Swampland Conjectures for $N = 1$ Orientifolds

MASTER THESIS

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ABSTRACT

Despite the incalculable effort over the last half century, there is no available theory describing the quantum gravity regime at present, although advances have been made in the field. From the point of view of String Theory, the greatest hope lies in the Swampland Program, initiated in the first decade of this century, and whose proceeding lies in the rigorous establishment of conjectures dealing with those features that have to verify the Quantum Field Theories (QFTs) for being compatible with gravity in the UV. Recently, extraordinary connections have been discovered between several of the conjectures with major transcendence, which point to a possible microscopic description of the observed infrarred (IR) dynamics as fundamentally emerging from the ultraviolet (UV) renormalization group (RG) flow.

Studies with an acceptable degree of precision have been carried out in relation to the two conjectures par excellence: Weak Gravity Conjecture and Swampland Distance Conjecture. Among them, the one carried out by [5] stands out, where both conjectures are tested in the framework of the complex structure moduli space of $N = 2$ Type IIB Superstring Theory compactified on Calabi-Yau threefolds. In such work the conjectures are verified and a duality between both is established as microscopically coming from the Emergence proposal.

Our aim goes further and we examine the same conjectures and proposal in a more exotic setting: complex structure moduli space of $N = 1$ Type IIB Superstring Theory compactified in Calabi-Yau threefold orientifolds. There are numerous motivations to carry out this study, among which two stand out: $N = 1$ supergravity is much more linked to any potentially phenomenological model than $N = 2$, and this scenario presents a series of very rare properties constituting a challenging test for the current paradigm of the Swampland.

Throughout our work, the reader will appreciate that our object of study is far from being simple and, therefore, we need to introduce a vast number of technical concepts and information. The first three sections are devoted to this, containing a concise, but thorough, theoretical introduction. It will be appreciated when we encounter numerous subtleties, disregarded in $N = 2$, which take on a primordial role now.

In the next couple of sections, we present our setting and proceed to carry out the appropriate calculations and analysis. There we notice that the bulk supergravity spectrum is determinant and modifies accordingly the $N = 2$ results for $N = 1$. All this is perfectly logical, with the exception of an apparent paradox, which we name “Puzzle", which puts the Emergence proposal at risk, deserving our remaining performance.

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Allow me to welcome you to this thesis with a simple motto, describing my utopian lifestyle, in the four languages that have accompanied me along this period.

Soñando en vida, vivindo en soños.

Soñando en vida, viviendo en sueños.

Dreaming in life, living in dreams.

Im Leben träumen, in Träumen leben.

Contents

1 Introduction

The present project is framed in the *String Theory*. It started as an unfruitful attempt to describe the strong nuclear force, by means of strings, and its current status is that of being the most promising theory of everything, considering extended objects as the most fundamental pieces building our universe. Fairly speaking, our four dimensional universe plus (compact) extra dimensions.

More concretely, our work fits in the vast field of *String Phenomenology*, whose current aim is to describe the understood (Standard Model of Particle Physics and Cosmology ...), the not understood (Neutrino Masses, Hierarchy Problem, Inflation, Dark Energy and Matter ...) and the unknown Physics surrounding us, using String Theory.

A further sharpening is required. From a Physics perspective, this Master thesis deals with a highly non-trivial test of a network of conjectures relating what we observe (Effective Field Theories) with what awaits us (Quantum Gravity), the so-called *“Swampland Program"*.

Thus, roughly speaking, one obtains the following picture:

Physics ⊇ String theory ⊃ String phenomenology ⊃ “Swampland Program" ⊃ Master thesis.

Nevertheless, overcoming our Physics bias, plenty of *Mathematics* are involved around these topics. More precisely, we are lead to *Calabi-Yau internal compact manifolds* in order to approach the observable four dimensional non-compact space-time where we live. Being punctilious, if we want to meet the proper phenomenology in this four dimensional universe, we need to go further. One possible alternative for that is to quotient the internal Calabi-Yau by rather restricted discrete transformations, which lead us to the *Orientifolds*. From the mathematical perspective one may say that the following nested sequence of sets is rather appropriate:

Mathematics ⊃ Calabi-Yau manifolds ⊃ Orientifolds ⊃ Master thesis.

Like everything in life, taste lies in variety, or, in this case, in the intersection. We can thus conclude that:

“Swampland program" ∩ Orientifolds ⊃ Master thesis.

The ⊃ instead of = will become transparent when the reader reaches the end of this text. We advise the reader to keep this approximate picture in mind with some suspicion and guarantee her or him that the accuracy will increase along the contents of the text.

The starting point will be to provide us with a concise *Review of Compactifications* in section 2. Specifically, in subsection 2.1 we will sketch the standard compactification method, as well as introduce the four dimensional resulting supergravity spectrum and focus on the relevant $N = 2$ moduli space geometry regarding Type II Calabi-Yau compactifications. Whereas the previous is a purely closed string subsection, we will continue by discussing Dp-branes from an space-time perspective, focusing on a crucial example and collecting later the necessary mass formulae, in subsection 2.2. To close this section, we will delve into Type II orientifolds in subsection 2.3.

The following topic to address cannot be other than a *Review of the Swampland* in section 3. A fleeting revision of the “Swampland program" short history, motivation and current state can be found in subsection 3.1. Subsections 3.2, 3.3 and 3.4 will be devoted respectively to the Weak Gravity conjecture, Swampland Distance conjecture and Emergence proposal, which constitute the set of conjectures we will focus on. Along the aforementioned subsections the variety of versions they present and the reasons motivating such statements are introduced. The "Swampland program" is actually larger, and so we complement this text with other conjectures, finding outstanding and useful connections, in subsection 3.5. Subsection 3.6 consists of a short recapitulation of the more impactful (surely biased) tests, reserving a prominent place for $N = 2$ Type IIB Calabi-Yau threefold complex structure moduli. That is because our main setting will consist of its orientifold truncation to $N = 1$, where many subtleties arise. The remaining subsections 3.7 and 3.8 contain specialized discussions regarding, respectively, two fundamental distance singularities in moduli space and stability. While the former is well established in the literature, the latter gives the impression of being a complicated topic, about which is difficult to find information. As a consequence, we tried to join there the little information we found about the different stability concepts which are relevant for this project and those parts which we did not find anywhere, we intend to motivate from an intuitive perspective.

Despite the fact that some original ideas where included in the previous sections, one can regard the section 4 as the beginning of our original research. It consists of an exposition of the *General Case* within our orientifold scope.

We will notice that it would be a herculean task to extract information from the most general situation, which leads us to tackle concrete *Examples* in section 5. From a String theory perspective it results more natural to approach toroidal compactifications and their orientifolds, as we briefly recall in subsection 5.1. Nevertheless, we will soon realize that the "rara avis" of twisted sector, which encodes those bizarre/puzzling aspects absent in the parent $N = 2$, cannot be geometrically treated with the existing studies. Consequently, we are forced to lower the level of demand and test simple supergravity abstract models in subsection 5.2. The question about if such models can actually happen in String theory is open, although we are going to motivate a possible answer in the next section.

Along the previous section an intricate situation arose, with respect to the fundamental origin of the global symmetry limit (if any) in the gauge coupling, when we approach infinite distance singularities in orientifold complex structure moduli space. We call the "Puzzle" to such a paradoxical picture. Therefore, results mandatory an *Analysis of the "Puzzle"* and that is the topic of the section 6. In our attempt to resolve it, we start with the most straightforward options which seem to fail one after another in subsections 6.1 and 6.2. This leads us, ultimately, to two possible solutions, being explored in subsection 6.3.

Finally, we conclude with the *Summary of Results and Outlook* in section 7. There, we collect our findings and motivated proposals, briefly summarize our setting, propose some future research lines related to our work, which deserve to be studied, and, especially, we emphasize the need for studies in depth concerning those complicated aspects that hide us the full knowledge of a setting equally difficult, interesting, useful, and we would even dare to say that determinant for the future of the "Swampland Program".

2 Review of Compactifications

In this section we briefly describe Type II compactifications, focusing on the main features that are relevant for us, as well as introducing some useful results we will use later.

2.1 Type IIB and IIA Calabi-Yau compactifications

Our framework is Type II Superstring Theory. Actually, there exist two Type II inequivalent theories depending on the GSO projection[1]: IIA (odd RR forms) and IIB (even RR forms). However, these theories live in ten dimensions and what we observe is four dimensional. Then to make the connection, the most promising way is to compactify 6 dimensions, obtaining an effective theory in 4d through which we could potentially contact low energy phenomenology. This can be done, a priori, in (at least) a large number of ways because depends mainly on the choice of compact manifold and the choice of vacua (AdS, dS or Minkowski depending on the selection of branes and fluxes). A technical treatment leads to Calabi-Yau manifolds when fluxes are turned off, when fluxes are included a warp factor [2] appears [18]. Regarding flux considerations, we will usually set them to zero because, as will be clarified later, they are not always necessary and we want to test the conjectures in full generality. Finally, permit us mention that Type II theories are related through T-duality (c-map) and mirror symmetry (see figure in page 8). Because we do not really need to include all the details, we will just introduce the necessary data. This and much more can be found anywhere in the literature. The main reference is [18] and the notation is nearer to [14] and [15] because it will be convenient when we study the orientifold version of these theories. A clear discussion including many details is also provided by [23].

2.1.1 Type II and compactification procedure

In the beginning of this section we mentioned that our theories are formulated in ten spacetime dimensions whereas in our daily experience we just observe three space dimensions added to the temporal one. As a consequence, the remaining six dimensions must be such that no current experiment can detect them. The most followed guideline to address this issue is to require the extra dimensions to be small and compact, such that cannot be detected below a certain energy scale (or equivalently above a certain length scale). This is equivalent to assume the Kaluza-Klein ansatz[3]:

$$\mathcal{M}^{1,9} = \mathcal{M}^{1,3} \times Y \ , \tag{2.1}$$

where $\mathcal{M}^{1,3}$ corresponds to some maximally symmetric space with four non-compact dimensions representing our 4D world (e.g. Minkowski) and Y is the compact internal manifold. The metric corresponding to (2.1) is of block diagonal form:

$$G_{MN}(x,y) = \begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & g_{mn}(y) \end{pmatrix}. \tag{2.2}$$

[1]The GSO projection is an ingredient used in constructing a consistent model in superstring theory, which typically eliminates the tachyonic ground state of the string and preserves spacetime supersymmetry. It consists of a selection of a subset of possible vertex operators in the worldsheet CFT, concretely those with specific worldsheet fermion number and periodicity conditions. Such a projection is necessary to obtain a consistent worldsheet CFT and can be matched directly to spacetime supersymmetry and modular invariance for the partition function of the theory.

[2]Negligible in the large volume/large complex structure regime but on the conifold creates a warped throat [22].

[3]In the absence of fluxes. Otherwise we would encounter a warped product.

Before analyzing the consequences of imposing (2.1) and (2.2), we need to discuss the field content of IIA/IIB supergravity.

The Type IIA/B massless spectrum in ten spacetime dimensions can be determined by using representation theory over the massless (closed string) excitations, coming from the (consistent) quantization of the superstring in the worldsheet with the appropriate GSO projection [18]. To obtain the closed string spectrum, we take the tensor product of two open string spectra, one for the left- and one for the right-movers, obeying physical state constraint. We have to distinguish between four sectors: (NS,NS) and (R,R) leading to space time bosons and (NS,R) and (R,NS) to space time fermions [4]. The resulting space-time (bosonic [5]) spectra correspond to the so-called Type IIA/B SUGRA in 10D:

$$\mathscr{G}_{IIA}(10) = \{G_{MN}, B_{MN}, (C_3)_{MNP}, (C_1)_M, \Phi\}, \tag{2.3}$$

$$\mathscr{G}_{IIB}(10) = \left\{G_{MN}, B_{MN}, (C_2)_{MN}, (C_4^\dagger)_M, C_0, \Phi\right\}, \tag{2.4}$$

where the metric G_{MN}, the antisymmetric tensor B_{MN} and the dilaton Φ appear in the NS-NS sector, while the C_0, C_1, C_2, C_3 and $C_4^\dagger$ (self-dual[6]) forms are in the RR sector. We see that both share the same NS-NS sector but differ in the RR sector (that's precisely the role of the GSO projection). Remarkably the Type IIA is a non-chiral $N=2$ theory ($2^{D/2}=32$ supercharges), while the Type IIB is chiral (both gravitini have the same chirality).

Now, we can continue with the KK reduction. The easiest part corresponds to the p-forms, whose equation of motion is $d\star dC_p = 0$, which together with the gauge condition $d\star C_p = 0$ permits us to write it as:

$$(dd^\dagger + d^\dagger d)C_p = \triangle^{(10)}C_p = 0\,. \tag{2.5}$$

To obtain a four-dimensional effective theory, the p-form is expanded as a product $C_p(x,y) = C^a(x)\phi_a(y)$. Moreover, by means of the ansatz (2.2), the ten-dimensional Laplacian decomposes into a sum of the 4-dimensional and the 6-dimensional Laplacian. As a consequence, schematically:

$$\triangle^{(10)}C_p(x,y) = (\triangle^{(4)}C^a(x))\phi_a(y) + (\triangle^{(6)}\phi_a(y))C^a(x) = 0\,, \tag{2.6}$$

where the second term often can be regarded as a mass term such that we have an (usually infinite) tower of massive modes and the massless modes in 4 dimensions ($\triangle^{(4)}C^a(x)=0$) require $\triangle^{(6)}\phi_a(y)=0$, so the expansion is in terms of harmonic forms on the internal manifold. The latter constitute the effective spectrum we can measure [7] and correspond to the supergravities in 4D we are going to discuss in the following subsection.

