Flux-Scaling Scenarios for Moduli Stabilization in String Theory and Axion Monodromy Inflation

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Abstract

In this thesis we study tree-level moduli stabilization via flux compactification of type IIB string theory on Calabi-Yau 3-folds. At first, we review general facts about orientifolds as well as various moduli fields and present eventually the complete Kähler potential. We also investigate the 10d effective supergravity action and introduce the fluxes necessary for moduli stabilization. Special emphasis is placed on geometric and non-geometric fluxes, which finally lead us to a convenient expression for the full superpotential. Thereby we obtain several scalar potentials for the moduli, showing interesting properties in their stable non-supersymmetric flux minima. On the one hand, we come up with a concrete model where all possible fluxes have been turned on, and on the other hand every scenario obeys a novel flux-scaling behaviour. As all of our models keep one axionic modulus unstabilized, in the end we are able to apply these concepts to the discussion of inflation in string theory, more precisely to axion monodromy inflation. It turns out that some backreacted, uplifted F-term scalar potentials interpolate between quadratic and Starobinsky-like inflation. The latter appears to belong no longer to single-field inflation, which motivates further investigations of the inflationary trajectory.

Chapter 1

Introduction

Why String Phenomenology?

As a matter of fact, modern high precision experiments are in astonishing agreement with predictions derived from rather abstract and mathematical concepts of theoretical physics. On the one hand, the standard model of particle physics (SM) embedded in the more general framework of quantum field theory, is a successful theory to investigate small length scales and in celebrated accordance with data from the CERN collaboration. On the other hand, Einstein's renowned general relativity (GR) determines the large scale physics of our universe and gives rise to the standard model of cosmology. Nevertheless, so far there is no "Theory of Everything" incorporating the SM and GR. However, a proper description of black holes or the big bang demands a full theory of quantum gravity, i.e. a unification of quantum physics and gravity. Arguably the currently best candidate for a quantized theory of gravity is given by string theory which is not just appealing due to its totally new features in physics, but also because of its aesthetic formulation in various branches of mathematics.

String theory is constructed quite differently from the SM in the sense that it is a top-down approach. To be more precise, string theory is constructed as an ultraviolet (UV) complete theory and people try to deduce an effective theory for lower energy scales. In contrast, the SM is a theory at a relatively low energy scale (IR) and people try to enhance it towards the UV. The basic idea of string theory is to take tiny open and closed strings as fundamental objects sweeping out a 2d worldsheet. Mapping the worldsheet to a physical target space, all particles arise from excitations of a string after quantization. A remarkable feature is the natural appearance of a spin two particle identified with the graviton. Apparently, String theory masters the idea of unification and represents a promising theory in the direction of quantum gravity. However, there are even more striking properties following from elementary assumptions in string theory. For instance, cancellation of Weyl anomaly requires a certain number of dimensions, which turns out to be 10 if we take supersymmetry into account. One finds five consistent superstring theories in 10d, all connected by a web of string dualities. In this thesis we will focus on type IIB superstring theory.

Although the pure 10d string theory is conceptually already very attractive, it must eventually be connected to particle physics and cosmology in 4d in order to be a relevant theory for describing nature. This part of string theory research is known as *string phenomenology*. Nowadays this is a fairly broad field of research since modern experiments might be able to probe string theoretical predictions. Cosmic inflation, which will be investigated later on, seeks an embedding in an ultraviolet complete theory and hence is an excellent opportunity for string phenomenology.

To get from 10d superstring theory to the 4d standard model, one obviously has to get rid of the redundant 6d. This can be achieved by compactifying them on a so-called Calabi-Yau manifold. In practice, the extra dimensions were made very small and "rolled up", such that we perceive solely four large dimensions. A major issue in that procedure is the unavoidable appearance of numerous massless scalar fields called *moduli* after any possible string compactification. Moduli correspond to "free parameters" of the compactification as for example size and shape of the Calabi-Yau manifold. However, there exist various reasons why moduli fields are unwanted in the 4d effective theory [1]: First, they would lead to fifth forces, second, destroy successful predictions of Big Bang Nucleosynthesis and third, change the gauge and Yukawa couplings. These problems can be solved by giving the moduli a mass which is large enough to overcome conflicts with observational facts. Generating a mass term for the a priori massless moduli is know as *moduli stabilization* and a central task in string phenomenology. Thus we introduce background fluxes originating from field strengths of the effective string action. We will show that fluxes generate a scalar potential for moduli and thereby fix their vacuum expectation values (vev). Besides, fluxes additionally induce new tadpoles via Chern-Simons terms and source terms in the Einstein equation which can spoil the Calabi-Yau property. We will not consider those issues in this thesis, instead let us face another important consequence of fluxes: they are able to either preserve supersymmetry (susy) or break it spontaneously. Here, we focus on supersymmetry breaking scenarios and as a characteristic result, susy breaking happens at a high scale. Since the LHC experiment has not detected any evidence for supersymmetry yet, high-scale susy breaking might indeed be a fact we have to accept.

String phenomenology does not only take care of embedding the 4d standard model into the 10d string theory, but equally includes applications of string theory to cosmology. In recent years there has been particularly much effort to find a stringy formulation of cosmic inflation. Let us briefly explain why.

Why String Inflation?

In the last decades cosmological data improved enormously due to the WMAP and PLANCK missions measuring the cosmic microwave background (CMB). One crucial outcome is the fact that today's large scale structures in the universe must have originated from primordial quantum fluctuations stretched by an extremely rapid expansion of the universe. We call this early epoch of accelerated expansion of the universe inflation. Aside of primordial fluctuations, inflation solves further issues of standard cosmology, for instance the horizon problem which creates causality inconsistencies.

In physics, we approach inflation via a scalar inflaton field moving along a specific potential. Usually, the inflaton traverses a so-called slow-rolling phase of the potential where some of the inflaton's large potential energy is converted into an energy source driving the extremely rapid expansion of the universe. It is well-known that there exists a huge number of scalar potentials enabling inflation and some experimental input is needed to choose the one realized in nature.

In the beginning of 2014 BICEP2 [2] provided us with such an astonishing discovery by publishing their observations of a large tensor-to-scalar ratio $r \sim 0.2$. This was the initial motivation for the project summarized in this thesis. Due to the Lyth bound [3]

$$\frac{\Delta\phi}{M_{\rm Pl}} = O(1)\,\sqrt{\frac{r}{0.01}}\ , \tag{1.0.1}$$

a large value $r \sim 0.2$ implies obviously $\Delta\phi > M_{\rm pl}$ which corresponds to the class of large-field inflation. Note that $\Delta\phi$ is the distance the inflaton ϕ moves between the creation of the CMB and the end of inflation. In 2015 improved data from the PLANCK collaboration [4–6] released a much smaller upper bound for the tensor-to-scalar ratio $r < 0.113$. Hence large-field inflation became more unlikely, but let us stress that it is nevertheless not ruled out! The fact which makes large-field inflation quite interesting for string theory is its high sensitivity to Planck scale physics. Potentials of large-field inflation are hugely affected by non-renormalizable interactions at a high scale and should therefore be handled via an UV complete theory. Thus, it is an exceptional chance for string theory to embody large-field inflation in a consistent way.

One popular method to exclude the dangerous non-renormalizable operators is to employ axions as inflaton field, whose shift symmetry prevents harmful UV contributions. In string theory axions appear naturally during compactification. Hence people worked out several scenarios of string inflation via axions and one quite intuitive possibility is *natural inflation* [7]. As a drawback, large-field natural inflation requires a large axion decay constant which contradicts a controllable string compactification. We will instead consider *axion monodromy inflation* which was developed in [8,9]. A more detailed derivation from string theory is postponed to later sections. Axion monodromy can for instance be realized via F-term potentials of the moduli scalar potential. This has the advantage that supersymmetry is broken spontaneously by the very same effects which stabilizes the moduli. To summarize, in this thesis we will realize inflation in the context of moduli stabilization by using an axionic inflaton field, which is natural in string theory. As a remark, this continues the work of [10] by extending the discussion to Kähler moduli. A major challenge of our string inflation scenarios is to guarantee a correct hierarchy of mass scales. One can easily ensure an appropriate mass hierarchy for controlled moduli stabilization or for a desirable single-field inflation model by choosing the above mentioned background fluxes accordingly. However, a mass hierarchy suitable for both turns out to be rather tricky.

Overview

The structure of this thesis can be roughly divided into three parts. First, we introduce essential and necessary concepts of string theory, second we construct various models of moduli stabilization and finally we present an application to axion monodromy inflation.

The string theoretical framework of moduli stabilization requires a concrete definition of moduli fields, which is the core task of chapter 2. In string phenomenology one usually works in a 4d effective $\mathcal{N} = 1$ supersymmetric theory and thus we need to introduce orientifold projections. Afterwards we will show how moduli arise as fluctuations of the metric of the Calabi-Yau manifold on which we compactify the unwanted 6d. Variations of the shape of the Calabi-Yau manifold will correspond to complex structure moduli, while variations of the size are called Kähler moduli. A mathematical treatment in the context of Kähler geometry is illustrated as well, and in the end of chapter 2 we present a complete list of the involved moduli. Chapter 3 investigates the stabilization of moduli fields by employing background fluxes. Having clarified the origin of fluxes, we will state their quantization and the important connection to the Gukov-Vafa-Witten superpotential. Then, T- and S- duality lead us to new fluxes with geometric and even non-geometric characteristics. Chapter 3 ends with more comments on moduli stabilization, i.e. how these fluxes generate a scalar potential for the moduli and give them a mass.

The second part of my thesis consists of chapter 4, where we accurately analyze different models of moduli stabilization. The complexity of the models increases step by step as we add more moduli and fluxes. In the end we examine a model including all possible moduli and geometric plus non-geometric fluxes. A common feature is the appearance of a non-supersymmetric minimum of the moduli scalar potential. Consequently, we are able to avoid the no-go theorem by [11] and keep one modulus unfixed. If this modulus is axionic, one might try to apply the model to axion monodromy inflation. Furthermore, every scenario obeys a novel scaling behaviour with the fluxes, that is, the superpotential determines the flux dependence of the moduli at their minima. The scaling property is quite powerful and helps us to realize the correct mass hierarchy as demonstrated in the last section of chapter 4.

Last but not least, we want to combine our moduli stabilization scenarios with inflation in order to come back to our original motivation by BICEP2. This is the third part of my thesis and covered in chapter 5. To shed more light on basic concepts of inflation, we will begin with a general introduction and afterwards show an embedding into string theory via axion monodromy. In the very last section we investigate one particular scenario of chapter 4 and check whether one can realize single-field F-term axion monodromy inflation and moduli stabilization simultaneously in a controlled way.

The results of this thesis are based on the recent paper:

> *"A Flux-Scaling Scenario for High-Scale Moduli Stabilization in String Theory"*
> by R. Blumenhagen, A. Font, M. Fuchs, D. Herschmann, E. Plauschinn, Y. Sekiguchi, F. Wolf,
> published in $Nucl.\ Phys.\ B\mathbf{897}, 500(2015)$, arXiv:1503.07634 hep-th.

Chapter 2

Type IIB Orientifold Compactification on Calabi-Yau 3-folds

This introductory chapter defines the theoretical framework of my thesis. At first we discuss orientifold projections to obtain a suitable effective theory in 4d after compactification on a Calabi-Yau manifold. Compactification allows for certain fluctuations identified with massless moduli fields in 4d. The goal of this chapter is to clarify the origin of moduli and shed some light on a mathematical embedding in special Kähler geometry by deriving an explicit Kähler potential.

2.1 Orientifold Planes and D-Branes

Let us begin with type IIB superstring theory, which is described in 10d and contains exclusively closed strings. The massless spectrum can be found by imposing the so-called level matching constraint and same GSO projection for left- and right-movers on the closed string. Its bosonic fields are a scalar dilaton ϕ, an anti-symmetric 2-form B_2 and a traceless symmetric graviton g in the NS-NS sector as well as a 0-form axion C_0, a 2-form C_2 and a 4-form C_4 in the R-R sector. Spacetime fermions emerge from the NS-R and R-NS sector. A more detailed explanation can be found in the standard literature on string theory, see for instance [12, 13].

Moreover, the low energy effective theory of type IIB superstring theory corresponds to 10d type IIB supergravity equipped with 32 supercharges. Eventually string theory ought to give a complete description of nature which appears to be 4d Minkowski space $\mathbb{R}^{1,3}$. Therefore we have to compactify the 10d string theory on a 6d manifold $\mathcal{M}$ partly preserving supersymmetry. It turns out that this can be realized on a so-called *Calabi-Yau* 3-fold which is defined as follows:

Definition: *A Calabi-Yau n-fold is a 2n (real) dimensional compact Kähler manifold* *with vanishing first Chern class.*

As proposed by E. Calabi and proven by S.T. Yau, a vanishing first Chern class of a compact Kähler manifold is equivalent to the existence of a Ricci flat Kähler metric and holonomy group $SU(n)$. The topological structure of a Calabi-Yau manifold is fully encoded in terms of Dolbeault cohomology groups $H^{p,q}$ and their dimensions $h^{p,q} = \dim_{\mathbb{C}}(H^{p,q})$, called *Hodge numbers*. These are usually arranged in Hodge diamonds which are obliged to satisfy the following properties in the case of Calabi-Yau manifolds:

- $h^{n,0} = 1$ and $h^{p,0} = 0$ for $0 < p < n$
- complex conjugation $h^{p,q} = h^{q,p}$
- Hodge-$*$ duality $h^{p,q} = h^{n-q,n-p}$

For a Calabi-Yau 3-fold there remain solely the two non-trivial Hodge numbers $h^{1,1}$ and $h^{1,2}$. As a comment, let us remark that for a Calabi-Yau 3-fold $\mathcal{M}$ there exists a mirror manifold $\hat{\mathcal{M}}$ with $h^{1,1}$ and $h^{1,2}$ interchanged. It is essential for our work to mention that every Calabi-Yau 3-fold is endowed with

- a unique nowhere vanishing holomorphic (3,0)-form Ω_3
- a closed Kähler (1,1)-form J .

After compactification we are left with a $\mathcal{N} = 2$ supersymmetric theory in 4d with 8 supercharges. However, this is too much supersymmetry! Realistic 4d models of particle physics, for instance the MSSM, are based on $\mathcal{N} = 1$ supersymmetry which finally has to be (softly) broken entirely. Hence for phenomenological reasons the standard compactification scheme on Calabi-Yau 3-folds requests new ingredients. This can be achieved via *orientifold compactification*, which we will introduce in the following.

Orientifold Theory

Following [1, 14–16] and especially [17], we can constuct a new string theory by modding out the worldsheet parity Ω_P, i.e. we keep only states that are invariant under exchange of left- and right-movers. One then obtains an unoriented theory which is usually named orientifold theory and defined by the full orientifold projection, which we derive next.

In the case of type IIB string theory, we introduce in addition to Ω_p a geometric symmetry operator σ acting holomorphically on the internal manifold $\mathcal{M}$. σ does not change the Kähler form J, but its action on the holomorphic $(3,0)$-form Ω_3 of the Calabi-Yau 3-fold is not fixed yet. Denoting the action of σ on differential forms by its pull-back σ^*, throughout this thesis we will use the choice:

$$\sigma^* : J \to J, \qquad \sigma^* : \Omega_3 \to -\Omega_3. \tag{2.1.1}$$

Lastly, an operator $(-1)^{F_L}$ with the left-moving fermion number F_L is necessary for the orientifold action to square to the identity operator. Thus the full orientifold projection reads

$$\Omega_P(-1)^{F_L}\sigma. \qquad (2.1.2)$$

It can be shown that this projection indeed truncates the massless spectrum and modifies the couplings, such that the low energy effective theory becomes $\mathcal{N}=1$ supergravity which motivated the introduction of orientifolds.

For later usage let us briefly explain how the 10d bosonic fields of type IIB string theory transform under the orientifold action. Under the worldsheet parity Ω_P the fields ϕ, g and C_2 are even, whereas B_2, C_0 and C_4 are odd according to [17]. The left-moving fermion operator $(-1)^{F_L}$ leaves apparently all NS-NS fields ϕ, g and B_2 invariant, but produces a minus for the R-R fields C_0, C_2 and C_4. To summarize, we end up with the following transformation of the bosonic fields:

$$\Omega_{\mathrm{P}}(-1)^{F_{\mathrm{L}}} = \begin{cases} g,\, \phi,\, C_0,\, C_4 & \text{even}\,, \\ B_2,\, C_2 & \text{odd}\,. \end{cases} \qquad (2.1.3)$$

The involution σ leaves the 4d Minkowski spacetime unchanged and acts only on the Calabi-Yau manifold $\mathcal{M}$. Its action on the bosonic fields is easily derived as the fields have to be invariant under the full orientifold projection (2.1.2).

Next, consider points of the full 10d spacetime fixed under the internal symmetry σ. These points define the *orientifold planes* or Op-planes where p denotes the number of spatial dimensions. Op-planes wrap a compact $(p-3)$-cycle on the Calabi-Yau manifold $\mathcal{M}$ and cover the 4d Minkowski spacetime.

As σ must be an involution (i.e. $\sigma^2 = \mathrm{id}$) [16], all orientifold planes in type IIB are even dimensional (including the time direction) and hence non but $O3$-, $O5$-, $O7$- and $O9$-planes are possible. Choosing z^1, z^2 and z^3 to be complex coordinates of the Calabi-Yau manifold $\mathcal{M}$, one can always write $\Omega_3 \sim dz^1 \wedge dz^2 \wedge dz^3$. Recalling that σ acts only on the Calabi-Yau manifold, our choice $\sigma^*\Omega_3 = -\Omega_3$ implies then that the internal part of the orientifold plane is either a point or a surface of complex dimension two. Together with the 4d Minkowski spacetime, the orientifold planes in our type IIB setting are therefore $O3$- and $O7$-planes.

A general fact of orientifold planes is that they carry (negative) RR-charge corresponding to 1-point vertices of spacetime fields in the closed string sector; consider the disk diagrams in figure 2.1 from [14]. Diagrams of that kind are usually called *tadpoles*. In accordance with Gauss' law, flux lines of RR p-forms cannot escape on a compact manifold and thus RR-charge must in total add up to zero. This initiates tadpole cancellation conditions which are of great importance, but will not be investigated in detail in this thesis.

Figure 2.1: (a) Disk diagram leading to a tadpole for the R-R 10-form C_{10} in theories with a 10d Poincaré invariant sector of open strings. (b) Crosscap diagram leading to a tadpole for the R-R 10-form C_{10} in 10d Poincaré invariant unoriented theories.

D-Branes

The easiest way to cancel the RR-tadpoles from the orientifold planes is introducing Dp-branes, where again p stands for the number of spatial dimensions, since they are sources of (positive) RR-charges. It turns out that $O3$- and $O7$-planes require $D3$- and $D7$-branes in order to fulfill the tadpole cancellation conditions. Note that $D3$- and $D7$-branes also preserve 4d $\mathcal{N}=1$ supersymmetry and are consequently not in conflict with the original motivation for orientifold planes. Having Dp-branes with $p \neq 3,7$ is in general allowed, however, we are not going to take this situation into account.

Dp-branes must span Minkowski spacetime because of 4d Poincaré invariance. $D3$-branes are point-like regarding the compact Calabi-Yau manifold. More exciting, the preservation of supersymmetry forces the $D7$-branes to wrap a homological 4-cycle which can be different from the cycle wrapped by the $O7$-plane. Consider [18–20] for further information related to our setup.

2.2 Moduli Space and Special Geometry

The appearance of numerous moduli fields is a characteristic feature of string theory and the issue of stabilizing them one of the core tasks in string phenomenology. Generally speaking, moduli occur as free continuous parameters which in the case of string theory distinguish possible string backgrounds. We start by emphasizing that Calabi-Yau manifolds with certain Hodge numbers are not unique at all, since they may still vary in size and shape. This is embodied in the discussion of Kähler and complex structure moduli and will be presented next.

Given a Ricci-flat metric $g_{\mu\nu}$ of a Calabi-Yau manifold $\mathcal{M}$, it is natural to question which infinitesimal deformations $\delta g_{\mu\nu}$ do not spoil the Calabi-Yau conditions, cf. [13]:

$$R_{\mu\nu}(g+\delta g) \;=\; 0 \qquad \Longrightarrow \qquad \nabla^{\rho}\nabla_{\rho}\delta g_{\mu\nu} + 2R_{\mu\ \nu}^{\ \rho\ \sigma}\delta g_{\rho\sigma} \;=\; 0\,. \tag{2.2.1}$$

The final equation is also known as Lichnerowicz equation and constitutes all possible deformations of the metric preserving the Calabi-Yau property. Using the Riemann tensor on Kähler manifolds, it can be shown that there are two different types of solutions satisfying Lichnerowicz's equation separately: on the one hand, we have mixed components $g_{i\bar{j}}$ and on the other hand, those of pure type g_{ij}. These deformations are called *moduli*. $g_{i\bar{j}}$, g_{ij}

correspond to Kähler and complex structure moduli, respectively, as explained below. It is remarkable that the moduli space of a Calabi-Yau manifold splits locally into the direct product:

$$\text{moduli space of } \mathcal{M} \;=\; \text{Kähler moduli} \;\otimes\; \text{complex structure moduli}$$

For further investigation of the precise structure of these moduli spaces we at first review some concepts of special Kähler geometry. We refer to [21,22] and in particular to [23] for details.

Special Kähler Geometry

Recall that a Hermitian metric is a covariant tensor field of the form $ds^2 = 2\sum_{i,j=1}^{n} g_{i\bar{\jmath}}(z)dz^i \otimes d\bar{z}^j$ with $g_{j\bar{\imath}}(z) = \overline{g_{i\bar{\jmath}}(z)}$ and $g_{i\bar{\jmath}}(z)$ being positive definite[1]. One can always associate to such a Hermitian metric a real fundamental (1,1)-form $J = i\sum_{i,j=1}^{n} g_{i\bar{\jmath}}(z)dz^i \wedge d\bar{z}^j$. If the (1,1)-form J is closed, that is $dJ = 0$, it is called *Kähler form* and its associated Hermitian metric g *Kähler metric.* This gives rise to the following definition:

Definition: *A Kähler manifold is a complex manifold equipped with a Kähler metric.*

The existence of a Kähler form J has already been stressed to be a crucial property of Calabi-Yau manifolds, which are Kähler manifolds by definition.

Spelling out the closure condition of J, we immediately find

$$\partial_i g_{j\bar{k}} = \partial_j g_{i\bar{k}}, \qquad \bar{\partial}_i g_{j\bar{k}} = \bar{\partial}_k g_{j\bar{\imath}} \ , \tag{2.2.2}$$

which motivates the local existence of a real *Kähler potential* $K = K(z,\bar{z})$:

$$g_{i\bar{\jmath}} = \partial_i\bar{\partial}_j K \qquad \text{or} \qquad J = i\partial\bar{\partial}K \ . \tag{2.2.3}$$

The Kähler potential is hugely important throughout this thesis as it is necessary to determine the manifold we are compactifying on.

By adding some extra structure on Kähler manifolds, we finally arrive at *special (Kähler) geometry.* Such extra structure basically boils down to independent holomorphic functions $X^I(z)$ and a holomorphic *prepotential* $F(X)$ which fix the Kähler potential on the moduli space in a uniform manner.

We explained above that 10d type IIB superstring theory resembles effectively 10d type IIB supergravity and its compactification on a Calabi-Yau 3-fold generates a 4d theory with $\mathcal{N}=2$ supersymmetry. If the $\mathcal{N}=2$ vector multiplet in 4d is seen to be a $\mathcal{N}=1$ vectormultiplet plus a $\mathcal{N}=1$ chiral scalar multiplet, the chiral superfields can be regarded as coordinates of a manifold which then is in fact Kähler. According to [24–26] this is not just a Kähler manifold, instead it turns out that $\mathcal{N}=2$ supersymmetric theories exhibit even special Kähler geometry.

[1]Positive definite means that $z^i g_{i\bar{\jmath}}\bar{z}^{\bar{\jmath}} \geq 0 \ \forall\{z^i\} \in \mathbb{C}$ and equality holds if and only if all $z^i = 0$.

There are two distinct special geometries:

$$\begin{aligned} &\textit{rigid} \text{ special geometry } \longleftrightarrow \mathcal{N}=2 \textit{ global} \text{ supersymmetry} \\ &\textit{local} \text{ special geometry } \longleftrightarrow \mathcal{N}=2 \textit{ local} \text{ supersymmetry } (\textit{supergravity}) \ . \end{aligned} \tag{2.2.4}$$

Since compactification of type IIB string theory on a Calabi-Yau manifold leads to moduli integrated in a 4d $\mathcal{N}=2$ supergravity theory, it is natural to assume that *local special geometry* accounts for the moduli spaces of Calabi-Yau manifolds. Before illustrating this relation in the next sections, there is a crucial remark we want to point out:

Orientifold compactifications lead to 4d $\mathcal{N}=1$ (not $\mathcal{N}=2$) supersymmetric theories and hence special geometry is not applicable at a first glance! This issue will be addressed later on in section 2.2.3.

2.2.1 Complex Structure Moduli

Above we defined moduli fields to be the solutions of the Lichnerowicz equation (2.2.1). Let us start with deformations of pure type $\delta g_{\bar{\imath}\bar{\jmath}}$ which are no longer Hermitian as the indices are not mixed. However, the deformed metric ought to be Hermitian in order to be Kähler. Hence a non-holomorphic transformation must remove $\delta g_{\bar{\imath}\bar{\jmath}}$, i.e. we need a new choice of complex coordinates implying a new complex structure. For this reason, $\delta g_{\bar{\imath}\bar{\jmath}}$ are declared to be complex structure deformations.

Employing the holomorphic (3,0)-form Ω_3, one associates a complex (2,1)-form to $\delta g_{\bar{\imath}\bar{\jmath}}$

$$\Omega_{ijk} g^{k\bar{l}} \delta g_{\bar{l}\bar{m}} \, dz^i \wedge dz^j \wedge d\bar{z}^{\bar{m}} \ . \tag{2.2.5}$$

This form is actually harmonic if and only if $\delta g_{\bar{\imath}\bar{\jmath}}$ satisfies the Lichnerowicz equation (2.2.1). Therefore, zero modes of the Lichnerowicz equation related to $\delta g_{\bar{\imath}\bar{\jmath}}$ are in 1-to-1 correspondence with elements[2] of $H^{2,1}$.

Consider a basis $e^a_{ij\bar{m}}$, with $a=1,\dots,h^{2,1}$, of harmonic (2,1)-forms and expand the complex structure deformations

$$\Omega_{ijk} g^{k\bar{l}} \delta g_{\bar{l}\bar{m}} = \sum_{a=1}^{h^{2,1}} U^a e^a_{ij\bar{m}}. \tag{2.2.6}$$

Later on we will refer to the complex parameters U^a, $a=1,\dots,h^{2,1}$, as *complex structure moduli*.

