density perturbations are fully imprinted. It follows that:

$$N = N\left(\hat{t}, t_f, \vec{x}\right) = \int_{\hat{t}}^{t_m} H dt = \int_{\hat{t}}^{t_f} H dt + \int_{t_f}^{t_m} H dt \quad (4.65)$$

with t_f being the moment when inflation finishes. We want to find the dependence of N on the decay rate Γ. When reheating takes place we have $\Gamma = H(t_{reh})$. Take t_{reh} to be the moment when reheating begins. The last term in (4.65) can be further decompose into the matter and the radiative part:

$$N \supset \int_{a_f}^{a_{reh}} d\ln a + \int_{a_{reh}}^{a_m} d\ln a \quad (4.66)$$

since $a \propto \rho^{-1/3}$ in a matter universe, while $a \propto \rho^{-1/4}$ in a radiation dominated universe, and $\rho_{reh} \propto \Gamma^2$, it follows:

$$N \supset -\frac{1}{3}\ln\rho_{reh} + \frac{1}{4}\ln\rho_{reh} = -\frac{1}{12}\ln\rho_{reh} = -\frac{1}{6}\ln\Gamma(S(t_{reh})) \quad (4.67)$$

In the derivation we are totally ignoring the standard fluctuations of the inflaton that might ruin our modulation. However this approximation works fine for low scale inflationary model where the natural fluctuations of the massless field are proportional to the Hubble parameter H.

Static S case We are now ready to compute f_{NL} and to recover the result of [26]. We assume that S does not evolve while imprinting density perturbations. Then:

$$\left.\frac{\partial N}{\partial S}\right|_{reh} = \left.\frac{\partial N}{\partial \Gamma}\frac{\partial \Gamma}{\partial S}\right|_{reh} = \left.-\frac{1}{6}\frac{\Gamma'}{\Gamma}\right|_{reh} \tag{4.68}$$

where $\Gamma' \doteq \partial\Gamma/\partial S$. The second derivative is

$$\left.\frac{\partial^2 N}{\partial S^2}\right|_{reh} = -\frac{1}{6}\frac{\partial}{\partial S}\left.\left(\frac{\Gamma'}{\Gamma}\right)\right|_{reh} = -\frac{1}{6}\left.\left(\frac{\Gamma''}{\Gamma} - \frac{\Gamma'^2}{\Gamma^2}\right)\right|_{reh} \tag{4.69}$$

From (4.61) we get:

$$f_{NL} = \frac{5}{6}\cdot\frac{1}{6}\left(\frac{\Gamma'^2}{\Gamma^2} - \frac{\Gamma''}{\Gamma}\right)\frac{36\Gamma^2}{\Gamma'^2} = 5\left.\left(1 - \frac{\Gamma''\Gamma}{\Gamma'^2}\right)\right|_{reh} \tag{4.70}$$

which is the result that can be found, for example, in [12]. In our modulated reheating scenario we had $\lambda = \frac{S}{M}$ and $\Gamma \propto \lambda^2 m^2$ implying:

$$f_{NL} = \frac{5}{2} \tag{4.71}$$

In the case where, for example, the mass depends on a scalar field $m = m(\tilde{S})$ linearly (for example through a Yukawa coupling) we have

$$f_{NL} = 5 \tag{4.72}$$

the bottom line is that $f_{NL} \sim O(1)$ has been obtained, independently of the model.

S evolving As explained in the basic mechanism, there is a window of time where the two different universes have a different evolution. This is the origin of our perturbations. In principle S might have a non trivial evolution during this window changing the resulting f_{NL}. This might either lower or increase the amount of non Gaussianities.

We rewrite

$$\left.\frac{\partial N}{\partial S}\right|_{t_m} = \frac{\partial N}{\partial S_{reh}}\frac{\partial S_{reh}}{\partial S_m} = -\frac{1}{6}\frac{\Gamma'}{\Gamma}\frac{\partial S_{reh}}{\partial S_m} \tag{4.73}$$

with t_m being the moment when the imprint of density perturbations is over. $S_m \doteq S(t_m)$.

For the second derivative we have:

$$\frac{\partial^2 N}{\partial S_m^2} = -\frac{1}{6}\left(\left(\frac{S_{reh}}{\partial S_m}\right)^2\left(\frac{\Gamma''}{\Gamma} - \frac{\Gamma'^2}{\Gamma^2}\right) + \frac{\Gamma'}{\Gamma}\frac{\partial^2 S_{reh}}{\partial S_m^2}\right) \tag{4.74}$$

Finally this gives, for f_{NL}

$$f_{NL} = 5\left(1 - \frac{\Gamma''\Gamma}{\Gamma'^2} - \frac{\Gamma}{\Gamma'}\frac{\partial^2 S_{reh}}{\partial S_m^2}\left(\frac{\partial S_{reh}}{\partial S_m}\right)^{-2}\right) \tag{4.75}$$

From (4.75) it follows that, depending on the evolution of S during reheating, f_{NL} can deviate from the value of the static case. Notice that in the static limit we recover the original solution (4.70)

Two-point function

The mechanism of modulated reheating also happens to modify the value of the two point function. Again, here, a new correction due to the dynamics of the modulating field is obtained.

Under slow roll condition, we already computed the fluctuations of a field in a quasi De Sitter Universe. The result was:

$$|\delta\phi_k| \simeq \frac{H}{\sqrt{2k^3}} \left(\frac{k}{aH}\right)^{\tilde{\mu}} \tag{4.76}$$

with $\tilde{\mu} \simeq \eta - \epsilon$, η, ϵ being the slow roll parameters. Then the power spectrum is $P \simeq \left(\frac{H_i}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{2\tilde{\mu}}$, H_i being the scale factor during inflation. From equation (4.54) it follows, for constant S:

$$P_\zeta = N_I N_J \delta^{IJ} P = \left(\frac{1}{6}\frac{\Gamma'}{\Gamma}\right)^2 P \tag{4.77}$$

This is the standard literature result (see for example [28]).

Implementing again S's evolution taking the derivative at t_m it follows:

$$P_\zeta = N_I N_J \delta^{IJ} P = \left(\frac{1}{6}\frac{\Gamma'}{\Gamma}\frac{\partial S_{reh}}{\partial S_m}\right)^2 \left(\frac{H_i}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{2\tilde{\mu}} \tag{4.78}$$

Hence a small correction is obtained due to the prefactor in (4.78).

Chapter 5

Locked inflation

One field inflationary scenarios fail to be embedded in supergravity and string theory. This is because even though spontaneously broken supergravity lifted flat directions are natural inflationary candidates, the various moduli have masses of order H or even bigger during inflation violating the slow roll condition. This is the so called η-problem. Locked inflation proposes a way out of this problem [2].