[4]NS (R) comes from Neveu-Schwarz (Ramond) antiperiodic (periodic) boundary conditions for the worldsheet fermions. Recall that, contrary to 4D space time fermions, the 2D CFT fermions can be also periodic because what really distinguish them from the bosons is their central charge $c=0.5$ (leading to a fermionic-like two point function). It can be shown that the string states in the NS (R) sector are space-time (bosons) fermions.

[5]We will always focus on the bosonic spectrum, taking into account that their fermionic superpartners fill also the multiplets.

[6]Condition required in order to have the same number of fermionic and bosonic degrees of freedom and so to match supersymmetry.

[7]This is called KK scale and obviously is lower than the string scale aforementioned.

The above compactification procedure is also valid for the NS-NS fields except for the metric, which is not a form but a symmetric 2-tensor. It will be briefly sketched soon.

Until know, we do not know what to choose as the compactification manifold. However, from a phenomenological perspective, we need to reduce the amount of supercharges in order to preserve the lowest possible supersymmetry. It turns out that the condition for unbroken SUSY is that the ten dimensional Majorana-Weyl (MW) spinor must be covariantly constant with respect to the background metric. If the spinors are covariantly constant on Y, they are left invariant under parallel transport around any closed loop (i.e. they transform as a singlet under the holonomy group $\mathrm{Hol}(Y) \subset SO(6) \cong SU(4)$). Taking $\mathrm{Hol}(Y) = SU(3)$, one can show that we arrive at one covariantly constant spinor in 4-dimensions for each ten dimensional MW spinor. As a consequence, we get 8 supercharges, which is equivalent to $N = 2$ SUSY in four dimensions.

From the existence of covariantly constant spinors and the requirement of the non-compact spacetime to be maximally symmetric, we arrive at the Ricci-flat condition. Again by using the covariantly constant condition for the spinor, we can construct an almost complex structure with vanishing Nijenhuis tensor field (so Y is complex manifold), and taking into account that the metric is also covariantly constant, one can see that the Kähler form is closed. Then, Y is a compact Kähler n-fold with $\mathrm{Hol}(Y) \subset SU(n)$ (equivalently Y Ricci-flat and also equivalently its first Chern class vanishes), that is a *Calabi-Yau*. Concretely, in our case, a Calabi-Yau 3-fold. For these, the Hodge diamond (necessary to determine the number of massless modes though the different classes of harmonic forms given by the Hodge numbers) is (fortunately) restricted to the following structure:

$$\begin{array}{ccccccc} & & & h^{0,0} & & & \\ & & h^{1,0} & & h^{0,1} & & \\ & h^{2,0} & & h^{1,1} & & h^{0,2} & \\ h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\ & h^{3,1} & & h^{2,2} & & h^{1,3} & \\ & & h^{3,2} & & h^{2,3} & & \\ & & & h^{3,3} & & & \end{array} \quad = \quad \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{2,1} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \tag{2.7}$$

To sum up: In order to preserve a minimal amount of SUSY, the six dimensional compact space is required to be a *Calabi-Yau threefold*, which enjoys the above simple Hodge diamond.

Finally, considering perturbations $g \to g + \delta g$ of the Kähler metric that preserve Ricci-flatness (and so preserve the Calabi-Yau condition), the variations δg need to satisfy the Lichnerowicz equation $\triangle^k \triangle_k \delta g_{ab} + 2R_{a\ b}^{\ p\ q} \delta g_{pq} = 0$, through which we can obtain the reduction of the metric. In complex coordinates, the previous equations split into two independent equations for perturbations $\delta g_{i\bar{j}}$ with mixed indices and those with pure indices δg_{ij} and $\delta g_{\bar{i}\bar{j}}$. Without going into too many details we are going to analyze both a bit.

The mixed variations correspond to a real (1,1)-form, permitting us to expand the perturbated mixed components of the metric in terms of the basis w_A of $H^{1,1}(Y)$. Since the manifold is Kähler, the Kähler form can be expressed in terms of the *Kähler moduli* v^A as:

$$J = v^A(x) w_A \ , \qquad A = 1, ..., h^{1,1} \ . \tag{2.8}$$

Then we see that the Kähler moduli determine the volume of the compact manifold. This follows from (can be straightforwardly extended to a CY n-fold):

$$\frac{1}{3!}\int_Y J\wedge J\wedge J=\int_Y \sqrt{g}dx^1\wedge dx^2\wedge dx^3\wedge dy^1\wedge dy^2\wedge dy^3=Vol(Y)\ . \tag{2.9}$$

As a final note on the mixed sector, we would like to highlight that the metric deformations preserving the positivity of the volume form a cone, which can be complexified when adding the $b^A(x)$, coming from the expansion of the B-field, to the previous Kähler moduli. Thus, the complex scalars

$$t^A := b^A + iv^A(x)\ \ , \qquad A=1,...,h^{1,1} \tag{2.10}$$

form the so-called complexified Kähler cone $\mathscr{M}^k$ [8].

Regarding to the pure variations, they cannot be directly expanded due to the fact that there are no (2,0)-forms on a CY threefold. Instead, making use of the holomorphic three-form Ω they fit to a complex (2,1)-form:

$$\delta g_{ij}=\frac{i}{||\Omega||^2}\bar{z}^K(\bar{\eta}_K)_{i\bar{k}\bar{l}}\Omega^{\bar{k}\bar{l}}{}_j\ , \qquad ||\Omega||^2 := \frac{1}{3!}\Omega_{ijk}g^{i\bar{l}}g^{j\bar{m}}g^{k\bar{n}}\bar{\Omega}_{\bar{l}\bar{m}\bar{n}}\ , \qquad K=1,...,h^{2,1} \tag{2.11}$$

in terms of the $h^{2,1}$ *complex structure (CS) moduli* $\bar{z}^K$ and the basis $\bar{\eta}_K$ for $H^{1,2}(Y)$. The scalars receive such a name because the deformations (2.11) usually break the hermiticity condition, and the coordinate transformation required to make the metric hermitian again, respect to the new complex coordinates, is not holomorphic such that a new complex structure arises. These CS scalars span the so-called complex moduli space $\mathscr{M}^{cs}$.

2.1.2 4D SUGRA spectrum

As previously explained, using the Calabi-Yau ansatz for the metric, we can expand the fields in terms of their moduli deformation harmonic modes [18], such that the supergravity spectrum corresponds to just taking the massless level in the expansions, which requires to assume that our energy regime is far below the KK scale and the string scale [9]. The number of legs in Minkowski and in CY3, as well as the Hodge numbers determine the non-vanishing terms in the expansion.

Once performed the expansion, the moduli fields fit perfectly in four-dimensional $N=2$ multiplets.

Type IIB

In this case, the field expansions for (2.4) in the harmonic-form basis are:

$$\begin{aligned} &J=v^A(x)\omega_A\ , && \delta g_{ij}\propto \bar{z}^K(\bar{\eta}_K)_{i\bar{k}\bar{l}}\Omega^{\bar{k}\bar{l}}{}_j\ , && B_2=B_2(x)+b^A(x)\omega_A\ ,\\ &C_2=C_2(x)+c^A(x)\omega_A\ , && C_4=D_2^A(x)\wedge\omega_A+V^{\hat{K}}(x)\wedge\alpha_{\hat{K}}-U_{\hat{K}}(x)\wedge\beta^{\hat{K}}+\rho_A(x)\tilde{\omega}^A\ , \end{aligned} \tag{2.12}$$

where $\mu,\nu=0,...,3,\ i,j=1,2,3,\ A=1,...,h^{1,1},\ K=1,...,h^{2,1}$ and the hat denotes we include the index value 0. Also keep in mind that half of the degrees of freedom regarding to C_4 must be eliminated "by hand" due to self-duality condition (in the next table we implicitly eliminate D_2^A and

[8] We will see soon that, through mirror symmetry, it can be related to the complex structure moduli, but for such a comparison we require both to be complex and $h^{1,1}(Y)=h^{2,1}(\tilde{Y})$. This is why this complexification is important.

[9] Energy scale at which the massive string modes cannot be integrated out.

$U_{\hat{K}}$) and that the two forms in four dimensions are dual to scalars. Moreover, we also have the axion $C_0 = l(x)$ and the dilaton $\phi(x)$ trivial expansions.

Table 1 contains the (bosonic) supergravity spectrum in 4D for Type IIB:

gravity multiplet	1	$(g_{\mu\nu}, V^0_\mu)$
vector multiplets	$h^{2,1}$	(V^K_μ, z^K)
hypermultiplets	$h^{1,1}$	(v^A, b^A, c^A, ρ_A)
double-tensor multiplet	1	(B_2, C_2, ϕ, l)

Table 1: $N=2$ multiplets for Type IIB on CY3.

Type IIA

Now, the field expansions for (2.3) in the harmonic-form basis are:

$$\begin{aligned} &J = v^A(x)\omega_A\,, \quad \delta g_{ij} \propto \bar{z}^K(\bar{\eta}_K)_{i\bar{k}\bar{l}}\Omega^{\bar{k}\bar{l}}{}_j\,, \qquad B_2 = B_2(x) + b^A(x)\omega_A\,, \\ &C_1 = A^0(x)\,, \quad C_3 = A^A(x)\wedge\omega_A + \xi^{\hat{K}}(x)\alpha_{\hat{K}} - \tilde{\xi}_{\hat{K}}(x)\beta^{\hat{K}}\,. \end{aligned} \tag{2.13}$$

Besides, the dilaton ϕ trivial expansion is also present.

Table 2 contains the (bosonic) supergravity spectrum in 4D for Type IIA:

gravity multiplet	1	$(g_{\mu\nu}, A^0_\mu)$
vector multiplets	$h^{1,1}$	(A^A_μ, v^A, b^A)
hypermultiplets	$h^{2,1}$	$(z^K, \xi^K, \tilde{\xi}_K)$
tensor multiplet	1	$(B_2, \phi, \xi^0, \tilde{\xi}_0)$

Table 2: $N=2$ multiplets for Type IIA on CY3.

2.1.3 Rudiments of $N=2$ moduli space geometry and SUGRA

In this subsection, we use the conventions of [23] (it contains an extended discussion including detailed computations) and [3] in order to write the relevant equations. This section will just contain the necessary expressions required for our derivations.

Firstly, we just state that for Type IIB $N=2$ CY compactifications the moduli space is locally a direct product of the form $\mathcal{M} = \mathcal{M}^{cs} \times \mathcal{M}^{k}$ such that the CS moduli space factorizes forming an *Special Kähler manifold* [10] and the Kähler moduli space (containing not only (2.8) but also the rest of the scalar moduli) is a *Quaternionic Kähler manifold* [11]. For type IIA $N=2$ CY compactifications the situation is reversed such that the Kähler moduli and CS moduli exchange their roles.

For the purposes of this thesis we mostly focus on the complex structure moduli space. We take the real basis defined by:[12]

[10] The word special comes from the fact that the Kähler potential is completely determined by a prepotential F, which is a holomorphic homogeneous function of degree two, whose derivatives constitute the F_I periods.

[11] A Riemannian manifold is called quaternionic Kähler if $\mathrm{Hol}(M) \subset Sp(n)\cdot Sp(1)$.

[12] Note that v_0 is a six dimensional fixed CY reference volume introduced for future convenience.

$$\int_Y \alpha_I \wedge \alpha_J = \int_Y \beta^I \wedge \beta^J = 0\,, \quad \frac{1}{\nu_0}\int_Y \alpha_I \wedge \beta^J = \delta_I^J \quad \left(\Rightarrow \int_{A_I} \alpha_J = \sqrt{\nu_0}\,\delta_J^I = -\int_{B^J} \beta^I\right) \quad \text{(Basis)} \tag{2.14}$$

in terms of which, Ω can be expanded as:

$$\Omega = X^I \alpha_I - F_I \beta^I = \frac{1}{\sqrt{\nu_0}}\left(\alpha_I \int_{A_I} \Omega - \beta^I \int_{B^I} \Omega\right) \quad \text{(Holomorphic 3-form expanded on the periods)} \tag{2.15}$$

The coordinates X^I are projective (because Ω is homogeneous of degree one), such that we can choose $(1, x^i) = \frac{X^I}{X^0}$ and there is an special case we will use where we set $X^0 = 1$ called *special coordinates*. The indices run as follows: $I = 0, 1, ..., h^{2,1}$, $i = 1, ..., h^{2,1}$.