To gain more insight into the complex structure moduli space, we parametrise it by w^a with $a=1,\dots,h^{2,1}$ and redefine the harmonic (2,1)-forms on the Calabi-Yau manifold $\mathcal{M}$

$$\chi_a = \frac{1}{2}(\chi_a)_{ij\bar{k}} \, dz^i \wedge dz^j \wedge d\bar{z}^{\bar{k}}, \qquad (\chi_a)_{ij\bar{k}} = -\frac{1}{2}\Omega_{ij}{}^{\bar{l}} \frac{\partial g_{\bar{l}\bar{k}}}{\partial w^a} \tag{2.2.7}$$

[2]By Hodge' theorem $H^{p,q}$ is isomorphic to the space of harmonic (p,q)-forms.

with the inverse

$$\delta g_{\bar{l}\bar{k}} = -\frac{1}{||\Omega||^2}\overline{\Omega}^{ij}{}_{\bar{l}}(\chi_a)_{ij\bar{k}}\delta w^a, \qquad ||\Omega||^2 = \frac{1}{3!}\Omega_{ijk}\overline{\Omega}^{ijk}. \tag{2.2.8}$$

Next, one has to plug this into the general ansatz for the metric of the Calabi-Yau moduli space which is known as Weil-Petersson metric:

$$ds^2 = \frac{1}{2\mathcal{V}}\int_{\mathcal{M}} d\mathcal{V}\; g^{i\bar{\jmath}}g^{k\bar{l}}\;\delta g_{ik}\delta g_{\bar{\jmath}\bar{l}} \tag{2.2.9}$$

where $\mathcal{V}$ is the volume of the Calabi-Yau manifold, which we will explain more precisely later on. If we want the line element to be of the form $ds^2 = 2G_{a\bar{b}}\delta w^a\delta\overline{w}^{\bar{b}}$, one can show that the metric of the complex structure moduli space is given by

$$G_{a\bar{b}} = -\frac{\int \chi_a \wedge \bar{\chi}_{\bar{b}}}{\int \Omega_3 \wedge \overline{\Omega}_3}\;. \tag{2.2.10}$$

It turns out that this metric is indeed Kähler and the corresponding Kähler potential for complex structure moduli can be determined to be

$$K_{U^i} = -\ln\left(i\int_{\mathcal{M}} \Omega_3 \wedge \overline{\Omega}_3\right)\;. \tag{2.2.11}$$

The subindex U^i will become clear in equation (2.2.15). At this point, it is appropriate to relate the discussion at hand to the context of special geometry.

We begin with introducing a basis of 3-cycles $\{A^\Lambda, B_\Sigma\}$ for H_3 of the Calabi-Yau 3-fold $\mathcal{M}$ where $\Lambda, \Sigma = 0, \ldots, h^{2,1}$. This basis can be chosen such that the intersection numbers are

$$A^\Lambda \bigcap B_\Sigma = -B_\Sigma \bigcap A^\Lambda = \delta^\Lambda{}_\Sigma, \qquad A^\Lambda \bigcap A^\Sigma = B_\Lambda \bigcap B_\Sigma = 0. \tag{2.2.12}$$

Since the Calabi-Yau 3-fold $\mathcal{M}$ has real dimension 6, the Poincaré dual space of H_3 is of the similar form H^3. The dual cohomology basis denoted by the real 3-forms $\{\alpha_\Lambda, \beta^\Sigma\}$, where again $\Lambda, \Sigma = 0, \ldots, h^{2,1}$, obeys the following relations

$$\begin{aligned} &\int_{A^\Sigma} \alpha_\Lambda = \int_{\mathcal{M}} \alpha_\Lambda \wedge \beta^\Sigma = \delta_\Lambda{}^\Sigma, \qquad \int_{B_\Sigma} \beta^\Lambda = \int_{\mathcal{M}} \beta^\Lambda \wedge \alpha_\Sigma = -\delta^\Lambda{}_\Sigma, \\ &\int_{A^\Lambda} \beta^\Sigma = \int_{B_\Lambda} \alpha_\Sigma = 0. \end{aligned} \tag{2.2.13}$$

Moreover, we define the so-called *periods* of the holomorphic 3-form Ω_3:

$$X^\Lambda = \int_{A^\Lambda} \Omega_3 = \int_{\mathcal{M}} \Omega_3 \wedge \beta^\Lambda, \qquad F_\Lambda = \int_{B_\Lambda} \Omega_3 = \int_{\mathcal{M}} \Omega_3 \wedge \alpha_\Lambda. \tag{2.2.14}$$

One might suggest to take X^Λ as coordinates of the $h^{2,1}$-dimensional space of complex structure deformations. But they span a $(h^{2,1}+1)$-dimensional space and hence, form an over-complete basis. So, we redefine the coordinates of the complex structure moduli space to be

$$U^i = v^i + iu^i \equiv -i\frac{X^i}{X^0}, \qquad i = 1, \dots, h^{2,1} \ , \tag{2.2.15}$$

where U^i are again the complex structure moduli. Comparing equations (2.2.13) and (2.2.14), one can easily read off an useful expression for the holomorphic (3,0)-form

$$\Omega_3 = X^\Lambda \alpha_\Lambda - F_\Lambda \beta^\Lambda \ . \tag{2.2.16}$$

Then, we can rewrite the Kähler potential for the complex structure moduli by plugging expression (2.2.16) into definition (2.2.11)

$$K_{U^i} = -\ln\left(i\int_{\mathcal{M}} \Omega_3 \wedge \overline{\Omega}_3\right) = -\ln\left[-i\left(X^\Lambda \bar{F}_\Lambda - \bar{X}^\Lambda F_\Lambda\right)\right] \ . \tag{2.2.17}$$

It is quite remarkable that there exists a *prepotential* $F(X^\Lambda)$, which is homogeneous of degree two, that fulfills the following relation

$$F_\Lambda = \frac{\partial}{\partial X^\Lambda} F \tag{2.2.18}$$

where X^Λ and F_Λ are the periods defined above. Thus, all we need to describe the holomorphic $(3,0)$-form Ω_3 and the Kähler potential K_{U^i} are the complex structure moduli U^i and the prepotential F! For this reason local special geometry provides a very useful framework to investigate the complex structure moduli space.

Since the prepotential F is homogeneous of degree two, it makes sense to assume the general ansatz

$$F = -\frac{1}{3!}\kappa_{ijk}\frac{X^i X^j X^k}{X^0} + \frac{1}{2}a_{ij}X^i X^j + b_i X^i X^0 + \frac{i}{2}\gamma(X^0)^2 \tag{2.2.19}$$

which does not incorporate non-perturbative contributions. The constants κ_{ijk} are symmetric in their indices and will be defined later in equation (2.2.24).

In this thesis we always work in the so-called *large complex structure limit* $\mathrm{Re}\, U^i \gg 1$, where the prepotential takes the simple form[3]

$$F = -\frac{1}{3!}\kappa_{ijk}\frac{X^i X^j X^k}{X^0}, \qquad i = 1, \dots, h^{2,1} \ . \tag{2.2.20}$$

[3]The other terms in the prepotential (2.2.19) are not sub-leading, but their effects in the superpotential can be absorbed in the fluxes.

2.2.2 Kähler Moduli

Similarly to the complex structure moduli, let us consider the metric deformations $\delta g_{i\bar{\jmath}}$ of mixed type. They can obviously be viewed as components of a (1,1)-form and thus these variations lead to a cohomological non-trivial change of the Kähler (1,1)-form J. As a consequence, $\delta g_{i\bar{\jmath}}$ correspond to deformations of the Kähler structure.

To be more precise, for the $(1,1)$- and $(2,2)$-cohomologies of $\mathcal{M}$ we introduce bases of the form

$$\begin{aligned} \{\omega_{\mathsf{A}}\} &\in H^{1,1}(\mathcal{M}) \,, \\ \{\tilde{\omega}^{\mathsf{A}}\} &\in H^{2,2}(\mathcal{M}) \,. \end{aligned} \qquad \mathsf{A} = 1, \ldots, h^{1,1} \,, \tag{2.2.21}$$

For later convenience we define in addition $\{\omega_A\} = \{1, \omega_{\mathsf{A}}\}$ and $\{\tilde{\omega}^A\} = \{d\mathrm{vol}_6, \tilde{\omega}^{\mathsf{A}}\}$, where $A = 0, \ldots, h^{1,1}$, such that

$$\int_{\mathcal{M}} \omega_A \wedge \tilde{\omega}^B = \delta_A{}^B \,. \tag{2.2.22}$$

Employing this basis, we define the complex *Kähler moduli* T_α by the following expansion of the (1,1)-form:

$$(i\delta g_{i\bar{\jmath}} + \delta B_{i\bar{\jmath}})\ dz^i \wedge d\bar{z}^{\bar{\jmath}} = i \sum_{\mathsf{A}=1}^{h^{1,1}} T_{\mathsf{A}} \omega_{\mathsf{A}} \ . \tag{2.2.23}$$

The contribution $\delta B_{i\bar{\jmath}}$ has its origin in internal components of the antisymmetric NS-NS 2-form[4] B_2. The real part τ_{A} of the Kähler moduli T_{A} belongs to the deformations $\delta g_{i\bar{\jmath}}$ of the Kähler structure. Equation (2.2.23) is often called complexification of the *Kähler cone*[5], which is the subspace of $\mathbb{R}^{h^{1,1}}$ spanned by the τ_{A}.

Furthermore the overall volume $\mathcal{V}$ of the Calabi-Yau manifold $\mathcal{M}$ is typically defined as

$$\mathcal{V} = \frac{1}{3!}\int_{\mathcal{M}} J \wedge J \wedge J = \frac{1}{3!}\kappa_{\mathsf{ABC}} t^{\mathsf{A}} t^{\mathsf{B}} t^{\mathsf{C}} \qquad \text{with} \qquad \kappa_{\mathsf{ABC}} = \int_{\mathcal{M}} \omega_{\mathsf{A}} \wedge \omega_{\mathsf{B}} \wedge \omega_{\mathsf{C}}. \tag{2.2.24}$$

The parameters κ_{ijk} are triple intersection numbers and $J = t^{\mathsf{A}}\omega_{\mathsf{A}}$ the Kähler form expanded in 2-cycle volumina t^{A} (compare to equation (2.2.29)).

There is an interesting connection between the real parts τ_{A} of the Kähler moduli T_{A} and the t^α in the expansion of the Kähler form J (2.2.29):

$$\tau_{\mathsf{A}} = \frac{1}{2}\int_{\gamma_{\mathsf{A}}} J \wedge J = \frac{1}{2}\kappa_{\mathsf{A}jk} t^j t^k = \frac{\partial \mathcal{V}}{\partial t^{\mathsf{A}}} \tag{2.2.25}$$

[4]In the gauge $d^* B_2 = 0$, the equation of motion reads $\Delta B_2 = 0$. Thus the solutions correspond to harmonic 2-forms on the Calabi-Yau manifold.

[5]The Kähler form J associated with the metric involves certain conditions guaranteeing positive definiteness of the deformed metric. If J fulfills these conditions, then $\lambda J\ \forall \lambda \in \mathbb{R}^+$ do so either. For this reason it is a cone.

where we integrated over a 4-cycle $\gamma_{\mathsf{A}} \in H_4$. That is why τ_i are basically 4-cycle volumina, which in turn suggests to imagine Kähler moduli as fluctuations of the size of the Calabi-Yau manifold we are compactifying on.

Finally, we want to consider the metric of the Kähler moduli space. Although we will not present a derivation here, it can be computed putting (1,1)-forms into the Weil-Petersson metric ansatz and rewriting it in terms of the Kähler form J as well as the bases introduced above. Applying then equation (2.2.3), we obtain the Kähler potential for the Kähler moduli

$$K_{T_{\mathsf{A}}} = -2\ln\mathcal{V}\ . \tag{2.2.26}$$

It is worth mentioning that alike the complex structure case, there exist also complex coordinates $X^0 = 1, X^\Lambda = t^i$ and a holomorphic prepotential $F(X^\Lambda)$, such that

$$K_{T_{\mathsf{A}}} = -\ln\left[-i\left(X^\Lambda\bar{F}_\Lambda - \bar{X}^\Lambda F_\Lambda\right)\right]\ . \tag{2.2.27}$$

Concluding, the moduli space of Kähler deformations of the Calabi-Yau manifold is again encapsulated by local special Kähler geometry. However, this statement is not true if we include orientifolds!

2.2.3 Axio-Dilaton, G-Moduli and the Full Kähler Potential

A phenomenologically reasonable string compactification must end up in a 4d theory endowed with $\mathcal{N}=1$ supersymmetry. In section 2.1 we demonstrated how this goal can be achieved via orientifold planes. But $\mathcal{N}=1$ supersymmetry makes in turn the application of special geometry questionable. In fact, a more involved analysis is needed to understand the structure of the moduli space.

We start with compactifying the massless string spectrum on a Calabi-Yau 3-fold including orientifolds. The holomorphic involution σ (2.1.1) splits the cohomology groups $H^{p,q}$ into even eigenspace $H^{p,q}_+$ as well as an odd eigenspace $H^{p,q}_-$:

$$H^{p,q}(\mathcal{M}) = H^{p,q}_+(\mathcal{M}) \oplus H^{p,q}_-(\mathcal{M})\,, \qquad\qquad h^{p,q} = h^{p,q}_+ + h^{p,q}_-\,. \tag{2.2.28}$$

Recalling earlier definitions of ω_A, $\tilde{\omega}^A$, χ_a, α_Λ and β^Σ, all non-trivial cohomologies and their bases are summarized in table 2.1.[6].

Note that the volume form $d\mathrm{vol}_6$ is invariant under the action of σ because it is proportional to $\Omega_3 \wedge \overline{\Omega}_3$ and Ω_3 transforms according to equation (2.1.1).

Moreover, the Kähler form J is not affected by the involution σ, cf. equation (2.1.1), hence odd Kähler deformations are projected out and one can expand J in the basis of $H^{(1,1)}_+$

$$J = t^\alpha\omega_\alpha, \qquad \alpha = 1,\dots,h^{1,1}_+. \tag{2.2.29}$$

[6]In order to determine the dimensions of the bases for H^3_+ and H^3_-, one should notice that the action of σ on Ω_3 implies $h^{3,0}_+ = h^{0,3}_+ = 0$ and $h^{3,0}_- = h^{0,3}_- = 1$.

cohomology group		dimension		basis	
$H^{1,1}_+$	$H^{1,1}_-$	$h^{1,1}_+$	$h^{1,1}_-$	ω_α	ω_a
$H^{2,2}_+$	$H^{2,2}_-$	$h^{1,1}_+$	$h^{1,1}_-$	$\tilde{\omega}^\alpha$	$\tilde{\omega}^a$
$H^{2,1}_+$	$H^{2,1}_-$	$h^{2,1}_+$	$h^{2,1}_-$	$\chi_{\hat{\lambda}}$	χ_λ
H^3_+	H^3_-	$h^{2,1}_+$	$h^{2,1}_- + 1$	$\{\alpha_{\hat{\lambda}}, \beta^{\hat{\lambda}}\}$	$\{\alpha_\lambda, \beta^\lambda\}$

Table 2.1: Cohomology groups and their basis elements from [17].

As all fields must survive the full orientifold projection (2.1.2), B_2, C_2 transform odd under the action of the involution σ and C_4 even as implied by equation (2.1.3). Therefore the internal components of these fields are given by

$$\begin{aligned} B_2 &= b^a \omega_a, \qquad C_2 = c^a \omega_a, \qquad && a = 1, \ldots, h^{1,1}_- \\ C_4 &= \rho_\alpha \tilde{\omega}^\alpha, && \alpha = 1, \ldots, h^{1,1}_+ \end{aligned} \tag{2.2.30}$$

where b^a, c^a and ρ_α are real scalar fields forming together with t^α two chiral multiplets (b^a, c^a) and (t^α, ρ_α).

Having encountered the truncated massless spectrum of the orientifold compactification, we are now able to return to the discussion of the moduli space. Deformations of the 10d metric with respect to the Calabi-Yau conditions correspond to moduli fields, i.e. scalar fields in 4d. The correct metric is actually obtained by a Kaluza-Klein reduction of the 10d low-energy effective supergravity action using the field content we just revealed. For details of this rather elaborate computation we refer to the literature [17, 27]. The metric derived from Kaluza-Klein reduction is nevertheless not obviously Kähler. Therefore we have to find good Kähler coordinates on the space of scalar moduli fields in 4d.

At first we define the so-called *axio-dilaton*

$$S = e^{-\phi} - iC_0 := s + ic \tag{2.2.31}$$

with the dilaton ϕ and R-R 0-form axion C_0. Then we introduce G-moduli according to [17] as a combination of b^a and c^a

$$G^a = Sb^a + ic^a := \psi^a + i\eta^a \tag{2.2.32}$$

and redefine the Kähler moduli as follows

$$T_\alpha = \frac{1}{2}\kappa_{\alpha\beta\gamma} t^\beta t^\gamma + i\left(\rho_\alpha - \frac{1}{2}\kappa_{\alpha ab} c^a b^b\right) - \frac{1}{4} e^\phi \kappa_{\alpha ab} G^a (G + \overline{G})^b. \tag{2.2.33}$$

Note that the intersection numbers κ_{ijk} given in (2.2.24) are invariant under the holomorphic involution σ.

The complex structure moduli U^i from equation (2.2.15) appear to be already good Kähler coordinates.

To summarize, the full moduli space $\mathcal{M}_{\text{mod}}$ of $\mathcal{N}=1$ Calabi-Yau orientifold compactifications is again a direct product $\mathcal{M}_{\text{mod}} = \mathcal{M}_{\text{cs}} \times \mathcal{M}_{\text{rest}}$ of the complex structure moduli space $\mathcal{M}_{\text{cs}}$ and the space $\mathcal{M}_{\text{rest}}$ of the remaining moduli. Both factors are Kähler manifolds and $\mathcal{M}_{\text{cs}}$ is even a special Kähler manifold. Thus the difference to the 4d $\mathcal{N}=2$ supersymmetric theories is the fact that $\mathcal{M}_{\text{rest}}$ is not described by special geometry, so there is no prepotential for the Kähler moduli.

All of this depends heavily on the specific orientifold planes which are in our case $O3/O7$-planes! If one uses $O5/O9$-planes instead, the truncated spectrum and consequently the moduli fields will look completely different.

Let us now figure out how the Kähler potential reads in terms of the new good Kähler coordinates. Apparently, the part of the Kähler potential sourced by the complex structure moduli stays the same as there was no redefinition of the complex structure moduli. We restrict our calculation to one Kähler modulus T as well as one axionic-odd modulus G for simplicity. Next, consider the part arising from the volume of the Calabi-Yau manifold, see equation (2.2.26), and plug in the new expression for the 2-cycle volume t^α from definition (2.2.33):

$$\begin{aligned} &(t)^2 = \left(T+\overline{T}\right) \; + \frac{1}{8} e^{\phi}\kappa \; \left(G+\overline{G}\right)^2 \\ &\Rightarrow K_{T_\alpha} = -2\ln\mathcal{V} = -2\ln\left(t^3\right) = -3\ln\left((T+\overline{T}) \; + \; \frac{\kappa}{4\;(S+\overline{S})}\;(G+\overline{G})^2\right) \;, \end{aligned} \tag{2.2.34}$$

where we have set for convenience $\kappa := 2\kappa_{\alpha ab}$ for $\alpha = a = b = 1$.

It remains to identify the Kähler potential for the axio-dilaton modulus S. Guided by [14], the kinetic part of the 4d effective action is found to be of the form $\int d^4x \, \frac{\partial S \; \bar{\partial}\bar{S}}{2(\text{Re}(S))^2}$. In standard supergravity formalism this term is supposed to be $K^{S\bar{S}} \; \partial S \partial \bar{S}$, where $K^{i\bar{\jmath}}$ is the inverse of $K_{i\bar{\jmath}} = \partial_i \partial_{\bar{\jmath}} K$ in compliance with equation (2.2.3). Thereby, it is simple to deduce the Kähler potential for the axio-dilaton:

$$K_S = -\ln(S+\overline{S}). \tag{2.2.35}$$

number	modulus	name
1	$S \ = e^{-\phi} - iC_0$	axio-dilaton
$h_-^{2,1}$	$U^i = v^i + iu^i$	complex structure
$h_+^{1,1}$	$T_\alpha = \tau_\alpha + i\rho_\alpha + \ldots$	Kähler
$h_-^{1,1}$	$G^a = Sb^a + ic^a$	axionic odd

Table 2.2: Moduli in type IIB orientifold compactifications from [18].

Summary

At the end of this chapter we briefly want to put the most important results together. In order to acquire a phenomenologically attractive $\mathcal{N} = 1$ supersymmetric theory in 4d after compactifying type IIB string theory on Calabi-Yau manifolds, we introduced $O3$- and $O7$-planes. The associated orientifold projection restricted the structure of the moduli space, such that it is eventually described by the moduli fields collected in table 2.2.

The convention of the real part of the moduli was chosen such that for all moduli holds

$$\text{Modulus} = \text{Saxion} + i \cdot \text{Axion}. \tag{2.2.36}$$

Let us postpone the justification for speaking of axions and saxions to section 5.2. Moreover, adding up equations (2.2.17), (2.2.34) and (2.2.35), the full Kähler potential (for $h_+^{1,1} = h_-^{1,1} = 1$) is given by

$$\begin{aligned} K &= -\ln\left(i\int_{\mathcal{M}} \Omega_3 \wedge \overline{\Omega}_3\right) - \ln(S+\overline{S}) - 2\ln\mathcal{V} \\ &= -\ln\left[-i\left(X^\Lambda \bar{F}_\Lambda - \bar{X}^\Lambda F_\Lambda\right)\right] - \ln(S+\overline{S}) - 3\ln\left((T+\overline{T}) \ + \ \frac{\kappa}{4\,(S+\overline{S})}\,(G+\overline{G})^2\right) . \end{aligned} \tag{2.2.37}$$

F is the prepotential 2.2.19 since the complex structure part of the Kähler potential can be expressed via special geometry.

Chapter 3

Fluxes and Moduli Stabilization

Having explained the concept of moduli, let us now discuss the idea of moduli stabilization and how to make the moduli massive. Hence, we introduce background fluxes that generate a scalar potential containing a mass term for the moduli. Compactifications including field strength fluxes or their generalizations are known as flux compactifications. Fluxes obey a useful quantization condition and can be described via the famous Gukov-Vafa-Witten superpotential. Employing T- and S-duality leads to even more fluxes which have an interesting geometric or non-geometric interpretation. In the end of this chapter, we will comment more on the scalar potential and the different properties of its flux minima. Finally we state the precise objective in our moduli stabilization models and explain the difference to other scenarios which involve perturbative and non-perturbative corrections.

3.1 10d Effective Supergravity Action and 3-form Fluxes

Recalling the bosonic field content of type IIB string theory as listed in section 2.1, the 10d low-energy effective type IIB supergravity action reads [12, 13]

$$\begin{aligned}\mathcal{S}_{\mathrm{IIB}} =& \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\left(\mathcal{R} - \frac{\partial_M S\partial^M \bar{S}}{2\,(\mathrm{Re}(S))^2} - \frac{1}{2}\frac{|G_3|^2}{\mathrm{Re}(S)} - \frac{1}{4}|\tilde{F}_5|^2\right)\\ &+ \frac{1}{2\kappa_{10}^2}\int \frac{1}{4i\ \mathrm{Re}(S)}\ C_4\wedge G_3\wedge \bar{G}_3 + \mathcal{S}_{\mathrm{local}}\ .\end{aligned} \tag{3.1.1}$$

Here we are working in Einstein frame, thus we have redefined the 10d metric $\tilde{g}_{MN}$ in string frame with help of the dilaton ϕ

$$g_{MN} = e^{-\frac{\phi}{2}}\tilde{g}_{MN}\ . \tag{3.1.2}$$

The second integral is arising from Chern-Simons interactions and the last part $\mathcal{S}_{\mathrm{local}}$ contains local source terms of 10d supergravity fields present in the compactification, e.g. $D3$-branes or $O3$-planes [14].

Moreover, recall our definition (2.2.31) of the axio-dilaton S and the gravitational coupling $2\kappa_{10}^2 = (2\pi)^7(\alpha')^4$ with the fundamental string tension $1/(2\pi\alpha')$. $\mathcal{R}$ is the 10d Ricci curvature scalar.

The field strengths $F_{p+1} = dC_p$ are combined with the NS-NS 2-form B_2, cf. [13]:

$$\begin{aligned} F_1 &= dC_0, \qquad \tilde{F}_3 = dC_2 - C_0\, dB_2, \\ \tilde{F}_5 &= dC_4 - \frac{1}{2}C_2 \wedge dB_2 + \frac{1}{2}B_2 \wedge dC_2. \end{aligned} \tag{3.1.3}$$

Note also that we have employed the notation $|F_p|^2 = \frac{1}{p!}F_{M_1\dots M_p}F^{M_1\dots M_p}$ and imposed the important self-duality constraint[1] $\tilde{F}_5 = *_{10}\tilde{F}_5$ in type IIB by hand.

Our next task is to clarify the origin of G_3 in the effective action (3.1.1). At this point it is suitable to introduce fluxes:

Definition: *A field strength with non-trivial vacuum expectation value is called background flux.*

In the end we will deal with several fluxes showing quite different features, but let us begin with the NS-NS 3-form flux $H = \langle dB_2 \rangle$ and the R-R 3-form flux $\mathfrak{F} = \langle dC_2 \rangle$. These two fluxes are usually joined together to

$$G_3 = \mathfrak{F} - iSH. \tag{3.1.5}$$

By turning on fluxes, i.e. choosing $\mathfrak{F}$ and H unequal to zero, a mass term for the moduli fields can be generated. This procedure belongs to moduli stabilization and will be discussed later on. Useful references for flux compactification and moduli stabilization are [15, 28].

We want to remark that there is alternatively a so-called *democratic* formulation of the type IIB supergravity action (3.1.1). This approach includes for every C_p form a C_{8-p} dual form and hence treats electric and magnetic branes on equal footing. Even though this formulation is convenient for flux compactification, we refer the reader to the literature, e.g. [29].

3.1.1 Quantization of Fluxes

An important property of fluxes is the fact that they are quantized and their charges satisfy a Dirac quantization condition. This can be proven as follows [14]:

Start with a q-cycle Σ_q containing a trivial $(q-1)$-cyclce Π_{q-1} as depicted in figure 3.1. Π_{q-1} splits the q-cycle Σ_q in the two parts Σ_+ as well as Σ_-, obeying the conditions

[1] $*_{10}$ denotes the 10d Hodge-star operator. It is useful to know the symmetric inner product

$$\int_{\mathcal{M}} F_p \wedge *_{10}F_p = \frac{1}{p!}\int_{\mathcal{M}} d^{10}x\, \sqrt{-g}F_{\mu_1\dots\mu_p}F^{\mu_1\dots\mu_p} \equiv \int_{\mathcal{M}} d^{10}x\, |F_p|^2\, . \tag{3.1.4}$$

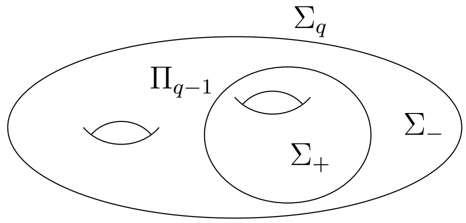


Figure 3.1: Dirac quantization of fluxes from the appendix of [14]

$\partial\,\Sigma_+ = \partial\,\Sigma_- = \Pi_{q-1}$ and $\Sigma_+ - \Sigma_- = \Sigma_q$ where the latter minus sign is caused by the orientation flip to glue both pieces.