5.1 Main Idea

Consider the following two weakly coupled fields potential

$$V(\Phi,\phi) = M_\Phi^2\Phi^2 + \lambda\Phi^2\phi^2 + \frac{\alpha}{4}(\phi^2 - M_\star^2)^2 \tag{5.1}$$

with $\alpha \sim M^4/M_p^4$, $m_\Phi^2 \sim M^4/M_p^2$, $M_\star \sim M_p$ with M being some intermediate scale of order of supersymmetric scales and $\lambda \sim 1$. The above potential is a typical potential for two moduli fields that parametrize supersymmetric flat directions, after the vacuum degeneracy is lifted by supersymmetry breaking effects. A closer look at the potential reveals the presence of a global minimum at $\phi = M_\star$ and $\Phi = 0$ and the presence of a saddle point at $\phi = 0$ and $\Phi = 0$. Suppose that the field has the following initial conditions:

$$\phi = 0, \qquad \Phi \gg \alpha \frac{M_\star}{\lambda} \tag{5.2}$$

Later we will explain how to obtain such initial conditions. Initially, when $\phi = 0$, the energy configuration has two main contributions: one coming from the vacuum energy of ϕ and one coming from the oscillating field Φ around zero i.e.

$$\rho_0 = \alpha M_\star^4, \qquad \rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}M_\Phi^2 \Phi^2 \tag{5.3}$$

The equation for Φ field, ignoring ϕ's dynamic, is the previously derived KG equation

$$\ddot{\Phi} + 3H\dot{\Phi} + M^2\Phi = 0 \tag{5.4}$$

from now on we will ignore all prefactors of $O(1)$.
Looking at the characteristic equation

$$r^2 + 3rH + M^2 = 0 \tag{5.5}$$

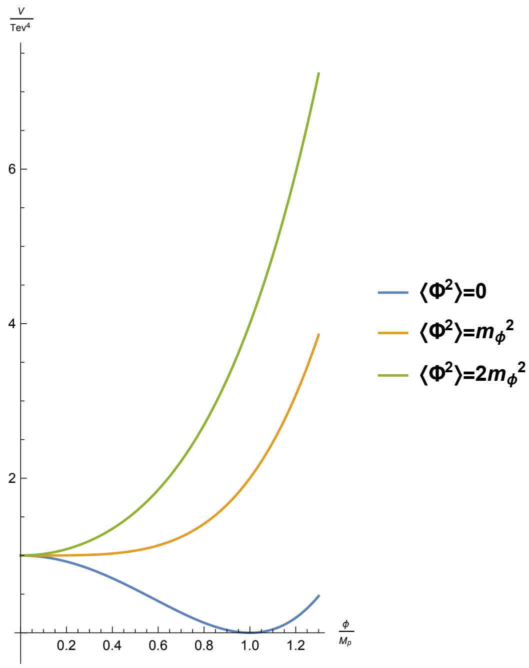
it follows that, for $M \geq H$, Φ will oscillate around its minimum, with modulus decreasing exponentially due to the friction term. Approximately the solution can be written as:

$$\Phi(t) \simeq \Phi_0 e^{-3/2Ht} \cos M_\Phi t \tag{5.6}$$

Hence, through the λ coupling, an effective mass for the ϕ field is produced:

$$m_{eff}^2(t) = \left\langle \Phi^2 \right\rangle(t) - \alpha M_\star^2 \tag{5.7}$$

Figure 5.1: Dependance of ϕ's potential w.r.t. the averaged $\langle \Phi^2 \rangle$ for different values.



where $\langle ... \rangle$ denotes time average. Because of Φ's initial conditions the effective mass in (5.7) will, at early times, be positive i.e. the Φ field is locking ϕ in its saddle point at zero as it is possible to see in fig. 5.1. Under this assumption, if $\rho_0 \geq \rho_\Phi$, then the dominating energy of the university is constant, and is ρ_0. This will generate a state of exponential expansion through Friedmann's equation of the form

$$H^2 = \frac{1}{M_p^2}\rho_0 = \frac{\alpha}{M_p^2}M_\star^4 \tag{5.8}$$

However, since Φ is oscillating around its minimum, and hence decreasing in a matter-like way (see (4.8)) it follows that eventually m_{eff}^2 becomes negative and ϕ's starts rolling towards its true minimum. We assume that at this moment inflation ends.

Let us now worry about the initial conditions necessary for this to happen. Inflation will start, approximately, when $\rho_0 \simeq \rho_\Phi$. We know from (4.6) that:

$$\langle\rho_\Phi\rangle(t) \simeq M_\Phi^2 \left\langle\Phi^2\right\rangle(t) \tag{5.9}$$

setting the two energy density to be equal at $t = 0$ implies:

$$\langle\rho_\Phi\rangle(0) \simeq M_\Phi^2 \Phi_0^2 \sim \alpha M_\star^4 \tag{5.10}$$

giving the initial condition for $\Phi_0^2 = \alpha M_\star^4 / M_\Phi^2$.

Now setting $m_{eff}^2 = 0$ will give an estimate of the number of e-folds:

$$m_{eff}^2(t_f) = 0 = \Phi_0^2 e^{-3Ht_f} - \alpha M_\star^2 \tag{5.11}$$

Since H is constant during inflation, Ht_f corresponds to the number of e-folds. Then we have

$$Ht_f = N = -\frac{1}{3}\ln\left(\frac{\alpha M_\star^2}{\Phi_0^2}\right) = \frac{1}{3}\ln\left(\frac{M_\star^2}{M_\Phi^2}\right) \tag{5.12}$$

For $M \sim O(Tev)$ this gives $N \simeq 50$.

Another constraint one should worry about is weather the approximation with the average time of Φ in (5.7) is appropriate. To check that, it is sufficient to require that the time Φ takes to cross zero while oscillating is smaller than the inverse mass $m^2 = \alpha M_\star^2$. To do so we linearize Φ around the zero value. Since $M_\Phi \geq H$ by hypotesis, it follows that

$$\frac{d\Phi}{dt} \sim \Phi_0 e^{-3/2Ht} M_\Phi \tag{5.13}$$

implying the condition:

$$\Delta t \sim \Delta\Phi e^{3/2Ht} \frac{1}{\Phi_0 M_\Phi} \leq m^{-1} = \frac{1}{\sqrt{\alpha} M_\star} \tag{5.14}$$

Taking $\Delta\Phi \sim \sqrt{\alpha}M_\star$ and inserting Φ_0 we get a condition that needs to hold all the way until the end of inflation. If such a condition breaks, then ϕ will be able to slow down the hill, terminating inflation early. Setting $t = t_f$ in (5.14) we get that the number of e-folds depends only on α:

$$N \sim -\frac{1}{3}\ln\alpha \tag{5.15}$$

imposing M to be of order Tev and $\alpha \sim 10^{-65}$ very small. This condition was just checked at the beginning of inflation in [2] and is equivalent (5.12) with the choice of parameter of [2]. However this condition will play a fundamental role later, when we will take some of the parameters to be free in order to avoid some phenomenological constraints.
Given such a period of inflation, the upper bound for the reheating temperature is given by (4.14):

$$T_r \simeq \sqrt{HM_p} \simeq M \sim O(Tev) \tag{5.16}$$

As for the size of the observable universe, if we take the initial homogeneous patch to be of size of order $1/H$ we have that today's patch is:

$$a_{today} \sim \frac{1}{H}e^N\frac{T_r}{T_{today}} \sim 10^{37}cm \tag{5.17}$$

we will see later why the initial size of the patch is $1/H$. Notice that the obtained value is much larger than the actual horizon size $\simeq 10^{28}cm$. Notice also that one should not worry that $N \leq 50$ since in this model, the horizon and flatness problems are partially solved by the fact that preceding to the period of "locked inflation" , there is a stage of inflation in the original false vacuum before the instanton guides the phase transition.

5.2 The complete model

In [2] a complete model is described. We would like to go through it once more in order to analyze better how the initial condition for our locked inflation are produced.