Let us now define the Kähler potential for the CS moduli space as [13]:

$$e^{-K_{cs}} := \frac{i}{\nu_0}\int_Y \Omega \wedge \bar{\Omega} = \frac{\nu}{\nu_0}||\Omega||^2 = i(X^I \bar{F}_I - \bar{X}^I F_I)\,, \quad G_{i\bar{j}} = \partial_{x^i}\partial_{x^{\bar{j}}} K_{cs} \quad \text{(Kähler potential and metric)}\,. \tag{2.16}$$

From [6] and [16], we know that the special geometry (determined by a holomorphic prepotential F such that $F_I := \partial_{X^I} F$) of the complex structure moduli space of Type IIB $N=2$ CY3 has no corrections, in the sense that the analytic expression (2.16) is not modified. However, the exact expression for the prepotential includes perturbative and worldsheet instanton corrections (α'), which can be computed through *Mirror Symmetry*. This is a well based conjecture roughly stating that for every Type IIB CY3 exists a mirror Type IIA which exchanges $h^{2,1} \longleftrightarrow h^{1,1}$ and such that the moduli spaces are equivalents in pairs. The next picture, minorly modified from [6], is clear and concise.

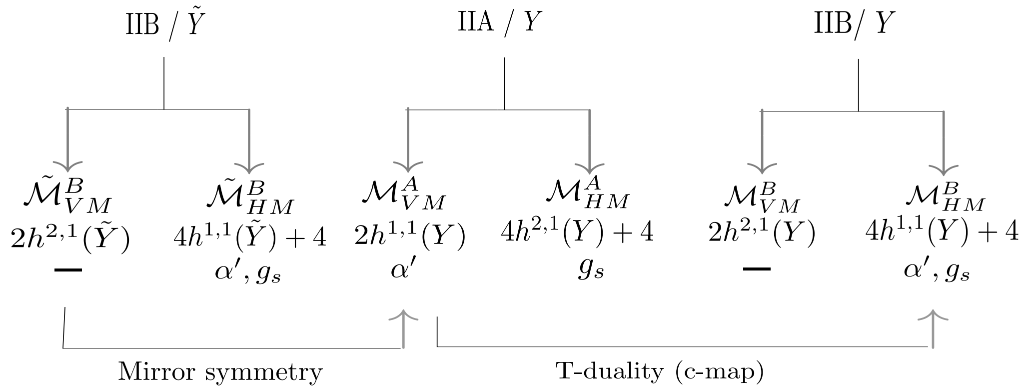

Figure 1: Dualities connecting vector and hypermultiplet moduli spaces in $N=2$ Type II CY compactifications. The moduli spaces for vector and hypermultiplets are denoted by $\mathscr{M}_{VM}$ and $\mathscr{M}_{HM}$, respectively, with their dimensions below in terms of the Hodge numbers $h^{2,1}$ and $h^{1,1}$ of the Calabi-Yau Y or its mirror $\tilde{Y}$ with $h^{1,1}(\tilde{Y}) = h^{2,1}(Y)$. The line below indicates the kind of quantum corrections and the arrows point out some of the dualities that relate the various moduli spaces. Mirror symmetry claims that $\tilde{\mathscr{M}}^B_{VM/HM} = \mathscr{M}^A_{VM/HM}$, whereas the c-map determines, at string tree-level, $\mathscr{M}^{B/A}_{HM}$ in terms of $\mathscr{M}^{A/B}_{VM}$.

Effectively, it means that we can write the prepotential in terms of the Kähler geometry of Type IIA mirror such that:

[13] The second equality follows by means of (2.11) taking into account the antisymmetry of the holomorphic three-form on its indices, such that $\Omega_{ijk} = f(z)\epsilon_{ijk} \Rightarrow ||\Omega||^2 = |f|^2(\sqrt{g}^{-1})$.

$$F_{IIB}(Y)=F_{IIA}(\tilde{Y})=-\frac{1}{6}d_{ijk}\frac{X^iX^jX^k}{X^0}+\frac{1}{2}a_{ij}X^iX^j+b_iX^iX^0+\frac{1}{2}c(X^0)^2+i(X^0)^2\sum_{k_i}n_{k_i}\mathrm{Li}_3(e^{2\pi i k_i X^i/X^0}) \tag{2.17}$$

where n_{k_i} are the genus zero Gopakumar-Vafa invariants, the triple intersection numbers d_{ijk} are the ones of the mirror CY3 (that's the reason why we use d_{ijk} instead of the usual κ_{ijk}), $c=-\frac{i}{(2\pi)^3}\chi_E(Y)\zeta(3)$, $\mathrm{Im}(a_{ij})=\mathrm{Im}(b_i)=0$ requiring Peccei-Quinn symmetry in the absence of instanton corrections (so c, being imaginary, is the only relevant contribution of a, b, c to the Kähler potential, as one can check in base of (2.16)) and $\mathrm{Li}_s(x)=\sum_{n>0}n^{-s}x^n$ are the polylogarithms (these are quite complicated but fortunately we will profit from some identities to manipulate them). The first term in the sum corresponds to the large CS regime (LCS), while the next two are relevant when we are not in LCS and the last two correspond respectively to the perturbative and instanton α' corrections on the Type IIA Kähler moduli space.

The simplest computations are carried out in the *LCS regime*, corresponding to $\kappa := d_{ijk}v^iv^jv^k >> \mathrm{Im}(c)$, where in *special coordinates* $(X^0,X^i)=(1,u^i+iv^i)\approx(1,iv^i)$ (the last approximation follows because in this regime the metric in LCS regime will not depend on the the axions (see later)). In this regime, just the first term in (2.17) contributes. We will work most of the time in LCS, so we will give, following our conventions, some crutial results which can be straightforwardly checked:

$$G_{ij}=-\frac{3}{2}\left(\frac{d_{ij}}{d}-\frac{3}{2}\frac{d_id_j}{d^2}\right) \qquad \text{(LCS metric)} \tag{2.18}$$

$$G^{ij}=-\frac{2d}{3}\left(d^{ij}-3\frac{v^iv^j}{d}\right) \qquad \text{(LCS inverse metric)} \tag{2.19}$$

$$K_{cs}=-\ln\left(\frac{4}{3}d\right) \qquad \text{(LCS Kähler potential)} \tag{2.20}$$

where $d_{ijk}:=\int_{\tilde{Y}}w_i\wedge w_j\wedge w_k$, $d_{ij}:=d_{ijk}v^k$, $d_i:=d_{ij}v^j$ and $d:=d_iv^i$ come from the mirror Kähler moduli space.

Next, we will introduce the general form of $N=2$ SUGRA (gravity and) vector multiplet and some crucial identities regarding its symplectic structure because it will be important later. It is relevant because, as we discussed before, Type IIA and IIB CY3 have precisely this kind of SUGRA with the appropriate Hodge numbers. [14] [15] [16]

$$\mathscr{L}=\frac{R}{2}-g_{ij}\partial_\mu x^i\partial^\mu x^j+\mathscr{I}_{IJ}F^I_{\mu\nu}F^{J,\mu\nu}+\mathscr{R}_{IJ}F^I_{\mu\nu}(*F^{J,\mu\nu}) \quad (N=2 \text{ gravity-vector multiplet effective action}) \tag{2.21}$$

$$Z=e^{\frac{K}{2}}(X^Iq_I-F_Ip^I) \quad \text{(Central Charge)} \tag{2.22}$$

$$D_iZ^J:=(\partial_i+(\partial_iK))Z^J \quad \text{(Kähler Covariant Derivative)} \tag{2.23}$$

[14] We will be primarily interested in Type IIB CS moduli space, whose metric appears in front of the CS moduli in (2.21).

[15] $\mathscr{R}_{IJ}\neq 0$ means that we are considering the CP-violating θ-angle matrix coming from dyonic solutions.

[16] The central charge corresponds to the graviphoton charge at ∞.

$$\mathcal{N}_{IJ} = \mathcal{R}_{IJ} + i\mathcal{I}_{IJ} = \bar{F}_{IJ} + 2i\frac{\text{Im}(F_{IK})X^K\text{Im}(F_{JL})X^L}{\text{Im}(F_{MN})X^M X^N} \ , F_{IJ} = \partial_I F_J \quad \text{(Gauge coupling matrix)} \tag{2.24}$$

$$\mathcal{M} = \begin{pmatrix} -(\mathcal{I} + \mathcal{R}\mathcal{I}^{-1}\mathcal{R}) & \mathcal{R}\mathcal{I}^{-1} \\ \mathcal{I}^{-1}\mathcal{R} & -\mathcal{I}^{-1} \end{pmatrix}, \mathcal{Q} = \begin{pmatrix} p^I \\ q_I \end{pmatrix}, \mathcal{Q}^2 = \frac{1}{2}\mathcal{Q}^T\mathcal{M}\mathcal{Q} \quad \text{(Symplectic structure)} \tag{2.25}$$

$$\boxed{\mathcal{Q}^2 = |Z|^2 + g^{i\bar{j}} D_i Z \bar{D}_{\bar{j}} \bar{Z} = m^2 + 4g^{i\bar{j}} D_i m \bar{D}_{\bar{j}} m} \quad \textbf{(“no force" condition} \equiv \textbf{WGCSF, for BPS states)} \tag{2.26}$$

$$|Z||Z'| + \text{Im}(4g^{ij}\partial_i|Z'|\bar{\partial}_j|Z|) = \mathcal{Q}\mathcal{Q}'\text{Re}\left(\frac{Z\bar{Z}'}{|Z\bar{Z}'|}\right) - \frac{1}{2}(q_I p'^I - q'_I p^I)\,\text{Im}\left(\frac{Z\bar{Z}'}{|Z\bar{Z}'|}\right) \quad \text{(Interaction BPS states)} \tag{2.27}$$

The last two equations (especially (2.26)) will be of capital importance within this thesis and correspond to $N = 2$ SUGRA identities with a clear physical meaning. The first one represents the equilibrium between the gauge force (repulsive) and the gravity and scalar forces (both attractive) between two identical BPS states [17] [18]. The second one reflects in the same way the equilibrium of forces between two different BPS states, from it we can infer that, surprisingly, when the states are locally mutual (i.e. $q_I p'^I - q'_I p^I = 0$, such that the second term in RHS vanishes) and do not gauge interact (i.e. $\mathcal{Q}\mathcal{Q}' = 0$), then their scalar force is repulsive.

Again the simplest computations correspond to the *LCS regime*, and we will mostly work in this regime, so we write the coupling formulae (in the chosen conventions).

$$\mathcal{N} = \mathcal{R} + i\mathcal{I} = \begin{pmatrix} -\frac{1}{3}d_{ijk}u^i u^j u^k & \frac{1}{2}d_{ijk}u^j u^k \\ \frac{1}{2}d_{ijk}u^j u^k & -d_{ijk}u^k \end{pmatrix} + i\frac{-d}{6}\begin{pmatrix} 1 + 4G_{ij}u^i u^j & -4G_{ij}u^j \\ -4G_{ij}u^j & 4G_{ij} \end{pmatrix} \text{(LCS gauge coupling)} \tag{2.28}$$

$$\mathcal{I}^{-1} = -\frac{6}{d}\begin{pmatrix} 1 & u^i \\ u^i & \frac{G^{ij}}{4} + u^i u^j \end{pmatrix} \quad \text{(LCS Inverse electric gauge coupling matrix)} \tag{2.29}$$

Finally, even if we are not going to go into details regarding the Kähler moduli space, we would like to present its Kähler potential in our conventions:

$$K_k := -\ln\frac{4}{3}\kappa \ , \quad \kappa = \kappa_{ABC} v^A v^B v^C \ . \tag{2.30}$$

Recall that, in the case of Type IIB CY3 compactifications, where the Kähler moduli forms part of the Quaternionic Kähler moduli space, (2.30), just reproduces the Kähler moduli metric, such that

[17] Massive representations of an extended supersymmetry algebra called BPS states have mass equal to the supersymmetry central charge $m = |Z|$. Their importance arises as the multiplets shorten for generic massive representations, with stability and mass formula exact.

[18] The WGCSF will be defined in subsection 3.2.1, where it becomes clear that is equivalent to the BPS “no force" condition.

more terms are required for obtaining the full quaternionic kähler metric. The same happens for CS moduli and (2.16) in Type IIA CY3 compactifications.

2.2 DBI & CS Dp-brane actions

These actions are well-known within the community, and we borrow them from [13] because, besides providing the formulas, also motivates them, discussing the extensions to the supersymmetric case (where there are many subtleties), abelian and non-abelian stacks of Dp-branes in its section 2.3 and several appendices. Just remark that (in this subsection) we are not going to extend the analysis to the $N > 1$ Dp-brane case and also we are not going to consider any backreaction, taking as a consequence the *probe limit*. We avoid both issues because taking them into account is an extraordinary technical problem, although we will note later that the latter is very important. The following expressions correspond to a single Dp-brane in Type II superstring background:

$$S_{DBI}^{sf} = -T_p \int_{W_{p+1}} d^{p+1}\xi e^{-\varphi^\star \phi_{10}} \sqrt{-\det(\varphi^\star(g_{10}+B)_{ab} - 2\pi\alpha' F_{ab})} \,, \tag{2.31}$$

$$S_{CS} = \mu_p \int_{W_{p+1}} e^{2\pi\alpha' \mathscr{F}_{ab}} \wedge \sqrt{\frac{\hat{A}(4\pi\alpha' \mathscr{R}_T)}{\hat{A}(4\pi\alpha' \mathscr{R}_N)}} \wedge \bigoplus_q C_q|_{p+1} \quad , \qquad \mathscr{F}_{ab} = \varphi^\star(B)_{ab} + 2\pi\alpha' F_{ab} \,, \tag{2.32}$$

where (2.31) is the so-called Dirac-Born-Infeld action in the string frame [19] representing the coupling of the Dp-brane to the bulk NS-NS fields and the U(1) field strenght F, which describes the U(1) gauge theory of the worldvolume gauge boson (to leading order in F, the gauge theory reduces to a U(1) Yang-Mills theory on the worldvolume W of the brane). On the other side, (2.32) is the Chern-Simons action, giving us the coupling to the appropriate bulk RR forms.