Now consider a $(q-2)$-brane[2] electrically charged under C_{q-1} and sweeping out a $(q-1)$-dimensional space W_{q-1} as it evolves in time. Then its electric coupling is given by

$$\mathcal{S}_{\text{electric}} = Q_e \int_{W_{q-1}} C_{q-1} \tag{3.1.6}$$

with the electric charge Q_e. If such a $(q-2)$-brane wraps the $(q-1)$-cycle of figure 3.1, the quantum amplitude of this process yields

$$\exp\left(iQ_e \int_{\Sigma_{q-1}} C_{q-1}\right) = \exp\left(iQ_e \int_{\Sigma_\pm} F_q\right) . \tag{3.1.7}$$

Equality holds due to Stokes theorem with the flux $F_q = dC_{q-1}$ and consequently $dF_q = 0$. However, there is an ambiguity in this expression as the two parts Σ_+ and Σ_- can differ by a phase:

$$Q_e \left(\int_{\Sigma_+} F_q - \int_{\Sigma_-} F_q\right) = 2\pi\mathbb{Z} . \tag{3.1.8}$$

In order to guarantee that Σ_+ and Σ_- give the same amplitude (3.1.7), we are apparently led to the flux quantization condition

$$Q_e \int_{\Sigma_q} F_q \in 2\pi\mathbb{Z} . \tag{3.1.9}$$

For the flux F_q there exists a dual field strength $F_{d-q} = dC_{d-q-1}$ in d-dimensions according to Poincaré duality. A $(q-2)$-brane is said to be magnetically charged under C_{d-q-1} with charge Q_m. Using equation (3.1.9) with Σ_q being a sphere $\mathcal{S}^{d-q}$, one can easily derive a Dirac quantization condition relating electric and magnetic charge, see [14]:

$$Q_e Q_m \in 2\pi\mathbb{Z}. \tag{3.1.10}$$

[2] p-branes are non-perturbative objects with p-spatial dimensions plus time and correspond to solutions to the effective supergravity equations of motion describing the interaction of Dp-branes and closed strings.

To apply the quantization to the 3-form fluxes $\mathfrak{F}$ and H from definition (3.1.5), we have to take for Σ_q the 3-cycles $\{A^\Lambda, B_\Lambda\}$ ($\Lambda = 0, \dots, h^{2,1}$) representing a basis of H_3 as defined in (2.2.12). In this case the flux quantization condition (3.1.9) reads

$$\begin{aligned} \int_{A^\Lambda} \mathfrak{F} &= 2\pi\tilde{\mathfrak{f}}^\Lambda \in 2\pi\mathbb{Z}, \qquad & \int_{B_\Lambda} \mathfrak{F} &= 2\pi\mathfrak{f}_\Lambda \in 2\pi\mathbb{Z}, \\ \int_{A^\Lambda} H &= 2\pi\tilde{h}^\Lambda \in 2\pi\mathbb{Z}, \qquad & \int_{B_\Lambda} H &= 2\pi h_\Lambda \in 2\pi\mathbb{Z}. \end{aligned} \tag{3.1.11}$$

Employing in addition the dual cohomology 3-form basis $\{\alpha_\Lambda, \beta^\Lambda\}$ ($\Lambda = 0, \dots, h^{2,1}$) set by equations (2.2.13), the fluxes $\mathfrak{F}$, H can be expanded

$$\mathfrak{F} = -\tilde{\mathfrak{f}}^\Lambda\alpha_\Lambda + \mathfrak{f}_\Lambda\beta^\Lambda\,, \qquad\qquad H = -\tilde{h}^\Lambda\alpha_\Lambda + h_\Lambda\beta^\Lambda\ . \tag{3.1.12}$$

Throughout this thesis one should keep in mind that $\tilde{\mathfrak{f}}^\Lambda$, $\mathfrak{f}_\Lambda$, $\tilde{h}^\Lambda$, $h_\Lambda \in \mathbb{Z}$. It will turn out that geometric and non-geometric fluxes are quantized in a similar way.

3.1.2 Gukov-Vafa-Witten Superpotential

Our task is now to deduce the flux superpotential as it will be the starting point for the construction of the various models discussed in chapter 4. Thus we have to perform a dimensional reduction from the 10d type IIB to a 4d supergravity.

Consider again the 10d low-energy effective supergravity action (3.1.1) and focus on the first integral[3]. Assuming a constant axio-dilaton $S = e^{-\phi} - iC_0$, we have $F_1 = 0$ and $\partial_M S = 0$. We may also neglect so-called *warping effects* as they are subleading at large volume and thus $\tilde{F}_5 = 0$ [4]. The interesting part of the supergravity action that is left over reads

$$\frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\left(\mathcal{R} - \frac{1}{2}\frac{|G_3|^2}{\mathrm{Re}(S)}\right) \tag{3.1.14}$$

The next step is to decompose the 10d metric in a 4d part $g^{(4)}$ and a metric $g^{(6)}$ of the 6d internal manifold. Since fluxes carry tension and charge they act similar to a distribution of D3-brane charges and hence suggest a supergravity solution ds^2_{10} which would involve a warp factor [14]:

$$ds^2_{10} = e^{2A(y)}\underbrace{g^{(4)}_{\mu\nu}dx^\mu dx^\nu}_{\text{4d spacetime}} + e^{-2A(y)}\underbrace{g^{(6)}_{mn}dy^m dy^n}_{\text{6d Calabi-Yau}}. \tag{3.1.15}$$

[3] We restricted our work solely to the first integral of the action (3.1.1). New interesting features might arise from including the open string sector via D-branes, see for instance [30].

[4] It can be shown that $\alpha = e^{2A(y)}$ expresses not only the warping, but additionally the size of the 5-form flux:

$$\tilde{F}_5 = (1 + *_{10})\left[d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3\right]. \tag{3.1.13}$$

Therefore, neglecting warping effects implies a vanishing 5-form $\tilde{F}_5$.

Here, $e^{2A(y)}$ parametrises the warping and is determined by Laplace equations sourced by fluxes. For completeness let us mention that one could derive the *imaginary self-dual* condition $*_6 G_3 = iG_3$ at this point. However, we will neglect the warp factor in the following as emphasized above.

By using the metric (3.1.15) we can now perform the dimensional reduction of the action (3.1.14) and obtain [31]

$$\int d^4x \, \sqrt{-g^{(4)}} \mathcal{R}^{(4)} \mathcal{V} - \underbrace{\int d^4x \, \sqrt{-g^{(4)}} \left(\int d^6x \, \sqrt{g^{(6)}} \, \frac{|G_3|^2}{2\mathrm{Re}(S)} \right)}_{V_{\text{flux}}} \tag{3.1.16}$$

with the 4d Ricci curvature scalar $\mathcal{R}^{(4)}$ and the volume $\mathcal{V}$ of the Calabi-Yau manifold. After a redefinition of $g^{(4)}$ and $\mathcal{R}^{(4)}$, the first integral corresponds to the well-known Einstein-Hilbert action $\mathcal{S}_{\text{EH}} \sim \int d^4x \, \sqrt{-g_E} \, \mathcal{R}_E$.

The second integral of (3.1.16) describes a 4d supergravity action and is denoted by V_{flux}. Compare this to the standard form of a 4d $\mathcal{N} = 1$ supergravity action which is entirely governed by a Kähler potential K, a holomorphic superpotential W and a holomorphic gauge-kinetic function f [17]:

$$V_{\text{flux}} \overset{!}{=} \int d^4x \, \sqrt{-g_E} \, \left[e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right) + \frac{1}{2} \left((\text{Re } f)^{-1} \right)^{\kappa\lambda} \hat{D}_\kappa \hat{D}_\lambda + V_{\text{tad}} \right] . \tag{3.1.17}$$

Let us postpone further explanations of this scalar potential to section 3.4. Nevertheless note that the first piece of (3.1.17) is called *F-term potential* and the latter *D-term potential* in accordance with their origin from auxiliary fields. In addition we have the usual NS-NS tadpole contribution denoted by V_{tad}.

So, the question is whether there exists a superpotential W, such that equality holds in equation (3.1.17). Remarkably the famous *Gukov-Vafa-Witten* superpotential W_{GVW} [32, 33] does indeed satisfy this equation with a vanishing D-term $\hat{D}_\kappa = 0$:

$$W_{\text{GVW}} = \int_{\mathcal{M}} G_3 \wedge \Omega_3 \tag{3.1.18}$$

where Ω_3 is the holomorphic $(3, 0)$-form defined in (2.2.16) and we are integrating over the Calabi-Yau 3-fold $\mathcal{M}$. Alternatively to the dimensional reduction, W_{GVW} can as well be found via domain walls with NS5/D5-brane charge.

Employing equations (3.1.5)and (2.2.16), together with the expansion (3.1.12) and the symplectic basis (2.2.13), one can expresses the Gukov-Vafa-Witten superpotential in terms moduli and fluxes:

$$\begin{aligned} W_{\text{GVW}} &= \int_{\mathcal{M}} G_3 \wedge \Omega_3 = \int_{\mathcal{M}} (\mathfrak{F} - iSH) \wedge \left(X^\Lambda \alpha_\Lambda - F_\Lambda \beta^\Lambda \right) = \\ &= - \Big(\, \mathfrak{f}_\Lambda X^\Lambda - \tilde{\mathfrak{f}}^\Lambda F_\Lambda \, \Big) \; + \; iS \Big(\, h_\Lambda X^\Lambda - \tilde{h}^\Lambda F_\Lambda \, \Big) \; . \end{aligned} \tag{3.1.19}$$

3.2 Geometric and Non-Geometric Fluxes

One of the most astonishing concepts of string theory are string dualities as for instance T-duality. Thus it is natural to propose the question what happens with our fluxes if we perform T-duality transformations. It will turn out that string dualities, in particular T-duality produces new fluxes of totally different character. This is going to be explained in sections 3.2 and 3.3 following closely the approach by [34–36].

3.2.1 T-duality and Fluxes

The idea of T-duality is easy to visualize by considering the mass spectrum of a bosonic string theory compactified on a circle $\mathcal{S}^1$ with radius R. Surprisingly, there is a physically equivalent $\mathcal{S}^1$ compactification, i.e. the same mass spectrum, with a new radius $\frac{\alpha'}{R}$ and simultaneous exchange of winding and momentum modes. Taking furthermore the mode expansion of the strings into account, T-duality acts like an asymmetric $\mathbb{Z}_2$ reflection of the right-moving states on the world-sheet, leaving the left-moving states invariant. In superstring theories of type II, T-duality alters the sign of the right-moving GSO-projection in the Ramond sector and we obtain the following relation:

$$\text{type IIB on } \mathcal{S}^1 \text{ with radius } R \quad \overset{\text{T-duality}}{\longleftrightarrow} \quad \text{type IIA on } \mathcal{S}^1 \text{ with radius } \tfrac{\alpha'}{R} \ .$$

To make T-duality more practicable, it must be extended to more general backgrounds. In fact, a T-duality transformation is possible for any background if there is an isometry, which we choose to be in θ-direction. Here we just state the results without presenting the prove [13].

After applying T-duality in θ-direction, the new background $G'_{\mu\nu}$, $B'_{\mu\nu}$, ϕ' in terms of the original background $G_{\mu\nu}$, $B_{\mu\nu}$, ϕ reads

$$\begin{aligned}
&G'_{\theta\theta} = \frac{1}{G_{\theta\theta}}, \qquad G'_{\theta i} = \frac{1}{G_{\theta\theta}} B_{\theta i}, \qquad B'_{\theta i} = \frac{1}{G_{\theta\theta}} G_{\theta i}, \\
&G'_{ij} = G_{ij} - \frac{1}{G_{\theta\theta}}(G_{\theta i}G_{\theta j} - B_{\theta i}B_{\theta j}), \qquad B'_{ij} = B_{ij} - \frac{1}{G_{\theta\theta}}(G_{\theta i}B_{\theta j} - B_{\theta i}G_{\theta j}), \\
&\phi' = \phi - \frac{1}{4}\ln\left|\frac{G_{\theta\theta}}{G'_{\theta\theta}}\right| .
\end{aligned} \tag{3.2.1}$$

The generalized T-duality transformations (3.2.1) have first been developed by T.H. Buscher [37] and are therefore called *Buscher rules*. Next, we want to connect T-duality via Buscher rules with flux compactification.

First of all, R-R forms are not affected by Buscher rules and hence we focus merely on NS-NS fluxes.

A didactic example to T-dualize is a torus T^3 with H-flux. This is indeed a reasonable model because a Calabi-Yau manifold can in a certain limit be view as a T^3 fibred over some base manifold according to [38]. In this limit mirror symmetry acts like T-duality on the T^3 fibre without changing the base.

Begin with a simple metric on T^3:

$$ds^2 = dx^2 + dy^2 + dz^2 \tag{3.2.2}$$

and $N \in \mathbb{Z}$ units of H-flux, $H_{xyz} = N$. In order to satisfy the quantization condition $\int_{T^3} H = N$, we may choose the gauge $B_{xy} = Nz$. As nothing depends on the x, y-coordinates, they represent isometries we are allowed to T-dualize. Applying Buscher rules at first in x-direction modifies the background to

$$ds^2 = (dx - Nzdy)^2 + dy^2 + dz^2 \qquad \text{and} \qquad B = 0\ , \tag{3.2.3}$$

where N is usually renamed F^x_{yz}. This metric is globally well-defined and produces a space topologically distinct from T^3: a so-called *twisted torus*. We can figuratively imagine a twisted torus as follows: View a T^3 as a torus T^2 in x, y-directions fibred over $\mathcal{S}^1$ in z-direction. The T-dualized metric (3.2.3) implies that the fibre T^2 undergoes a shift in complex structure $\tau \to \tau + F^x_{yz}$ while one circles around the base $\mathcal{S}^1$. Due to the fact $F^x_{yz} \in \mathbb{Z}$, we certainly get an equivalent fibre after traversing $\mathcal{S}^1$.

To conclude, these F^x_{yz} give rise to additional rigid structure of the background metric and hence non-trivial changes of the manifold. For this reason, F^x_{yz} is called *(geo)metric flux F*.

Back to the torus T^3 where we applied so far one T-duality in x-direction. Since the y-direction is an isometry as well, let us check what the Buscher rules yield when T-dualizing also this direction. For the once T-dualized background (3.2.3), the Buscher rules (3.2.1) lead to

$$ds^2 = \frac{1}{1+N^2z^2}\,(dx^2 + dy^2) + dz^2 \qquad \text{and} \qquad B_{xy} = \frac{Nz}{1+N^2z^2}\ . \tag{3.2.4}$$

Again, N is usually renamed and here we have $N = Q^{xy}_z$. Note that this transformation mixes metric and B-field. Moreover, it is apparently only locally well-defined, i.e. locally geometric , but globally no longer a manifold. That is why we speak of non-geometric backgrounds characterized by a *non-geometric flux Q*. The space we have achieved is knows as *T-fold*.

So what about a third T-duality that acts in z-direction? Analogously, to the procedure so far, one might guess an additional T-duality transformation in z-direction raises another index of Q^{xy}_z and we end up with a final mysterious quantity that we call R^{xyz}. However, great care has to be taken of the existence of this R-flux as there is actually no isometry for the final T-duality and Buscher rules are not applicable at a first glance! R-fluxes are not yet completely understood and are part of current research. Nevertheless we will be working with R-fluxes in this thesis without studying their physical origin in more details. [39] demonstrates why R-fluxes lack even locally of any geometric description and are thus truly non-geometric objects. A more mathematical investigation of R-flux culminates in so-called *non-associative geometry*, which is studied for instance in [40, 41].

To summarize, we found new fluxes via applying T-duality (Buscher rules) on a 3-torus as depicted in the following:

$$
\begin{array}{ccccccc}
H_{abc} & \overset{T_c}{\longleftrightarrow} & F^{c}_{ab} & \overset{T_b}{\longleftrightarrow} & Q^{bc}_{a} & \overset{T_a}{\longleftrightarrow} & R^{abc} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
\text{NS-NS flux} & & \text{geometric flux} & & \text{non-geom. flux} & & \text{non-geom. flux} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
\text{flux background} & & \text{twisted torus} & & \text{T-fold} & & \text{non-associative}
\end{array}
$$

3.2.2 Generalized Superpotential

It remains to include geometric and non-geometric fluxes in the superpotential (3.1.18) or to put it in another way, find a superpotential invariant under T-duality. For this purpose, we introduce the formalism for generalized $\mathcal{N}=1$ orientifold compactifications proposed in [42, 43]:

$$W = \int_{\mathcal{M}} \Big[\mathfrak{F} + d_H \Phi^{\text{ev}}_{\text{c}}\Big]_3 \wedge \Omega_3 \,, \tag{3.2.5}$$

where in present conventions, the complex multi-form of even degree $\Phi^{\text{ev}}_{\text{c}}$ is defined by

$$\Phi^{\text{ev}}_{\text{c}} = iS - iG^a \omega_a - iT_\alpha \tilde{\omega}^\alpha \,. \tag{3.2.6}$$

The subscript on the parentheses in (3.2.5) means that the 3-form part of a multi-form should be selected, and the operator d_H is defined as $d_H = d - H\wedge$. Evaluating then (3.2.5) leads to the familiar Gukov-Vafa-Witten superpotential [33].

Given this formalism, the authors of [36] suggested a natural extension of the differential operator d_H to incorporate geometric and non-geometric fluxes:

$$d_H \longrightarrow \mathcal{D} = d - H\wedge\ - F\circ\ - Q\bullet\ - R\llcorner \tag{3.2.7}$$

where the operators appearing in (3.2.7) implement the mapping

$$
\begin{aligned}
H\wedge\ &:\quad p\text{-form} \ \to\ (p+3)\text{-form}\,,\\
F\circ\ &:\quad p\text{-form} \ \to\ (p+1)\text{-form}\,,\\
Q\bullet\ &:\quad p\text{-form} \ \to\ (p-1)\text{-form}\,,\\
R\llcorner\ &:\quad p\text{-form} \ \to\ (p-3)\text{-form}\,.
\end{aligned} \tag{3.2.8}
$$

Another motivation for this generalized differential arises from tadpole cancellation constraints and Bianchi identities on fluxes in general compactifications, see [39].

If we act with the extended differential $\mathcal{D}$ from (3.2.7) on the multiform $\Phi^{\text{ev}}_{\text{c}}$ and keep eventually only 3-forms, the superpotential invariant under T-duality reads

$$
\begin{aligned}
W &= \int_{\mathcal{M}} \Big[\mathfrak{F} + \mathcal{D}\Phi^{\text{ev}}_{\text{c}}\Big]_3 \wedge \Omega_3 \\
&= \int_{\mathcal{M}} \Big[\mathfrak{F} - iSH + iG^a (F\circ\omega_a) + iT_\alpha \big(Q\bullet\tilde{\omega}^\alpha\big)\Big]_3 \wedge \Omega_3 \,.
\end{aligned} \tag{3.2.9}
$$

One should observe that there is no R-flux in the superpotential (3.2.9) since $R \llcorner \Phi_c^{ev}$ cannot yield a 3-form.

In order to actually carry out the integral in (3.2.9), we need one more valuable ingredient. Recalling the cohomology bases in table 2.1, due to the operator mapping (3.2.8) it is obvious to make the general ansatz:

$$
\begin{aligned}
\mathcal{D}\alpha_\Lambda &= q_\Lambda{}^A\omega_A + f_{\Lambda A}\tilde{\omega}^A\,, & \mathcal{D}\beta^\Lambda &= \tilde{q}^{\Lambda A}\omega_A + \tilde{f}^\Lambda{}_A\tilde{\omega}^A\,,\\
\mathcal{D}\omega_A &= \tilde{f}^\Lambda{}_A\alpha_\Lambda - f_{\Lambda A}\beta^\Lambda\,, & \mathcal{D}\tilde{\omega}^A &= -\tilde{q}^{\Lambda A}\alpha_\Lambda + q_\Lambda{}^A\beta^\Lambda\,.
\end{aligned}
\tag{3.2.10}
$$

Here, $f_{\Lambda A}$ and $\tilde{f}^\Lambda{}_A$ denote the geometric fluxes, while $q_\Lambda{}^A$ and $\tilde{q}^{\Lambda A}$ are the non-geometric ones. Moreover, we use the following convention for the H- and R-flux

$$
\begin{aligned}
f_{\Lambda 0} &= h_\Lambda\,, & \tilde{f}^\Lambda{}_0 &= \tilde{h}^\Lambda\,,\\
q_\Lambda{}^0 &= r_\Lambda\,, & \tilde{q}^{\Lambda 0} &= \tilde{r}^\Lambda\,.
\end{aligned}
\tag{3.2.11}
$$

Imposing then a nilpotency condition of the form $\mathcal{D}^2 = 0$ leads to the well-known Bianchi identities for the fluxes [35]

$$
\begin{aligned}
0 &= \tilde{q}^{\Lambda A}\tilde{f}^\Sigma{}_A - \tilde{f}^\Lambda{}_A\tilde{q}^{\Sigma A}\,, & 0 &= q_\Lambda{}^A f_{\Sigma A} - f_{\Lambda A}q_\Sigma{}^A\,,\\
0 &= q_\Lambda{}^A\tilde{f}^\Sigma{}_A - f_{\Lambda A}\tilde{q}^{\Sigma A}\,, & 0 &= \tilde{f}^\Lambda{}_A q_\Lambda{}^B - f_{\Lambda A}\tilde{q}^{\Lambda B}\,.\\
0 &= \tilde{f}^\Lambda{}_A f_{\Lambda B} - f_{\Lambda A}\tilde{f}^\Lambda{}_B\,, & 0 &= \tilde{q}^{\Lambda A}q_\Lambda{}^B - q_\Lambda{}^A\tilde{q}^{\Lambda B}\,.
\end{aligned}
\tag{3.2.12}
$$

With help of these relations and definition of the Ω_3 (2.2.16), the explicit calculation of the superpotential including geometric and non-geometric fluxes is straightforward.

3.3 S-Dual Completion of Non-Geometric Fluxes and Full Superpotential

In the last section we took advantage of a string duality to acquire new fluxes. Now we will extend this idea to the non-perturbative S-duality which was first proposed by A. Sen [44] in 1994. In short, 10d type IIB string theory with string coupling g_s is S-dual to the same 10d type IIB string theory with coupling $\frac{1}{g_s}$. Due to the inversion of the coupling, we speak of a non-perturbative duality. Note that S-duality is not limited to type IIB string theory, but also applicable to other string theories.

Before we are more precise about S-duality in type IIB string theory, let us stress that an extensive discussion of the following is presented in [45]. It is well-known that the 10d low-energy supergravity action[5] (3.1.1) in Einstein frame is invariant under the transformation[6]

$$
S \to \frac{aS - ib}{icS + d} \qquad \text{and} \qquad \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} C_2 \\ B_2 \end{pmatrix}
\tag{3.3.1}
$$

[5] We are not taking the Chern-Simons term and $\mathcal{S}_{\text{local}}$ into account at this point.

[6] The unusual factors of i in the transformation of S result from our definition $S = e^{-\phi} - iC_0$.

with $ad - bc = 1$. The matrix with components a, b, c, d belongs to the discrete symmetry group $SL(2,\mathbb{Z})$. Notice that it cannot be the continuous group $SL(2,\mathbb{R})$ due to charge quantization. The RR scalar C_0, 4-form C_4 and (Einstein) metric do not transform. $SL(2,\mathbb{Z})$ is actually a generalization of the S-duality transformation and not merely a symmetry of the effective supergravity action, but indeed of the full IIB string theory.

We encountered in section 3.1 that the 10d type IIB supergravity action (3.1.1) leads to a 4d scalar potential. Hence, if the 10d supergravity action is invariant under S-duality transformations (3.3.1), the 4d F-term scalar potential should be invariant as well. Since the scalar potential can be expressed in pure supergravity formalism, see equation (3.1.17), we better check how the Kähler potential K and superpotential W transform.

One can readily guess the transformation of the Kähler potential (2.2.37)

$$K \to K + \ln\left(|icS + d|^2\right) \ . \tag{3.3.2}$$

Recalling the scalar potential $V_F = e^K\left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2\right)$, the transformation of the Kähler potential 3.3.2 renders the superpotential W to behave as follows under S-duality [46]

$$W \to \frac{1}{icS + d} W. \tag{3.3.3}$$

Concerning the moduli, it is easy to deduce their transformations from definitions (2.2.32) and (2.2.33):

$$G^a \to \frac{1}{icS + d}\, G^a, \qquad\qquad T_\alpha \to T_\alpha + \frac{i}{2}\, \frac{c}{icS + d}\, \kappa_{\alpha bc} G^b G^c \ . \tag{3.3.4}$$

In spite of those transformations, $\ln \mathcal{V}$ in the Kähler potential does not change. That was expected as $\mathcal{V} = \frac{1}{3!}\kappa_{\mathsf{ABC}} t^{\mathsf{A}} t^{\mathsf{B}} t^{\mathsf{C}}$ depends solely on the invariant 2-cycle volumina t^{A}.

However, this is not the full story if we include the geometric flux F and the non-geometric fluxes Q, R in the superpotential. Then the superpotential does in fact not transform covariantly under these fluxes and in particular Q-flux spoils the invariance of the scalar potential! This issue is resolved by introducing a so-called *P-flux* [45] similar to the Q-flux:

$$P\bullet \ : \quad p\text{-form} \to (p-1)\text{-form} \ . \tag{3.3.5}$$

The transformation under $SL(2,\mathbb{Z})$ is given by

$$\begin{pmatrix} Q \\ P \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} Q \\ P \end{pmatrix} \tag{3.3.6}$$

and analogous to the Q-flux, we extend (3.2.10)

$$\begin{aligned} -P \bullet \alpha_\Lambda &= p_\Lambda{}^A\, \omega_A\,, & -P \bullet \beta^\Lambda &= \ \ \tilde{p}^{\Lambda A}\, \omega_A\,, \\ -P \bullet \omega_A &= 0\,, & -P \bullet \tilde{\omega}^A &= -\tilde{p}^{\Lambda A}\, \alpha_\Lambda + p_\Lambda{}^A\, \beta^\Lambda\,. \end{aligned} \tag{3.3.7}$$

We require the superpotential including geometric and non-geometric fluxes to transform like equation (3.3.3), which results in additional terms as suggested by [47] and [48]:

$$\begin{aligned} W = \int_{\mathcal{M}} \Big[\quad & \mathfrak{F} - iSH \\ & + iG^a \left(F \circ \omega_a\right) \\ & + iT_\alpha \left([Q - iSP] \bullet \tilde{\omega}^\alpha\right) + \frac{1}{2}\kappa_{\alpha bc} G^b G^c \left(P \bullet \tilde{\omega}^\alpha\right) \quad \Big]_3 \wedge \Omega_3 \, . \end{aligned} \tag{3.3.8}$$

Plugging in Ω_3, (3.2.10), (3.3.7), we finally obtain a convenient form of the superpotential:

$$\begin{aligned} W = & - && \left(\mathfrak{f}_\lambda \, X^\lambda - \tilde{\mathfrak{f}}^\lambda \, F_\lambda \right) \\ & + iS && \left(h_\lambda \, X^\lambda - \tilde{h}^\lambda \, F_\lambda \right) \\ & - iG^a && \left(f_{\lambda a} X^\lambda - \tilde{f}^\lambda{}_a F_\lambda \right) \\ & + iT_\alpha && \left(q_\lambda{}^\alpha X^\lambda - \tilde{q}^{\lambda\alpha} F_\lambda \right) \\ & + \left(ST_\alpha + \tfrac{1}{2}\kappa_{\alpha bc} G^b G^c \right) && \left(p_\lambda{}^\alpha X^\lambda - \tilde{p}^{\lambda\alpha} F_\lambda \right) . \end{aligned} \tag{3.3.9}$$

We want to remark some interesting features regarding the full superpotential (3.3.9):

- There appears no R-flux in W.
- Geometric fluxes F couple to odd moduli G^a.
- Aside of P-flux, all terms in W depend only linearly on moduli S, G^a, T_α.
- P-flux involves non-linear terms ST_α as well as $G^a G^b$.
- ST_α and $G^a G^b$ come always together when P-fluxes are turned on.
- The term involving the geometric flux F transforms covariantly under $SL(2,\mathbb{Z})$, and therefore no additional flux parameters have to be introduced. This observation is particularly interesting, because it contradicts the common expectation that for every known flux one has to introduce a dual flux, when constructing a duality-invariant theory. One explanation might be that the geometric flux involves solely the metric which does not transform under S-duality.