Consider the following generic potential for ϕ and Φ:

$$V(\Phi, \phi) = \alpha_0 \Phi^2 (\Phi - M_0)^2 + M_\Phi^2 \Phi^2 + \lambda \phi^2 \Phi^2 + \alpha(\phi^2 - M_\star^2)^2 \tag{5.18}$$

with $M_\Phi^2 \sim \frac{M^4}{M_p^2}$, $\alpha_0 \sim \frac{M^4}{M_p^4}$, $M_0 \sim M_p$. The new potential introduces a new false vacuum at $\Phi \simeq M_0$ and $\phi = 0$, and since the fields are weakly coupled, the other minima are substantially unaffected. As in Guth's inflation, the idea here, is that the universe starts in a false vacuum and then ends in the true one via an instanton process, nucleating a bubble. Inside this bubble the initial conditions are those required by locked inflation as we are going to show now. In this false vacuum, the mass of the ϕ field is $\sim \lambda M_p$ and hence its dynamic can be ignored in the nucleation because it is very heavy compared to the Hubble scale

$$H_0^2 \simeq M^4 / M_p^2 \tag{5.19}$$

Consider, then only Φ's potential

$$V(\Phi) = \alpha_0 \Phi^2 (\Phi - M_0)^2 + M_\Phi^2 \Phi^2 \tag{5.20}$$

the global minimum is at zero and there is a local minimum at M_0. A maximum is found at approximately $M_p/2$ with $V(\Phi = 1/2M_0) \sim 1/16\alpha_0 M_0^4$. In principle one might worry that the transition can via a normal fluctuation of the Φ field in a De Sitter universe, rather than an instanton. That would be the case if the barrier was too low. Indeed Φ can be regarded as massless and hence its fluctuation $\delta\Phi \sim H_0 \sim M^2/M_p$. On the other hand, the height of the barrier per unit volume is

$$V(\Phi = 1/2M_0) \cdot H_0^{-3} \sim \frac{M_p^3}{M^2} \tag{5.21}$$

where H_0^{-3} is the natural volume, implying that a transition via quantum tunneling is necessary.

Notice here, however, how the problem of initial condition is not really a problem: the field is evolving in a De Sitter universe. This means that different bubbles, that do not interact with each other ($M_\Phi \ll H$), will be nucleated. One of this bubble eventually will have the correct initial condition, generating our universe according to our locked mechanism. However we want to show in the next section that the most probable configuration for each bubble is exactly the one of our universe.

5.2.1 Hawking-Moss Instanton

We want now to understand the formula proposed by Hawking and Moss [19] for the probability of bubble nucleation in a De Sitter universe (1.36). As already said this has a clear explanaition if one considers a stocasthic description of inflation. To understand this we will follow [20] adapting it to our situation.

The brownian motion in De Sitter The fluctuation of a mode in a De Sitter Universe was computed in (4.15). In order to obtain the two point correlator $\langle\phi^2\rangle$ we have to sum over all possible modes contributing. Moreover we know that the only modes behaving differently from a Minkowski perturbation, are those still within the horizon. In principle it means that we will have a cutoff at $k \geq k_0 e^{-Ht}$. This implies that

$$\langle \phi^2 \rangle \sim H^2 \int_{He^{-Ht}}^{H} \frac{1}{|k|^3} d^3k \sim H^2 \int \frac{d|k|}{|k|} = \frac{H^3 t}{4\pi^2} \tag{5.22}$$

where the fact that H is constant has been used, as well as the prefactors have been reintroduced.

The process of generating a classical field in an inflationary universe can be thought as a Brownian motion due to the conversion of the quantum modes into classical modes at horizon crossing i.e. when the physical mode is of the same order as H. This happens because the modes freeze at horizon crossing, but with different phases between each other, generating a mismatch for the classical field ϕ (they all contribute with different signs). This is the source of the variance of the random distribution. As in the standard diffusion problem for a particle undergoing Brownian motion, the mean squared particle distance from the origin is directly proportional to the duration of the process (5.22).

Define the probability distribution $P(\phi, t)$ as the distribution to find the field at ϕ, at time t. The evolution of such a distribution can be written as a diffusion equation of the form(in the case of a massless field)

$$D \frac{\partial^2 P}{\partial \phi^2} = \frac{\partial P}{\partial t} \tag{5.23}$$

The diffusion coefficient D can be found as follows:

$$\frac{d\langle \phi^2 \rangle}{dt} = \frac{d}{dt} \int \phi^2 P d\phi = \frac{H^3}{4\pi^2} \tag{5.24}$$

where (5.22) has been used. Now, passing the derivative sign of (5.24) and using the fact that $\langle \phi\dot{\phi} \rangle = 0$ and imposing (5.23) we have:

$$\frac{d\langle \phi^2 \rangle}{dt} = D \int d\phi \phi^2 \frac{\partial P}{\partial t} = 2D = \frac{H^3}{4\pi^2} \tag{5.25}$$

where it was integrated twice by parts and the fact that $\int dP = 1$ has been used. This implies that the diffusion is

$$D = \frac{H^3}{8\pi^2} \tag{5.26}$$

For example taking the initial condition $P(\phi, 0) = \delta(\phi)$. The solution is readily given as:

$$P(\phi, t) = \sqrt{\frac{2\pi}{H^3 t}} e^{-\frac{2\pi^2 \phi^2}{H^3 t}} \tag{5.27}$$

It is easy to check that the solution is properly normalized and that it satisfies the diffusion equation with dispersion $\Delta^2 = \langle \phi^2 \rangle = \frac{H^3 t}{4\pi^2}$.

D can be interpreted as the coefficient describing the rate of the transition from $k > H$ to $k < H$.

In the case of a massive field (5.23) has to be generalized with an extra term as follows [29]:

$$\frac{\partial P}{\partial t} = D \frac{\partial^2 P}{\partial \phi^2} + b \frac{\partial}{\partial \phi}\left(P \frac{dV}{d\phi} \right) \tag{5.28}$$

where the mobility coefficient b satisfies $\dot{\phi} = -b\frac{dV}{d\phi}$.

The KG eom are

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi} \tag{5.29}$$

and since under slow roll conditions $\ddot{\phi} \ll 3H\dot{\phi}$ it follows:

$$\frac{\partial P}{\partial t} = \frac{H^3}{8\pi^2} \frac{\partial^2 P}{\partial \phi^2} + \frac{\partial}{\partial \phi}\left(\frac{P}{3H} \frac{dV}{d\phi} \right) \tag{5.30}$$

This result was first derived by Starobinsky [30] in a more rigorous way and will be our starting point to understand Hawking Moss formula.
Since in our case the height of the wall is much bigger then the mass of the Φ field and its fluctuations, we can assume a quasi-stationary distribution:

$$\frac{\partial P}{\partial t} = -\frac{\partial j}{\partial \Phi} = 0 \tag{5.31}$$