Finally, we just remark that for BPS Dp-branes the energy density, i.e. the Dp-brane tension T_p, is entirely determined by its RR charge μ_p. Two static and parallel BPS Dp-branes of the same charge do not feel a net force. Therefore the attraction due to the exchange of closed string (bulk) NS-NS modes must be canceled by the repulsion resulting from closed string RR modes. This requirement relates the Dp-brane tension T_p to its RR coupling μ_p:

$$\mu_p^2 = T_p^2 = \frac{\pi}{\kappa_{10}^2}(4\pi\alpha')^{3-p} \,. \tag{2.33}$$

2.2.1 A particular example: D3-particle

The D3-particle configuration consist of a D3-brane wrapping a 3-cycle on a Type IIB Calabi-Yau threefold. In subsection 2.2.2 we will specify for the case of a D3-BPS[20] particle (which requires to wrap a special-Lagrangian (sLag) 3-cycle in order to preserve supersymmetry). By now, we simply apply (2.31) and (2.32) for a general 3-cycle, taking into account that we are in the probe limit. Moreover, we take the first order in the pullback's expansion (that is to freeze the open string modes)

[19] It will be necessary to go to Einstein frame through a Weyl rescaling such that we can keep M_P constant when testing the conjectures.

[20] Briefly recall that the BPS bound is saturated when some fraction of the SUSY generators are unbroken. This happens when the mass equals the central extension, which is typically a topological charge.

because we are just measuring the effect of considering a probe on our setting and so the direct couplings to the bulk [21].

We start with the *DBI action*. Firstly, we will give a handwavy argument of why for this setting the B and F fields are irrelevant and can be setted to zero:

$$S_{DBI}^{D3} \sim \int_{t\times\Gamma_3} \sqrt{-det(G+\mathscr{F})} \sim \int_{\Gamma_3} \sqrt{-g_3}(1+(\alpha')^2\mathscr{F}\wedge\mathscr{F}+...)\int_t \sqrt{g_{tt}}d\tau \ . \tag{2.34}$$

Here one can observe that the first term just involves the metric, while in the second is where the gauge fields could come in. The higher order terms clearly always involve forms of higher rank, such that, when integrated, give us zero (from the following reasoning it will be clear). Now, in the second term we need forms with one leg in Minkowski (already covered by $d\tau$) and three legs in the CY. As the internal part of the metric just can contribute through the Kähler (1,1)-form or the holomorphic 3-form, $h^{1,0}=h^{0,1}=0$ for CY3 and [22] $\mathscr{F}$ is a two form, clearly there is no possible match and the second terms as well as the higher order ones vanish.

As a consequence, what remains is:

$$S_{DBI}^{D3} = -T_3\int_{t\times\Gamma_3} d^4\xi e^{-\phi_{10}}\sqrt{-det(G)} \ . \tag{2.35}$$

We observe that $T_3=\frac{\sqrt{\pi}}{\kappa_{10}}$, $\kappa_{10}=\kappa_4\sqrt{\nu_0}$, $\kappa_4=8\pi G_4=M_P^{-2}$ and that, taking the dilaton constant, it gives an overall prefactor. In any case, we go to the Einstein frame through a Weyl rescaling with $e^{-\phi_{10}/2}$ to avoid the dilaton and with $(\nu_0/\nu)^{-1/4}$ for future convenience we get:

$$S_{DBI}^{D3} = -\sqrt{\pi}M_P^2\sqrt{\frac{1}{\nu}}V(\Gamma_3)\int_t \sqrt{g_{tt}}d\tau \Rightarrow M_{D3}=\sqrt{\pi}M_P^2\sqrt{\frac{1}{\nu}}V(\Gamma_3) \ . \tag{2.36}$$

Now we continue with the *Chern-Simons* part. The starting point is (2.32) from which we obtain:

$$S_{CS}^{D3} = \mu_3\int_{t\times\Gamma_3} C_4 \ . \tag{2.37}$$

This can be easily obtained by expanding the different fields (2.12) and matching legs in (2.32), taking into account that just the even RR forms contribute in Type IIB and that the second term (between the wedges) is a curvature term, which once expanded contributes just as unity because the following terms have too many legs and would introduce a curvature term on the Minkowski space, such that due to flatness must vanish.

Expanding C_4 as in (2.12) just survive the vectors in four dimensions, because they have 3 legs in CY and one in Minkowski. Taking the cycle $\Gamma_3=q_IA^I+p^IB_I$, where A^I, B_I represent a symplectic basis of 3-cycles, and using Poincaré duality with the conventions introduced before, we get:

[21] The situation is different for the space-time filling case, where the D3/D7 are included in the setting and so we need to take the open string modes into account as a part of it. Said in other words, these are $\mathscr{O}(1)$ probes such that we can neglect their backreaction but not the open string modes coming from the strings attached to them.

[22] Really with this last argument is enough.

$$S_{CS}^{D3} = \mu_3 \sqrt{\nu_0}\left(q_I \int V^I + p^I \int U_I \right) \tag{2.38}$$

taking into account that self duality of F_5 still needs to be imposed reducing half of the degrees of freedom (d.o.f.).

Finally, we just remark that for the BPS case verifying (2.33) we obtain:

$$S_{CS}^{D3} = q_I \int V^I + p^I \int U_I \quad \text{(in units of } \sqrt{\pi} M_P^2) \; . \tag{2.39}$$

2.2.2 BPS mass formulae

In this subsection we will discuss a couple of the action/mass/tension formulae and then just take from [10] the rest for different D-domain walls, D-strings and D-particles (also the E2 instanton from [23]). Although we had already found some of these mass expressions, others were unknown for us, so we are grateful to Ibáñez et al. for making them explicit such that we borrow them directly from their paper [23].

We start with the BPS D3-particle in Type IIB CY3. It verifies (2.33) and takes the lower bound in the following inequality [24]:

$$V(\Gamma_3) \geq \frac{1}{\sqrt{||\Omega||}} \left| \int_{\Gamma_3} \Omega \right| = \sqrt{\frac{\nu}{\nu_0}} e^{\frac{1}{2}K_{cs}} \left| \int_{\Gamma_3} \Omega \right| \tag{2.40}$$

(in the equality we employed (2.16)). Now we can use the expansion of the holomorphic 3-form, $\Gamma_3 = q_I A^I + p^I B_I$ and Poincaré duality such that:

$$\int_{\Gamma_3} \Omega = \sqrt{\nu_0}(q_I X^I - p^I F_I) \; . \tag{2.41}$$

As a consequence replacing in (2.36) we get:

$$M_{D3}^{BPS} = e^{\frac{1}{2}K_{cs}} |q_I X^I - p^I F_I| \quad \text{(in units of } \sqrt{\pi} M_P^2) \tag{2.42}$$

and we realize that the mass just depends on the CS moduli, the chosen sLag 3-cycle and the wrapping number!

In a similar way, the BPS E2-instanton in Type IIA CY3 is discussed in [23]. In this case, we wrap a Euclidean E2-brane (which couples to C_3) around sLag 3-cycles (again lower bound of (2.40)) obtaining a (-1)-dimensional object in four dimensions. The procedure is analogous to the BPS D3-particles such that [25] we obtain:

$$S_{E2-\text{brane}}^{BPS} = -e^{\frac{1}{2}K^{cs}} |q_I X^I - p^I F_I| + q_I \xi^I + p^I \tilde{\xi}_I \; . \tag{2.43}$$

[23] Note that there is some mismatch between their and our conventions which leads to different prefactors but that will have no practical effects.

[24] The BPS condition is linked to wrap an sLag 3-cycle corresponding to the lower bound in the inequality (2.40)

[25] In the appropriate units.

This is the action of an object localized in space and time, i.e. an instanton, with action:

$$S^{BPS}_{E2} = e^{\frac{1}{2}K^{cs}}|q_I X^I - p^I F_I| \tag{2.44}$$

coupled to the axions of the hypermultiplets.

In this case, we decided to include also the Chern-Simons part to note that it's analogous to the previous D3-particle such that the future discussions and results regarding to the D3-particle could be mostly extended to the E2-instanton.

Now we present a survey of mass/tension formulae provided by [10]. Their conventions are a bit different than ours, but using Planck units and fixing the M_P we see that the the analytic dependence on the moduli turns out to be exactly the same. Please, note that here we will just present the formulae, for details on the derivation look at the aforementioned reference.

We would like to remark that, while for the Dp-branes (also NS5-brane [26]) wrapping 3-cycles, the BPS condition is achieved for the sLag 3-cycles with the lower bound on (2.40), the Dp-branes (also NS5-brane) wrapping even cycles are BPS if the cycle is holomorphic (and calibrated when fluxes are present). That is, the Kähler form J is a calibration such that the volumes of the supersymmetric cycles are given by integrating powers of J [27] along any cycle in the same homology class. Bear in mind that the dependence on the CS moduli for the BPS objects comes always from wrapping 3-cycles, while the ones wrapping even cycles just can depend on the Kähler moduli.

Moreover, the formulae we are going to present are general and so not restricted to the LCS and/or LV (large volume) regimes, where the CS and/or Kähler moduli are taken to be large. They represent the basis masses, i.e. with every brane wrapping just once the cycle.

Type IIA CY3 BPS D-particles

These masses have remarkably no dependence on the CS moduli, just on the complexified Kähler moduli t^A, defined in (2.10).

Brane	Cycle	Tension (in Planck units)
D0	-	$\sqrt{8\pi}e^{K_k/2}$
D2	2-cycle $[w^A]$	$\sqrt{8\pi}e^{K_k/2}\|t^A\|$
D4	4-cycle $[\tilde{w}_A]$	$\sqrt{8\pi}e^{K_k/2}\|\frac{1}{2}\kappa_{ABC}t^B t^C\|$
D6	Y	$\sqrt{8\pi}e^{K_k/2}\|\frac{1}{6}\kappa_{ABC}t^A t^B t^C\|$

Table 3: DBI masses for D-particles in Type IIA CY3

Type IIB CY3 BPS D-particles

This case has been computed explicitly before with our conventions and will be made more explicit for the LCS regime later. We just take the result from [10] so that the reader can compare conventions.

Cycle	A^0	A^i	B_i	B_0
Mass (units of $2\sqrt{2\pi}M_P$)	$e^{K_{cs}/2}$	$e^{K_{cs}/2}\|X^i\|$	$e^{K_{cs}/2}\|\frac{1}{2}d_{ijk}X^i X^j\|$	$e^{K_{cs}/2}\|\frac{1}{6}d_{ijk}X^i X^j X^k\|$

Table 4: DBI masses for D3-particles in Type IIB CY3

[26]These branes differ from the D5-branes in the fact that they couple to the B-field instead of the RR fields and their tension requires an additional $1/g_s = e^{-\phi_{10}}$ factor.
[27]When the B-field is taken into account, just replace $J \to J + iB$.

Type IIA CY3 BPS D/NS-strings

Brane	Cycle	Tension (in Planck units)
D4	3-cycle $[\beta^K]$	$\frac{1}{2}e^{K_{cs}/2}e^{\phi_4}\lvert X^K\rvert$
D4	3-cycle $[\alpha_K]$	$\frac{1}{2}e^{K_{cs}/2}e^{\phi_4}\lvert F^K\rvert$
NS5	4-cycle $[\tilde{w}_A]$	$8e^{K_k/2}\lvert\frac{1}{2}\kappa_{ABC}t^Bt^C\rvert$

Table 5: DBI masses for D/NS-strings in Type IIA CY3

Where ϕ_4 is the four dimensional dilaton defined by $e^{\phi_4} = e^{\phi_{10}}/\sqrt{\nu}$.

Type IIB CY3 BPS D/NS-strings

These tensions have remarkably no dependence on the CS moduli, just on the complexified Kähler moduli t^A.