3.4 Scalar Potential and Flux Vacua

Let us now be more concrete about moduli stabilization. In chapter 2 we compactified 10d type IIB string theory on orientifolds to end up with a 4d $\mathcal{N} = 1$ supergravity theory. As a consequence the massless spectrum in 4d contained a large number of moduli. We have already emphasized in the introduction that any realistic string theory model has to face a serious challenge [1]:

Problem: Many quantities (e.g. Yukawa couplings) of the low energy effective theory depend on the moduli but should be fixed. Besides, moduli would give rise to fifth forces or modify successful predictions of Big Bang Nucleosynthesis.

Solution: Generate a scalar potential fixing the vacuum expectation value (vev) of the moduli fields, that is, giving them a mass large enough to overcome the problems above. This technique is know as *moduli stabilization*.

Fixing the vevs of moduli fields can be realized via turning on non-trivial backgrounds of the other 10d fields. These backgrounds are apparently given by the fluxes introduced in this chapter.

F-term Scalar Potential

Recall the scalar potential from equation (3.1.17) coinciding with the standard form of 4d $\mathcal{N} = 1$ supergravity. In all of the models presented in this thesis we choose a vanishing D-term and restrict ourselves to the F-term scalar potential:

$$V_F = \frac{M_{\mathrm{Pl}}^4}{4\pi}\, e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \overline{W} - 3\left|W\right|^2 \right), \tag{3.4.1}$$

where W is the holomorphic superpotential (3.3.9), K the real Kähler potential (2.2.37) and we sum over all moduli of table 2.2. Besides, we employed the Kähler-covariant derivative

$$D_I W = \partial_I W + (\partial_I K)\, W. \tag{3.4.2}$$

$G^{I\bar{J}}$ in the scalar potential is the inverse of the positive definite Hermitian Kähler metric $G_{I\bar{J}} = \partial_I \partial_{\bar{J}} K$ defined in section 2.2. We speak of a F-term potential because it involves the F-terms [49]

$$F^I = e^{\frac{K}{2}}\; K^{I\bar{J}} D_{\bar{J}} \overline{W}. \tag{3.4.3}$$

We furthermore observe that the Kähler potential (2.2.37) satisfies a so-called *no-scale* relation [17]

$$K^{I\bar{J}} (\partial_I K)(\partial_{\bar{J}} K) = 4\,, \tag{3.4.4}$$

where the sum runs over the axio-dilaton S, and the even and odd moduli T_α and G^a. This relation plays an important role in supersymmetry breaking and mediation to the standard model [51]. However, perturbative corrections from loop effects or from the Kähler potential, see subsection 3.4.1, would spoil this no-scale structure.

Moduli stabilization corresponds to fixing the vevs of the moduli by turning on fluxes and hence our next task is to compute the minima of the scalar potential. So, *a moduli field M^I is stabilized if there exists a solution to $\partial_I V = 0$.* Such solutions are called *flux vacua* and categorized by different features. More precisely a flux vacuum is said to be

- **supersymmetric** if $D_I W = 0$ for all moduli M^I.
  This is a general condition in supergravity [52].

- **tachyonic** if there are negative mass eigenstates.
  As we will encounter later on, the mass matrix is $M^I_J = K^{IJ} V_{IJ}$ with the second derivative of the scalar potential $V_{IJ} = \partial_I \partial_J V$. K^{IJ} is positive definite and thus $V_{IJ} < 0$ corresponds to tachyons. Minima without tachyons are often called stable minima.

- **AdS** if $V|_{\text{Minimum}} < 0$.
  AdS space has by definition a negative cosmological constant which is proportional to the vacuum energy density and consequently to the minimum of the scalar potential [53, 54]. If we instead have $V|_{\text{Minimum}} = 0$ or $V|_{\text{Minimum}} > 0$, we obtain a Minkowski or dS space, respectively.

Let us remark that supersymmetric minima have $D_I W = 0$ for all moduli fields M^I, such that the scalar potential reduces to $V_F = -e^K\,|W|^2 \leq 0$. Thus supersymmetric vacua are always AdS.

Objective

Following our original motivation as stated in the introduction, we are trying to implement moduli stabilization while keeping one axion massless. Afterwards we apply our models of moduli stabilization to axion monodromy inflation, see chapter 5. We basically intend to continue the work of [10]. However, there is a powerful *no-go theorem* found by J. Conlon in [11]:

No-Go theorem: *There does not exist any tachyon-free supersymmetric minimum of the F-term potential consistent with stabilized moduli and unfixed axions.*

Although Conlon mentions a few loopholes to this no-go theorem, for instance corrections to the Kähler potential or D-term contributions, we will not discuss those possibilities further. Therefore, in order to apply our models to string phenomenology and string cosmology, we focus on flux vacua equipped with the following properties:

1. Vacua should be non-supersymmetric and tachyon-free, so that after uplifting they can lead to stable de Sitter vacua.

2. The moduli should be stabilized in the perturbative regime, i.e. at weak string coupling and large radius.

3. All saxionic moduli should be stabilized with axions providing candidates for the inflaton and possibly dark radiation.

Proposition 3 takes care of the existence of massless (light) axions allowing for axion monodromy inflation. Moreover, due to proposition 1 we are able to circumvent the no-go theorem by Conlon. Further information about the uplift to stable dS vacua in presented in [18].

3.4.1 Corrections and the LARGE Volume Scenario

Another essential condition on our models is proposition 2. Postulating weak string coupling $g_s = e^{\phi} = 1/s \ll 1$, requires the real part s of the axio-dilaton to be large in all of our flux vacua. Consequently, we do not have to worry about string loop calculations.

Let us concisely comment on other possible corrections to the setup we are considering. The leading contributions with effects on the scalar potential are given by:

- **perturbative $(\alpha')^3$-corrections to the Kähler potential** [31, 55, 56]

$$K_{\text{Kähler}} = -2 \ln \left(\mathcal{V} + \frac{\xi}{2} \left(\frac{S + \bar{S}}{2} \right)^{3/2} \right) \tag{3.4.5}$$

  with $\xi = \frac{\chi(\mathcal{M})\ \zeta(3)}{2(2\pi)^3}$. Moreover, $\chi(\mathcal{M})$ is the Euler number of the manifold $\mathcal{M}$ and $\zeta(3) \approx 1.202$ Apéry's constant. These corrections have their origin in the dimensional reduction of the 10d curvature term $\mathcal{R}$.

- **non-perturbative corrections to the superpotential**

$$W = W_0 + \sum_{i=1}^{h_+^{1,1}} A_i\ e^{-a_i T_i} \tag{3.4.6}$$

  where W_0 is the standard superpotential, A_i are model dependent constants and T_i Kähler moduli. These corrections can be generated either by $D3$-brane instantons ($a_i = 2\pi$) [57] or gaugino condensation from wrapped $D7$-branes ($a_i = \frac{2\pi}{N}$) [58, 59].

Such corrections have been successfully used for moduli stabilization. In the well-known *KKLT scenario* [60] the authors took advantage of non-perturbative corrections to the superpotential, as we briefly describe now. The KKLT approach is actually a two-step procedure. First, they stabilize complex structure moduli and the axio-dilaton via G_3-fluxes and second they utilize non-perturbative effects to fix the Kähler moduli. The minimum is then supersymmetric as well as AdS, but can be uplifted to a positive cosmological constant by adding anti-$D3$-branes. The KKLT scenario is definitely impressive, even though it involves also some shortcomings. For instance, α'-corrections are entirely neglected and it has been pointed out that this two-step procedure is not always justified.

Some of the problems of KKLT are cured in the so-called *LARGE volume scenario* (LVS) [61, 62]. Most important, in this scenario α'-corrections are indeed taken into account. Furthermore, the overall volume $\mathcal{V}$ of the compactification manifold is assumed to be very large compared to the string length:

$$\frac{1}{\mathcal{V}} \ll 1. \tag{3.4.7}$$

This limit suggests to expand various quantities in powers of $1/\mathcal{V}$. Hence, the LVS approach starts similar to KKLT by stabilizing complex structure moduli and the axio-dilaton with the usual Gukov-Vafa-Witten superpotential at order $1/\mathcal{V}$ in the scalar potential. Afterwards they consider order $1/\mathcal{V}^3$-contributions of the F-term scalar potential including non-perturbative corrections of W and α'-corrections to K which finally stabilizes the Kähler moduli.

Let us be more specific about the compactification manifold. [31] assumed that the overall volume $\mathcal{V}$ can be expressed in terms of 4-cycle volumina τ_i:

$$\mathcal{V} = a_b \tau_b^{3/2} - \sum_{i=1}^{h_+^{1,1}-1} a_i \tau_i^{3/2} \tag{3.4.8}$$

with $\{a_b, a_i\}$ being constants. If we take the well-defined limit $\tau_b \gg \tau_i \; \forall i = 1, \ldots, (h_+^{1,1}-1)$, τ_b may be approximately thought of as the overall volume and τ_i as internal little holes. For this reason, we speak of *swiss-cheese manifolds*. However, the LVS is also possible for a certain class of more general geometries, as demonstrated in [63].

Back to our models: In the following we demand the overall volume of the internal manifold, i.e. the real part τ_α of the Kähler moduli, to be large. Therefore, perturbative and non-perturbative corrections to our models are suppressed and may be neglected. Note that there is no need to insert these corrections since we are able to stabilize the Kähler moduli without taking subleading orders of the scalar potential into account.

An advantage of not using non-perturbative effects is the fact that Kähler moduli can be stabilized at the same mass scale as all other moduli. This is crucial for the mass hierarchy explained in section 5.3.

Chapter 4

Flux-Scaling Scenarios with Non-Supersymmetric Vacua

In this chapter we want to illustrate several models of moduli stabilization in detail. However, we will not be completely specific about the concrete Calabi-Yau 3-fold, but rather work at the level of supergravity. In the beginning we present a systematic way to construct new scenarios by using a certain scaling property. Thereby various vacua of the broad flux landscape are investigated with increasing complexity. We evaluate the induced mass hierarchy that is necessary for a controlled string compactification in the final section. Note that the names 'model A' and 'model C' have been chosen to stay in accordance with [18].

4.1 Construction Strategy

What we need for moduli stabilization is a flux induced scalar potential as explained in section 3.4. However, in our work the scalar potential was not derived from a specific Calabi-Yau manifold including orientifolds, since such a procedure would require a better understanding of the backreaction of the fluxes on the geometry we are compactifying on. We rather make a supergravity ansatz, investigate various scalar potentials and may question its stringy motivation afterwards. At the level of supergravity the scalar potential of a certain amount of moduli consists of only two ingredients: Kähler potential and superpotential. See section 3.4 for explanations.

Analyzing a few exemplary models, one realizes soon that it is often quite difficult to solve for the minima of the scalar potential analytically. Furthermore, in many cases there are actually no solutions to the minima conditions. Having found some working models, an interesting commonality became apparent: All of the models obey a novel scaling behaviour with the fluxes. Employing this flux-scaling property, it becomes straightforward to construct new models and even easy to read off from the superpotential how the minima scale with the fluxes.

To be more precise, the defining property of the models is a common flux scaling of the superpotential W which in turn implies a common flux scaling of the scalar potential

V and the moduli masses. So the strategy is to choose a superpotential dictated by a particular scaling. In practice this means that only a subset of the allowed fluxes is turned on to ensure that the moduli vevs scale in a simple way. Let us consider some concrete examples in order to shed more light on the scaling property. If we demand that every term in the superpotential scales the same, we are immediately led to two useful observations:

- One can simply read off how the moduli scale at the extremum, e.g.:

$$\begin{aligned} W = i\,\tilde{\mathfrak{f}}^0 + i\,h_0\,S \quad &\Longrightarrow \quad \text{scaling at the extremum:} \quad W_0 \sim \tilde{\mathfrak{f}}^0 \\ &\Longrightarrow \quad s \sim \frac{\tilde{\mathfrak{f}}^0}{h_0} \quad \text{at the extremum} \end{aligned} \tag{4.1.1}$$

  We will also take the imaginary part c of the axio-dilaton S into account, when we revisit this superpotential in section 4.2.
  Using this technique one can step by step determine the full minimum solution of the scalar potential up to some numerical prefactors.

- It also implies a strong restriction on the possible terms in the superpotential (3.3.9). For instance, take a superpotential containing complex structure moduli

$$\begin{aligned} W = -\,\mathfrak{f}^0 + 3\,\tilde{\mathfrak{f}}^1\,U^2 \quad &\Longrightarrow \quad \text{scaling at the extremum:} \quad W_0 \sim \mathfrak{f}^0 \\ &\Longrightarrow \quad v^2 \sim \frac{\mathfrak{f}^0}{\tilde{\mathfrak{f}}^1} \quad \text{at the extremum} \end{aligned} \tag{4.1.2}$$

  The general form of the superpotential (3.3.9) allows as well for the term $(-i\,\mathfrak{f}^1\,U)$. But, such a term breaks the scaling because of

$$-i\,\mathfrak{f}^1\,U \quad \xrightarrow{v^2 \sim \frac{\mathfrak{f}^0}{\tilde{\mathfrak{f}}^1}} \quad -i\,\mathfrak{f}^1 \cdot \sqrt{\frac{\mathfrak{f}^0}{\tilde{\mathfrak{f}}^1}} \;\nsim\; \mathfrak{f}^0 \;\sim\; W_0 \quad \text{for arbitrary } \mathfrak{f}^1 \in \mathbb{Z}. \tag{4.1.3}$$

  Consequently we exclude such a term in the superpotential. This is a powerful constraint on the construction of flux-scaling scenarios.

Generically, for n complex moduli it suffices to switch on $n+1$ flux parameters. For instance, to stabilize T^α we include one flux of type q^α_λ or one of type $\tilde{q}^\alpha_\lambda$. Similarly, for S we take one h_λ or one $\tilde{h}_\lambda$. For the complex structure moduli we need one R-R flux of type $\mathfrak{f}_\lambda$ and one $\tilde{\mathfrak{f}}_\lambda$. Of course, we have to be careful that the chosen NS-NS, R-R and non-geometric fluxes satisfy the Bianchi identities. We observe that in the studied examples an off diagonal Kähler metric did not spoil the scaling property.

Aside from that, there is in fact another valuable guideline for constructing new scaling scenarios when including complex structure moduli. It seems like only superpotentials with every term containing either an even or odd number of moduli work. Considering again the superpotential (4.1.2), all terms are apparently even in the number of moduli. Thus the term $(-i\, \mathfrak{f}^1\, U)$, which has an odd number of moduli, is not allowed. Notice that also superpotentials like

$$W = i\hat{\mathfrak{f}}_1 U - i\tilde{\mathfrak{f}}^0 U^3 + 3i\tilde{h}^1 U^2 S + 3i\tilde{q}^1 U^2 T \tag{4.1.4}$$

are perfectly solvable as all terms have an odd number of moduli. This constraint was found on an empirical basis and its conceptual explanation remains an open question.

Let us stress that the flux scaling property is not only helpful to engineer new scenarios, but in particular useful to achieve parametric control over the hierarchies among the relevant scales $M_{\rm s}$, $M_{\rm KK}$ and the moduli masses. On the other hand, parametrically controlled hierarchies among the different moduli masses are then impossible. We will try to circumvent this problem by introducing additional fluxes in W that break the scaling, cf. section 4.6.

Note that in all our models the fluxes allow for the existence of supersymmetric AdS vacua which often contain tachyons above the Breitenlohner-Freedman bound. Moreover we have found scenarios with non-supersymmetric, non-tachyonic AdS vacua. Those can be applied to axion monodromy inflation, see chapter 5. For other string phenomenological studies concerning particle physics predictions we refer to [18]. A detailed computation of the tadpoles and constraints from so-called *Freed-Witten conditions*[1] are part of [18], too.

4.2 Model A: No Complex Structure Moduli

This model should be considered as our simplest prototype example and we will come back to it throughout this thesis. Furthermore, we will show in this section that tachyonic vacua can still be stable (Breitenlohner-Freedman bound) and in the end emphasis a possible extension of this model to two Kähler moduli.

Setup:

Let us assume a particularly simple Calabi-Yau manifold with Hodge numbers $h^{1,1}_- = h^{2,1}_- = 0$ and $h^{1,1}_+ = 1$, i.e. with a single Kähler modulus and neither complex structure nor axionic-odd moduli. Such a geometry can be viewed as an isotropic 6-torus T^6 with frozen complex structure modulus. In this situation the Kähler potential (2.2.37) takes the succinct form

$$K = -3\ln(T+\overline{T}) - \ln(S+\overline{S})\,. \tag{4.2.1}$$

Furthermore, we turn on only three fluxes: the NS-NS flux $h_0 = h$, the non-geometric flux ${q_0}^1 = q$, and the R-R 3-form flux $\tilde{\mathfrak{f}}^0 = \tilde{\mathfrak{f}}$. These fluxes satisfy the Bianchi identities (3.2.12),

[1] Freed-Witten anomalies arise if fluxes and D-branes are used simultaneously and ensures that a cycle wrapped by a D-brane is still closed in the geometry deformed by the fluxes.

and are subject to the quantization condition $\tilde{\mathfrak{f}}, h, q \in \mathbb{Z}$ according to section 3.1. From (3.3.9) we determine the corresponding superpotential:

$$W = i\tilde{\mathfrak{f}} + ihS + iqT\,, \tag{4.2.2}$$

where we have set $X^0 = 1$ and $F_0 = i$.

Flux Vacua of the Scalar Potential:

It is easy to compute the inverse Kähler metric from the Kähler potential K above:

$$\left(K_{I\bar{J}}\right)^{-1} = \left(\partial_I \bar{\partial}_J K\right)^{-1} = \begin{pmatrix} \frac{4\tau^2}{3} & 0 \\ 0 & 4s^2 \end{pmatrix}. \tag{4.2.3}$$

In the case of more involved models one has to pay attention to obtain the correct, real and positive definite matrix.

Together with the Kähler potential K and superpotential W, the resulting scalar potential (3.4.1) is given by

$$V = \frac{M_{\rm Pl}^4}{4\pi \cdot 2^4} \left[\frac{(hs - \tilde{\mathfrak{f}})^2}{s\tau^3} - \frac{6hqs + 2q\tilde{\mathfrak{f}}}{s\tau^2} - \frac{5q^2}{3s\tau} + \frac{1}{s\tau^3}\left(hc + q\rho\right)^2 \right]. \tag{4.2.4}$$

Notice that this scalar potential depends merely on the following linear combination of axions

$$\theta = hc + q\rho\,. \tag{4.2.5}$$

Hence, the orthogonal linear combination $(qc - h\rho)$ of the axions c and ρ is not stabilized by the potential (4.2.4). We will try to stabilize this modulus later on by introducing additional fluxes. The goal is to keep it parametrically lighter than s, τ and θ in order to make this scenario applicable to inflation. It will turn out to be not possible here, but in fact doable in other models.

The extremal points of (4.2.4) are obtained by solving for $\partial_s V = \partial_\tau V = \partial_\theta V = 0$, and we find the three solutions shown in table 4.1. Note that the fluxes must be chosen so that the values of s and τ are inside the physical domain $s, \tau > 0$.

Evaluation of the Flux Vacua:

- The **first solution** in table 4.1 is the supersymmetric one, since in this case $D_T W = D_S W = 0$. As W does not depend on one axionic direction, the no-go theorem of [11] implies that the saxionic partner is tachyonic, which can indeed be confirmed for this minimum.

  The extremum corresponds to AdS space as it was expected for the supersymmetric case. However, this extremum is stable even if the Hessian of the potential has

solution	(s, τ, θ)	susy	tachyons	Λ
1	$(-\frac{\tilde{\mathfrak{f}}}{2h}, -\frac{3\tilde{\mathfrak{f}}}{2q}, 0)$	yes	yes	AdS
2	$(\frac{\tilde{\mathfrak{f}}}{8h}, \frac{3\tilde{\mathfrak{f}}}{8q}, 0)$	no	yes	AdS
3	$(-\frac{\tilde{\mathfrak{f}}}{h}, -\frac{6\tilde{\mathfrak{f}}}{5q}, 0)$	no	no	AdS

Table 4.1: Extrema of the scalar potential (4.2.4) for model A. Λ denotes the cosmological constant, which indicates whether we end up in dS, Minkowski or AdS space.

negative eigenvalues. Indeed, as it is well known, for AdS vacua tachyonic fluctuations are stable provided they satisfy the *Breitenlohner-Freedman bound* [64]

$$m^2\, M_{\mathrm{Pl}}^2 \geq \frac{3}{4}\, V_0\,, \tag{4.2.6}$$

where V_0 is the value of the potential at the extremum and m^2 is the physical mass. For supersymmetric extrema the bound is always guaranteed, which can be verified in our scenario.

- **Solution three** of table 4.1 is non-supersymmetric and has no tachyonic directions. This is precisely the strictly stable vacua we are looking for!

  As the minimum belongs to AdS space, it is necessary to eventually come up with an uplift mechanism to dS space. Moreover, for $|\tilde{\mathfrak{f}}/h| \gg 1$ and $|\tilde{\mathfrak{f}}/q| \gg 1$ we obtain weak string coupling and large radius, so that it is justified to ignore higher-order corrections to the scalar potential.

  Note that the scaling of the stabilized moduli with the fluxes implies that all terms in the superpotential are of the same order.

  For $\theta = 0$, the potential has the shape shown in figure 4.1.

Masses of the Moduli:

We finally compute the mass eigenvalues and eigenstates for the moduli and compare them in section 4.6 to the string and Kaluza-Klein scales. The squared physical masses for the canonically normalized fields are given by the eigenvalues of the mass matrix

$$(M^2)^i_{\ j} = K^{ik} V_{kj} \tag{4.2.7}$$

with $V_{kj} = \frac{1}{2}\partial_k \partial_j V$, evaluated at the extremum of the potential V, cf. [65].

Computing then the physical mass matrix for the non-supersymmetric tachyon-free

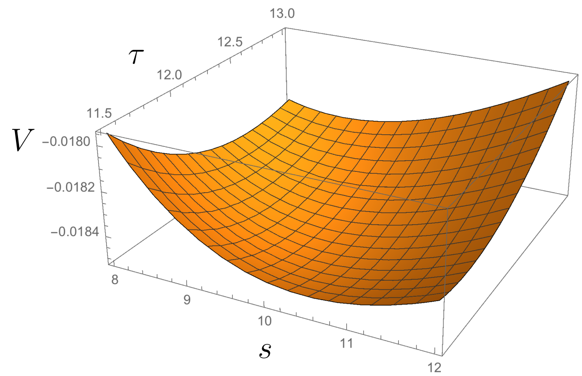


Figure 4.1: The scalar potential V in units of $\frac{M_{\rm Pl}^4}{4\pi\cdot 2^4}$ for $h=q=1$, $\tilde{\mathfrak{f}}=10$, showing the expected stable minimum at $s_0=10$ and $\tau_0=12$.

minimum (solution 3 of table 4.1) gives

$$M^2 = \frac{M_{\rm Pl}^2}{4\pi\cdot 2^4}\,\frac{5\,q^2}{54\tilde{\mathfrak{f}}^2}\begin{pmatrix} 60hq & 12h^2 & 0 & 0 \\ 25\,q^2 & 25hq & 0 & 0 \\ 0 & 0 & 12hq & 12h^2 \\ 0 & 0 & 25q^2 & 25hq \end{pmatrix}. \tag{4.2.8}$$

The mass eigenvalues can be written as

$$M^2_{{\rm mod},i} = \mu_i\,\frac{hq^3}{\tilde{\mathfrak{f}}^2}\,\frac{M_{\rm Pl}^2}{4\pi\cdot 2^4}\,, \tag{4.2.9}$$

with the numerical values

$$\mu_i = \left(\frac{25(17+\sqrt{97})}{108},\frac{25(17-\sqrt{97})}{108}\,;\,\frac{185}{54},0\right) \approx (6.2, 1.7\,;\,3.4, 0)\,. \tag{4.2.10}$$

The eigenvectors of the first (last) two masses are combinations of saxions (axions) and the massless state corresponds to the unfixed axionic combination $(qc - h\rho)$.

A remarkable feature, which we observe in all our models, is the fact that all moduli are parametrically of the same mass. Therefore we are able to control all moduli masses at once, which is useful to achieve the correct mass hierarchy (4.6.1).

Let us stress that one can also compute the gravitino mass $M_{\frac{3}{2}}$:

$$M_{\frac{3}{2}}^2 = e^{K_0}\,|W_0|^2\,\frac{M_{\rm Pl}^2}{4\pi}\,, \tag{4.2.11}$$

with K_0 and W_0 denoting the value of the Kähler potential and superpotential at the minimum. The gravitino mass is useful as it indicates the scale of supersymmetry breaking [66]. Interestingly, in our model A the gravitino shares the same flux dependence as the moduli (4.2.9) with the numerical prefactor $\mu_{\frac{3}{2}} = \frac{5}{6} \approx 0.833$. The supersymmetry breaking scale is therefore only slightly below the moduli masses. Due to the idea of moduli stabilization, the moduli masses and are obliged to be at a high scale. Consequently, our flux-scaling scenario predicts high-scale supersymmetry breaking. This is one of the major results of our work.

It remains to generate a parametrically-light mass for the axionic combination $(qc-h\rho)$ that has not been stabilized so far. If that can be achieved, this axion is a good candidate for realizing F-term axion monodromy inflation. The way of proceeding is to turn on supplementary fluxes causing new terms in the superpotential.

From equation (3.3.9) we determine the most general superpotential (without P-flux) as follows

$$W = -\mathfrak{f} + i\tilde{\mathfrak{f}} + i(h - i\tilde{h})\, S + i(q - i\tilde{q})\, T\,, \tag{4.2.12}$$

with $\mathfrak{f} = \mathfrak{f}_0$, $\tilde{h} = \tilde{h}^0$ and $\tilde{q} = \tilde{q}^{01}$. The only non-trivial Bianchi identity following from (3.2.12) reads

$$\tilde{h}\, q - h\, \tilde{q} = 0\,, \tag{4.2.13}$$

so that the superpotential (4.2.12) reduces to

$$W = -\mathfrak{f} + i\tilde{\mathfrak{f}} + \left(1 - i\,\frac{\tilde{q}}{q}\right) i(hS + qT)\,. \tag{4.2.14}$$

Therefore, W still depends only on the linear combination of axions (4.2.5), so that the orthogonal direction remains unfixed! In fact, the vacua of the superpotential (4.2.14) can be determined analytically and share the same qualitative structure of the three extrema shown in table 4.1. Consequently model A cannot enable F-term axion monodromy inflation via this procedure.