This implies that the current j defined as

$$-j = -\frac{H^3}{8\pi^2}\frac{\partial P}{\partial \Phi} + \frac{P}{3H}\frac{dV}{d\Phi} \tag{5.32}$$

is constant. In principle this current extends over all space. Hence it makes sense to assume it is zero. This readily gives the following equation:

$$\frac{d \ln P}{d\Phi} = \frac{8\pi^2}{9H^4}\frac{dV}{d\Phi} = \frac{M_p^4}{8V(\phi)^2 \frac{dV}{d\Phi}} \tag{5.33}$$

implying that the probability is

$$P = N e^{\frac{3M_p^4}{8V(\Phi)}} \tag{5.34}$$

where N is a normalization constant. The solution has a clear cut-maximum at $\Phi = M_p$. Hence, up to a sub exponential prefactor we obtain:

$$P = e^{-3\frac{M_p^4}{8}\left(\frac{1}{V(\Phi)} - \frac{1}{V(\Phi \simeq \frac{1}{2}M_p)}\right)} \tag{5.35}$$

which is Hawking Moss instanton formula. Before going back to locked inflation another remark is necessary. Notice that for massless field in a De Sitter universe, fluctuations are of order H. These fluctuations, at horizon crossing, generates a brownian like behaviour on the expectation value of the field. Eventually the field reaches the top of the hill. However, since the gradient term in a De Sitter universe is suppresed by the scale factor squared, it follows that it can roll down the hill towards its true minimum. The initial size of the patch where this happens (or the size of the nucleated bubble) is most probably of size H^{-1} (the size of fluctuations) justifying our assumptions in the locked inflation description. However, even if the initial size of the patch is much smaller thant H^{-1}, it is shown in [2] that the final size of our universe horizon is not much affected. The reason is that if the patch is much smaller than $1/H$ then the curvature term in Friedmann's equation will dominate the expansion of the universe while the amplitude of Φ stays constant until the curvature of the bubble becomes comparable to M_Φ^2 (the Φ field is strongly overdamped and frozen). But since $M_\Phi \sim H$ it follows that Φ will start oscillating only when the most probable initial size is reached.

Moreover, it follows from the previous analysis that the field Φ starts on the top of the hill. And so also its initial condition $\Phi \simeq O(M_p)$ is justified.

Chapter 6

Problems and constraints of locked inflation

There are mainly three problems that seem to fully rule out the parameters space of locked inflation. First of all, the transition of ϕ to its true vacuum might generate a period of extra inflation baptized as saddle inflation in [7]. This inflationary stage is a slow roll type of inflation. There are three possibilities: 1) this slow roll inflation lasts longer than fifty e-folds, thus washing out all possible imprints of locked inflation, 2) it lasts less than fifty e-folds. However it is shown by the authors of [7] that this would produce an unacceptable number of black holes within our horizon or 3) Saddle inflation does not take place because of the choice of our parameters. This is the phenomenological viable situation we will assume.

Another problem comes from radiative corrections pointed out in [2] and [8]. This has to do with a Weinberg-Colemann correction to the potential at one loop that might move the vacuum of Φ from zero making the locking actually

last forever.
The last problem, indicated in [2] and expanded in [8] is that of parametric resonances that might ruin inflation within the first e-folds. It is important to point out that in this article the analysis of parametric resonances is performed via simulations and the study does not take the full non linearities into account. In this sense the obtained constraints cannot be fully trusted. In particular it is not clear how the background locking field backreacts on the inflationary field when the production of particles due to adiabaticity violation becomes non negligible. Moreover, even if we assume the analysis to be reliable, still a window for the model to work is left for $10\,H \leq M_\Phi \geq H$. In this regime fully analytical considerations can be done and the inflationary expansion is enough to suppress the particle production.
In the following sections these problems are introduced and contextualized. After that, possible solutions to make the model again viable are presented.

6.1 Saddle Inflation

At the end of inflation the classical field ϕ is sitting at $\phi = 0$ and is ready to roll down to its true vacuum. Let us assume we can ignore the Φ field dynamic (which is, however, relaxing at zero). Since ϕ is very small wrt its vacuum value we can expand the potential as follows:

$$V(\phi) = \alpha(\phi^2 - M_\star^2)^2 \sim \alpha M_\star^4 - \alpha \frac{\phi^2}{M_\star^2} \tag{6.1}$$

where as usual factors of order 1 are ignored. The eom are now:

$$\ddot{\phi} + 3H\dot{\phi} - \alpha M_\star^2 \phi \simeq 0 \tag{6.2}$$

Now defining $N = \ln a$ i.e. $dN = \frac{1}{a}da$ we can rewrite

$$\dot{\phi} = \frac{d\phi}{dN}H, \qquad \ddot{\phi} = \frac{\partial^2\phi}{\partial N^2}H^2 \tag{6.3}$$

implying that (6.2) can be rewritten as

$$\phi'' + 3\phi' - \frac{M_p^2}{M_\star^2}\phi \doteq \phi'' + 3\phi' - 3\eta\phi \simeq 0 \tag{6.4}$$

where $\phi' = \partial\phi/\partial N$ and we defined $\eta \doteq \frac{M_p^2}{M_\star^2}$. As for the initial conditions for ϕ we can assume that $\phi_i \sim H$ since that is the order of fluctuations of the field. We will assume $\phi_i' = 0$

A solution to (6.4) is

$$\phi(N) = A\exp\left\{\left(-\frac{3}{2}(1-\delta)N\right)\right\} + B\exp\left\{\left(-\frac{3}{2}(1+\delta)N\right)\right\} \tag{6.5}$$

with $\delta \doteq \sqrt{1 + \frac{4}{3}\eta}$. Imposing initial conditions we have

$$\begin{aligned}\phi(N) = \frac{\phi_i}{2\delta}(\delta+1)\exp\left\{\left(\frac{3}{2}(\delta-1)N\right)\right\}+ \\ \frac{\phi_i}{2\delta}(\delta-1)\exp\left\{\left(-\frac{3}{2}(\delta+1)N\right)\right\}\end{aligned} \tag{6.6}$$

Since we are interested in the case where $N \geq 1$ we can drop the exponentially decaying solution i.e.

$$\phi(N) \approx \frac{\phi_i}{2\delta}(\delta+1)\exp\{(f(\eta)N)\}, \qquad f(\eta) \doteq \frac{3}{2}(\delta-1) \tag{6.7}$$

its derivative being

$$\phi' = \frac{\phi_i}{2\delta}(\delta+1)f(\eta)\exp\{(f(\eta)N)\} \tag{6.8}$$

it follows that the equation of state has the parameter [1]

$$\omega \doteq \frac{\dot{\phi}^2 - 2V}{\dot{\phi}^2 + 2V} \tag{6.9}$$

since $\phi_i' \sim H$ it follows that $\phi' \ll V \sim \alpha M_\star^4$. Thus (6.9) can be rewritten as:

$$\omega \sim -1 + \frac{H^2 \phi'^2}{V} = -1 + \frac{\phi(N)^2 f(\eta)^2}{M_p^2} \approx -1 \tag{6.10}$$

so the equation of state $p \approx -\rho$ holds true at the beginning of saddle inflation. Inflation will end when $\omega \sim 0$ or when $\phi^2 \approx \frac{M_p^2}{f(\eta)^2}$. Inserting this back in (6.7) we get the approximate number of e-folds:

$$\frac{M_p}{f(\eta)} \approx \frac{\phi_i}{2\delta}(\delta + 1) \exp\left(f(\eta) N_{saddle}\right) \tag{6.11}$$