Brane	Cycle	Tension (in units of $M_P^2/2$)
D1	-	$\frac{1}{g_s^{1/2}\nu}$
D3	2-cycle $[w^A]$	$\frac{t^A}{\nu}$
D5	4-cycle $[\tilde{w}_A]$	$\frac{g_s^{1/2}}{\nu}\left(\frac{1}{2}\kappa_{ABC}t^Bt^C\right)$
NS5	4-cycle $[\tilde{w}_A]$	$\frac{1}{g_s^{1/2}\nu}\left(\frac{1}{2}\kappa_{ABC}t^Bt^C\right)$
D7	Y	g_s

Table 6: DBI tensions for D/NS-strings in Type IIB CY3

Type IIA CY3 BPS D/NS-domain walls

Brane	Cycle	Tension (in units of $M_P^3/2$)
D2	-	$2e^K$
D4	2-cycle $[w^A]$	$2e^K\lvert t^A\rvert$
D6	4-cycle $[\tilde{w}_A]$	$2e^K\lvert\frac{1}{2}\kappa_{ABC}t^Bt^C\rvert$
D8	Y	$2e^K\lvert\frac{1}{6}\kappa_{ABC}t^At^Bt^C\rvert$
NS5	3-cycle $[\beta^K]$	$e^{K/2}\lvert N^K\rvert$

Table 7: DBI tensions for D/NS-domain walls in Type IIA CY3

Where the CS moduli are encoded in the complexified 3-form $\Omega_c = \tilde{C}_3 + 2\sqrt{2}i\mathrm{Re}(C\Omega)$ (with C a compensator determined by dimensional reduction to be $C = e^{-\phi_4}e^{K_{CS}/2}$ [15]) [28], such that $N^K = -i\int_Y \Omega_c \wedge \beta^K$, and K is the total Kähler potential including both CS and Kähler moduli contributions.

Type IIB CY3 BPS D-domain walls

Regarding to the CS moduli dependence, it is analogous to the D3-particles. However, there is an extra component depending on the Kähler moduli appearing though the CY3 volume ν. Besides, for the NS5-domain walls, we just need to add the $1/g_s$ factor.

Cycle	A^0	A^i	B_i	B_0
Mass (units of $M_P^3/\sqrt{2\pi}$)	$\frac{g_s^{1/2}}{\nu}e^{K_{cs}/2}$	$\frac{g_s^{1/2}}{\nu}e^{K_{cs}/2}\lvert X^i\rvert$	$\frac{g_s^{1/2}}{\nu}e^{K_{cs}/2}\lvert\frac{1}{2}d_{ijk}X^iX^j\rvert$	$\frac{g_s^{1/2}}{\nu}e^{K_{cs}/2}\lvert\frac{1}{6}d_{ijk}X^iX^jX^k\rvert$

Table 8: DBI tensions for D5-domain walls in Type IIB CY3

[28] Compare later with our conventions in (2.69) and (2.70).

2.3 Type IIB Orientifolds, Type IIA Orientifolds & Mirror symmetry

Phenomenologically the most interesting cases are compactifications which lead to (spontaneously broken) $N = 1$ supergravity in four space-time dimensions. Such theories naturally arise as CY compactifications of heterotic or type I string theories. In Type II string compactifications on Calabi-Yau threefolds one obtains instead $N = 2$ theory in 4D. However, this $N = 2$ can be further broken to $N = 1$ by introducing appropriate BPS Dp-branes and/or orientifold planes. Turning on additional background fluxes in such compactifications generically breaks (spontaneously) the left over $N = 1$. As a consequence, these constructions deserve full attention, such that this thesis focuses on their Swampland analysis in unfluxed configurations.

Regarding this subsection our discussion is based mainly on [14] and [13] (Type IIB), and [15] (Type IIA and $N = 1$ Mirror Symmetry).

2.3.1 Type IIB $N = 1$ CY3 Orientifolds

We start from type IIB string theory and compactify on CY3. In addition, we mod out by orientation reversal of the string world-sheet Ω_p together with an "internal" symmetry σ, which acts solely on CY3 but leaves the 4d Minkowskian spacetime (ST) untouched. Consistency requires σ to be an isometric and holomorphic involution (so leaves the metric and the complex structure invariant, and $\sigma^2 = 1$) of CY3 but, within this class, the threefolds are left arbitrary. It can be shown that for such threefolds σ leave the *Kähler form J invariant* but can act non-trivially on the holomorphic three-form Ω. Depending on the transformation properties of Ω two different symmetry operations $\mathcal{O}$ are possible. One can have either:

$$\mathcal{O}_{(1)} = (-1)^{F_L}\Omega_p\sigma^\star \ , \ \sigma^\star\Omega = -\Omega \qquad \text{(O3/O7)} \tag{2.45}$$

or

$$\mathcal{O}_{(2)} = \Omega_p\sigma^\star \ , \ \sigma^\star\Omega = \Omega \qquad \text{(O5/O9)} \tag{2.46}$$

where F_L is the spacetime fermion number and $\sigma^\star$ denotes the pullback of σ.

Modding out by $\mathcal{O}_{(1)}$ leads to O3- and/or O7- planes, while modding out by $\mathcal{O}_{(2)}$ leads to O5- and/or O9- planes (always space-time filling). This can be easily seen from the fact that one can always take in local coordinates $\Omega \propto dz^1 \wedge dz^2 \wedge dz^3$ and that the Minkowski space is left invariant by σ.

Let us mention before continuing that consistency, through tadpole cancellation conditions (comes from the requirement of existence of a beginning and end for charge lines in compact space, that is, a generalization of the Gauss law), requires the presence of background D-branes and/or turning on fluxes when Orientifold planes are introduced (and viceversa). Firstly, in our analysis we freeze all of the associated massless fluctuations and solely concentrate on the couplings of the orientifold bulk. Later, we will add to the bulk the open string sector coming from the space-time filling branes [13] just for the O3/O7 case.

The aforementioned tadpole cancellation conditions which are relevant for us are provided by [17]:

$$\sum_{D7_a} N_{D7_a}\left([\Gamma_{D7_a}] + [\Gamma'_{D7_a}]\right) = 8\sum_{O7_i}[\Gamma_{O7_i}] \tag{2.47}$$

$$N_{D3}+2N_{\text{flux}}=\frac{N_{O3}}{4}+\sum_{D7_a}N_{D7_a}\frac{\chi_0(\Gamma_{D7_a})}{24}+\sum_{O7_i}\frac{\chi(\Gamma_{O7_j})}{12}+\text{flux term} \tag{2.48}$$

where the prime denotes the image under the orientifold, N_{D7_a} is the total number of D7-branes wrapping the four-cycle Γ_{D7_a}, χ denotes the Euler characteristic and $\chi_0(\Gamma)=\chi(\Sigma)-n_{pp}$ with n_{pp} the number of pinch-points and Σ an appropriate blow-up (in case of being necessary, for smooth D7 follows $\chi_0(\Gamma)=\chi(\Gamma)$) . Moreover, we do not focus on the flux terms because we want to analyze the most possible general setting and, as we can deduce from (2.47) and (2.48), fluxes are not forced to come into it[29].

O3/O7

Taking into account (2.45), that under world-sheet parity ϕ, g, C_2 are even and B_2, l, C_4 odd and that $(-1)^{F_L}$ leaves NS-NS fields invariant while changes sign for RR fields, the non-projected (invariant) states have to obey:

$$\begin{array}{ll}\sigma^\star\phi=\phi\ , & \sigma^\star l=l\ ,\\ \sigma^\star g=g\ , & \sigma^\star C_2=-C_2\ ,\\ \sigma^\star B_2=-B_2\ , & \sigma^\star C_4=C_4\ .\end{array} \tag{2.49}$$

Using some Hodge technology, together with the above information, one can check that: $h^{2,1}_\pm=h^{1,2}_\pm$, $h^{1,1}_\pm=h^{2,2}_\pm$, $h^{3,0}_+=h^{0,3}_+=h^{0,0}_-=h^{3,3}_-=0$ and $h^{3,0}_-=h^{0,3}_-=h^{0,0}_+=h^{3,3}_+=1$.

From (2.12), Table 1 and (2.49) we obtain the (bosonic) spectrum in Table 9:

gravity multiplet	1	$g_{\mu\nu}$
vector multiplets	$h^{2,1}_+$	$V^{k_+}_\mu$
chiral multiplets	$h^{2,1}_-$	z^{k_-}
chiral multiplet	1	(ϕ, l)
chiral multiplets	$h^{1,1}_-$	(b^{a_-}, c^{a_-})
chiral/linear multiplets	$h^{1,1}_+$	(v^{a_+}, ρ_{a_+})

Table 9: $N=1$ spectrum for Type IIB O3/O7

The non-vanishing of c^{a_-}, b^{a_-} and V^{k_+} is closely related to the appearance of O7-planes. This can be seen by realizing that O3 planes appear when the fix point set of σ is zero-dimensional in CY3 or, in other words, all tangent vectors at this point are odd under the action of σ. This in turn implies that locally two-forms are even under $\sigma^\star$, while three-forms are odd. However, this is incompatible with the expansions for non-vanishing c^{a_-}, b^{a_-} and V^{k_+}. For a setup also including O7-planes we locally get the correct transformation behavior, so that harmonic forms in $H^{1,1}_-$ and $H^{2,1}_+$ can be supported. Remarkably, tadpole cancellation conditions (2.47) force us in this case to have D7 branes [30].

Moreover, the fact that Ω is an element of H^3_- forces: $X^{k_+}=0$ and $F_{k_+}|_{z^{k_+}=0}=0$ [14].

Up to know, he have presented the closed string (bulk) sector. However, as we have already discussed, tadpole cancellation requires to include space time filling D3 and/or D7 branes representing the open string sector. These couple to the bulk and a highly non-trivial analysis is required in order

[29]The so-called non-geometric fluxes might be also present in (2.47) and (2.48) but again are not indispensable [29].

[30]If non-geometric fluxes are allowed, these modify the tadpole cancellation conditions making possible to avoid D7 branes [29]. Anyway, we are not going to consider them as we will restrict to a simple setting, where requiring this fluxes would actually imply to lose generality.

to obtain the full action including bulk, open string sector (the D3/D7 and its open string moduli), as well as the coupling to the bulk of the latter. Such an analysis has been realized in [13] for a stack [31] of space-time filling D3 and then for a space-time filling D7. Next, we will give a brief overview and highlight those relevant facts concerning our project.

First of all, we introduce the open (bosonic) string spectrum in the following tables, taking always into account that, for a stack of N D3-branes, the U(1) gauge theory is enhanced to $U(N)$, and the gauge boson $A_\mu(x)$ (and the gaugino) transform in the adjoint representation of $U(N)$.

bosonic fields	**multiplet**	**multiplicity**
A_μ	vector	1
$\phi^i, \bar{\phi}^{\bar{j}}$	chiral	3

Table 10: Massless D3-brane spectrum in $N = 1$ multiplets

bosonic fields	**multiplet**	**multiplicity**
A_μ	vector	1
$\zeta^A, \bar{\zeta}^{\bar{A}}$	chiral	dim $H_-^{(2,0)}(S)$
$a_I, \bar{a}_{\bar{I}}$	chiral	dim $H_-^{(0,1)}(S)$

Table 11: Massless ST filling D7-brane spectrum in 4D $N = 1$ multiplets

Where S is the 4-cycle resulting form the union of cycle wrapped by the space-time filling D7-brane and the image cycle in the CY3 (covering space), such that $\sigma(S) = S$.

The full (bosonic) action including the bulk, open string sector and couplings for an stack of N space-time filling D3-branes is presented in [13] (equation (4.36), in Einstein frame). It consist of the bulk theory already described in this text, the kinetic terms for the open string moduli of Table 10 and the couplings between both sectors. It fits, as expected, in the 4D $N = 1$ SUGRA structure and here we will present the most remarkable facts:

- Good Kähler Coordinates: In order to specify the Kähler potential in the standard form, we must first identify the proper Kähler coordinates such that in these coordinates the metric of the scalar fields becomes manifestly Kähler. These are:

$$z^{k_-} \ , \ \phi^i \ , \ \tau = l + ie^{-\phi} \ , \ G^{a_-} = c^{a_-} - \tau b^{a_-} \ , \ T_{a_+} = T_{a_+}(\rho, c, b, \phi, z, \tau, G) \ . \tag{2.50}$$

- CS does not factorize: Contrary to the bulk theory case, when including the open string sector and the common couplings we find that the CS does not factorize in the Kähler potential:

$$K(\tau, G, T, z, \phi) = K_{cs}(z) - \ln[-i(\tau - \bar{\tau})] - 2\ln\left[\frac{1}{6}\kappa(\tau, G, T, z, \phi)\right] \tag{2.51}$$

  where $\kappa \equiv \kappa_{a_+ b_+ c_+} v^{a^+} v^{b^+} v^{c^+}$. The Kähler potential reproduces all the kinetic terms of the full action. However, it is given as an implicit expression since κ is explicitly only known in terms of v^{a_+} which are no Kähler coordinates. Instead they are determined in terms of the other coordinates by solving (2.50). Unfortunately, this solution cannot be given explicitly in general [32].

[31] If the stack of D3-branes resides on an orientifold fixed point then the gauge group on the worldvolume of the D3-brane is either $SO(N)$ or $USp(N)$. Otherwise the gauge group of the stack of N D3-branes is $U(N)$. The case we present is the latter and further assume that the D3-branes are separated far enough from their image D3-branes, such that no additional light modes arise from open strings stretching from the D3-branes to the image D3-branes.

[32] For a simple model with one radial Kähler modulus see equation (4.41) of [13]

- The gauge kinetic coupling for the bulk $V_\mu^{k_+}$ remains as in the bulk case.