4.2.1 Extension to Models with Two Kähler Moduli

New intriguing features arise when adding one more Kähler modulus, in particular, we obtain new tachyons in the flux vacua. However, we will solely present the basic idea and conceptually new results in this thesis. The reader is referred to [18] for a detailed investigation of these scenarios. Let us consider two different models with $h^{1,1}_{+} = 2$, while keeping $h^{1,1}_{-} = h^{2,1}_{-} = 0$ for simplicity:

K3-fibration The most easy extension of model A is given by $\mathbb{P}_{1,1,2,2,2}[8]$ (cf. [67]) whose intersection numbers of the Kähler sector are such that the Kähler potential splits into sums

$$K = -2\ln(T_1 + \overline{T}_1) - \ln(T_2 + \overline{T}_2) - \ln(S + \overline{S})\,. \tag{4.2.15}$$

Swiss cheese Alternatively, one may start with a swiss-cheese manifold $\mathbb{P}_{1,1,1,6,9}$[18] that we biefly discussed in section 3.4. Its Kähler potential is of the following form [31]

$$K = -\ln(S+\overline{S}) - 2\ln\Big((T_1+\overline{T}_1)^{3/2} - (T_2+\overline{T}_2)^{3/2}\Big)\,. \tag{4.2.16}$$

For both cases we enlarge the superpotential (4.2.2) only minimally by turning on one more flux:

$$W = i\tilde{\mathfrak{f}} + ihS + iq_1T_1 + iq_2T_2\,. \tag{4.2.17}$$

For the K3-fibration as well as the swiss-cheese manifold the scalar potential shows four AdS vacua, where three of them generalize the solutions of the ordinary model A collected in table 4.1. The stabilized axion is in both cases $\theta = q_1\rho_1 + q_2\rho_2 + hc$, leaving the two orthogonal axionic combinations unfixed.

The new attribute comes from the two saxions τ_1 and τ_2. Calculating the physical mass states we find a saxionic combination that corresponds to a tachyon below the Breitenlohner-Freedman bound. To make this scenario plausible, it is inevitable to get rid of this tachyonic state. Fortunately, there exists a novel uplift mechanism based on adding a D-term contribution to the scalar potential. See [18] for details.

Having lifted the tachyon, extensions of model A by an additional Kähler modulus lead to suitable flux scaling scenarios.

4.3 Model C: Including One Complex Structure Moduli

In this this section we revisit model A including one complex structure modulus. There will be again a strictly stable, non-supersymmetric flux vacua of the scalar potential with one modulus unfixed. Besides, we will encounter a useful transformation one may employ in order to construct new scenarios.

Setup:

Consider again a Calabi-Yau manifold with $h^{1,1}_+ = 1$ and $h^{1,1}_- = 0$, but unlike model A set $h^{2,1}_- = 1$. Thus model C contains one Kähler modulus T and one complex structure modulus U, which can once again be viewed as an isotropic 6-torus T^6. Following section 2.2.1 and the conventions in [28], the complex structure moduli space of the isotropic limit of toroidal orbifold models is described by homogeneous coordinates $X^\Lambda = (1, iU)$. In the large complex structure limit the prepotential is given by equation (2.2.20)

$$F = -\frac{1}{3!}\kappa_{ijk}\frac{X^iX^jX^k}{X^0} = -\frac{(X^1)^3}{X^0} = iU^3, \tag{4.3.1}$$

where U^3 denotes $(U)^3$. The derivatives of the prepotential are then found to be

$$F_\Lambda = \big(\ -iU^3\ ,\ 3U^2\ \big) \tag{4.3.2}$$

and consequently equation (2.2.17) leads us to the Kähler potential for the complex structure part:

$$K_{U^i} = -\ln\left[-i\left(X^\Lambda \bar{F}_\Lambda - \bar{X}^\Lambda F_\Lambda\right)\right] = -\ln\left[\left(U+\bar{U}\right)^3\right]. \tag{4.3.3}$$

Taking also the Kähler modulus and axio-dilaton into account, the full Kähler potential reads

$$K = -3\ln(T+\overline{T}) - \ln(S+\overline{S}) - 3\ln\left(U+\overline{U}\right). \tag{4.3.4}$$

In the superpotential there are now more fluxes available, which of course have to satisfy the Bianchi identities. For the flux superpotential (3.3.9) we choose

$$W = -\mathfrak{f}_0 - 3\tilde{\mathfrak{f}}^1 U^2 - hUS - qUT\,, \tag{4.3.5}$$

where $h := h_1$ and $q := q_1$.

Flux Vacua of the Scalar Potential:

Apparently the superpotential depends only on the linear combination $\theta = hc + q\rho$ of the axions c, ρ and hence the scalar potential will only depend on this combination as well. Analogously to model A its orthogonal complement $(qc - h\rho)$ remains unstabilized. If it is possible to generate a parametrically-light mass for the latter by turning on additional fluxes, this axionic combination is likely to embody the inflaton field. We will examine this possibility in section 5.3. In the end it will turn out that model C is indeed appropriate to realize axion monodromy inflation.

Analyzing the scalar potential following from (4.3.4) and (4.3.5), we find two interesting extrema shown in table 4.2. There are actually more extrema, but they either have a non-vanishing imaginary part or are related to table 4.2 by adding a minus sign to the fluxes. Moreover, we have to emphasize that there might be further extrema with $u \neq 0$, however, those exceeded our computational capabilities.

solution	$(s, \tau, v^2, u, \theta)$	susy	tachyons	Λ
1	$(-6v\frac{\tilde{\mathfrak{f}}^1}{h}, -18v\frac{\tilde{\mathfrak{f}}^1}{q}, \frac{1}{9}\frac{\mathfrak{f}_0}{\tilde{\mathfrak{f}}^1}, 0, 0)$	yes	yes	AdS
2	$(-12v\frac{\tilde{\mathfrak{f}}^1}{h}, -15v\frac{\tilde{\mathfrak{f}}^1}{q}, \frac{1}{3\sqrt{10}}\frac{\mathfrak{f}_0}{\tilde{\mathfrak{f}}^1}, 0, 0)$	no	no	AdS

Table 4.2: Extrema of the scalar potential for model C.

Evaluation of the Flux Vacua:

- The **first solution** is a supersymmetric AdS vacuum with one axion unfixed. As expected from the no-go theorem by Conlon [11], this extremum contains tachyons. The vacuum is nevertheless stable since the tachyons are above the Breitenlohner-Freedman bound, cf. equation (4.2.6).

- The **second minimum** possesses the desired properties, that is, it is non-supersymmetric and tachyon-free with one axionic modulus unstabilized. For $h, q < 0 < \mathfrak{f}_0, \tilde{\mathfrak{f}}^1$, we have s, $\tau > 0$ and thus we are inside the physical regime. By scaling up $\mathfrak{f}_0$, one easily ensures to have perturbative control. Similar to model A all terms in the superpotential exhibit the same scaling order $\mathfrak{f}_0$, such that one could have guessed the scaling of the moduli vevs already from W.

Masses of the Moduli:

The physical masses of the non-supersymmetric non-tachyonic vacuum (solution 2) in the canonically normalized basis is given by

$$M^2_{\text{mod},i} = \mu_i \, \frac{hq^3}{(\mathfrak{f}_0)^{\frac{3}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}} \, \frac{M^2_{\text{Pl}}}{4\pi \cdot 2^7} \,, \tag{4.3.6}$$

with numerical values

$$\mu \approx \big(2.1,\, 0.37,\, 0.25\,;\, 1.3,\, 0.013,\, 0 \big)\,. \tag{4.3.7}$$

The first three eigenstates are saxions, the last three axions and the massless eigenstate is the axionic combination $(qc - h\rho)$. In particular, it is interesting that the lightest massive mode is axionic, and although not parametrically light, its mass is numerically light. In fact, it is by a factor of 1/5 smaller than the second-lightest massive state, which is purely saxionic.

For the gravitino mass the flux dependence is the same as for the moduli masses and the numerical prefactor is given by $\mu_{\frac{3}{2}} \approx 0.152$.

4.3.1 New Models from the Transformation $U \to 1/U$

In [18] we analyzed also a scenario with the same Kähler potential as model C, but a different superpotential

$$W = i\hat{\mathfrak{f}}_1 U + i\tilde{\mathfrak{f}}^0 U^3 + 3i\tilde{h}^1 U^2 S + 3i\tilde{q}^1 U^2 T\,, \tag{4.3.8}$$

where we set $\hat{\mathfrak{f}}_1 = -\mathfrak{f}_1$ for notational convenience. This scenario was called model D. Astonishingly, model D and model C are related via the transformation $U \to 1/U$ of the superpotentials:

$$W_D \to -\frac{i}{U^3}\left[-\hat{\mathfrak{f}}_1 U^2 - \tilde{\mathfrak{f}}^0 - 3\,\tilde{h}^1 U\, S - 3\,\tilde{q}^1 U\, T\right] = -\frac{i}{U^3} W_C. \tag{4.3.9}$$

Hence, $e^K|W_D|^2 = e^K|W_C|^2$ and the resulting scalar potential is basically the same as in model C because W_C has the same form as the superpotential in (4.3.5). Indeed, one can show that the vevs in both models match under $U \to 1/U$ and appropriate redefinition of the fluxes involved. This kind of transformation was exploited in [68, 69] to classify the allowed superpotentials induced by non-geometric fluxes. Moreover, duality symmetries in moduli space allow to fix the moduli vevs, thereby simplifying the search for new vacua [70, 71].

4.4 A Model with Two Complex Structure Moduli

Next we will check that scenarios with more than one complex structure modulus show the same scaling property and obtain a non-supersymmetric vacuum with at least one modulus unfixed. As a drawback new tachyons will emerge and an uplift mechanism of those directions is still unknown.

Setup:

Let us assume to have a manifold with $h_+^{1,1} = 1$, $h_-^{1,1} = 0$ and $h_-^{2,1} = 2$, thus we extend model C by one additional complex structure modulus. The geometry matches a non-toroidal background and can be interpreted as the mirror dual of the Kähler sector of $\mathbb{P}_{1,1,2,2,2}[8]$, which was briefly mentioned in the end of section 4.2. We want to start again from the prepotential, guided by section 2.2.1 and the appendix of [10]. Employing the homogeneous coordinates $X^\Lambda = (1, iU_1, iU_2)$, consider a prepotential (2.2.20) (in the complex structure limit) of the form

$$F = -\frac{1}{3!}\kappa_{ijk}\frac{X^iX^jX^k}{X^0} = -\frac{(X^1)^2X^2}{X^0} = iU_1^2U_2, \tag{4.4.1}$$

where the index downstairs numerates the complex structure moduli and the index upstairs gives the power of the modulus. The derivatives of the prepotential are then found to be

$$F_\Lambda = \left(\ -iU_1^2U_2\ ,\ 2U_1U_2\ ,\ U_1^2\ \right) \tag{4.4.2}$$

and consequently equation (2.2.17) leads us after a brief computation to the Kähler potential for the complex structure part:

$$K_{2,1} = -\ln\left[-i\left(X^\Lambda\bar{F}_\Lambda - \bar{X}^\Lambda F_\Lambda\right)\right] = -\ln\left[\left(U_1+\bar{U}_1\right)^2\left(U_2+\bar{U}_2\right)\right]. \tag{4.4.3}$$

Hence the full Kähler potential can be written as

$$K = -2\ln(U_1+\overline{U}_1) - \ln(U_2+\overline{U}_2) - \ln(S+\overline{S}) - 3\ln(T+\overline{T}). \tag{4.4.4}$$

There are now many possibilities for the superpotential (3.3.9), which are soon rather complicated to calculate. The following choice appeared to be relatively simple:

$$W = -\mathfrak{f}_0 - \left(hS + qT + \tilde{\mathfrak{f}}_2U_1 + 2\tilde{\mathfrak{f}}_1U_2\right)U_1, \tag{4.4.5}$$

where we have set $h_1 = h$ and $q_1 = q$.

Flux Vacua of the Scalar Potential:

Now we can calculate the scalar potential from the superpotential and Kähler potential above and check for extrema. However, since the analytical computation of the minima appeared to be quite involved, we have set $u_1 = 0$ for simplification and solved for the remaining moduli. The superpotential depends consequently solely on the axionic combination $\theta = hc + q\rho + 2\tilde{\mathfrak{f}}_1 u_2$, while the two orthogonal axions $(2\tilde{\mathfrak{f}}_1 c - hu_2)$ and $(h\rho - qc)$ are not stabilized.

In fact, there exist numerous solutions which are in many cases related by a sign change of the fluxes. We state only one representative minimum in table 4.3. As usual the fluxes have to be chosen such that we are inside the physical regime.

solution	$(s, \tau, v_1^2, v_2^2, \theta, u_1)$	susy	tachyons	Λ
1	$(\frac{2}{3}\frac{\tilde{\mathfrak{f}}_2}{h} v_1, 2\frac{\tilde{\mathfrak{f}}_2}{q} v_1, \frac{\mathfrak{f}_0}{\tilde{\mathfrak{f}}_2}, \frac{1}{3}\frac{\tilde{\mathfrak{f}}_2}{\tilde{\mathfrak{f}}_1} v_1, 0, 0)$	no	yes	AdS

Table 4.3: Extrema of the scalar potential for the model of section 4.4 including two complex structure moduli.

Evaluation of the Flux Vacua:

This scenario has indeed non-supersymmetric vacua, but all of them contain at least one tachyon. The tachyons of the vacua in table 4.3 are given by the saxionic combination $(h\tau - qs)$ and $(2s\tilde{\mathfrak{f}}_1 - hv_2)$. As one can show these tachyons are below the Breitenlohner-Friedman bound and hence the minimum is clearly unstable. Unfortunately, lifting the tachyons via the D-term mechanism, that we briefly mentioned above, is not applicable in the case of multiple complex structure models [18]. How to get rid of the tachyons in such a scenario remains an open issue.

Moreover, we observe a supersymmetric minimum, which has two tachyons as expected from the no-go theorem by [11]. In this case, the tachyons lie above the Breitenlohner-Friedman bound and thus the minimum is stable.

Masses of the Moduli:

The physical masses of the non-supersymmetric, but tachyonic vacuum shown in table 4.3 are in the canonically normalized basis found to be

$$M^2_{\mathrm{mod},i} = \mu_i \, \frac{\tilde{\mathfrak{f}}_1 h q^3}{(\mathfrak{f}_0)^{\frac{3}{2}} (\tilde{\mathfrak{f}}_2)^{\frac{3}{2}}} \, \frac{M^2_{\mathrm{Pl}}}{4\pi \cdot 2^7} \,, \tag{4.4.6}$$

with numerical values

$$\mu_i = \big(18,\, 18,\, -2\ \ -2\,;\, 10,\, 10,\, 0,\, 0\big) \,, \tag{4.4.7}$$

where the first four eigenvalues are saxions and the others axions. Moreover, the gravitino mass has the numerical prefactor $\mu_{\frac{3}{2}} \approx 16$.

4.5 A Model with All Moduli and Non-Geometric P-Fluxes

So far no scaling scenario contained non-geometric P-fluxes. By adding them to our setup, all moduli will be stabilized. In accordance with [18] we will proceed in two steps, more precisely axionic-odd moduli will be considered only in the second step.

Step 1: $\mathbf{h^{1,1}_{-} = 0}$

Setup:

Our final flux scaling model starts out quite similar to model C with $h^{1,2}_{-} = 1$ and $h^{1,1}_{+} = 1$. The Kähler potential is given by the simple expression (4.3.4). The new ingredient is an additional P-flux.

For the superpotential we exchange the last term (qUT) of model C with a P-flux term, such that we end up with the following superpotential

$$W = \hat{\mathfrak{f}} - 3\tilde{\mathfrak{f}}U^2 - hSU + pST\,, \tag{4.5.1}$$

with $\hat{\mathfrak{f}} := -\mathfrak{f}_0$, $\tilde{\mathfrak{f}} := \tilde{\mathfrak{f}}^1$, $h := h_1$ and $p := p_0$.

Flux Vacua of the Scalar Potential:

We find the same structure of minima as in other examples with the same Hodge numbers.

Note that the superpotential is chosen in such a way that every modulus is stabilized. The reason for this fact is the term pST mixing both moduli S and T. It is an important result that in general all moduli can be stabilized if we include P-fluxes. Choosing $h < 0$ and $\tilde{\mathfrak{f}}, p > 0$, we are inside the physical regime and obtain several distinct minima. Two illustrative solutions are shown in table 4.4. Obviously the two solutions differ just by a sign in front of $\hat{\mathfrak{f}}$ in v^2. We want to remark that $\hat{\mathfrak{f}}$ has to be taken positive for solution 1 and negative for solution 2.

Evaluation of the Flux Vacua:

Solution 2 is a non-supersymmetric flux vacuum, however, now all moduli are stabilized in contrast to the models before. There is one tachyon which lies above the Breitenlohner-Friedman bound.

As shown in table 4.4 we obtain in addition a supersymmetric AdS minimum with all moduli fixed as well. It is stable since there are no tachyons appearing.

solution	$(s, \tau, v^2, \rho, c, u)$	susy	tachyons	Λ
1	$(-\frac{12}{5}\frac{\tilde{\mathfrak{f}}}{h}v, \frac{3}{2}\frac{h}{p}v, \frac{5}{9}\frac{\hat{\mathfrak{f}}}{\tilde{\mathfrak{f}}}, 0, 0, 0)$	yes	no	AdS
2	$(-\frac{12}{5}\frac{\tilde{\mathfrak{f}}}{h}v, \frac{3}{2}\frac{h}{p}v, -\frac{5}{9}\frac{\hat{\mathfrak{f}}}{\tilde{\mathfrak{f}}}, 0, 0, 0)$	no	yes	AdS

Table 4.4: Extrema of the scalar potential for the model of section 4.5 including P-fluxes.

Masses of the Moduli:

The superpotential (4.5.1) and the Kähler potential (4.3.4) lead to the following mass eigenvalues for the non-supersymmetric solution 2 in the canonically normalized basis

$$M^2_{\mathrm{mod},i} = \mu_i \, \frac{p^3\, (\tilde{\mathfrak{f}})^{\frac{5}{2}}}{h^2\, (\hat{\mathfrak{f}})^{\frac{3}{2}}} \, \frac{M^2_{\mathrm{Pl}}}{4\pi \cdot 2^7}\,, \tag{4.5.2}$$

where the numerical prefactors take the values

$$\mu_i = (52,\ 31,\ 15\,;\ 45,\ -3.4,\ 19). \tag{4.5.3}$$

Here the first three eigenvalues correspond to saxionic states, whereas the last three to axionic states. In particular, the tachyonic state is given by a linear combination of c, ρ and u. Computing the gravitino mass, we find the numerical prefactor $\mu_{\frac{3}{2}} = 10.7$.

Step 2: $\mathbf{h^{1,1}_{-} = 1}$

Finally let us construct a flux scaling scenario including all possible moduli of table 2.2, that is, extending the previous case with $h^{1,1}_{-} = 0$ by one axionic-odd modulus $G = \psi + i\eta$. Hence we have to deal with the full Kähler potential (2.2.37):

$$K = -3\ln\Big((T+\overline{T}) + \frac{\kappa}{4(S+\overline{S})}\,(G+\overline{G})^2\Big) - \ln(S+\overline{S}) - 3\ln\big(U+\overline{U}\big), \tag{4.5.4}$$

where $\kappa = 2\kappa_{\alpha ab}$ with $\alpha = a = b = 1$. When including an axionic-odd modulus, the superpotential (4.5.1) becomes by default

$$W = \hat{\mathfrak{f}} - 3\tilde{\mathfrak{f}}U^2 - hSU + p\left(ST + \frac{\kappa}{4}\,G^2\right)\,, \tag{4.5.5}$$

with κ as in the Kähler potential. This superpotential can be easily derived from the general ansatz (3.3.9). In doing so, it is important to notice that the G^2 and ST terms are generated by the same P-flux.

Since only the differences between the Kähler potential and superpotential from the previous example with $h^{1,1}_{-} = 0$ depend on terms with G, there are supersymmetric and non-supersymmetric minima with the axionic-odd moduli stabilized at $\psi = \eta = 0$. The remaining moduli still take the values of table 4.4, where the supersymmetric minimum and its non-supersymmetric counterpart differ again by a minus in front of $\hat{\mathfrak{f}}$.

The G modulus decouples from S, T, U moduli in the canonically normalized mass matrix, thereby leading to the same masses in the S, T, U sector as before. On the other hand, η and ψ turn out to be eigenstates of the canonically normalized mass matrix. The corresponding mass eigenvalues are of the form (4.5.2) with numerical prefactors $(\mu_\psi, \mu_\eta) = (0, -3.4)$ and $(0, 17)$, for the supersymmetric and non-supersymmetric extrema respectively. Therefore both cases are now plagued with a tachyon and a massless saxion. There exist additional extrema with unstabilized $\psi \neq 0$ showing the same qualitative behavior.

To conclude, we have not been able to detect a non-supersymmetric stable flux vacuum with one axion unfixed for the final scenario and hence this model is not suitable for realizing axion monodromy inflation. Nevertheless we in fact presented a flux scaling scenario stabilizing all possible moduli at tree level.

4.6 Moduli Spectroscopy

An absolutely crucial necessity for any flux-scaling vacuum to be a realistic scenario is to guarantee the correct mass hierarchy:

$$M_{\rm Pl} \ > \ M_{\rm s} \ > \ M_{\rm KK} \ > \ M_{\rm mod}\,, \tag{4.6.1}$$

where $M_{\rm Pl}$ is the Planck mass, $M_{\rm s}$ the string scale, $M_{\rm KK}$ the Kaluza-Klein scale and $M_{\rm mod}$ are the moduli masses which we computed above. It will be explained how to calculate these mass scales later on in this section. Notice that we are not taking inflation into account at this point, otherwise there would be even more scales which have to be in proper order. We postpone this discussion to chapter 5.

$M_{\rm mod}$ has to be smaller than $M_{\rm s}$ and $M_{\rm KK}$ in order to make the four-dimensional supergravity approximation trustable. The first two hierarchies in (4.6.1) are evident by construction of string theory.

In this section we will calculate the mass scales for our scaling models and check whether they have the ability to satisfy the hierarchy (4.6.1).

The Planck mass in (4.6.1) is $M_{\rm Pl} = (8\pi G)^{-1/2} \approx 2.435 \cdot 10^{18}$GeV in our conventions. As usual the string mass is $M_{\rm s} = (\alpha')^{-\frac{1}{2}}$, and in terms of $M_{\rm Pl}$ the string and Kaluza-Klein scales can be expressed as

$$M_{\rm s} = \frac{\sqrt{\pi} M_{\rm Pl}}{s^{\frac{1}{4}}\, \mathcal{V}^{\frac{1}{2}}}\,, \qquad\qquad M_{\rm KK} = \frac{M_{\rm Pl}}{\sqrt{4\pi}\, \mathcal{V}^{\frac{2}{3}}}\,, \tag{4.6.2}$$

where $s = e^{-\phi}$, cf. e.g. [72] for details. Recall that $\mathcal{V}$ is the volume of the Calabi-Yau manifold in Einstein frame measured in string units, namely $\mathcal{V} = \mathrm{Vol}/\ell_s^6$ with $\ell_s = 2\pi\sqrt{\alpha'}$. In the flux-scaling models we had large fluxes f_L guaranteeing that the moduli are in their perturbative regime and other fluxes f_S that we usually choose to be of order one. Moreover, there are further order one coefficients entering the Kähler potential, once we specify a concrete Calabi-Yau manifold. The hope is now to achieve parametric control over the mass scales, such that we can fulfill the hierarchy (4.6.1) by tuning the fluxes appropriately. Let us now formalize what we mean by parametrical control: A scale M_1 is called *parametrically larger* than a scale M_2, denoted as $M_1 \underset{p}{\gtrsim} M_2$, if it occurs that $M_2/M_1 \to 0$ for $f_L \to \infty$. The two scales are called *parametrically equal*, $M_1 \underset{p}{\simeq} M_2$, if $M_2/M_1 \to O(1)$ for $f_L \to \infty$. This distinguishes the case where one has parametric control over the relative size of two mass scales from the case when their relative size is just a numerical coincidence. It can happen that even though $M_1 \underset{p}{\simeq} M_2$ one of the order one fluxes f_S can guarantee parametric control. If that is the case we mention it explicitly. It is also possible that in all our examples it just happens that the numerical prefactors are such that $M_1 > M_2$. In this case, we say that M_1 is numerically larger than M_2 and denote it as $M_1 \underset{n}{\gtrsim} M_2$.

Due to the definitions (4.6.2), $M_{\rm Pl}$, $M_{\rm s}$ and $M_{\rm KK}$ are highly depending on the concrete model. So, next we want to investigate two representative scaling models.