Giving for N_{saddle}:

$$N_{saddle} = \frac{1}{f(\eta)} \ln\left(\frac{M_p \delta}{(\delta+1)\phi_i f(\eta)}\right) = \frac{1}{3/2(\delta - 1)} \ln\left(\frac{M_p \delta}{(\delta+1)\phi_i 3/2(\delta+1)}\right) \tag{6.12}$$

now using $\phi_i \sim H_{locked}$ and taking $M \sim O(Tev)$ numerically one can obtain $N_{saddle} \leq 1$ for $\eta = M_p^2/M_\star^2 \geq 10^3$. This means that we need to push down the vacuum of ϕ down to at least $10^{-2} M_p$ to be safe. Such a constraint has a two-fold utility. It not only avoids the extra period of inflation, but it also avoids overproduction of black-holes phenomenologically unacceptable as it was shown in [31]

Consequences for locked inflation It immediately follows that, in order to have the desired fifty e-folds, also M_Φ needs to be changed accordingly because of the e-folds masses constraint (5.12). In this sense also M_Φ becomes a free parameter of the theory. Moreover, because of the constraint on the e-folds coming from the oscillatory approximation with the average (5.15) we have that M cannot be moved from the Tev scale in order to have a small enough α. In this context the question of small $M_\star$ can be arises naturally. A possible bound could come from the reheating temperature. In the natural lore, no temperature lower than $10^2 Gev$ or so can be accepted because of baryogenesis. However our ϕ field, could be coupled via Yukawa coupling to some other degrees of freedom, which, at the phase transition, becomes extremely heavy and then decay in the known degrees of freedoms of the standard model. Hence, in this sense, a more natural lower bound for the reheating temperature would be around $10Mev$ in order not to spoil nucleisynthesis and neutrino decoupling. Imposing this lower bound one has:

$$T_r \sim 10^{-2} Gev \sim \sqrt{HM_p} \sim \alpha^{1/4} M_\star \sim 10^{-16} M_\star \tag{6.13}$$

implying $M_\star \geq 10^{-5} M_p$.

6.2 One loop correction

As already pointed out in [5] the potential gets a loop correction of the Coleman-Weinberg type due to the inflaton breaking of supersymmetry:

$$\Delta V = \frac{m_\phi^2}{64\pi^2} \Phi^2 \ln \left(\frac{\Phi^2}{Q^2} \right) \tag{6.14}$$

This type of correction to the Kähler potential, arising from ϕ loops, does not cancel in a supersymmetric scenario even though all other corrections are absent (for example the ones proportional to Φ^4) [32]. Such a correction might move the minimum Φ from zero and if the minimum is bigger than Φ_c, Φ_c being the value of Φ for which m_{eff}^2 becomes zero, then locked inflation will last forever. Hence we are going to check for which parameters this is the case.

In order to derive (6.14) we will consider perturbations on top of the classical solution and then integrate them back adapting the analysis in [1]. The same result can be obtained also computing the effective one loop action through a functional determinant (see [21] for an example). To do so consider the following field expansion:

$$\phi(x) = \phi(t) + \delta\phi(x) \tag{6.15}$$

The equation of motion become:

$$\phi(x)^{;\alpha}_{;\alpha} + V' = \phi(t)^{;\alpha}_{;\alpha} + \delta\phi(x)^{;\alpha}_{;\alpha} + V' + V''\delta\phi + \frac{1}{2}V'''(\delta\phi)^2 + O(\delta\phi^3) = 0 \tag{6.16}$$

Now we take spatial average over (6.16):

$$\phi(t)^{;\alpha}_{;\alpha} + V' + \frac{1}{2}V'''\langle\delta\phi^2\rangle = 0 \tag{6.17}$$

To evaluate $\langle\delta\phi^2\rangle$ we quantize the perturbation $\delta\phi \to \delta\hat{\phi}$. From now on we will drop the hat.

Notice that $\delta\phi$ has the following linear equation of motion:

$$\delta\phi(x)^{;\alpha}_{;\alpha} + V''\delta\phi = 0 \tag{6.18}$$

Assume that $V'' \doteq m_\phi^2 \geq 0$. Hence we expand the field as

$$\delta\phi(x) = \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_k}} \left(a_k e^{-ikx} + a_k^\dagger e^{ikx}\right) \tag{6.19}$$

with $\left[a_k, a_{k'}^\dagger\right] = \delta^3(k - k')$ and

$$\omega_k^2 = k^2 + V''$$

Now define the $|n_k\rangle$ as:

$$|n_k\rangle = \frac{(a_k^\dagger)^n}{\sqrt{n!}}|0\rangle \tag{6.20}$$

It follows from the definitions that:

$$\langle a_k^\dagger a_{k'} \rangle_Q \doteq \frac{\langle n_k | a_k^\dagger a_{k'} | n_k \rangle}{\langle n_k | n_k \rangle} = n_k \delta^3(k - k') \tag{6.21}$$

and similarly $\langle a_k^\dagger a_{k'}^\dagger \rangle_Q = \langle a_k a_{k'} \rangle_Q = 0$.

We are now ready to evaluate the quantum average $\langle \delta\phi^2 \rangle_Q$:

$$\langle \delta\phi^2 \rangle_Q = \int_k \int_{k'} \left\langle \left(a_k e^{-ikx} + a_k^\dagger e^{ikx} \right) \left(a_{k'} e^{-ik'x} + a_{k'}^\dagger e^{ik'x} \right) \right\rangle_Q \tag{6.22}$$

where we denote $\int_k \doteq \int d^3k/(2\pi)^3\sqrt{2\omega_k}$. Using (6.21) in (4.52) it follows:

$$\langle \delta\phi(\vec{x})^2 \rangle_Q = \frac{1}{2\pi^2} \int dk \frac{k^2}{\sqrt{k^2 + V''}} \left(\frac{1}{2} + n_k \right) \tag{6.23}$$

Since we are interested in the one loop effective potential we can ignore the n_k term and focus only on the vacuum contribution. In view of (6.23) we can rewrite the term appearing in (6.17) as

$$\frac{1}{2} V''' \langle \delta\phi^2 \rangle = \frac{1}{4\pi^2} \left(\int dk k^2 \sqrt{k^2 + V''} \right)' \doteq \tilde{V}' \tag{6.24}$$

The integral in (6.24) is divergent. We regularize it with a cutoff M. Evaluating the integral gives:

$$\frac{1}{8} \left[M(2M + m^2)\sqrt{M^2 + m^2} + m^4 \ln \frac{m}{M + \sqrt{M^2 + m^2}} \right] \tag{6.25}$$

where we remind that $m^2 \doteq V''$. Now we multiply and divide the argument of the logarithm by a mass scale Q and separate the terms in (6.25) in finite and diverging part, obtaining:

$$V_{eff} = V + V_\infty + \frac{m^4}{64\pi^2} \ln \frac{m^2}{Q^2} \tag{6.26}$$

with the divergent part being:

$$V_\infty = \frac{M^4}{4\pi^2} + \frac{m^2}{16\pi^2} M^2 - \frac{m^4}{32\pi^2} \ln \frac{2M}{e^{3/4} Q} + O\left(\frac{1}{M^2} \right) \tag{6.27}$$

Our original potential for ϕ is of the form:

$$V(\phi) = \alpha\phi^4 + (\lambda\Phi^2 - \alpha M_\star^2)\phi^2 \tag{6.28}$$

Hence $m^2 = \lambda\Phi^2 - \alpha M_\star^2$ The divergent term V_∞ can be reabsorbed in a redefinition of the parameters in the potential, while the finite part in (6.26) becomes:

$$V_{finite} = \frac{\left(\lambda\Phi^2 - \alpha M_\star^2\right)^2}{64\pi^2} \ln \frac{\lambda\Phi^2 - \alpha M_\star^2}{Q^2} \tag{6.29}$$