- Holomorphic coupling constant for the 4D $U(N)$ gauge theory of the stack of D3-branes: It can be shown to be:

$$f^{D3} = -i\kappa_4^2 \mu_3 l^2 \tau \ . \tag{2.52}$$

- D-term potential: for a stack of N D3-branes the "matter fields" ϕ^i transform in the adjoint representation of $U(N)$ and hence gives rise to a D-term potential.

$$D = -\frac{6i}{\kappa} v^{a^+} \omega_{a_+ i\bar{j}} [\phi^i, \bar{\phi}^{\bar{j}}] \ . \tag{2.53}$$

  This D-term gives rise to a non-Abelian scalar potential, which, can be identified with some of the quartic couplings in the action.

- Non-Abelian superpotential: More quartic terms are traced back to F-terms, which appear from the non-Abelian superpotential:

$$W(\phi) = \frac{\mu_3}{l} \Omega_{ijk} \ \mathrm{tr} \phi^i \phi^j \phi^k \ . \tag{2.54}$$

- Mass term for the ϕ^i: The ϕ^i couple to CS moduli and the stack $U(N)$ gauge fields manifestly through covariant derivatives. Remarkably, the coupling to the CS happens is a derivative coupling. As a consequence these ϕ^i obtain a mass term dependent, among other moduli, on (derivatives of) CS moduli of the form:

$$\frac{6i}{\kappa} \mu_3 v^{a_+} \omega_{a_+ i\bar{j}} \ \mathrm{tr} \mathscr{D}\phi^i \wedge \star_4 \mathscr{D}\bar{\phi}^{\bar{j}} \propto \frac{1}{\kappa} \partial_\mu z^{k_-} \partial_\mu \bar{z}^{k^-} v^{a_+} \omega_{a_+ i\bar{j}} \ \mathrm{tr} \phi^i \bar{\phi}^{\bar{j}} \ . \tag{2.55}$$

- No mass term for the CS moduli: Without fluxes, there is neither mass term nor potential for the CS moduli.

Finally, the full (bosonic) action including the bulk, open string sector and couplings for an space time filling D7-brane appears in [13] (equation (4.51), in Einstein frame). It consist of the bulk theory already described in this text, the kinetic terms for the open string moduli of Table 11 and the couplings between both sectors. It fits, as expected, in the 4D $N = 1$ SUGRA structure and here we will briefly mention some facts mainly related to the CS which are related to our setting:

- Good Kähler Coordinates: Again we search for the appropriate Kähler variables in order to identify the complex structure of the Kähler manifold. For us, it is just relevant that again z^{k_-} is already a good coordinate. Remark that now no one of the other "good" coordinates depends on CS moduli, this already signals its decoupling.

- CS factorizes: As in the case of D3 we can use the proper Kähler variables to obtain the Kähler potential and remarkably the CS part is diagonal, without coupling to any other part of the potential. So now we have a complete decoupling, contrary to the D3 case. We just mention, that, again, the volume term is defined implicitly.

- The gauge kinetic coupling for the bulk $V_\mu^{k_+}$ remains as in the bulk case (as before).

- The U(1) D7-brane gauge coupling does not depend on the CS moduli (as before).

- D-term potential: The D-term potential does not depend on the CS moduli. We just comment, that it minimizes for $b^{a_-}=0$, where the D-term and V_D itself vanish and hence one obtains a supersymmetric ground state.
- No mass term for the CS moduli: Without fluxes, there is no mass term for the CS moduli (as before).

Before continuing, we would like to add some remarks (c.f. section 4.7 of [13]) which apply to both (D3 and D7) cases:

- Structure of the Kähler potential: We observe that in all cases the generic form

$$K=-\ln[-i\int_Y \Omega\wedge\bar{\Omega}]-\ln[-i(\tau-\bar{\tau})]-2\ln \text{vol}(Y) \tag{2.56}$$

  follows independently of the presence of D3- and/or D7-branes. However, what is non trivial for a specific setup is to determine the Kähler coordinates and their precise relation to the holomorphic three-form Ω, axiodilaton τ and volume vol(Y). Even though, for the CS still remains like in the bulk case.
- Gravitino mass: The previous (2.56) can be motivated (look at [13]) by the fact that:

$$m_{3/2}\sim e^{K/2}W \tag{2.57}$$

  is known by supersymmetry arguments.
- No corrections included: The Kähler potentials have been discussed in the lowest order in α' and without including any quantum correction. Including such effects is expected to modify the Kähler potential in (2.56), taking into account that, contrary to $N=2$ Type IIB, in $N=1$ Type IIB Orientifolds there is no non-renormalization theorem fully protecting the Kähler potential.

O5/O9

Taking into account (2.46) and that under world-sheet parity ϕ, g, C_2 are even and B_2, l, C_4 odd, the non-projected (invariant) states have to obey:

$$\begin{array}{ll} \sigma^\star\phi=\phi\ , & \sigma^\star l=-l\ ,\\ \sigma^\star g=g\ , & \sigma^\star C_2=+C_2\ ,\\ \sigma^\star B_2=-B_2\ , & \sigma^\star C_4=-C_4\ . \end{array} \tag{2.58}$$

Using some Hodge technology, mixed with the above information, one can check that: $h^{2,1}_\pm=h^{1,2}_\pm$, $h^{1,1}_\pm=h^{2,2}_\pm$, $h^{3,0}_-=h^{0,3}_-=h^{0,0}_-=h^{3,3}_-=0$ and $h^{3,0}_+=h^{0,3}_+=h^{0,0}_+=h^{3,3}_+=1$.

From (2.12), Table 1 and (2.58) we obtain the (bosonic) spectrum in Table 12:

gravity multiplet	1	$g_{\mu\nu}$
vector multiplets	$h_-^{2,1}$	$V_\mu^{k_-}$
chiral multiplets	$h_+^{2,1}$	z^{k_+}
chiral multiplets	$h_+^{1,1}$	(v^{a_+}, c^{a_+})
chiral/linear multiplets	$h_-^{1,1}$	(b^{a_-}, ρ_{a_-})
chiral/linear multiplets	1	(ϕ, C_2)

Table 12: $N = 1$ spectrum for Type IIB O5/O9

Compared to the spectrum of the first projection given in Table 9 we see that the vectors and complex structure deformations have switched their role with respect to the decomposition in H^3. Furthermore, different real fields combine into the complex scalars of the chiral/linear multiplets or, in other words, the complex structure on the moduli space has changed.

In this case of O5/O9 we are not going to discuss the open string sector because we are not aware of any reference where it has been extensively studied and also does not seem to be relevant for our purposes. Nonetheless, similar features are clearly expected.

Again, the fact that Ω is an element of H^3_+ forces: $X^{k_-} = 0$ and $F_{k_-}|_{z^{k_-}} = 0$ [14].

As shared by both projections, (2.45) and (2.46) (and also for Type IIA one), and already mentioned before, we will write the generic form of the bosonic part of a $N = 1$ SUGRA action in four dimensions with its manifestly positive definite potential (c.f. [14]) :

$$S = -\int \left[\frac{1}{2} R \star 1 + \frac{1}{2} \mathrm{Re}(f_{\kappa\lambda}) F^\kappa \wedge \star F^\lambda + \frac{1}{2} \mathrm{Im}(f_{\kappa\lambda}) F^\kappa \wedge F^\lambda + \mathcal{G}_{I\bar{J}} DM^I \wedge \star D\bar{M}^{\bar{J}} + V \star 1 \right] , \quad (2.59)$$

$$V = e^K (K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2) + \frac{1}{2} (\mathrm{Re} f)^{-1\ \kappa\lambda} D_\kappa D_\lambda \ , \quad (2.60)$$

where $K^{I\bar{J}}$ is a Kähler moduli space metric, which in our case factorizes the CS from the rest, and W is the superpotential. The first term is called F-term and the second D-term potential. An inspection of (2.60) permits us to observe that the moduli fields can receive mass, case in which we do not talk about moduli space but configuration space. That is generically the case of the complex structure moduli, which receive mass contributions from the fluxes. If not frozen, the open string moduli coming from the Dp-branes, can contribute to generate masses (not for CS) as observed before.

2.3.2 Type IIA $N = 1$ CY3 Orientifolds

The project focusses mainly on Type IIB, so we will discuss Type IIA case with much less detail than the Type IIB one. Also because the logic and relevant features equally apply. We will comment on the projection and the spectrum in the bulk theory, while there will be no explicit treatment for the open string sector.

In this case, we start from Type IIA string theory, compactify on CY3 and mod out by a discrete symmetry:

$$\mathcal{O} = \Omega_p (-1)^{F_L} \sigma \quad (2.61)$$

where σ is again an involutive symmetry of Y (i.e. $\sigma^2 = 1$), which acts trivially on the four flat dimensions. However, if we want to preserve $N=1$ supersymmetry σ has to anti-holomorphic and isometric (so it leaves the metric invariant but reverses the complex structure) such that the *Kähler form is odd*:

$$\sigma^* J = -J \ . \tag{2.62}$$

Compatibility of σ with the CY condition $\Omega \wedge \bar{\Omega} \propto J \wedge J \wedge J$ forces σ to act non-trivially on the holomorphic three form Ω as

$$\sigma^* \Omega = e^{2i\theta} \bar{\Omega} \tag{2.63}$$

with $e^{2i\theta}$ a constant phase, where the factor 2 is included for convenience.

Type IIA orientifolds with anti-holomorphic involution generically admit **O6** planes. This is because the fixed point set of σ in Y are 3-cycles Λ supporting the internal part of the orientifold planes. Let us just state, that these cycles are sLag submanifolds of Y as a consequence of (2.62) and (2.63), which implies [33]:

$$J|_\Lambda = 0 \ , \qquad \mathrm{Im}(e^{-i\theta}\Omega)|_\Lambda = 0 \ . \tag{2.64}$$

Taking into account (2.61), that under world-sheet parity ϕ, g, C_1 are even and B_2, C_3 are odd and that $(-1)^{F_L}$ leaves NS-NS fields invariant while changes sign for RR fields, the non-projected states have to obey:

$$\begin{aligned} &\sigma^\star \phi = \phi \ , && \sigma^\star C_1 = -C_1 \ , \\ &\sigma^\star g = g \ , && \sigma^\star C_3 = C_3 \ , \\ &\sigma^\star B_2 = -B_2 \ , && \end{aligned} \tag{2.65}$$

while the deformations of the CY metric are constrained by (2.62) and (2.63).

Using Hodge technology and the previous information, it is not difficult to check that: $h^{1,1}_\pm = h^{2,2}_\mp$, $h^{3,3}_+ = h^{0,0}_- = 0$, $h^{3,3}_- = h^{0,0}_+ = 1$ and, that H^3 can be decomposed independently of the complex structure as $H^3 = H^3_+ \oplus H^3_-$, where the real dimensions of both are equal and given by $h^3_+ = h^3_- = h^{2,1} + 1$ (consequence of Hodge duality and oddness of volume-form).

Remarkably, for Type IIA we can recover the complexified Kähler form J_c, that is a good Kähler coordinate, because now both B_2 and J are odd so can combine together as in $N=2$ but only in the odd cohomology, such that $t^{a_-} = b^{a_-} + i v^{a_-}$.

The number of CS deformations is also reduced since (2.63) constrains the possible deformations. To see that, we perform a symplectic rotation in order to expand Ω in the aforementioned “real" basis:

$$\Omega = X^K a_K - F_K b^K \tag{2.66}$$

and insert in (2.63), finding

[33] In other words, they are calibrated with respect to $\mathrm{Re}(e^{-i\theta}\Omega)$.

$$\mathrm{Im}(e^{-i\theta}X^K) = 0\ , \qquad \mathrm{Re}(e^{-i\theta}F_K) = 0 \tag{2.67}$$

where the first set of equations are $h^{2,1}+1$ real conditions for $h^{2,1}$ complex scalars x^k. One of these equations is redundant due to the scale invariance of Ω. As a consequence, half of the CS deformations are projected out. The second set of equations play a similar role for the F_K periods.