Model A Let us start with model A of section 4.2, where solution 3 of table 4.1 corresponded to a non-supersymmetric stable vacuum. The volume of an isotropic 6-torus is given by the Kähler potential (4.2.1) and can be rewritten in terms of the real part τ of the Kähler moduli:

$$\mathcal{V} = \left(T + \overline{T}\right)^{\frac{3}{2}} = (2\tau)^{\frac{3}{2}}. \tag{4.6.3}$$

Then the Kaluza-Klein and string scale for the non-supersymmetric tachyon-free minimum follow directly from definitions (4.6.2):

$$M_{\rm s}^2 = \mu_{\rm s}\, \frac{h^{\frac{1}{2}} q^{\frac{3}{2}}}{\tilde{\mathfrak{f}}^2}\, \frac{M_{\rm Pl}^2}{4\pi \cdot 2^7}\,, \qquad\qquad M_{\rm KK}^2 = \mu_{\rm KK}\, \frac{q^2}{\tilde{\mathfrak{f}}^2}\, \frac{M_{\rm Pl}^2}{4\pi \cdot 2^7}\,, \tag{4.6.4}$$

with $\mu_{\rm s} = 21$ and $\mu_{\rm KK} = 22$. In order to check the mass hierarchy, consider at first the ratio of the Kaluza-Klein scale to the string scale

$$\frac{M_{\rm KK}^2}{M_{\rm s}^2} = \frac{1}{4\pi^2}\left(\frac{s}{2\tau}\right)^{\frac{1}{2}} = \frac{1}{4\pi^2}\left(\frac{5}{12}\right)^{\frac{1}{2}}\left(\frac{q}{h}\right)^{\frac{1}{2}}\,. \tag{4.6.5}$$

Therefore, to make the string scale parametrically higher than the Kaluza-Klein scale, we would need to require $h > q$. This means $\tau > s$ so that α'-corrections to the tree-level Kähler potential are indeed subleading. The ratio of the Kaluza-Klein scale to the moduli mass scale comes out as

$$\frac{M_{\rm mod}^2}{M_{\rm KK}^2} = 0.36\ \mu_i\, hq\,. \tag{4.6.6}$$

One might think that the mass hierarchy for this model is not satisfied at all since both ratios do not depend on the very large flux $\hat{\mathfrak{f}}$ and we have $M_{\rm s} \underset{p}{\simeq} M_{\rm KK} \underset{p}{\simeq} M_{\rm mod}$. However, by choosing for the order one fluxes $h > q$ we can at least guarantee a parametrical separation between string and Kaluza-Klein scale. As a summary, the non-supersymmetric stable vacuum of model A leads to the following mass hierarchy

$$M_{\rm Pl} \;>\; M_{\rm s} \underset{p}{\gtrsim} M_{\rm KK} \underset{p}{\simeq} M_{\rm mod}\,. \tag{4.6.7}$$

Model C Let us eventually show that in contrast to model A it is indeed possible to realize the correct mass hierarchy in model C. Since the geometry is again described by an isotropic 6-torus, the overall volume of the internal manifold reads $\mathcal{V} = (2\tau)^{\frac{3}{2}}$. Consequently, the definitions (4.6.2) for the Kaluza-Klein and string scale yield

$$M_{\rm s}^2 = \mu_{\rm s}\,\frac{h^{\frac{1}{2}}q^{\frac{3}{2}}}{\mathfrak{f}_0\tilde{\mathfrak{f}}^1}\,\frac{M_{\rm Pl}^2}{4\pi\cdot 2^7}\,, \qquad\qquad M_{\rm KK}^2 = \mu_{\rm KK}\,\frac{q^2}{\mathfrak{f}_0\,\tilde{\mathfrak{f}}^1}\,\frac{M_{\rm Pl}^2}{4\pi\cdot 2^7}\,, \tag{4.6.8}$$

with $\mu_{\rm s} = 84$ and $\mu_{\rm KK} = 1.4$, for the non-supersymmetric stable vacuum of model C, that is solution 2 in table 4.2. For the ratio of the Kaluza-Klein and the string scale we obtain

$$\frac{M_{\rm KK}^2}{M_{\rm s}^2} = 0.016\left(\frac{q}{h}\right)^{\frac{1}{2}}\,, \tag{4.6.9}$$

whereas the ratio of the moduli masses and the Kaluza-Klein scale is

$$\frac{M_{\rm mod}^2}{M_{\rm KK}^2} \sim \frac{h\,q\,(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}{\mathfrak{f}_0^{\frac{1}{2}}}\,. \tag{4.6.10}$$

A separation between the moduli and Kaluza-Klein states can obviously be ensured by making $\mathfrak{f}_0$ large enough. Thus for the non-supersymmetric stable minimum of model C we can actually guarantee the desired mass hierarchy:

$$M_{\rm Pl} \underset{p}{\gtrsim} M_{\rm s} \underset{p}{\gtrsim} M_{\rm KK} \underset{p}{\gtrsim} M_{\rm mod}\,. \tag{4.6.11}$$

Since all scales differ only by a relative factor of $O(10)$, they are very sensitive to numerical prefactors. For concreteness let us make the choice

$$\mathfrak{f}_0 = 3200\,, \qquad \tilde{\mathfrak{f}}^1 = 1\,, \qquad h = -2\,, \qquad q = -1\,, \tag{4.6.12}$$

and analyze the moduli around the minimum with values

$$\tau = 275\,, \qquad s = 110\,, \qquad v = 18\,, \qquad u = c = \rho = 0\,. \tag{4.6.13}$$

Using $M_{\rm Pl} = 2.44\cdot 10^{18}\,$GeV, the string and Kaluza-Klein scale come out as

$$M_{\rm s} \sim 1.17\cdot 10^{16}\,{\rm GeV}\,, \qquad\qquad M_{\rm KK} \sim 1.25\cdot 10^{15}\,{\rm GeV}\,. \tag{4.6.14}$$

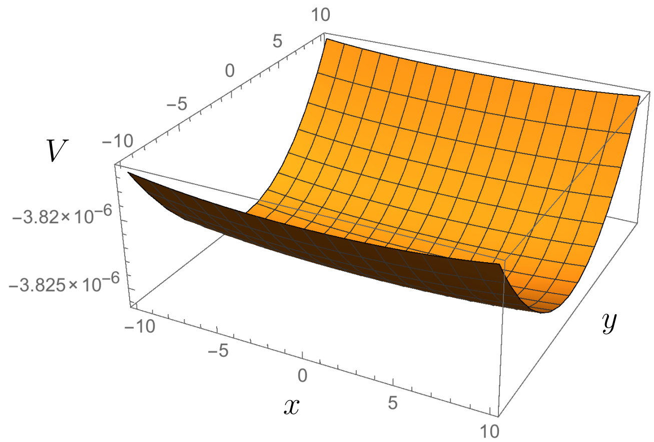


Figure 4.2: The potential $V(x, y)$ around the minimum, where x is pointing in the direction of the lightest axionic modulus and y in the direction of the lightest saxionic modulus.

The masses of the saxion moduli are

$$M_i^{\text{sax}} \sim \big(2.9,\ 1.2,\ 1.0\big) \cdot 10^{14}\,\text{GeV}\,, \tag{4.6.15}$$

and the masses of the two heavy axions are

$$M_i^{\text{ax}} \sim \big(2.5,\ 0.23\big) \cdot 10^{14}\,\text{GeV}\,. \tag{4.6.16}$$

Note that the second axion is the lightest (massive) axion and therefore might be a possible inflaton candidate. We will investigate this further in chapter 5. In figure 4.2 we show the potential around the minimum, in the directions of the lightest and the second-lightest modulus.

Chapter 5

Application to Inflation in String Theory

The final chapter applies the flux-scaling scenarios constructed above to inflation. We will start with a summary of inflation including a short overview of different types of inflation. Then, the need for an embedding in an UV-complete theory leads us to string inflation via axion monodromy. Note that the first two section follow mainly the excellent reviews [73, 74]. In the third section, the explicit inflaton potential induced by model C is analysed analogously to [75]. Therefore we have to take backreaction effects into account and add an uplift term. As the central result of this thesis, the final inflaton potential varies with the backreaction. More concretely, we obtain an interpolation between polynomial and Starobinsky-like inflation.

5.1 A Brief Review of Inflation

In the last decades cosmology fundamentally changed from a rather speculative field to one based on high precision measurements. Important milestones are the Hubble Space Telescope fixing the current Hubble parameter [76] that eventually motivated the introduction of a small ($\sim 10^{-122} M_{\rm Pl}^4$) cosmological constant. This effect is interpreted as dark energy and causes late-time accelerated expansion of the universe [77, 78]. Moreover, the satellite missions WMAP [79] and PLANCK [4, 5] as well as ground-based telescopes ACT [80] and SPT [81] provide us with a sensational resolution of the temperature fluctuations of the cosmic microwave background (CMB). These fluctuation must have a primordial origin in the very early universe ($\sim 10^{-34}s$) which presumably arise from quantum fluctuations stretched by an inflationary expansion of the universe [82]. Such an extremely rapid early-time accelerated expansion of our universe driven by the vacuum energy of a so-called inflaton field is consistent with the recent experimental results and will be explained next.

5.1.1 Horizon Problem and Cosmic Inflation

As an observational matter of fact, the universe is amazingly isotropic and homogeneous on large scales. Moreover, its geometry matches the spatially flat Friedmann-Robertson-Walker (FRW) metric quite well. The FRW metric employs a universal *scale factor* $a(t)$ with cosmic time t. This scale factor describes the distance between two comoving points, that is, between points moving simultaneously with the expansion of the universe. In standard Big Bang cosmology, tracing back the expansion of the universe, i.e. $a(t) \longrightarrow 0$, we eventually reach a spacetime singularity called Big Bang. We choose the Big Bang to happen at cosmic time $t = 0$. A very important definition in this context is the *Hubble parameter*

$$H = \frac{\dot{a}}{a} \, . \tag{5.1.1}$$

In accordance with usual conventions, here and in the following $\frac{d}{dt} \equiv \dot{}$ denotes the time derivative. The Hubble parameter quantifies the expansion rate of the universe and is positive for an expanding universe. Besides, H^{-1} is the *Hubble time* which corresponds to the *Hubble length* in units where $c = 1$. The latter is commonly known as *horizon* because it estimates the size of the observable universe.

In order to discuss causal structure, we define the *conformal time* τ as well as the *comoving distance* that a particle may traverse during τ. Starting at the initial singularity, τ is defined by

$$\tau = \int_0^t \frac{dt'}{a(t')} = \int_{-\infty}^{\ln a(t)} \frac{d\ln a}{aH} . \tag{5.1.2}$$

This is equivalent to the maximal comoving distance a particle could have traveled since the Big Bang and thus often call *particle horizon*. Definition (5.1.2) gives immediately rise to the following issue:

Horizon problem: While the universe expands, the *comoving Hubble radius* $(aH)^{-1} = (\dot{a})^{-1}$ grows in time. Hence the particle horizon, i.e. the integral in (5.1.2), receives its dominating contributions from late times. Therefore at sufficiently early time, all observable length scales have been outside the horizon and consequently not in causal contact! In case of the Cosmic Microwave Background (CMB), patches separated roughly by more than one degree were causally disconnected at their creation. However, the CMB looks sensationally uniform everywhere! To shed more light upon this miraculous coincidence, consider figure 5.1 which depicts past light cones that do not overlap before the Big Bang. There must be a reason why once causally disconnected regions of space ended up in perfect agreement later on. Solving this puzzle is knows a horizon problem.

Notice that the standard Big Bang cosmology leads to even more open issues, as for instance the *flatness problem* which means that a nearly flat universe today would require

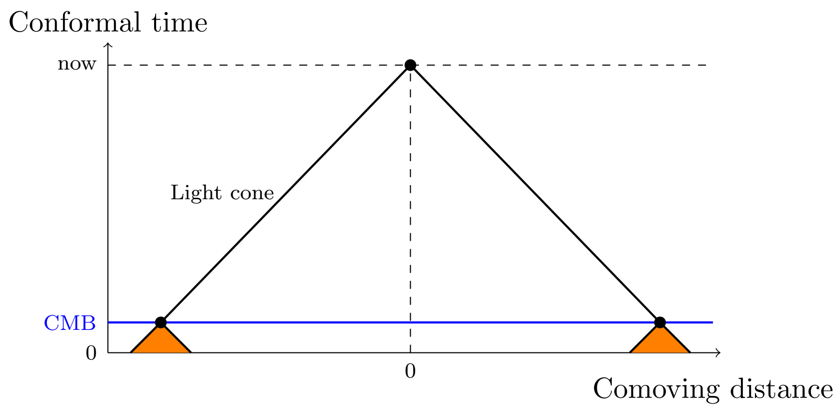


Figure 5.1: Spacetime diagram adapted from [74] illustrating the horizon problem in comoving coordinates. Our past light cone contains points clearly separated at CMB creation. As the light cones (orange) of these points do not overlap, the points have never been able to be in causal contact.

an extreme fine-tuning at very early times. Another striking problem are unwanted relics like magnetic monopoles (in Grand Unified Theories (GUT)), which should have been produced in the early universe.

Let us step back to the horizon problem and figure out its solution.

Solution: Introduce an early time period where the comoving Hubble radius $(aH)^{-1}$ was decreasing while the scale factor $a(t)$ was still increasing.

Thus comoving length scales have left the horizon at some early time and re-entered again when the comoving Hubble radius changed to increase. If the comoving Hubble radius was decreasing sufficiently long, the whole CMB spectrum could have been in causal contact at some very early time, see figure 5.2. In terms of equations, a decreasing comoving Hubble radius in an expanding universe corresponds to

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \qquad \text{for a very early time period.} \tag{5.1.3}$$

Using the definition of the Hubble parameter (5.1.1), it is easy to compute

$$\frac{d}{dt}\left(\frac{1}{aH}\right) = -\frac{1}{a}\left(1 + \frac{\dot{H}}{H^2}\right) = -\frac{1}{(aH)^2}\,\ddot{a} \tag{5.1.4}$$

Introducing then the so-called *(Hubble) slow-roll parameters*

$$\varepsilon = -\frac{\dot{H}}{H^2} \qquad \text{and} \qquad \tilde{\eta} = \frac{\dot{\varepsilon}}{H\varepsilon}, \tag{5.1.5}$$

finally leads to:

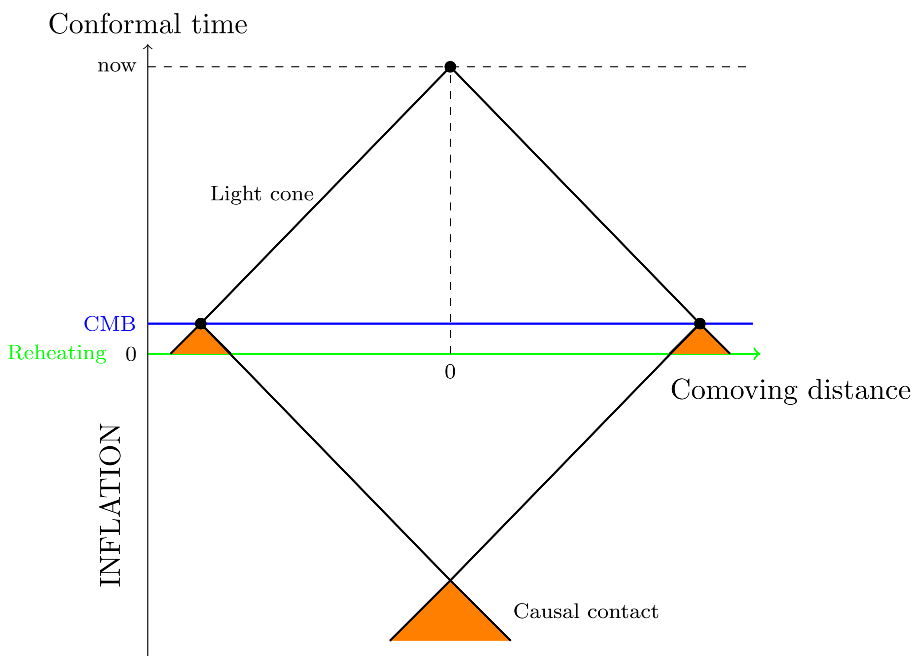


Figure 5.2: Spacetime diagram adapted from [74] illustrating how inflation solves the horizon problem. Now all points of the CMB have overlapping light cones and thus originate from causally connected regions of space. Note that the standard Big Bang at $\tau = 0$ has been replaced by the end of inflation, i.e. a reheating phase.

Definition: *Inflation is a time period with $\varepsilon < 1$ or equivalently $\ddot{a} > 0$. So, a period of accelerated expansion of the very early universe.*

Inflation does not only solve the horizon problem, but also other issues like the flatness and magnetic monopole problems. For details we refer to the literature, for instance [83].

It remains to clarify how long inflation needed to last in order to redress these problems. The duration of inflation is commonly quantified in the number N_e of e-foldings specifying the growth of the scale factor:

$$a(t_{\text{end of inflation}}) \;\simeq\; e^{N_e}\; a(t_{\text{begin of inflation}}). \tag{5.1.6}$$

Comparing the largest visible cosmological scales and the GUT scale as smallest one, it turns out that we must have $N_e \approx 60$ to overcome the issues above. The end of inflation is known as *reheating* as we will briefly explain now, cf. [83] for more information.

Our next task is to make out an energy source driving inflation and eventually fading away to culminate in the standard model spectrum.

5.1.2 Realizing Inflation in Effective Field Theory

In the end we would like to achieve inflation within sting theory or more precisely we look for an object in 10d string theory that effectively causes inflation in 4d. Here, we describe inflation by field theory and postpone its stringy origin to sections 5.2 and 5.3.

Start with a single scalar field ϕ whose potential energy is displayed in figure[1] 5.3. As ϕ moves towards the minimum, it passes a slow-rolling phase giving rise to inflation. Eventually, when ϕ reaches the minimum, its kinetic energy becomes comparable to its potential energy and inflation terminates. The remaining, large energy density of ϕ decays into standard model particles and in particular radiation, i.e. reheating occurs. Quantum fluctuations $\delta\phi$ at the beginning of inflation are responsible for later fluctuations in the CMB spectrum. In the following we will often refer to the scalar field ϕ as *inflaton field*. This is the most prevalent approach to inflation. More specifically assume a scalar field ϕ minimally coupled to Einstein gravity in 4d

$$\mathcal{S}_{\text{Inflaton}} = \int d^4x\; \sqrt{-g}\; \left[\frac{1}{2}\mathcal{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \tag{5.1.7}$$

where the inflaton potential $V(\phi)$ has not to be of the form shown in figure 5.3, but may look rather arbitrary.

The Friedmann equation and Klein-Gordon equation, cf. for instance [87], for the scalar field $\phi(t)$ are given by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \qquad \text{and} \qquad 3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{5.1.8}$$

[1] We will explain this potential more precisely when we discuss small-field inflation.

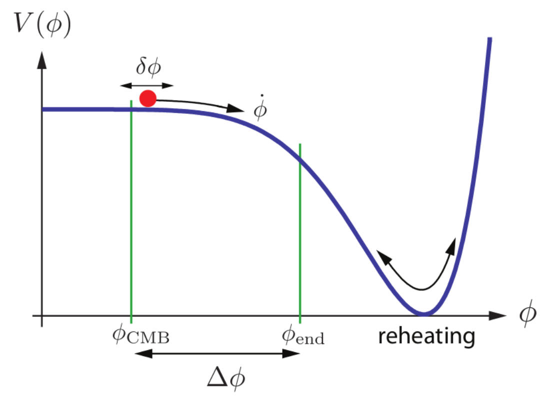


Figure 5.3: Example of an inflaton potential $V(\phi)$ from [74].

with $()' \equiv \frac{\partial}{\partial\phi}()$. Performing now a time derivative of the Klein-Gordon equation and plugging in the Friedmann equation, enables us to rewrite the condition for inflation. The Hubble slow-roll parameter then has to satisfy:

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2}\dot{\phi}^2}{H^2} < 1. \tag{5.1.9}$$

This result leads to slow-roll conditions for the scalar field (again with help of equations (5.1.8)):

$$\dot{\phi}^2 \ll V(\phi) \qquad \text{and} \qquad |\ddot{\phi}| \ll |3H\dot{\phi}|,\ |V'(\phi)|\ . \tag{5.1.10}$$

The condition on the left guarantees that the potential energy of ϕ dominates over the kinetic energy and thus motivates the label 'slow-roll'. Apart from that the condition on the right implies small acceleration and ensures thereby a long enough slow-roll period.

Alternatively the slow-roll conditions for inflation can be expressed in terms of conditions on the potential $V(\phi)$. For this reason we introduce the *potential slow-roll parameters*

$$\epsilon := \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \qquad \text{and} \qquad |\eta| := \frac{|V''|}{V} \ll 1 \tag{5.1.11}$$

which are related to the Hubble slow-roll parameters as follows [73]

$$\epsilon \approx \varepsilon \qquad \text{and} \qquad \eta \approx 2\varepsilon - \frac{\tilde{\eta}}{2}\ . \tag{5.1.12}$$

It is convenient to formulate the slow-roll conditions as restrictions on the inflaton potential because it is primarily the potential that distinguishes between various models of inflation. In this sense, one redefines also the number N_e of e-foldings (5.1.6)

$$N_e(\phi) \equiv \ln\frac{a(t_{\text{end of inflation}})}{a(t_{\text{begin of inflation}})} = \int_{t_0}^{t_{\rm end}} dt'\ H = \int_{\phi_0}^{\phi_{\rm end}} d\phi\ \frac{H}{\dot{\phi}} \approx \int_{\phi_{\rm end}}^{\phi_0} d\phi\ \frac{V}{V'}\ . \tag{5.1.13}$$

5.1.3 Models of Inflation

Let us now discuss some characteristic (but not completely distinct) classes of inflaton potentials:

- **Single-field slow-roll models**: Such models of inflation are usually the most simple ones. The subsequent examples on small- and large-field inflation belong to this category.

- **Modified gravity models:** Additional terms in the Einstein-Hilbert action lead to new types of inflation. For instance, one could consider higher curvature terms or expressions non-minimally coupled to gravity. We will investigate Starobinsky inflation as an example later in this section and encounter a similar behaviour again in section 5.3.

- **Non-slow-roll models:** These models are often called K-Inflation models because their characteristic property is a non-canonical kinetic term driving inflation [84]. As a consequence inflation can also be possible for steep potentials not obeying the slow-roll conditions. However, we will not consider these models further.

- **Multi-field models:** In this case there are several fields inflating which easily allow for a large number of possibilities for the inflationary dynamics. The final model presented in section 5.3 will turn out to be equipped with multi-field features.

Let us now consider single-field slow-roll inflation described by the Einstein-Hilbert action (5.1.7). It is convenient to distinguish between small and large distances that the inflaton is allowed to move from the creation of the CMB ϕ_{CMB} to the end of inflation ϕ_{end}. In this sense we define $\Delta\phi := \phi_{\text{CMB}} - \phi_{\text{end}}$ measured in Planck units. ϕ_{end} is set by the slow-roll conditions (5.1.11) and hence by the inflaton potential $V(\phi)$. Next, we want to spell out typical potentials for small- and large-field inflation:

Small-field inflation: $\Delta\phi < M_{pl}$ sub-Planckian field evolution
An example for a small-field inflation potential has already been shown in figure 5.3, which is also know as *hilltop inflation.*

Such small-field scenarios originate often from spontaneous symmetry breaking and motivate a Higgs-like potential. Usually the inflaton potential for small-field inflation takes locally the form ($p > 0$; V_0 and μ are model-dependent parameters)

$$V(\phi) \;=\; V_0 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right] + \;\ldots \tag{5.1.14}$$

with the dots denoting higher-order terms that become important for large values of ϕ, i.e. at the end of small-field inflation. Those are indeed relevant for the exact value of ϕ_{end} and determine the cosmological constant at the global minimum after inflation.

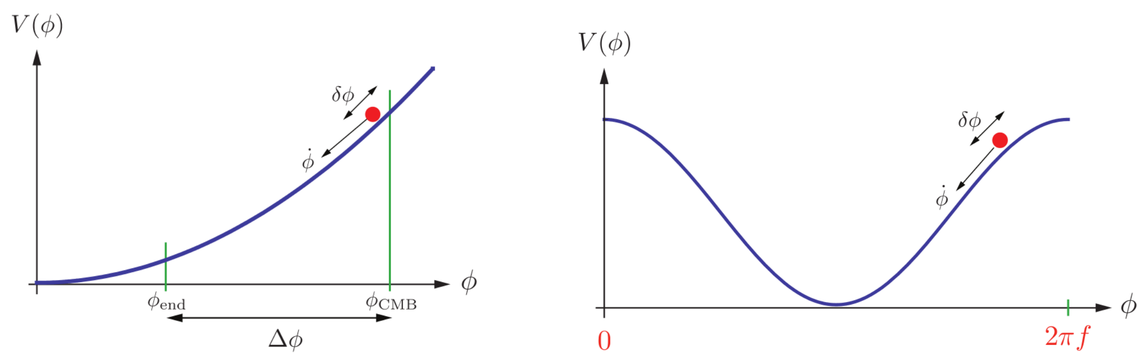


Figure 5.4: Potentials for large-field inflation from [74]. The figure on the left displays a monomial potential corresponding to *chaotic inflation*. The figure on the right shows a potential for *natural inflation* depending on the perodicity $2\pi f$.

Large-field inflation: $\Delta\phi > M_{pl}$ super-Planckian field evolution

Two prototypical examples of large-field inflation are depicted in figure 5.4. The potential on the left corresponds to *chaotic inflation* and is formally described by a simple monomial ansatz ($p > 0$)

$$V(\phi) \;=\; \mu^{4-p}\phi^p. \tag{5.1.15}$$

Note that the original idea of chaotic inflation proposed by Linde [85] was actually more general than our monomial potential.

We will encounter this model of inflation again in section 5.3, so let us in addition state the slow-roll parameters (5.1.11) for this monomial inflaton potential

$$\epsilon \;=\; \frac{1}{2}\left(\frac{p}{\phi}\right)^2 \qquad \text{and} \qquad \eta \;=\; \frac{p(p-1)}{\phi^2}. \tag{5.1.16}$$

Obviously ϵ and η do not depend on the scale μ. The number of e-foldings starting from the CMB creation is roughly given by $N_e \approx \frac{\phi^2_{\mathrm{CMB}}}{2p}$, cf. equation (5.1.13), which is often used to rewrite ϵ and η.

The potential on the right of figure 5.4 is called *natural inflation* and described by the following potential

$$V(\phi) \;=\; V_0\left[1 + \cos\left(\frac{\phi}{f}\right)\right]. \tag{5.1.17}$$

Natural inflation is especially attractive if an axion is driving inflation. In this case the parameter f represents the so-called *axion decay constant* which we will explain later on[2]. However, our scenarios in section 5.3 will not belong to this elegant class of inflationary

[2]Notice that natural inflation boils down to chaotic inflation if the decay constant fulfills $f \gg M_{pl}$.

models.

For later usage we briefly have to introduce another specific model of inflation which blends into the group of large-field models:
Starobinsky inflation: In the 1980s Starobinsky considered one-loop corrections to the Einstein-Hilbert action and focused merely on the additional R^2-term [86]:

$$\mathcal{S}_{\text{Starobinsky}} = \frac{1}{2}\int d^4x\ \sqrt{-g}\ \left[\mathcal{R} + \frac{\alpha}{2}\mathcal{R}^2\right] \tag{5.1.18}$$

where α is a normalization constant that is found to be $\alpha = 2.2 \times 10^8$. After performing a conformal transformation and defining a scalar field $\phi := \sqrt{\frac{2}{3}}\ln(1+\alpha\mathcal{R})$, we arrive at an action of the familiar form (5.1.7) with a minimally coupled scalar field ϕ and the following inflaton potential (cf. [74])

$$V(\phi) \ = \ \frac{1}{4\pi}\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\phi\right)\right]^2 . \tag{5.1.19}$$

This potential leads us immediately to the slow-roll parameters[3] (5.1.11):

$$\eta \ = \ -\frac{4}{3}\exp\left(-\sqrt{\frac{2}{3}}\phi\right) \qquad \text{and} \qquad \epsilon \ = \ \frac{3}{4}\eta^2 . \tag{5.1.20}$$

5.1.4 Quantum Fluctuations

The next step is to consider quantum mechanical fluctuations of the light scalar field driving inflation. Due to the extremely fast expansion during inflation, the wavelengths of these fluctuations were stretched to lengths larger than the Hubble horizon. The amplitude 'freezes' at this super-horizon stage since adiabatic fluctuations do not evolve outside the horizon. After inflation has ended, the frozen scalar field fluctuations, which are now large, will eventually re-enter the horizon and cause variations in the gravitational potential. These curvature perturbations $\mathcal{R}$ are responsible for density fluctuations in the matter distribution right after inflation and culminate in the large-scale structure formation of our observable universe.

Further information and an explicit computation of quantum fluctuations during inflation is most likely part of any text book on inflation. We recommend the books [74,87] and the TASI lecture notes [73]. We restrict ourselves to a brief summary of the most relevant results.

A crucial statistical measure of primordial fluctuations is the *power spectrum* $P_{\mathcal{R}}$ of the curvature perturbations $\mathcal{R}$:

$$\langle\mathcal{R}_{\mathbf{k}}\ \mathcal{R}_{\mathbf{k}'}\rangle = (2\pi)^3\ \delta(\mathbf{k}+\mathbf{k}')\ P_{\mathcal{R}}(k). \tag{5.1.21}$$

[3] Also in Starobinsky inflation one usually rewrites ϵ and η in terms of the number of e-foldings N_e.