Using the fact that $\lambda \sim 1$ and that $\alpha M_\star^2$ is always smaller than Φ^2 in our region of interest we get the following corrected potential for Φ:

$$V(\Phi) \simeq \Phi^2 \left(\frac{1}{2} M_\Phi^2 + \frac{m_\phi^2}{64\pi^2} \ln\left(\frac{\Phi^2}{Q^2}\right)\right) \tag{6.30}$$

where we dropped the quartic term since we are only interested in seeing how this one loop correction moves the minimum of Φ's potential from the origin. We want now to fix the parameter Q. Taking first derivative yields

$$V_{,\Phi} = M_\Phi^2 \Phi + \frac{m_\phi^2 \Phi}{32\pi^2}\left(\ln\left(\frac{\Phi^2}{Q^2}\right) + 1\right) \tag{6.31}$$

For the curvature we have

$$V_{,\Phi\Phi} = M_\Phi^2 + \frac{m_\phi^2}{32\pi^2}\left(\ln\left(\frac{\Phi^2}{Q^2}\right) + 3\right) \tag{6.32}$$

we impose the following renormalization condition

$$V_{,\Phi\Phi}(\Phi = \Phi_c) = M_\Phi^2 \tag{6.33}$$

giving $Q^2 = e^3\Phi_c^2$. To find now the vacuum of Φ we consider the first derivative of the running potential:

$$V_{,\Phi}(\Phi) = M_\Phi^2 \Phi + \frac{m_\phi^2}{32\pi^2}\Phi\left(\ln\left(\frac{\Phi}{\Phi_c}\right) - 2\right) \tag{6.34}$$

evaluating at $\Phi = \Phi_c$ it follows:

$$V_{,\Phi}(\Phi_c) = \Phi_c \left(M_\Phi^2 - \frac{m_\phi^2}{16\pi^2} \right) \geq 0 \qquad (6.35)$$

which is the condition under which the minimum occurs at values lower than Φ_c allowing locked inflation to end.

This implies that:

$$M_\Phi^2 \geq \frac{m_\phi^2}{16\pi^2} \sim 10^{-2}\alpha M_\star^2 \qquad (6.36)$$

which, for $M_\star \leq M_p$ naturally satisfies the constraint $M_\Phi \geq H$ necessary to have Φ oscillating during locked inflation.

6.3 Parametric resonances

Consider the equation of motion for ϕ:

$$\ddot{\phi} + 3H\dot{\phi} + \left(\lambda\Phi(t)^2 - m_\phi^2\right)\phi = 0 \qquad (6.37)$$

define $\tau \doteq m_\phi t$ and$\chi \doteq \exp\{(3/2Ht)\}\phi$. Then we have

$$\dot{\phi} = -m_\Phi \frac{3}{2} h e^{-\frac{3}{2}h\tau}\chi + m_\Phi e^{-\frac{3}{2}h\tau}\chi' \qquad (6.38)$$

where $h \doteq H/m_\Phi$ and the prime denotes derivative wrt τ. The second derivative is

$$\ddot{\phi} = m_\Phi^2 \left(\frac{9}{4} h^2 e^{-3/2h\tau}\chi - 3he^{-3/2h\tau}\chi' + e^{-3/2h\tau}\chi'' \right) \qquad (6.39)$$

inserting these back into (6.37)

$$\chi'' + (2q(\tau)(1 - \cos 2\tau) - b)\,\chi = 0 \qquad (6.40)$$

with $b \doteq \frac{m_\phi^2}{m_\Phi^2} + \frac{9}{4}h^2$, $q(\tau) = q_0 e^{-3h\tau}$, $q_0 = \frac{\lambda\Phi_0^2}{4m_\Phi^2}$.

Since the dependance on time of $q(\tau)$ is through the exponential of h, which changes very slowly wrt the oscillatory motion, it is possible to threat q as constant and to use the usual known results for Mathieu's equation.

The equation for χ's modes can be derived from (6.40) just remembering that it is sufficient to add a squared gradient term diluted by the scale factor a:

$$\chi_k'' + (2q(\tau)(1 - \cos 2\tau) - b(k,\tau))\,\chi_k = 0 \qquad (6.41)$$

where $b(k,\tau) = b - \frac{k^2}{m_\Phi^2} e^{-2h\tau}$.

Solutions to the Matheiu's equation are well known and they indeed depend on the parameters q and b. In particular a general solution has the form [33]

$$\chi(\tau) = e^{s\tau} f(\tau) \qquad (6.42)$$

with f with period of π in τ and s is known as the Floquet exponent which can be complex.

The physical interpretation of (6.40) is clear in our context. The amplitude of the oscillatory term $q(\tau)$ is much bigger than b by construction. Let us assume for a moment that b is zero. It follows immediately that whenever the oscilaltory part approaches zero, a violation of the adiabaticity condition is generated. To understand this consider the following condition on the diespersion relation for the mode χ_k:

$$\omega(k)^2 = m_\Phi^2(2q(t)(1 - \cos 2m_\Phi t) - b(k,t)) \qquad (6.43)$$

where the mass dimension of ω has been restored.

Now the process is adiabatic as long as the oscillatory part is equal to zero.

As long as the process is adiabatic there is no change in the occupation number, but the situation changes under violation. This can be estimated by

$$\left|\frac{\dot{\omega}}{\omega^2}\right| \geq 1 \tag{6.44}$$

The meaning of (6.44) is clear: it will sort out those modes that will be enhanced by a factor $e^{s\tau}$. Setting b to zero, and assuming Φ_0^2 much bigger than M_Φ, m_ϕ^2 and H^2 it follows

$$k_{max}^2 \leq \Phi_0^2 e^{-h\tau} \tag{6.45}$$

it is clear that what we are describing is a broad resonance, and as the universe keeps expanding more and more modes are amplified. However, as a consistency check, it is interesting to check on which scale these modes resonate.

We already found Φ_0^2 at the beginning of locked inflation by imposing an equality condition between Φ's energy and the vacuum energy: ρ_Φ and ρ_0 which implied $\Phi_0^2 = \frac{\alpha M_\star^4}{M_\Phi^2}$ implying that

$$k_{max}^{-2} \sim \frac{e^{Ht}}{\Phi_0^2} = \frac{e^{Ht} M_\Phi^2}{\alpha M_\star^4} \tag{6.46}$$

now if we compare this length with our Hubble patch size $1/H^2$ we get

$$\frac{e^{Ht}}{\Phi_0^2} \simeq \frac{1}{H^2} = \frac{M_p^2}{\alpha M_\star^4} \tag{6.47}$$

which for our Φ_0 gives

$$e^{Ht} \sim \frac{M_p^2}{M_\Phi^2} \tag{6.48}$$

For this to be true, a time much bigger than our inflationary period is required ($M_\star \leq M_p$). This means that all the modes for $k \leq k_{max}$ are affected by this type of resonance making our claim consistent.

From our computation of the one loop correction we have from equation (6.36) that $b \leq 100$. The problem can be splitted in two regimes: $b \leq O(1)$ and $b \geq O(1)$. From the known theory of Matheiu's equation [34] we know that the first case presents an unstable system, while the second case a stable one. In the second case we are going to obtain simply a small correction to the number of e-folds, but the essence of locked inflation will not be changed. Notice that one of the powerful features of locked inflation was the fact that $m_\phi \sim M_\Phi$. In this sense, these two masses were fixed by only one parameter. In principle this is a property we would like not to loose.