The complex rescaling invariance of Ω is reduced by (2.63) to a real rescaling. As a consequence, together with the discussion in the above paragraph, we observe that Ω just depends on $h^{2,1} = 1$ real deformation parameters. However, it usually turns out to be more convenient to maintain that gauge freedom and define a complex "compensator" $C = re^{i\theta}$, where r is related to the inverse of the four dimensional dilaton. Then, the scale invariant function $C\Omega$ depends on $h^{2,1}+1$ real parameters and enjoys (using (2.67)) the expansion:

$$C\Omega = \mathrm{Re}(CX^K)a_K - i\mathrm{Im}(CF_K)b^L\ . \tag{2.68}$$

Now, from (2.13) and (2.65), we observe that C_1 is completely projected out, the even 4D vectors A^{a_+} and $h^{2,1}+1$ real scalars ξ^K from C_3 survive. It turns out that we can combine the dilaton, the ξ^K and the CS deformations into chiral multiplets and the good Kähler coordinate is:

$$\Omega_c = \tilde{C}_3 + 2i\mathrm{Re}(C\Omega)\ , \quad \tilde{C}_3 = \xi^K a_K \tag{2.69}$$

such that:

$$\Omega_c = 2N^K a_K\ , \qquad N^K = \frac{1}{2}\int \Omega_c \wedge \beta^K = \frac{1}{2}(\xi^K + 2i\mathrm{Re}(CX^K))\ . \tag{2.70}$$

Finally, we observe that the (bosonic) spectrum is then given by:

gravity multiplet	1	$g_{\mu\nu}$
vector multiplets	$h_+^{1,1}$	$A_\mu^{a_+}$
chiral multiplets	$h_-^{1,1}$	t^{a_-}
chiral multiplets	$h^{2,1}+1$	N^K

Table 13: $N = 1$ spectrum for Type IIA O6

and follows the $N = 1$ SUGRA structure (2.59), (2.60) with a Kähler potential:

$$K = K_k + K_q = -\ln\left[\frac{4}{3}\int_Y J\wedge J\wedge J\right] - 2\ln\left[2\int_Y \mathrm{Re}(C\Omega)\wedge\star\mathrm{Re}(C\Omega)\right]\ . \tag{2.71}$$

2.3.3 Mirror symmetry for Orientifolds

The (bulk) moduli space of both Type IIB and the Type IIA orientifolds is locally a direct product of a special Kähler manifold, which comes from reducing the parent $N = 2$ one, and a Kähler subspace of the parent $N = 2$ quaternionic Kähler manifold:

$$\tilde{\mathcal{M}} = \tilde{\mathcal{M}}^k \times \tilde{\mathcal{M}}^q \tag{2.72}$$

such that the matching of the coordinates spanning them follows the scheme:

Multiplets	$\mathrm{IIA}_Y O6$	$\mathrm{IIB}_{\tilde{Y}} O3/O7$	$\mathrm{IIB}_{\tilde{Y}} O5/O9$
vector multiplets	$h_+^{1,1}$	$h_+^{2,1}$	$h_-^{2,1}$
chiral multiplets in $\tilde{\mathcal{M}}^k$	$h_-^{1,1}$	$h_-^{2,1}$	$h_+^{2,1}$
chiral multiplets in $\tilde{\mathcal{M}}^q$	$h^{2,1}+1$	$h^{1,1}+1$	$h^{1,1}+1$

Table 14: Type II orientifold mirror symmetry matching

which requires an orientifold version of the usual mirror symmetry:

$$O3/O7 \quad : \qquad h_-^{1,1}(Y) = h_-^{2,1}(\tilde{Y})\,, \qquad h_+^{1,1}(Y) = h_+^{2,1}(\tilde{Y})\,, \tag{2.73}$$

$$O5/O9 \quad : \qquad h_-^{1,1}(Y) = h_+^{2,1}(\tilde{Y})\,, \qquad h_+^{1,1}(Y) = h_-^{2,1}(\tilde{Y})\,. \tag{2.74}$$

The details of mirror symmetry are in general much more subtle for orientifolds, mainly because we have O3/O7 and O5/O9 in Type IIB but only O6 in Type IIA. Fortunately, as the CS moduli factorizes (for the bulk theory) in the Kähler potential, it follows, at least in the LCS-LV regime, directly truncating from the parent $N = 2$. In this LCS-LV regime one can check complete agreement between the vector multiplet mirror couplings [15].

Mirror symmetry for the Kähler manifolds coming from the quaternionic Kähler in $N = 2$, requires that the triple intersection numbers match with respect to the mirror expectancies as checked in [15]. For example, in Type IIA O6 the orientifold projection forces that $\kappa_{a^+b^+c^+} = \kappa_{a^+b^-c^-} = 0$ and so, when we take the Type IIB O3/O7 orientifold mirror, the matching of both prepotentials (following the truncated version of (2.17)) together with (2.74) forces:

$$F_{O3/O7}(Y) = F_{O6}(\tilde{Y})\,, \quad \kappa_{a^+b^+c^+} = \kappa_{a^+b^-c^-} = 0 \quad \Rightarrow \quad d_{a^+b^+c^+} = d_{a^+b^-c^-} = 0\,. \tag{2.75}$$

2.3.4 Truncation $N = 2 \to N = 1$

Finally, let us briefly explain what we mean by truncation $N = 2 \to N = 1$, because we will use this terminology often within this text. We refer to taking the parent $N = 2$ and impose the orientifold projection conditions over its structure, couplings, Kähler potential and, in general, various formulae, providing us with the $N = 1$ orientifold quantities.

For example, taking the $N = 2$ Type IIB action in 4D and applying the O3/O7 (O5/O9) orientifold truncation gives us automatically the $N = 1$ Type IIB orientifold O3/O7 (O5/O9) action. The truncation is perfectly consistent for the vector multiplet (and so for its gauge coupling), because there the CS factorizes, and is a bit more subtle for the rest (mainly because the good Kähler coordinates need to be redefined an with them also those elements where they could be involved (like their contribution to the Kähler potential) but can also be performed.

This will motivate us for choosing the $N = 2$ basis as the even-odd basis (which can be done always, see e.g.[24]) for our later analysis in section 5. Besides, we will use several times terminology like “the appropriate field configuration", really meaning the orientifold truncation (of the spectrum) already in the parent $N = 2$ covering space.

It is extremely important to take into account that, in general, the moduli spaces are not described by an unique patch. As a consequence, we need several patches with their respective local coordinates.

Nothing about mapping between different patches is discussed neither in [14] and [15], nor in the literature within our reach. Nevertheless, this point is crucial because the previous references projection is framed in just one patch in LCS/LV, but consistent mapping between one such chart and the rest would surely require extra conditions on the truncation. We will come back to this point later, but keep in mind that the truncation method described in these references might be consistent only locally and not globally.

3 Review of the Swampland

In the previous section we introduced the object to study. The aim of this section is to present the different conjectures we want to test and the motivation for doing so, as well as the current state-of-art, challenges and specific theory which will be required in our project. While the Swampland is an incredibly vast topic, our focus will be on those conjectures, aspects and techniques which will be of primary importance for the thesis. For the further reading, we recommend a detailed review [11], which also motivated a great deal of the following discussion and from which we have taken the statements of the conjectures, in order to preserve the same formulation and not confuse the reader.

3.1 The “Swampland Program": motivation and purposes

In the first third of the 20th century, General Relativity (GR) and Quantum Mechanics (QM) were discovered and represented an incredible breakthrough regarding Classical Mechanics and Electrodynamics. However, their properties are fundamentally different and the regimes that each one describes accurately are opposed. That was not really a problem during that time; nevertheless, during the second third of the last century three of the fundamental forces: Electromagnetism, Weak and Strong nuclear forces have been found to be accurately described in the framework of Quantum Field Theory (QFT), that is roughly speaking Special Relativity (SR) plus QM (plus many-body field theory description), at least in the perturbative regime, in the so-called *Standard Model (SM) of Particle Physics*. At the same time, GR was profoundly studied and a new branch of physics appeared related to outstanding solutions to Einstein's equations of motion, called Black Holes (BH). These objects hint to the breakdown of GR itself, manifesting also quantum properties (e.g. Hawking radiation) in a semiclassical approach (QFT in Curved Spacetime). As a consequence, the expectancies of many theorist became more robust such that there exist almost unanimity regarding the fact that GR and QM should be joined in a unified theory, abstractly denoted by *Quantum Gravity* (QG), serving as a basis to describe the aforementioned three forces and Gravity. During these two thirds of century, experimentalists were able to test the Physics at energies around a TeV, which permitted, for example, to succesfully test the Stardard Model.

Unfortunately, the technology required in order to carry out experiments to test physics at very high energy scales, as such required for finding any insight on QG (Planck scale), is not yet developed. Notwithstanding, it is enough to detect several issues unexplained by means of the SM (nor the Cosmological models), that is Inflation, Dark Energy and Matter, Neutrino masses ... which must be adressed by a UV completion containing it. At the same time, high energy physics has evolved enormously in the last fifty years and promising theories like String theory have been developed. String theory has overcome many challenges during this way, among which we would like to highlight anomaly cancellation (thanks to the Green-Schwarz mechanism) and the realization of a web of dualities that points to the existence of a unique theory (M-theory) from which the apparently unrelated Superstring theories can be obtained. At this stage, it is fair to consider String theory as the main candidate, of which we have, for being a QG theory. At a minimum, it contains all the necessary ingredients and predicts (modified) GR in a QFT framework. Nowadays, it faces two main issues: lack of a QG space-time description in the ultraviolet and lack of predictive power in the infrarred [34]. One can regard the latter as the initial motivation for the *“Swampland program*", whose main idea was to exploit the possibility that self-consistency could be strong enough to already determine the UV theory. That is just a possibility but clearly deserves attention and, in the worst case, can provide us with many constraints for the UV completion of Effective theories [35] which arise, for

[34] That is due to the fact that, naively, we can compactify from 10D to 4D in a, at least, large number of ways.
[35] Theories which describe the Physics as far as the energy scale we can measure.

example, when trying to describe Inflation models compatible with the conjectures (see, e.g., [35], [36]) forming the “Swampland program" (the same happens for Dark Matter, Energy ...). It is remarkable that there is the aim to build the program in the more general way for any QG theory, not just String theory [36].

The program aims to identify those QFTs whose effective theory is consistent when gravity is included and distinguish them from those which seem perfectly valid when gravity is turned off, but cannot be completed into QG in the UV. The former belong to the *Landscape* and the latter to the *Swampland*. It is made of several conjectures, where is fair to highlight the (diverse) **Weak Gravity Conjectures** (WGCs) and the (refined) **Swampland Distance Conjecture** (SDC). Roughly speaking, the WGCs state that gravity is the weakest force. They are based primarily on Black Hole Physics arguments against Remnants and the forbiddance of Global Symmetries in Quantum Gravity [1]. The diverse tests and refinements are mainly based on $N = 2$ Supergravity (SUGRA) in the context of Black Hole Physics and/or String Theory [3]. On the other side, the SDC claims, also roughly speaking, that in the moduli space (a space parametrized by the vacuum expectation values of the fields in the effective theory) of a QFT compatible with gravity, when we approach an infinite geodesic distance locus, an infinite tower of states become massless exponentially on the distance [2]. While the WGCs have a profound physical basis, the origin of the SDC is yet unclear. However, the case of $N = 2$ seems to be special, as there a "duality" between both conjectures has been pointed out first in [4] and non-trivially checked in [5], which precisely ties both to the **Emergence Proposal** (so to speak, affirms that in the UV the coupling constants diverge and so points to “dynamics as an IR emergent effect caused by integrating out towers of states"). We will be much more precise and further details and analysis will be provided later on.

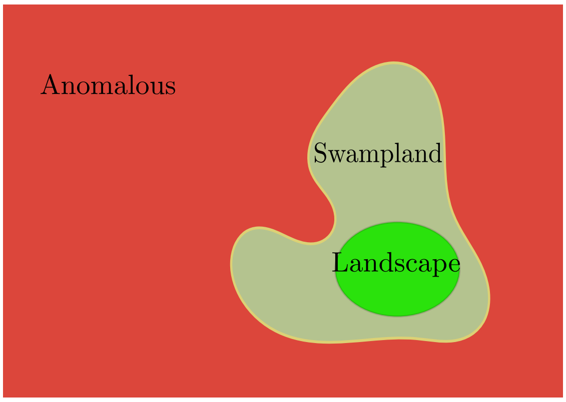

Figure 2: Universe of QFTs.

The “Swampland program" as such began with Cumrun Vafa’s seminal paper [30]. Even though the ideas were already circulating in the community, with [30] a new field was already openned with the main objective of transit from qualitative speculations to quantitative results.

It is quite fair to say that the current state of the program is in an intermediate stage. On one side, the different intuitions, piled up mainly from the previous decades of Black Hole Physics (e.g. the problems with remnants and their relation to BH information problem [31]), Kaluza-Klein (KK) compactifications (e.g. no-go theorems for KK and relation to Particle Physics [32]) and SUSY / String theory motivated exploration of moduli spaces (e.g. Seiberg- Witten theory [33] and Strominger conifold resolution [7]), have been stated as conjectures, subject to scrutiny and constant refinements, in a constructive way. On the other side, there is a growing number of conjectures that arise (some of them quite expected and sometimes almost tautological regarding the others) without

[36]Despite the fact that there is a logical bias because the program was born in an String theory framework and the scientists working on the topic are mostly string theorists.

having yet completely clear the role of the previous ones, and their possible interconnections have been sketched but few non-trivial tests have been carried out.

Maybe the greatest inconvenient with the broad way in which the conjectures are formulated is that usually it is much easier to test them in concrete cases than to false one of such conjectures, because in case of not being able to check the conjectures in a concrete setting, one could always argue that there exist some particle/new physics that we are not aware of or that the theory we are testing is in the Swampland but maybe what happens is that the conjecture is not accurate enough or true and the theory was perfectly fine (we have, in general, no objective way to distinguish between both possibilities without experiments). This could discourage researchers from the risk of testing the conjectures in the most difficult settings and lead them to better "go safe". Such a problem can be mostly corrected when when the conjectures are formulated and based in the more general possible setting and their evidence does not depend on particular properties like SUGRA, BPS condition ... then we can be pretty sure that they should be verified and, in case of tension, we can position ourselves in favor of the conjecture. Therefore, one should aim to relax these properties and our scenario (subsection 5.2.2) does it in a simple way.