We also define a *dimensionless scalar power spectrum* Δ_s^2 which is given by[4]

$$\Delta_s^2 \equiv \Delta_{\mathcal{R}}^2 \equiv \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) = \frac{H^2}{8\pi^2 \epsilon M_{\mathrm{Pl}}^2} , \tag{5.1.22}$$

where the right-hand side is evaluated at the horizon crossing $k = aH$. Scale dependence of the power spectrum is quantified by the scalar *spectral index*

$$n_s - 1 = \frac{d \ln \Delta_s^2}{d \ln k} . \tag{5.1.23}$$

Note that all statistical information is entirely determined by $\mathcal{R}$ only if $\mathcal{R}$ is Gaussian. Higher-order correlation functions, in particular the 3-point function $\langle \mathcal{R}_{\mathbf{k}} \, \mathcal{R}_{\mathbf{k}'} \, \mathcal{R}_{\mathbf{k}''} \rangle$, are encoded by so-called *non-Gaussianity* parameters and have to obey strong experimental limits by [5]. It turns out that non-Gaussianity is small for single-field slow-roll inflation, but crucial in the case of multi-field inflation.

Another elementary consequence of inflation is primordial gravitational waves. They correspond to tensor perturbations whose *dimensionless tensor power spectrum* reads

$$\Delta_t^2 \equiv \frac{k^3}{2\pi^2} P_t(k) = \frac{2H^2}{\pi^2 M_{\mathrm{Pl}}^2} . \tag{5.1.24}$$

One might now define a tensor spectral index n_t for the scale dependence of tensor modes analogously to n_s. Let us finally introduce an essential parameter highly constrained by observational data, the *tensor-to-scalar ratio*

$$r = \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon . \tag{5.1.25}$$

We also want to mention the *energy scale of inflation* $E_{\mathrm{inf}} = V_{\mathrm{inf}}^{1/4} = (3H_{\mathrm{inf}}^2 M_{\mathrm{Pl}}^2)^{1/4}$. One could now express the slow-roll parameters ϵ, η in terms of e-foldings N_e and calculate the spectral index n_s and the tensor-to-scalar ratio r, cf. the literature mentioned above. Table 5.1 summarizes $n_s = 1 + 2\eta - 6\epsilon$, r for polynomial and Starobinsky-like inflation as well as current experimental data.

5.2 Axion Monodromy Inflation

In contrast to most particle physics applications, the effective theory of inflation is highly sensitive to physics above the cutoff scale, i.e. ultraviolet (UV) corrections. As string theory is arguably the best theory to describe physical regimes approaching the Planck scale, it is natural to check whether inflation can be embedded in the context of string theory. We will now stress the need for a UV complete theory of inflation and afterwards show a prospective solution via axions from string theory generating a monodromy. We recommend [88] and once again [74].

[4] The definitions in this subsection actually contain in addition a 'speed of sound' parameter c_s that is approximately 1 in the slow-roll limit.

	n_s	r
Polynomial Inflation	$1 - \frac{p+2}{2N_e}$	$\frac{4p}{N_e}$
Starobinsky-like Inflation	$1 - \frac{2}{N_e}$	$\frac{8}{(\gamma N_e)^2}$
Experimental Data	0.9667 ± 0.004	< 0.113

Table 5.1: Spectral index n_s and tensor-to-scalar ratio r compared to experimental values.

5.2.1 Ultraviolet Sensitivity

UV corrections to the effective field theory of inflation contribute via two different effects. On the one hand, we have to renormalize the couplings of the inflaton field, and on the other hand, non-renormalizable higher-order operators must be addressed. Both issues lead to a so-called *eta problem* which is present in any model of slow-roll inflation.

Let us briefly consider radiative corrections to make the eta problem more precise. Start with a scalar field in a general effective field theory with cutoff scale Λ. The mass of the scalar field can run to the cutoff if there is no symmetry preventing this. Hence the inflaton mass receives a quantum correction $\Delta m^2 \sim \Lambda^2$. However, the cutoff for an effective field theory of inflation has to be greater than the Hubble scale, that is $\Lambda > H$, cf. the literature mentioned above for details. Recalling that $m^2 = V''$ and $3H^2 \approx V/M_{\rm Pl}^2$, the definition (5.1.11) of the slow-roll parameter η yields

$$\Delta\eta \;\approx\; \frac{\Delta m^2}{3H^2} \;\geq\; 1\ . \tag{5.2.1}$$

This is the well-known eta problem and contradicts the slow-roll condition! It turns out that a global symmetry, for instance of the form $\phi \to \phi + \text{const.}$, forbids a mass term of the inflaton field ϕ and is therefore able to avoid the eta problem. How such a shift symmetry arises from a consistent UV complete theory, however, is not obvious at a first glance.

Moreover, as stated in the introduction, we are focusing on large-field inflation due to the Lyth bound. But models of large-field inflation are plagued with an additional issue regarding UV sensitivity. Integrating out fields of mass Λ (with couplings to the inflaton of order 1) will generically deform the inflaton potential on scales of order Λ in the effective action. Hence the inflaton potential develops sub-Planckian structures as demonstrated in figure 5.5. In fact, there is an infinite series of non-renormalizable higher-dimensional operators contributing to the inflaton potential. Remember that large-field inflation requires a smooth inflaton potential over super-Planckian distances by definition. The problem of super-Planckian displacements is a serious drawback of large-field inflation whose solution is a crucial task. Again, the idea to prevent the problematic contributions is to invoke a shift symmetry:

$$\phi \;\to\; \phi \;+\; \text{const.}\ . \tag{5.2.2}$$

Note that this continuous shift symmetry holds to all orders of perturbation theory, but is

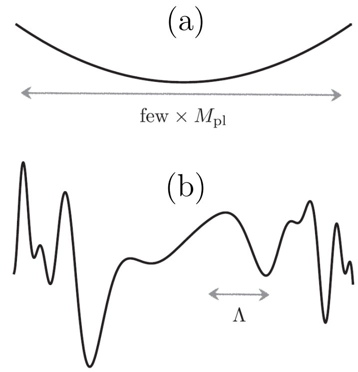


Figure 5.5: from [74]. (a) The potential of large-field inflation needs to be smooth over a super-Planckian range. (b) In the absence of symmetries, effective field theory predicts that generic potentials have structure on sub-Planckian scales $\Lambda < M_{\rm Pl}$.

non-perturbatively broken to a discrete shift symmetry[5] by instantons.

Remarkably, there are plenty of fields in string theory obeying such a shift symmetry. We call scalar fields equipped with the shift symmetry (5.2.2) *axions*[6]. In the next step, let us explain how axions appear in string theory.

5.2.2 Axions in String Theory

In general string compactifications axions arise from integrating p-form gauge potentials over p-cycles of the compact manifold. Considering type IIB superstring theory, for instance integration of the NS-NS 2-form B_2 over a 2-cycle Σ_2 gives rise to an axion b:

$$b \;=\; \int_{\Sigma_2} B_2 \;. \tag{5.2.3}$$

In addition we get axions associated to the R-R 2-form C_2, 4-form C_4 as well as the universal axions C_0 and 2 more from dualizing B_2, C_2. After orientifold projection we are left with the axion collected in table 2.2.

The reason for the continuous shift symmetry (5.2.2) of axions is the p-form gauge invariance of the 10d theory. More precisely, the 10d supergravity action (3.1.1) (with vanishing fluxes) does not depend on B_2, C_2, C_4, but instead on the corresponding field strengths. The continuous shift symmetry holds to all orders of perturbation theory.

Non-perturbative worldsheet instantons and/or D-branes break this continuous symmetry to a discrete shift symmetry $\phi \to \phi + (2\pi)^2$ [89].

The Lagrangian of an axion a is then determined by the discrete shift symmetry:

$$\mathcal{L}(a) \;=\; -\frac{1}{2} f^2 (\partial a)^2 - \Lambda^4 \left[1 - \cos\left(\frac{a}{2\pi}\right)\right] + \dots \;, \tag{5.2.4}$$

[5] A discrete shift symmetry allows for desirable periodic terms in axionic inflaton potentials.

[6] Some authors reserve the name 'axion' for the original QCD axion and speak of 'axion-like' fields otherwise. Note that we use 'axion' for all fields satisfying the properties above.

where Λ is a dynamically generated scale. The parameter f is the *axion decay constant*, which we have already introduced above. Canonically normalizing the field $\phi \equiv fa$ leads to the axion periodicity $(2\pi)^2 f$.

One might have noticed that this Lagrangian matches the potential of natural inflation (5.1.17). Thus, for a consistent model of large-field inflation we must have a super-Planckian axion decay constant. However, controlled string compactifications require unavoidably sub-Planckian decay constants. Current research tries to circumvent this issue by using several axions, each with a sub-Planckian decay constant, to generate an effective super-Planckian decay constant. Such models are called *aligned inflation* [90] and *N-flation* [91]. We choose a different approach to axion inflation via monodromy, which we will summarize in the next subsection.

5.2.3 Monodromy Inflation

The most easy example to visualize *monodromy* is the spiral staircase, where we reach a higher level after each circuit. Analogously, we speak of a monodromy when a system ends up in a new configuration after being transported around a closed loop in the (naive) configuration space [74]. This effect can be used in the context of axion inflation as first proposed by E. Silverstein and A. Westphal [8] and further developed in [9]. The basic idea is that the inflaton travels many cycles through a fundamental period, while the effective field evolution increases permanently. As a great advantage, the structure of the inflaton potential is preserved due to the protection of the axion shift symmetry during each individual cycle. In recent years, there was much progress in realizing an explicit scenario of *axion monodromy inflaton*, see for instance [10, 92–95]. Next, let us at least shed some light on the main concept of this kind of inflation.

Consider a 4d spacetime filling $D5$-brane in type IIB string theory that wraps a 2-cycle Σ_2 on the internal manifold. The wrapped $D5$-brane is described by the *Dirac-Born-Infeld action* [13] which leads to a 4d effective potential for the axion $b = \int_{\Sigma_2} B_2$ (see equation (5.2.3)) after integrating over the 2-cycle Σ_2

$$\mathcal{S}_{D5} \sim \int_{\mathcal{M}^4 \times \Sigma_2} d^6\sigma \, \sqrt{-\det(G_{ab} + B_{ab})} \sim \int_{\mathcal{M}^4} d^4x \sqrt{-g} \, \sqrt{(2\pi)^2 l_{\Sigma_2}^4 + b^2} \, , \tag{5.2.5}$$

where l_{Σ_2} is the size of the 2-cycle Σ_2 in string units. At first, note that the $D5$-brane has spoiled the discrete shift symmetry of the axion $b \to b + (2\pi)^2$. Furthermore, the potential energy is no longer a periodic function of the axion b, instead the axion exhibits a monodromy. Hence monodromy enables a super-Planckian field range of the axion as it circuits the fundamental domain over and over again.

For a large initial vev $b \gg l_{\Sigma_2}^2$, the potential boils down to a liner behaviour $V(b) \sim b$ and chaotic inflation can take place. The inflaton, i.e. the axion vev, decreases during inflation until the linear approximation of the potential fails. At this point, the inflaton begins to oscillate around the minimum of the potential, inflation ends and eventually reheating occurs. The important fact is that the axion shift symmetry does still protect the potential from UV corrections in each period and hence over super-Planckian displacements.

It remains to check whether moduli stabilization and compactification affects axion monodromy inflation. Constructing a concrete scenario of axion monodromy inflation is indeed rather involved and we refer to the literature (listed in the beginning of this subsection) for further information. In the scenarios investigated in this thesis axion monodromy inflation is realized via F-term scalar potentials induced by background fluxes following [96].

5.3 From Polynomial to Starobinsky-Like Inflation

In this final section we will check whether our flux-scaling scenarios of chapter 4 are able to realize axion monodromy inflation. Our initial motivation was to enable inflation in the context of moduli stabilization, hence this section represents the main result of our work. In order to make axion monodromy inflation possible, we have to handle constraints from string theory as well as inflation. From the string theory point of view, it is unavoidable to ensure that the moduli are lighter than Kaluza-Klein and string states. Our 4d gauged supergravity ansatz is only reliable if we can exclude those more massive states. Recall our discussion in section 4.6 and in particular the needful mass hierarchy (4.6.1). From the inflation point of view, it is necessary to attain an axionic modulus lighter than all other moduli such that we can indeed guarantee single-field inflation. The inflaton must also be below the energy scale of inflation $E_{\rm inf}$ and the Hubble constant during inflation $H_{\rm inf} \sim 9.14 \cdot 10^{13}$ GeV (for large field inflation). To summarize, a flux-scaling model is eminently suitable for realizing single-field F-term axion monodromy inflation in a controlled way if the following mass hierarchy can be substantiated

$$M_{\rm Pl} > M_{\rm s} > M_{\rm KK} > E_{\rm inf} \sim M_{\rm mod} > H_{\rm inf} > M_{\Theta}\,. \tag{5.3.1}$$

There are just four orders of magnitude between the Hubble-scale and the Planck-scale for all the other scales. It is clearly a major challenge for string theory to achieve and control such a sensitive hierarchy.

Let us now consider the flux vacua we investigated in chapter 4 of this thesis. We have already checked in section 4.6 which models satisfy the string theory part of the mass hierarchy. It was shown that model A is not able to separate the Kaluza-Klein scale from the moduli masses. Due to the simplicity of the superpotential of model A, there is no possibility to slightly extend this model without involving new moduli. Therefore we will not try to accomplish inflation in model A. Nevertheless, this was worked out in [75]. Instead we will analyze whether one can carry out axion monodromy inflation in model C of section 4.3. The advantage of model C is the fact that we have parametrical control over the string theory part of the mass hierarchy, see equation (4.6.11). Hence model C is a good candidate for realizing axion monodromy inflation and we will exclusively focus on this model during our final section of this thesis.

Let us next approach the inflation part of the mass hierarchy (5.3.1). So far the scalar potential of model C stabilizes just 5 out of the 6 real moduli and keeps one axionic direction Θ unfixed. As suggested in [10], the idea is to generate a non-trivial scalar potential for the axion Θ by turning on the additional flux $f_{\rm ax}$, while the former fluxes are scaled by a large number λ. Thus the total superpotential reads

$$W_{\rm inf} = \lambda W + f_{\rm ax}\, \Delta W\,. \tag{5.3.2}$$

If this leads to a parametrically lighter axion mass, the inflation part of the mass hierarchy (5.3.1) would be fulfilled and the axion Θ a strong candidate for the inflaton.

We want to remark that the deformation of the original superpotential W in the ansatz (5.3.2) is expected not to remove the minimum solution because first, the solutions from W were stable and second, we can regulate the deformation via the constant λ. But, one has to take care of the inflaton backreaction onto other moduli. Due to the fact that the inflaton is stabilized by the additional flux $f_{\rm ax}$, the parameter λ in the ansatz (5.3.2) allows to control the intensity of these backreaction effects.

Having added new fluxes to the superpotential and stabilized all moduli, while guaranteeing a proper mass hierarchy, the general procedure to realize inflation will be as follows:

1. Calculate the backreaction effect of the inflaton onto the other moduli.
2. Uplift the scalar potential to Minkowski space.
3. Evaluate the backreacted uplifted inflaton potential.

Unfortunately, it turned out that our computational capabilities are rather limited. In particular, it was not possible to take every backreaction into account. Aside of that, the expressions become much longer as soon as one includes additional fluxes according to ansatz (5.3.2). For this reason, we perform two different approaches to realize axion monodromy inflation for model C in the next two subsections:

- *Method 1:* We do not use the ansatz $W_{\rm inf}$ in equation (5.3.2), but instead take the normal superpotential W of model C. Hence one modulus remains unstabilized and we employ the axionic combination $(hc + q\rho)$ as inflaton. We consider backreaction effects only onto s, τ and keep u, v at the minimum.
- *Method 2:* In fact, here we turn on additional fluxes and implement ansatz (5.3.2). Thereby a parametrically small mass term for the so far unfixed axion $(qc - h\rho)$ can be generated and we interpret this modulus as inflaton field. However, regarding the backreaction we take only τ into account.

Although method 2 is the more appropriate way concerning the physical idea, method 1 elucidates the technical procedure more pedagogically as it is analytically better accessible. In the end both methods culminate in the same result. Notice that the incomplete backreaction is still an open issue of our scenarios.

All computations in this section are closely following [75]. However we want to stress that their scenario has two major drawbacks in contrast to the work presented here. On the one hand, the Kaluza-Klein scale is parametrically of the same order as the moduli masses. On the other hand, no additional fluxes are included to stabilize the a priori unfixed axion. Thus their analysis corresponds to our method 1 for model A.

5.3.1 Method 1: Without Additional Fluxes

For simplification we do not turn on additional fluxes in this method, but begin with the usual superpotential W of model C given in equation (4.3.5). The axionic combination $\theta = hc + q\rho$ plays the part of the inflaton field because one can show that it is relatively light. As we still want to control the backreaction and the parametrical mass difference between inflaton and all other moduli, let us mimic the effect of the deformed superpotential (5.3.2) by introducing a flux parameter λ. In practice this means to scale all terms in the scalar potential except the inflaton term with λ. When considering the backreaction later on, it will turn out that it is merely possible to take s and τ into account. Hence we further simplify the scalar potential by plugging in the values for u and v at the stable non-supersymmetric minimum, see solution 2 of table 4.2. Eventually, the scalar potential for model C reads

$$\begin{aligned} V \;=&\; \frac{5^{\frac{1}{4}}}{2^{\frac{19}{2}}3^{\frac{1}{2}}}\frac{(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}{(\mathfrak{f}_0)^{\frac{1}{2}}}\,\frac{\theta^2}{s\tau^3} + \\ &+ \lambda^2 \cdot \left(\frac{31\cdot 3^{\frac{1}{2}}10^{\frac{3}{4}}}{320}\frac{(\mathfrak{f}_0)^{\frac{1}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{3}{2}}}{s\tau^3} - \tilde{\mathfrak{f}}^1\frac{hs+q\tau}{16s\tau^3} + \frac{3^{\frac{1}{2}}10^{\frac{1}{4}}}{96}\frac{(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}{(\mathfrak{f}_0)^{\frac{1}{2}}}\frac{h^2s^2-4hqs\tau-q^2\tau^2}{s\tau^3}\right) \end{aligned} \tag{5.3.3}$$

The reader should keep in mind that the axionic combination $(qc - h\rho)$ orthogonal to θ remains unstabilized. The minimum of θ was 0 according to table 4.2. To ensure τ, $s > 0$ at the minimum, we require the fluxes to satisfy $h, q < 0 < \mathfrak{f}_0, \tilde{\mathfrak{f}}^1$. Moreover, $\mathfrak{f}_0 \gg \tilde{\mathfrak{f}}^1, h, q \sim O(1)$ guarantees weak string coupling and large radius, so we can ignore higher-order corrections to the scalar potential.

Let us now consider the backreaction of a slowly rolling and sufficiently light axion θ, i.e. we take into account that during the rolling the moduli s and τ adjust adiabatically. Hence one has to solve the extremum conditions for a non-vanishing value of θ:

$$\begin{aligned} s_0(\theta) &= -\frac{1}{h}\sqrt{24\sqrt{\frac{2}{5}}\mathfrak{f}_0\tilde{\mathfrak{f}}^1 + \frac{1}{2}\left(\frac{\theta}{\lambda}\right)^2} \\ \tau_0(\theta) &= -\frac{3^{\frac{1}{2}}}{10^{\frac{1}{4}}}\frac{(\mathfrak{f}_0)^{\frac{1}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}{q} - \frac{\sqrt{48\sqrt{10}\mathfrak{f}_0\tilde{\mathfrak{f}}^1 + 5\left(\frac{\theta}{\lambda}\right)^2}}{10^{\frac{1}{2}}q}. \end{aligned} \tag{5.3.4}$$

One can easily see that for $\theta = 0$ or very large values of λ, this corresponds exactly to the standard minimum of models C in table 4.2. Note also that for large θ, the values of τ_0 and s_0 stay in the perturbative regime and we may safely neglect higher-order α'- and g_s-corrections to the scalar potential.

Our next task is to uplift the scalar potential to vanishing cosmological constant in the minimum because so far all flux vacua belong to AdS space. In the KKLT scenario [60] and LVS [61], such an uplift was achieved by adding anti-D3-branes providing a positive-definite contribution to the potential

$$V_{\mathrm{up}} = \frac{\epsilon}{\mathcal{V}^{\alpha}}\ , \tag{5.3.5}$$

where $\alpha = 2$ for a $\overline{\text{D3}}$-brane in the bulk and $\alpha = 4/3$ for a brane located in a warped throat. However, following [75] we restrict our model to a constant uplift for simplification. Let us also remark that an uplift term like (5.3.5) does not work in our model at a first glance. Although the necessary uplift term for our model was found in [18], its physical motivation is not clear yet.

A constant uplift stands for computing the value of the scalar potential at the minimum V_0 and adding its absolute value to the scalar potential (5.3.3). Assuming $\theta_0 = 0$ at the minimum and using equation (5.3.4), the constant uplift term is given by

$$V_{\text{up}} = -V_0 = \frac{h\lambda^2 q^3}{120\sqrt{3}\sqrt[4]{10}\sqrt{(\mathfrak{f}_0)^3\tilde{\mathfrak{f}}^1}} \,. \tag{5.3.6}$$

Putting everything together, the backreacted uplifted inflaton potential is obtained by plugging s_0, τ_0 from eqation (5.3.4) into the scalar potential (5.3.3) and adding the uplift term (5.3.6). This yields in units of $(M_{\text{Pl}}^2/4\pi)$

$$\begin{aligned} V_{\text{back}}(\theta) =& \frac{\lambda^2 h q^3}{72\ 10^{3/4}(\mathfrak{f}_0)^{\frac{3}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}\sqrt{48\sqrt{10}\mathfrak{f}_0\tilde{\mathfrak{f}}^1\lambda^2+5\theta^2}\left(\sqrt{3}\sqrt[4]{10}\lambda\sqrt{\mathfrak{f}_0\tilde{\mathfrak{f}}^1}+\sqrt{48\sqrt{10}\mathfrak{f}_0\tilde{\mathfrak{f}}^1\lambda^2+5\theta^2}\right)^3} \times \\ & \times \left[3\theta^2\lambda\left(3\ 10^{3/4}\sqrt{\mathfrak{f}_0\tilde{\mathfrak{f}}^1\left(48\sqrt{10}\mathfrak{f}_0\tilde{\mathfrak{f}}^1\lambda^2+5\theta^2\right)}+100\sqrt{3}\mathfrak{f}_0\tilde{\mathfrak{f}}^1\lambda\right)-\right. \\ & \left. -432\sqrt[4]{10}\mathfrak{f}_0\tilde{\mathfrak{f}}^1\lambda^3\left(4\sqrt{3}\sqrt[4]{10}\mathfrak{f}_0\tilde{\mathfrak{f}}^1\lambda-\sqrt{\mathfrak{f}_0\tilde{\mathfrak{f}}^1\left(48\sqrt{10}\mathfrak{f}_0\tilde{\mathfrak{f}}^1\lambda^2+5\theta^2\right)}\right)+5\sqrt{30}\theta^4\right]. \end{aligned} \tag{5.3.7}$$

Obviously the original quadratic behaviour has changed drastically. The expected flattening of the potential becomes evident in figure 5.6. For small values of θ the potential is still of quadratic form, while it looks hyperbolic as θ gets larger. In the intermediate regime there is a turning point, where the potential grows linearly.

Evaluation of the Backreacted Uplifted Inflaton potential

The varying behaviour of the inflaton potential in figure 5.6 suggests to analyze different regimes of θ/λ and it will turn out that one receives different types of inflation. More precisely, the parameter λ, which controls the backreaction, interpolates between quadratic and Starobinsky-like inflation. As a great advantage of method 1, we are able to show how this becomes apparent from the formal point of view.

- **Quadratic Inflation for large λ**

  Assuming $\theta/\lambda \ll \sqrt{\mathfrak{f}_0\tilde{\mathfrak{f}}^1}$, the shift in the minimum (5.3.4) gets small such that one may neglect it. Therefore, the inflaton potential is approximately given by plugging the standard minimum of model C (solution 2 of table 4.2) into the potential (5.3.3)

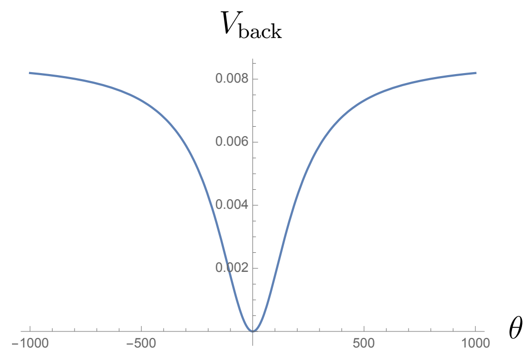


Figure 5.6: Backreacted uplifted inflaton potential $V_{\text{back}}(\theta)$ of equation (5.3.7) in units of $(M_{\text{Pl}}^2/4\pi)$ for the fluxes $h = -1$, $q = -1$, $\tilde{\mathfrak{f}}^1 = 1$, $\mathfrak{f}_0 = 10$ and $\lambda = 10$. Apparently, the initial quadratic behaviour changed.

and adding the uplift term (5.3.6):

$$V_{\text{back}}(\theta) \approx \frac{1}{144 \cdot 3^{\frac{1}{2}} \cdot 10^{\frac{7}{4}}} \frac{h\ q^3}{(\mathfrak{f}_0)^{\frac{5}{2}} (\tilde{\mathfrak{f}}^1)^{\frac{3}{2}}} \theta^2 \ . \tag{5.3.8}$$

Thus for large λ the inflaton potential is of the familiar quadratic form. In order to determine the physical inflaton field, we have to take canonical normalization into account. The Kähler metric can be read off from equation (4.3.4) and hence the total kinetic energy is given by

$$\mathcal{L}_{\text{kin}} = 3\left(\frac{\partial \tau}{2\tau}\right)^2 + \left(\frac{\partial s}{2s}\right)^2 + 3\left(\frac{\partial \rho}{2\tau}\right)^2 + \left(\frac{\partial c}{2s}\right)^2 \ . \tag{5.3.9}$$

To find the kinetic term for θ, we need to determine the orthogonal axionic combination σ. This can be fixed as

$$\partial\theta = h\partial c + q\partial\rho, \qquad \partial\sigma \sim -\frac{q}{s^2}\partial c + \frac{3h}{\tau^2}\partial\rho \ . \tag{5.3.10}$$

It is not necessary to normalize $\partial\sigma$ as one can show with the equations of motion that $\partial\sigma = 0$, so σ is not moving during the slow-roll of θ. The new basis (5.3.10) implies the following relations

$$\frac{\partial\rho}{\partial\theta} = \frac{q\tau^2}{3h^2s^2 + q^2\tau^2} \ , \qquad \frac{\partial c}{\partial\theta} = \frac{3hs^2}{3h^2s^2 + q^2\tau^2} \ , \tag{5.3.11}$$

so that the axionic terms in (5.3.9) add up to

$$\mathcal{L}_{\text{kin}}^{\text{ax},\theta} = \frac{3(\partial\theta)^2}{4(3h^2s^2 + q^2\tau^2)} \ , \tag{5.3.12}$$

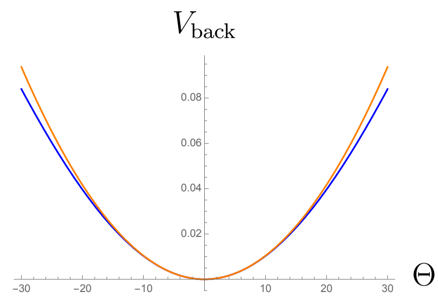


Figure 5.7: The exact inflaton potential $V_{\text{back}}(\Theta)$ is given by the blue curve and the quadratic approximation (5.3.8) for Θ by the orange curve (in units of $(M_{\text{Pl}}^2/4\pi)$) for the fluxes $h = -1$, $q = -1$, $\tilde{\mathfrak{f}}^1 = 1$, $\mathfrak{f}_0 = 10$ and $\lambda = 100$.

where we neglected the irrelevant part of $\partial\sigma$. The saxionic part of $\mathcal{L}_{\text{kin}}$ does not contribute because the backreacted moduli $s_0(\theta)$ and $\tau_0(\theta)$ are nearly constant in the large-field regime $\theta/\lambda \ll \sqrt{\mathfrak{f}_0\tilde{\mathfrak{f}}^1}$:

$$s_0 \approx -\frac{3^{\frac{1}{2}}2^{\frac{7}{4}}}{5^{\frac{1}{4}}}\frac{\sqrt{\mathfrak{f}_0\tilde{\mathfrak{f}}^1}}{h} \qquad \text{and} \qquad \tau_0 \approx -\frac{3^{\frac{1}{2}}5^{\frac{3}{4}}}{2^{\frac{1}{4}}}\frac{\sqrt{\mathfrak{f}_0\tilde{\mathfrak{f}}^1}}{q}\,. \tag{5.3.13}$$

Using these approximations, the kinetical term for θ can be calculated to be $\mathcal{L}^{\theta}_{\text{kin}} \approx \frac{1}{2}\frac{\sqrt{5}}{73\sqrt{2}\mathfrak{f}_0\tilde{\mathfrak{f}}^1}(\partial\theta)^2$. Consequently, the canonically normalized inflaton field takes the form

$$\Theta \approx \frac{5^{\frac{1}{4}}}{73^{\frac{1}{2}}2^{\frac{1}{4}}}\,\frac{\theta}{\sqrt{\mathfrak{f}_0\tilde{\mathfrak{f}}^1}}. \tag{5.3.14}$$

Notice that $\theta/\lambda \ll \sqrt{\mathfrak{f}_0\tilde{\mathfrak{f}}^1}$ implies $\Theta \ll \lambda$, i.e. our quadratic inflation is restricted to a small-field region. To sum up, the backreacted inflaton potential (5.3.7) is for sufficiently large values of λ well approximated by the quadratic potential (5.3.8) as shown in figure 5.7. This case is an example of the general group of chaotic inflation discussed in section 5.1.