Small b In the case of small b, the solution (6.42) has on average Floquet exponent $\overline{s} \sim 0.1$ (this value can be found numerically for large values of $q's$ [8] or it can be derived analytically [1]). This exponent is caused by the production of particles when adiabaticity is violated and in principle depends on k. For large q it can be taken constant. It follows for ϕ that

$$\phi(t) \sim e^{(\overline{s}-3h/2)\tau} \tag{6.49}$$

this implies that $H/M_\Phi \geq 10^{-1}$. This bound was already mentioned in [2] and was already found in different models of tachyonic reheating [35].

Now turning to the modes we have that each k mode is amplified by a k-indipendent amplification factor:

$$\langle\chi_k^2\rangle(\tau) \sim e^{2\overline{s}\tau}\langle\chi_k^2\rangle(0) \tag{6.50}$$

Now, assuming that at $\tau = 0$, χ_k is in its vacuum implies $\langle\chi_k\rangle(0) \sim k^{-1/2}$. To find $\langle\chi^2\rangle$ we have to sum over all modes being enhanced. Let us assume that all modes with $k \leq k_{max}$ are enhanced equally:

$$\langle \chi^2 \rangle(\tau) = \int \langle \chi_k^2 \rangle(\tau) d^3k \sim k_{max}^2 e^{2\bar{s}\tau} \tag{6.51}$$

this implies for ϕ:

$$\phi^2 \sim k_{max}^2 e^{(2\bar{s}-3h)\tau} \tag{6.52}$$

consider this factor after only one e-fold i.e. $Ht = h\tau = 1$:

$$\phi^2 \sim k_{max}^2 \sim \Phi_0^2 \tag{6.53}$$

where the fact that $H \sim M_\Phi$ has been used. Now consider the background equation for Φ. In our assumption we neglected the λ-coupling to derive our background equation. The full equation is:

$$\ddot{\Phi} + 3H\dot{\Phi} + (M_\Phi^2 + \lambda\phi^2)\Phi = 0 \tag{6.54}$$

but now $\langle \phi^2 \rangle \gg M_\Phi^2$ after only one e-folds, making questionable whether or not our background approximation is safe from backreaction. Indeed the claim that locked inflation finishes within one e-folds [8] seems not very reasonable since non linear effects are not fully taken into account. In particular it is not clear how Φ will backreact on ϕ as it grows.
However later it will be shown that when dealing with model building, it is sufficient to simply consider a complex scalar field as Φ to fully eliminate this problem.

Large b In the case of large b Matheiu's equation is in the stable regime [12]. This means that locked inflation proceeds normally. However, inevitably parametric resonances will end inflation earlier i.e. approximately when $q(\tau)$

becomes of order b and the usual tachyonic behaviour starts to dominate. This gives nothing but a small correction to the number of 50 e-folds [8] making this part of the parameter space still viable. Notice that dropping the constraint coming from loop corrections, b can be much bigger than 100 making the parameter space even larger.

6.4 Solutions

A way to eliminate parametric resonances is to add components to the Φ field. If the different components have different phases, then the oscillating components never happens to cross the zero value at the same time making the violation of the adiabaticity condition not possible. To understand this situation better consider the case where Φ is complex. The solution is going to have two components, both oscillating with the same frequency M_Φ and exponentially decreasing. However, since the phase difference between the two solution is exactly $\pi/2$ it follows immediately that the oscillatory term in (6.40) disappears and adiabaticity is always maintained. As a consequence no resonance takes place and the model works also for values of b of order one or smaller. But this is not the only advantage of considering a complex field. In fact, the λ coupling in the original potential is now

$$\lambda|\Phi|^2\phi^2 = \lambda\Phi_0^2 e^{-3Ht}\phi^2 \tag{6.55}$$

i.e. the oscillatory dependance is totally dropped. Instead now we have an effective mass term for ϕ which is nothing but a exponentially decreasing term. This in turn allows us to fully drop the requirement coming from the averaging of Φ (5.15) allowing, in principle for more natural values of α which now, in order to give $N \sim 50$ e-folds, does not require anymore $M \sim O(Tev)$. This means that a more natural value of α can be considered. As a consequence, also the lower bound on $M_\star$ coming from the reheating temperature (4.14) is relaxed. Indeed lowering $M_\star$ imposes also, as a consequence, a lowering of M_Φ (because of the required number of e-folds (5.12)) and the price to pay would be that M_Φ is no longer of the same order as m_ϕ.

If one takes into account the one loop constraints (6.36), however, the hierarchy between $M_\star$ and m_Φ, and hence the smallness of α, are reintroduced in the model (because of (5.12)). In this case M can be taken to be at most of O(10Tev) and anyway an enlargement of the parameter space is obtained. One may also take the more phenomenlogical view that it is not necessary to satisfy the one loop constraint since the underlying funamental physics is not known. Once this constraint is dropped it is possible to take $M \sim O(10^{-5}M_p)$ i.e. $\alpha \sim O(10^{-20})$, and the model can produce fifty e-folds of inflation even taking $M_\star \sim v_{Higgs} \sim 10^2 Gev$ satisfying the required reheating temperature bound $T_r \geq 10^{-3} Gev$. In this situation certainly one has to push down the scale of M_Φ a lot and in this sense the choice of M_Φ is no longer natural. However M_Φ is also bounded from below by the bound $M_\Phi \geq H$. Notice that in this situation the degrees of freedom to which ϕ is coupled could be directly the standard model degrees of freedom making this scenario phenomenologically appetible.

Due to the low scale of inflation it is not possible to produce the observed density perturbations. However, with the described mechanism of modulated reheating, which can be easily implemented [5], adiabatic perturbations are produced together with the extra feature of non gaussianities with

$f_{NL} \sim O(1)$.

6.5 Consistency check at the beginning of locked inflation

Inflation starts when ρ_0 becomes of order ρ_Φ. This corresponds to a transition from a matter dominated universe to an inflationary universe (or cosmological constant universe). The assumption that the universe starts immediately evolving like De Sitter has to be checked, the reason being that in the very first few e-folds, the contribution of matter might be non neglegible to the dynamics of Φ field, which is our background. To do this consider Friedmann's equation together with two fluids (ignoring spatial curvature):

$$H^2 = \frac{8}{3}\pi G(\rho_\Phi + \rho_\Lambda) \tag{6.56}$$

with $\rho_\Lambda = const$ and $\rho_\Phi \propto a^{-3}$. Assume that the two fluids have equal energy contribution at the moment when $a(t_\star) = 1$. Then

$$\left(\frac{\dot{a}}{a}\right)^2 = H_\Lambda^2 \left(\frac{a^3+1}{a^3}\right) \tag{6.57}$$

where $H_\Lambda^2 \doteq 8\pi G/3\rho_\Lambda$.