We still have to check the mass hierarchy (5.3.1) in order to guarantee a reliable model of single-field inflation. Recall from section 4.6 that it was indeed possible to satisfy the stringy part of this hierarchy if there is no λ parameter. However, with almost the same flux choice (only $\mathfrak{f}_0$ is larger):

$$\mathfrak{f}_0 = 40000\,, \qquad \tilde{\mathfrak{f}}^1 = 1\,, \qquad h = -2\,, \qquad q = -1\,, \tag{5.3.15}$$

and setting $\lambda = 100$, we obtain a different mass hierarchy:

$$M_{\text{Pl}} > M_{\text{s}} > M_{\text{mod}} > M_{\text{KK}} > H_{\text{inf}} > M_{\Theta}\,. \tag{5.3.16}$$

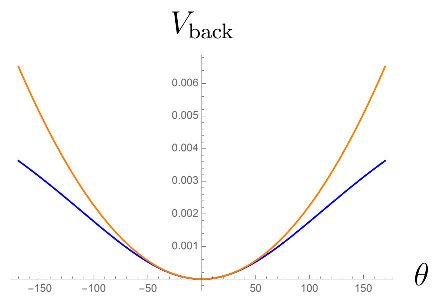


Figure 5.8: The figure shows the exact inflaton potential $V_{\text{back}}(\theta)$ in blue and the quadratic approximation (5.3.8) in orange (in units of $(M_{\text{Pl}}^2/4\pi)$) for the fluxes $h = -1$, $q = -1$, $\tilde{\mathfrak{f}}^1 = 1$, $\mathfrak{f}_0 = 10$ and $\lambda = 10$. Obviously, the exact solution separates from the quadratic approximation for smaller λ.

The good news is that we can easily ensure single-field inflation for large values of λ. Apparently, the disadvantages are the large masses of the moduli. In contrast to the situation without λ, the moduli are now heavier than the Kaluza-Klein scale and our supergravity ansatz is no longer trustworthy.

In order to estimate the agreement of the scenario with experimental data, let us compute the tensor-to-scalar ratio defined in (5.1.25) as well as table 5.1:

$$r = 16\epsilon \approx \frac{4p}{N_e} \approx 0.133 \,, \tag{5.3.17}$$

for quadratic inflation with a potential proportional to Θ^2 and $N_e = 60$ e-foldings. This value of r exceeds slightly the upper bound $r < 0.113$ by [4, 5].

- **Linear Inflation for intermediate λ**

  If we make λ smaller, the inflaton potential becomes flatter in the large-field region of θ. Figure 5.8 depicts the full inflaton potential (5.3.7) for $\lambda = 10$ in comparison with the quadratic approximation (5.3.8). The reason for this is that the backreaction effects of the inflaton onto the moduli s and τ increase.

  A more precise discussion would again require a canonical normalization of the inflaton in the intermediate regime $\theta/\lambda \approx 1$. Unfortunately, this exceeds our computational possibilities.

  Computing the mass hierarchy with the same flux choice (5.3.15), we obtain a similar relation as before. But now the moduli masses decrease and approach the Kaluza-Klein scale, i.e.

$$M_{\text{Pl}} > M_{\text{s}} > M_{\text{KK}} \backsimeq M_{\text{mod}} > H_{\text{inf}} > M_{\Theta} \,. \tag{5.3.18}$$

  The tensor-to-scalar ratio decreases, too, and one has $r \approx 0.067$ for linear inflation. Note that this is inside the experimental predictions.

- **Starobinsky-like Inflation for small λ**

Finally, let us consider the large-field regime $\theta/\lambda \gg \sqrt{\mathfrak{f}_0 \tilde{\mathfrak{f}}^1}$. In this case the backreaction onto s, τ is quite strong and thus the minimum (5.3.4) can be approximated as follows

$$s_0(\theta) \approx -\frac{\theta}{\sqrt{2}h\lambda}, \qquad \tau_0(\theta) \approx -\frac{\theta}{\sqrt{2}q\lambda} \ . \tag{5.3.19}$$

Applying these approximations to the potential (5.3.3) including the uplift (5.3.6), the inflaton potential for small values of λ approaches

$$V_{\text{back}} \approx \frac{1}{2^{\frac{13}{4}} \cdot 3^{(\frac{3}{2})} \cdot 5^{\frac{5}{4}}} \frac{h\lambda^2 q^3}{(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}(\mathfrak{f}_0)^{\frac{3}{2}}} \left(1 - 15\sqrt{10}\frac{\lambda^4 \mathfrak{f}_0 \tilde{\mathfrak{f}}^1}{\theta^2}\right) \ . \tag{5.3.20}$$

Next one must determine the canonically normalized inflaton. Therefore, we proceed analogous to the case of quadratic inflation and calculate the θ part of the the kinetic Lagrangian $\mathcal{L}_{\text{kin}}$. The difference is that we are now in the large-field regime where we may approximate $s_0(\theta)$ and $\tau_0(\theta)$ according to (5.3.19). If we plug these values into $\mathcal{L}_{\text{kin}}^{\text{ax},\theta}$, we derive

$$\mathcal{L}_{\text{kin}}^{\text{ax},\theta} = \frac{3}{8}\lambda^2 \left(\frac{\partial\theta}{\theta}\right)^2 \ . \tag{5.3.21}$$

In the large-field regime there is additionally a contribution from the saxionic part of (5.3.9). Using (5.3.19) leads immediately to $\mathcal{L}_{\text{kin}}^{\text{sax},\theta} = (\partial\theta)^2/\theta^2$, such that the overall kinetic term for θ is given by

$$\mathcal{L}_{\text{kin}}^{\theta} = \left(1 + \frac{3}{8}\lambda^2\right) \left(\frac{\partial\theta}{\theta}\right)^2 = \frac{2}{\gamma^2}\left(\frac{\partial\theta}{\theta}\right)^2 \tag{5.3.22}$$

with $\gamma^2 = 16/(8+3\lambda^2)$. Notice that γ is independent of the fluxes, but depends only on λ. Now, one can canonically normalize the inflaton via

$$\theta = \sqrt{15\sqrt{10}\lambda^4 \mathfrak{f}_0 \tilde{\mathfrak{f}}^1} \ \exp\frac{\gamma}{2}\Theta \tag{5.3.23}$$

and we arrive at a Starobinsky-like potential:

$$V_{\text{back}}(\Theta) \approx \frac{\lambda^2}{2^{\frac{13}{4}} \cdot 3^{(\frac{3}{2})} \cdot 5^{\frac{5}{4}}} \frac{hq^3}{(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}(\mathfrak{f}_0)^{\frac{3}{2}}} \left(1 - e^{-\gamma\Theta}\right) \ . \tag{5.3.24}$$

Let us remark that this potential becomes applicable in the large-field region and leads to an exponential behaviour in contrast to the polynomial scenario, where the backreaction was much weaker. Figure 5.9 displays a clear Starobinsky-like shape for $\lambda = O(1)$.

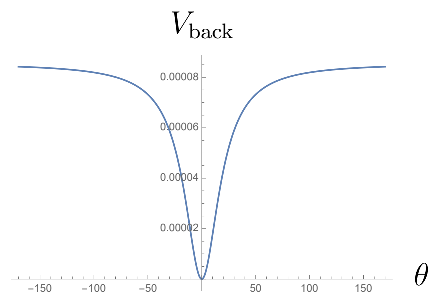


Figure 5.9: The exact inflaton potential $V_{\text{back}}(\theta)$ (in units of $(M_{\text{Pl}}^2/4\pi)$) for the fluxes $h = -1$, $q = -1$, $\tilde{\mathfrak{f}}^1 = 1$, $\mathfrak{f}_0 = 10$ and $\lambda = 1$ displays a Starobinsky-like shape.

Let us again consider the mass hierarchy of this scenario and compare it to the desirable relation (5.3.1). Employing the flux choice (5.3.15) and $\lambda = 1$ yields the following masses

$$M_{\text{s}} \approx 3.5 \cdot 10^{15}\text{GeV}, \qquad M_{\text{KK}} \approx 1.6 \cdot 10^{15}\text{GeV} \tag{5.3.25}$$

and the moduli are of order 10^{13}GeV. Assuming $N_e = 60$ e-foldings we find for the Hubble scale of inflation[7] $H_{\text{inf}} \approx 9.8 \cdot 10^{12}$GeV and the inflaton mass $M_\Theta \approx 2.2 \cdot 10^{12}$GeV. Altogether this scenario fulfills the mass hierarchy

$$M_{\text{Pl}} > M_{\text{s}} > M_{\text{KK}} > M_{\text{mod}} \gtrsim H_{\text{inf}} > M_\Theta\,. \tag{5.3.26}$$

The moduli masses are finally disconnected from the Kaluza-Klein scale as we have set λ to its minimal value 1. At a first glance our model seems to realize single-field axion monodromy inflation in a controlled way. However, if we compute the energy scale of inflation, cf. section 5.1.4, it turns out that $E_{\text{inf}} = V_{\text{inf}}^{1/4} \approx 6.4 \cdot 10^{15}$GeV. Unfortunately this value is even larger than the Kaluza-Klein scale M_{KK} and consequently the effective theory for inflation might not be reliable anymore.

Note that the Hubble scale agrees almost with the moduli masses and for a different flux choice we often achieve $H_{\text{inf}} > M_{\text{mod}}$. For this reason single-field inflation is actually no longer appropriate and we have to face a multi-field scenario instead.

Calculating the tensor-to-scalar ratio according to table 5.1, $\gamma^2 = 16/11$ and $N_e = 60$ e-foldings lead to $r = 0.0015$, which is well below the bound by the Planck results [4, 5].

5.3.2 Method 2: With Additional Fluxes

In the end of this thesis, let us consider the fully fledged approach to realize axion monodromy inflation in the framework of our flux-scaling model C. We will at first turn on

[7] H_{inf} can be obtained from the scalar power spectrum (5.1.22) for which we take the experimental value $2.142 \cdot 10^{-9}$.

additional fluxes in order to stabilize the so far unfixed axionic combination $\theta = (qc - h\rho)$ in accordance with ansatz 5.3.2. Notice the different definition of θ in contrast to method 1. The goal is to stabilize θ in such a way, that a parametrical mass hierarchy between θ and all other moduli is guaranteed. Then, θ is a suitable candidate for the inflaton and one can evaluate the backreacted uplifted potential.

The term we have to add to the superpotential stabilizing the unfixed axion θ should include c and τ moduli. A first guess might be to turn on an additional NS-NS H- and non-geometric Q-flux by adding the term $+i(h_0 S + q_0 T)$ to the superpotential (4.3.5). However, it can be shown that the backreaction on the old minimum is substantial. As a consequence, the effect of such a ΔW is not under parametric control and therefore not a good candidate for a deformation. We refer to [18] for a detailed investigation.

Instead we want to work with a different deformation of the superpotential originating from non-geometric P-flux:

$$W_{\rm inf} = \lambda W + \Delta W = \lambda\Big(- \mathfrak{f}_0 - 3\tilde{\mathfrak{f}}^1 U^2 - h_1 U\, S - q_1 U\, T\Big) - p_0\, S\, T\,. \tag{5.3.27}$$

Next we have to calculate the mass scales and check whether we are able to achieve the desired hierarchy (5.3.1). This is very involved for strong backreaction! Therefore, assume that the backreaction of ΔW on the values of the moduli in the old minimum is negligible and that the mass of the inflaton θ can be parametrically smaller than the masses of all the other moduli. Then we can analyze the problem by first integrating out all heavy moduli and computing an effective potential for the inflaton. We use $\theta \sim c$ to stay in accordance with the analysis in [18]. The enlarged superpotential (5.3.27) leads to a effective scalar potential of the form

$$V_{\rm eff} = \frac{1}{2^7}\Big(A\, c^4 + B\, c^2 + C\Big)\,, \tag{5.3.28}$$

with

$$A = \frac{2^{\frac{7}{4}}}{5^{\frac{5}{4}}\cdot 3^{\frac{1}{2}}}\,\frac{p_0^2 h_1^3 q_1}{\mathfrak{f}_0^{\frac{7}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}\,,\qquad B = \frac{2^{\frac{3}{4}}}{5^{\frac{9}{4}}\cdot 3^{\frac{1}{2}}}\,\frac{p_0 h_1 q_1(20\lambda h_1 q_1 + 73\sqrt{10}p_0\tilde{\mathfrak{f}}^1)}{\mathfrak{f}_0^{\frac{5}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}\,, \tag{5.3.29}$$

and

$$C = \frac{2^{\frac{15}{4}}}{5^{\frac{5}{4}}\cdot 3^{\frac{3}{2}}}\,\frac{q_1(-h_1^2 q_1^2\lambda^2 + 21 p_0 h_1 q_1\tilde{\mathfrak{f}}^1\lambda + 90 p_0^2(\tilde{\mathfrak{f}}^1)^2)}{h_1\mathfrak{f}_0^{\frac{3}{2}}(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}\,. \tag{5.3.30}$$

For all fluxes and λ being positive, the effective potential has a global minimum at $c = 0$. In this minimum the mass of the canonically normalized inflaton is computed as

$$M_\Theta^2 = \mu_\Theta\,\frac{p_0 q_1(\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}(20 h_1 q_1\lambda + 73\sqrt{10}p_0\tilde{\mathfrak{f}}^1)}{h_1\mathfrak{f}_0^{\frac{3}{2}}}\;\frac{M_{\rm Pl}^2}{4\pi\cdot 2^7}\,, \tag{5.3.31}$$

with $\mu_\Theta = 1.6$. Therefore, in the regime $20 h_1 q_1 \lambda \gg 73\sqrt{10} p_0 \tilde{\mathfrak{f}}^1$ the mass of the inflaton can be made parametrically smaller than the mass of the heavy moduli

$$\frac{M_\Theta^2}{M_{\text{mod}}^2} \sim \frac{p_0 \tilde{\mathfrak{f}}^1}{h_1 q_1 \lambda} \,. \tag{5.3.32}$$

The ratio of the Kaluza-Klein scale and the moduli masses behaves as

$$\frac{M_{\text{mod}}^2}{M_{\text{KK}}^2} \sim \frac{\lambda^2 h_1 q_1 (\tilde{\mathfrak{f}}^1)^{\frac{1}{2}}}{\mathfrak{f}_0^{\frac{1}{2}}} \,. \tag{5.3.33}$$

For large λ, it becomes impossible to keep the Kaluza -Klein scale larger than the heavy moduli mass, while still having a string scale of the order of the GUT scale. We can summarize these findings for realizing inflation by

$$M_{\text{mod}} \underset{p}{\gtrsim} M_\Theta \quad \Longrightarrow \quad M_{\text{mod}} \underset{p}{\gtrsim} M_{\text{KK}} \,. \tag{5.3.34}$$

Let us once again emphasis that we started the mass ratio discussion with the assumption of small backreaction effects.

Evaluation of the Backreacted Uplifted Inflaton Potential

Analogous to method 1, we now want to explicitly compute the backreacted uplifted inflaton potential. However, the terms are far too long to present them in this thesis, although they can in fact be solved analytically. Hence, we will only state the final results of our calculation. The proceeding is entirely identical to the one of method 1.

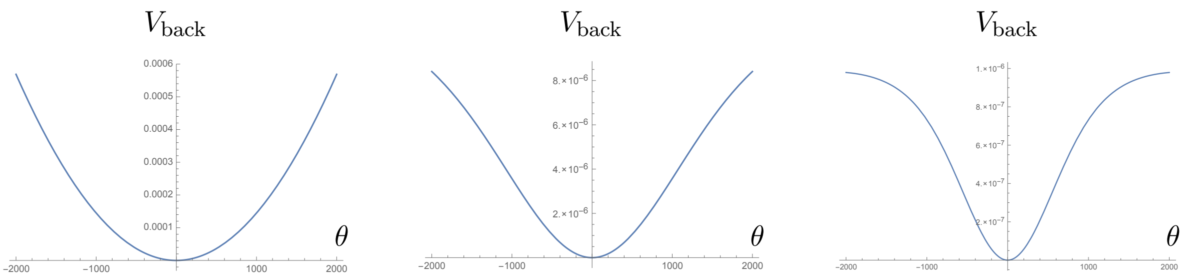
The major obstacle of our approach is the fact that we have been able take only one single modulus into account for backreaction effects. More precisely, we consider simply the backreaction of the inflaton θ onto the real Kähler modulus τ and set the other moduli $u, v, s, (hc + q\rho)$ to their values at the old minimum, see solution 2 of table 4.2.

After solving the extremum condition for a non-vanishing θ to find $\tau_0(\theta)$, one can perform a constant uplift by adding the term $V_{\text{up}} = -V(\theta = 0)$. Recall from the scalar potential (5.3.28) that $\theta = 0$ is a global minimum. We end up with the backreacted uplifted inflaton potential

$$V_{\text{back}}(\theta) = V(\theta)|_{\tau=\tau_0(\theta)} + V_{\text{up}} \,, \tag{5.3.35}$$

which is plotted in figure 5.10 for different values of λ.

Figure 5.10 shows at least graphically the same result as method 1. For large λ the inflaton potential looks quadratical, while it changes to Starobinsky-like shape for small λ. The intermediate regime of λ interpolates between these two types of inflation. Taking the mass hierarchy into account, the mass ratios (5.3.32) and (5.3.33) imply that polynomial inflation (i.e. large λ) guarantees single-field inflation, but makes a separation of the Kaluza-Klein and string states from the moduli questionable. On the other hand, the mass



$\lambda = 100$ $\lambda = 3$ $\lambda = 1$

Figure 5.10: The backreacted uplifted inflaton potential $V_{\mathrm{back}}(\theta)$ (in units of $(M_{\mathrm{Pl}}^2/4\pi)$) for the fluxes $h = -8$, $q = -8$, $\tilde{\mathfrak{f}}^1 = 1$, $p_0 = 1$, $\mathfrak{f}_0 = 6000$ and various values of λ. We find again an interpolation between polynomial and Starobinsky-like inflation regulated by the strength of the backreaction.

ratios (although they do truly apply for strong backreaction) suggest that Starobinsky-like inflation (i.e. small λ) ensures a disentanglement of the Kaluza-Klein and string states with the moduli. However, in this case a multi-field inflation scenario becomes more likely and a concrete computation of the inflationary trajectory might be necessary. A quantitative analysis of the mass ratios turns out to be quite involved.

For Starobinsky-like inflation, i.e. $\lambda = 1$, we computed in addition a plot of the backreacted inflaton potential in figure 5.11. Inflation takes place on the Starobinsky-like plateau.

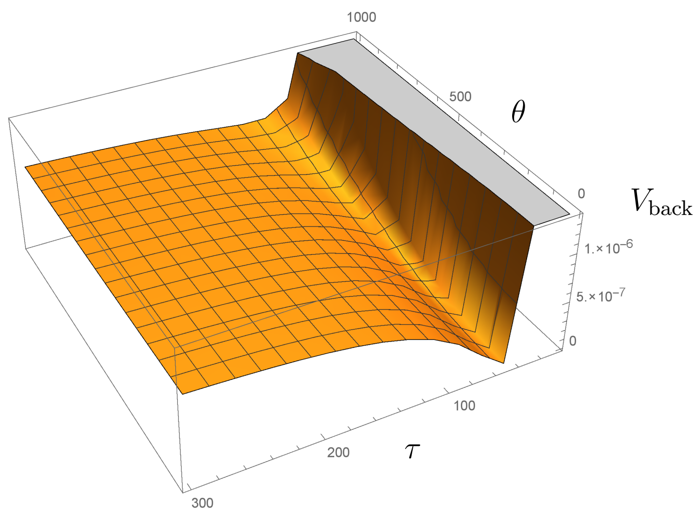


Figure 5.11: The exact inflaton potential $V_{\text{back}}(\theta)$ (in units of $(M_{\text{Pl}}^2/4\pi)$) with backreaction onto τ for the fluxes $h = -8$, $q = -8$, $\tilde{\mathfrak{f}}^1 = 1$, $p_0 = 1$, $\mathfrak{f}_0 = 6000$ and $\lambda = 1$. Inflation occurs on the Starobinsky-like plateau. When the inflaton moves further downwards the valley, its kinetic energy increases and eventually inflation ends.

Chapter 6

Summary and Outlook

In this thesis we studied certain scenarios of moduli stabilization in string theory. More concretely, after compactifying the 10d effective type IIB supergravity we ended up in a 4d orientifolded $\mathcal{N} = 2$ gauged supergravity framework. Non-trivial background fluxes including geometric and non-geometric fluxes generated a scalar potential for the moduli in 4d and subsequently various models of moduli stabilization have been analyzed.

As one of the special features of our models, all moduli have been fixed at tree level in contrast to the usual moduli stabilization scenarios KKLT and LARGE volume, which employ non-perturbative corrections. Let us emphasize that we stabilized Kähler moduli via non-geometric fluxes. This fact is the main reason to utilize the theoretically involved non-geometric fluxes. Another distinctive finding are the non-supersymmetric stable flux vacua that we discovered in most models. Such vacua allow to avoid the no-go theorem by [11] and thus to keep one axionic modulus unfixed, which we then used for inflation. Moreover, every minimum of the scalar potential fulfilled a novel scaling with the fluxes, enabling us to parametrically control many properties of the vacuum. Therefore it is straightforward to make the string coupling small and the internal volume large, which guarantees to be inside the perturbative regime and thus no string loop or α'-corrections are necessary. In addition we have been able to place all the moduli at one suitable scale since their generated masses turned out to be parametrically the same. Due to the fact that the gravitino mass (indicating the supersymmetry breaking scale) has the same flux dependence as the moduli and the orders of the numerical prefactors agree approximately, the supersymmetry breaking scale is really at a high-scale. Let us also stress that new tachyons appeared as soon as we employed multiple complex structure or Kähler moduli. Although this issue is not totally solved jet, a nice uplift mechanism for the case of two Kähler moduli is illustrated in [18].

In the final chapter we applied our moduli stabilization scenarios to inflation, more precisely to F-term axion monodromy inflation. The initial idea was to stabilize the so far unfixed axion of the non-supersymmetric stable flux vacua of model C in such a way that it is parametrically lighter than all other moduli. This is indeed possible, but technically rather involved. Instead we presented the analytical details of a similar method, where we used the lightest axion of the already stabilized moduli as inflaton. The major challenge was to realize a correct mass hierarchy that guarantees on the one hand a controlled string compactification and on the other hand single-field inflation. If we want the moduli masses strictly separated from the Kaluza-Klein and string scale, we have to face a scenario of multi-field inflation. However making the moduli more massive in order to ensure single-field inflation, they become usually even heavier than the Kaluza-Klein states. Increasing the moduli masses depends on a parameter regulating the backreaction of the inflaton onto the other moduli. This parameter turns out to interpolate between chaotic and Starobinsky-like inflation which is probably the most important result of our work. To summarize, weak backreaction leads to polynomial inflaton potentials, while strong backreaction causes the exponential Starobinsky-like behaviour. Note also that since the inflaton in our model is a linear combination of the universal axion and a Kähler axion, we can realize the stringy reheating mechanism proposed in [93].

It would be interesting to investigate the consequences of a wrong mass hierarchy for inflation. For instance, one could consider a multi-field inflation scenario and check the actual trajectory of the inflaton. This includes a computation of so-called non-Gaussianity parameters corresponding to higher-order CMB correlations, which are subject to strong experimental bounds. Alternatively, one could add some Kaluza-Klein or string states to the discussion from the very beginning. Another open issue of our work is given by the AdS flux vacua. Eventually they must be uplifted to dS space to be in agreement with cosmological observations. Perhaps, taking more fluxes into account might directly yield dS minima in the context of non-geometric flux models. Furthermore, we did not address the question whether the 4d effective theory with non-geometric fluxes can actually be uplifted to true solutions of the 10d string equation of motion. We refer to [97] for more information regarding this crucial problem.

To conclude, in the future we hope to get a better understanding of moduli stabilization with non-geometric backgrounds and of realizing inflation in string theory. Further progress in this field could enhance the chances of string theory to be a promising candidate towards a "Theory of Everything".

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Erklärung

Hiermit erkläre ich die vorliegende Masterarbeit selbstständig verfasst und nur die angegebenen Hilfsmittel verwendet habe.
Die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sind ausnahmslos als solche gekennzeichnet.

München, den 13. Oktober 2015

Florian Wolf