Equation (6.57) leads to the integrable equation

$$\frac{da\sqrt{a}}{\sqrt{a^3+1}} = H_\Lambda dt \tag{6.58}$$

after substituting $a \to a^{3/2}$ and integrating we obtain the result:

$$a(t) \propto \sinh\left(\frac{3}{2}H_\Lambda t\right)^{2/3} \tag{6.59}$$

in the limit of big t (small t) we recover $a \propto e^{H_\Lambda t}$ ($a \propto t^{2/3}$). It follows that

$$H^2(t) = H_\Lambda \coth^2\left(\frac{3}{2}H_\Lambda t\right) \tag{6.60}$$

but this means that also the eom for the Φ background has an extra time dependance:

$$\ddot{\tilde{\Phi}} + 3H(t)\dot{\tilde{\Phi}} + M_\Phi^2\tilde{\Phi} = 0 \tag{6.61}$$

where we wrote $\tilde{\Phi}$ to distinguish this solution by from the one used in locked inflation. To find how much $\tilde{\Phi}$'s evolution differs from the one assumed in locked inflation (with H constant), (6.61) has been solved numerically. Looking at (6.60) it follows that equality between the energies of the two fluids happens approximately when $t \sim O(H^{-1})$. Comparing the evolution of $\tilde{\Phi}(t + H^{-1})$ with $\Phi(t)$ ($\Phi(0)$ is the moment when the transition happens in locked inflation) it was found a difference in the oscillatory amplitudes of about 30% at later time (the correction is however very sensible on the shift of time and it is already practically zero for $\Delta t = 2H^{-1}$). This means that $\Phi_0 \sim 3/2\tilde{\Phi}_0$. This, in turn, gives a negligible correction in the final number of e-folds making the approximation $H = const$ in locked inflation correct.

6.6 Production of topological defects

Topological defects can be formed when a symmetry of a system is not respected by its vacuum manifold. Suppose, for example, a Lagrangian invariant under a $\mathbb{Z}^2$ i.e. invariant under $\phi \to -\phi$. If its vacuum value is not at zero, but it is due to a potential of the form

$$\lambda(\phi^2 - v^2)^2 \tag{6.62}$$

it follows that at the vacuum level the symmetry is broken. The ϕ field can choose as vacuum one of the two disconnected values $+v$ or $-v$ respecting the fact that $\Pi_0(\mathfrak{M})$ of the vacuum manifold is non trivial. For inflationary cosmology this means that the field, when transitioning to its vacuum, can choose either one of the two possible values. Suppose at $-\infty$ the value $-v$ is chosen, while at $+\infty$ the value $+v$ is chosen. Topological defects are nothing but field configurations interpolating between these two possible values. In the case at hand the topological defects is a domain wall of the form

$$\phi(x) = v \tanh(\lambda v x) \tag{6.63}$$

In general the type of topological defect is determined by the symmetry that is not respected by the vacuum manifold. For example in the case of a complex field the vacuum is a circle and the fundamental group Π_1 is now non trivial. This in turn can lead to the production of strings.

In the usual treatements of topological defects formation in the early universe [36] the phase transition is always understood to be thermal. This is because as the universe cools down and the temperature goes down, the vacuum due to the term such as (6.62) is ripristined [1]. Formation of topological defects in this context is usually understood in terms of Kibble mechanism [9]. The idea there is that there is a maximal length to which the field ϕ is correlated: ξ. This means that on such scale it is possible to safely assume that ϕ is choosing the same vacuum everywhere. However this is not true for bigger scales. An immediate cosmological consistency bound is that $\xi \leq H^{-1}$ because otherwise casuality would be violated by the field.

In our present scenario, at the end of locked inflation, however, due to the weak coupling of the theory, thermal effects do not substantially modify our potential and can be safely ignored. Hence it all boils down to evaluate the value of ξ. A first estimate from (6.63) tells us that this correlation length should be aproximately of order of the size of the correlator i.e. $\xi \sim (\lambda v)^{-1}$. For $\lambda = \alpha$ and $v = M_\star$ this turns out to be very big: $n_{mass} \sim \xi^3 \sim \alpha^{3/2} M_\star^3$ and in principle not a large number of topological defects is produced per hubble patch.

A better estimate could be imposing a non adiabaticity condition. The potential of ϕ is changing with time due to the relaxation of the Φ field. Thus a non adiabaticity condition of the form

$$\left| \frac{\dot{\omega}_k}{\omega^2} \right| \geq 1 \tag{6.64}$$

might sort out those wavelengths that do not have time to realize the change in the potential due to Φ's relaxation, and hence the correlation size of ϕ. Remember that the dispersion relation for ϕ is

$$\omega_k(t)^2 = k^2 + m^2(t) \tag{6.65}$$

with

$$m^2(t) = \lambda \langle \Phi^2 \rangle (t) - \alpha M_\star^2 \tag{6.66}$$

we will evaluate the non adiabaticity condition (6.64) at the critical moment $\Phi = \Phi_c = \frac{\alpha M_\star^2}{\lambda}$ i.e. when the minimum at $\phi = \pm M_\star$ appears and locked inflation ends. It follows, after some algebra:

$$k_{crit}^3 \sim \alpha H M_\star^2 \tag{6.67}$$

where factors of order one have been ignored. The H term is due to the derivative of $\langle \Phi^2 \rangle (t)$.

It follows that

$$n_{adiab} \sim k^{-3} \sim \xi^3 \sim \alpha H M_\star^2 \sim \alpha^{3/2} \frac{M_\star^4}{M_p} \sim n_{mass} \frac{M_\star}{M_p} \ll n_{mass} \qquad (6.68)$$

Thus we have a suppression of the number of topological defects produced (which was already small) due to the dynamicity of the potential. It is easy to check that n_{adiab} satisfy the casuality bound for our range of parameters. The negligible production of topological defects could have been deduced due to the low inflationary scale ($H \sim O(10^{-3}\text{eV})$. This is reflected by the presence of the coupling $\alpha \ll 1$ in (6.68).

Conclusion and Outlook

In this work the consistency problem for locked inflation was adressed. It turned out that the parameter space is not ruled out leaving a window of opportunitiy for $M_\star \sim 10^{-2}$ and $M_\Phi \sim 10\, H_0$ in order to have at least 50 e-folds. Notice that it was shown that such a number of e-folds is more than enough to solve the horizon problem. The flatness problem is also solved if one considers the period before locked inflation where a De Sitter stage nucleates bubbles via Hawking-Moss instantons with the required initial conditions. Such a mechanism is understood stochastically via a Fokker-Planck equation for the classical sup-horizon scalar field. Hence such parameters are elevated into a prediciton. The reasons for reducing the parameter space are mainly three: a possible period of inflation after locked inflation which might erase every possible signature of the model, quantum corrections to the potential which might make locked inflation ethernal, and broad parametric resonances, which might take place within the first e-fold of inflation making our background assumption unreliable. The latter problem turns out to be very difficult analytically and a fully non linear study would be required. Since the considerations made in the present work were made in a regime where analytic considerations were possible and where the behaviour of the

Mathieu's equation was known, a fully non linear study can only enlarge the above mentioned parameter space. Moreover, due to the weak coupling appearing in the theory, it has been shown that a neglegible production of topological defects is produced at the end of inflation.

However, due to the low inflationary scale, modulated reheating becomes necessary to produce the required density perturbations. A new feature is added to this known mechanism by considering the dynamical evolution of the modulating field. Such a correction, clearly model dependent, does not alter the prediction $f_{NL} \sim O(1)$ which is a general feature of modulated reheating.

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Declaration of authorship

I hereby declare that the submitted thesis is my own original work. All sources used are acknowledged as references.

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