It seems clear that further refinements are required and deeper studies on complicated scenarios are mandatory. That was actually one motivation for working $N = 1$ orientifolds, where the bulk theory is already $N = 1$, bringing into the game crucial stability issues, avoided in $N \geq 2$ by the BPS condition (see subsection 3.8). In general, as we mentioned several times along the text, things become harder when reducing SUGRA but also more phenomenologically appealing. In any case, the community's effort on the topic is being impressive and the evolution of the field is encouragingly fast.

3.2 Weak Gravity Conjecture (WGC)

3.2.1 Statement

Firstly, we proceed to present the statement of the different versions of the conjecture such that it serves as a reference for the reader along the project. Let us start with the original formulation [1].

Weak Gravity Conjecture in 4d.
Consider a theory, coupled to gravity, with a $U(1)$ gauge symmetry with gauge coupling g

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{4g^2} F^2 + \ldots \right] . \tag{3.1}$$

- *(Electric WGC) There exists a particle in the theory with mass m and charge q satisfying the inequality*

$$m \leq \sqrt{2} g q M_P . \tag{3.2}$$

- *(Magnetic WGC) The cutoff scale Λ of the effective theory is bounded from above approximately by the gauge coupling*

$$\Lambda \lesssim g M_P . \tag{3.3}$$

This was the first version but several extensions/refinements have appeared since that time. A relevant one for us is [5]:

Weak Gravity Conjecture with Scalar Fields.
A theory with scalar fields t^i, which have no potential, with general action as in (2.21), should have a particle with mass $m(t)$ satisfying the bound

$$\mathcal{Q}^2 M_P^2 \geq m^2 + g^{ij}(\partial_{t^i} m)(\partial_{t^j} m) M_P^2 \tag{3.4}$$

with $\mathcal{Q}^2$ defined as in (2.25).

We have seen that in $N = 2$ vector multiplets we have both $U(1)$ gauge vectors and CS moduli scalars together. Also we have observed that the $N = 2$ vector multiplets split into $N = 1$ vector and chiral multiplets when orientifolding. Thus, it seems clear that (3.4) will enjoy a prime role in our project.

Keep in mind that there exist modifications of both where "particle" is extended to "tower of (possibly extended) states", which will become relevant for us later. For completion, we will present (some of) them.

The tower versions started with the **Lattice WGC** [37] which was refined to

Sub-Lattice Weak Gravity Conjecture.
In a gauge theory with (possibly multiple) $U(1)$s, coupled to gravity, any spot on a charge sub-Lattice contains a super-extremal particle.

by [38]. The main difference is that the sub-lattice may have some spaces missing, while still preserving the lattice structure, coming from the charges consistent with Dirac quantization. There exists also the **Tower WGC** [39], which differs in that the towers of super-extremal particles [37] do not need to form a lattice of charges.

Finally, recalling the fact that a p-form couples naturally to a p dimensional object by integrating over the world-volume, the generalization seems obvious and was first introduced in [1] and explored in further detail by [37].

Weak Gravity Conjecture for p-forms in d-dimensions.
A d-dimensional theory with a p-form field with kinetic terms $\mathcal{L} \supset \frac{1}{2g_p^2}|F^{(p+1)}|^2$, should have a p dimensional object with (quantized) charge q_p and tension T_p satisfying

$$\frac{p(d-p-2)}{d-2} T_p^2 \leq q_p^2 g_p^2 (M_P^d)^{d-2} \tag{3.5}$$

[37] Super-extremal just means particles whose charge is larger than their mass (and scalar charge if any) in the sense of (3.2) and (3.4).

The case $p = 0$ corresponding to axions is rather special, and the most studied one, mainly because of its interest in Inflation models. It plays no role on this thesis but the interested reader can find more in [11] and [23].

3.2.2 Motivation

It would be out of the scope of this thesis to include all the journey and tests that led to the previous formulations of the WGC. Nevertheless, being this conjecture the most substained from a fundamental Physics point of view, we will try to convey the essence and those steps whose impact has been undeniable in our project.

In order to understand how [1] arrived at the original conjecture, we need to go back to the role of *Global symmetries* and *Completeness* in QG. In fact, they can be formulated [11] as two well-known conjectures before the WGC and one could say that the former (together with Black Hole/Entropy and the bound state [38] arguments) was the base for the WGC, whereas the latter may be regarded as the main motivation to look for its tower version.

The well-known fact that, in perturbative String theory, a global symmetry on the worldsheet gets always gauged in the target space was the first motivation to consider the absence of global symmetries as a QG constraint, which made into a conjecture reads:

No Global Symmetries Conjecture.
A theory with a finite number of states, coupled to gravity, can have no exact global symmetries.

The idea is pretty old and several attemps where made to generalize it to non-necessarily stringy frameworks, giving to it the most possible generality. As discussed before, such an attempt usually led to Black Hole (BH) Physics. A handwavy semiclassical argument is that, in order for a black hole to discharge ($U(1)$ symmetry) by emitting Hawking radiation, the symmetry must be local. The reason is the following: semiclassically the BH will undergo Hawking radiation and so lose mass; however, if the symmetry becomes[39] global there is no "electric field" creating a gradient in the horizon which could break the symmetry between positive and negative charge. As a consequence, in the case of a global symmetry, the (global) charge of the BH becomes independent of its mass and there is no way to determine it. This situation of total ignorance leads to an infinite entropy violating the Bekenstein bound $S_{BH} \sim M^2$. Such a situation is inconsistent and so by "reductio ad absurdum" global symmetries should be absent in QG. Of course, this argument has several caveats, the main one being to assume that a semiclassical approach could replace a proper (unknown) QG one.

A more precise formulation of the above argument uses *remnants*, that are objects which are stable thanks to their global charge, but which may be light (Planck mass). Following the previous paragraph argument, we can consider the case where the BH mass is small and so the Hawking radiation evaporation process ends in a QG regime [40]. In this case, by the time the BH mass reaches the Planck

[38] See subsection 3.8.

[39] It is essential to have in mind that in order the BH to have a charge it must have been formed with some kind of charged elements and for such "forming" process there must exist a (gauge) interaction, which at the same time requires dynamical degrees of freedom. Moreover, Hawking radiation process requires of vacuum pair production (e.g. electron-positron) which is a dynamical process. As a consequence, the symmetry had to have been local at some point and we are investigating just a limit where becomes global. That is totally different from considering an "invariant" global symmetry.

[40] In such a case, we cannot trust the semiclassical approach but some arguments exist in favor of a similar conclusion.

scale quantum regime, it no longer has enough mass to be able to emit enough of the charged particles (unless there exist always some particle with a larger charge to mass ratio inducing a decay) to shed its global charge. As a result, we have a (Planck-sized) remnant. Building the BH with N copies of one charged particle would lead to a remnant of charge N, but doing it with N+1 copies would lead to a remnant of charge N+1! As a consequence, we find an infinite number of remnant states, leading again to entropy inconsistency / BH information paradox among others. The issues related with remnants were already noticed by [31], but, as we can always argue that the remnants may lie at energy scales where gravity is strongly coupled, the QG derivation of inconsistencies is difficult to reach. Again, one cannot completely use semiclassical effective analysis to theoretically prove QG related conjectures by a general microscopic argument. That is ultimately which prevent us from any general proof and we just state well based conjectures.

We now see that the electric WGC (eWGC) can be motivated from the remnant's argument such that, if exists the particle verifying (3.2), then the BH has always open an (energetically favorable) decay channel during each stage of the process of Hawking evaporation. Then the eWGC particle avoids remnants!

Next, let us briefly motivate the magnetic WGC (mWGC), following the original arguments by [1]. They argue that, as well as there must be a light charged particle with $m_{\text{el}} \lesssim g_{\text{el}} M_P$, such an statement should also hold for magnetic monopoles:

$$m_{\text{mag}} \lesssim g_{\text{mag}} M_P \sim \frac{1}{g_{\text{el}}} M_P \ . \tag{3.6}$$

Monopole masses are, in some sense, a probe of the UV cutoff of a $U(1)$ gauge theory. They have a mass at least of order the energy stored in the magnetic field generated by themselves, which is linearly divergent, and if the theory has a cutoff Λ, it is of order

$$m_{\text{mag}} \sim \frac{\Lambda}{g_{\text{el}}^2} \ . \tag{3.7}$$

As a consequence, replacing (3.7) into (3.6) one obtains the mWGC (3.3).

Another heuristic argument, also provided in [1], consist of demanding that the minimally charged monopole is not a black hole [41]. Taking into account that the monopole mass is of order $M_{\text{mon}} \sim \Lambda/g^2$ and its size $R_{\text{mon}} \sim 1/\Lambda$. The condition of monopole not being a Black Hole:

$$\frac{M_{\text{Mon}}}{M_P^2 R_{\text{mon}}} \lesssim 1 \tag{3.8}$$

leads us again to (3.3).

Therefore, it is clear why it is called magnetic and can actually be regarded as a dual version of its electric partner conjecture.

Finally, let us present the conjecture motivating the tower WGC versions [40].

[41] This should not be the case, because the values of all charges carried by a Black Hole should be macroscopic (and continuous), as it corresponds to a classical concept.

Completeness Conjecture.
A theory with a gauge symmetry, coupled to gravity, must have states of all possible charges (consistent with Dirac quantization) under the gauge symmetry.

Without going into more detail, in [11] one can find a nice argument for it. Take an $U(1)$ gauge theory coupled to gravity (that is, (3.1) without the suspensive points) and consider a global limit for the generalized symmetry $A_\mu \to A_\mu + \sigma_\mu$ [42] in a non trivial topology $\mathbb{R}^{1,2} \times S^1$. A way to break it is to introduce charged matter, because its gauge covariant derivative is not invariant under it, only under its exact part $\partial_\mu \lambda$, that is, the usual local gauge transformation. Then in order to avoid the generalized global symmetry, we expect that the $U(1)$ gauge symmetry always has matter charged under it.

The above argument only needs a single charged state, but there is an extension of the previous argument [41] to discrete generalized global symmetries, which requires all possible charges in order to break also any possible discrete symmetry. We did not really understand the details within [41], so we will not be able to compare our future results and examples with it. This could be highly relevant, as will later realize that the requirement of "all possible charges" gets directly in tension with our findings for the $N=1$ orientifolds.

3.3 Swampland Distance Conjecture (SDC)

3.3.1 Statement

Again, we firstly present the statement of the conjecture and its best-known refinement, such that it serves as a reference for the reader along the project. Let us start with the original formulation [2].

Swampland Distance Conjecture.

- *Consider a theory, coupled to gravity, with a moduli space $\mathcal{M}$ which is parametrized by the expectation values of some field ϕ^i which have no potential. Starting from any point $P \in \mathcal{M}$ there exists another point $Q \in \mathcal{M}$ such that the geodesic distance between P and Q, denoted $d(P,Q)$, is infinite.*
- *There exists an infinite tower of states, with an associated mass scale M, such that*

$$M(Q) \sim M(P) e^{-\alpha d(P,Q)} \tag{3.9}$$

where α is some positive constant.

This was the first version, but some extensions appeared since that time. Even though it will not play any role in this thesis, we would like to provide the reader with its major refinement:

[42] With $\partial_{[\nu}\sigma_{\mu]} = 0$.

Refined Swampland Distance Conjecture.

- *Consider a theory, coupled to gravity, with a moduli space which is parametrized by the expectation values of some fields which have no potential. Let the geodesic distance between any point $P \in \mathcal{M}$ and another point $Q \in \mathcal{M}$ be denoted $d(P,Q)$. There exists an infinite tower of states with mass scale M such that*

$$M(Q) < M(P)e^{-\alpha \frac{d(P,Q)}{M_P}} \tag{3.10}$$

if $d(P,Q) \gtrsim M_P$.

- *The first statement holds even for fields with a potential, not just for moduli, where the moduli space is replaced with the field space in the effective theory.*

Which was proposed in [4], where tested in the context on weakly and strongly curved backgrounds under super-planckian spacial field variations and α was found to be $\mathcal{O}(1)$.

3.3.2 Motivation

The SDC was formally introduced by [2] among other geometrical conjectures [43]. It has been widely explored because of two reasons. Firstly, it is in general easier to test than other conjectures in [2] because one knows from the beginning what to search for. Secondly, its consequences are striking, hiding a great potential as it will become clear later.

The SDC behaviour (3.9) is far from new. It appeared since the first compactification which has been performed: Kaluza-Klein on a circle [44]. In fact, the SDC is linked to a moduli space and these moduli spaces are naturally associated to compactification (massless) modes as discussed before. As a matter of fact, we can regard the *compactification of a field theory on a circle* as the starting point for the SDC. Thus, we will briefly comment the crucial ideas regarding it and recommend [11] for the details.

We start with a $D = d + 1$ dimensional space-time, taking the last spatial dimension $X^d \simeq X^d + 1$ to be a compact circle with radius R. Our goal is to determine the effective theory in the remaining non-compact dimensions coming from compactifying a field theory containing D-gravity and a massless D-dimensional scalar field. Working in Planck units, one can write the D-dimensional metric in the Einstein frame as:

$$ds^2 = G_{MN}dX^M dX^N = e^{2\alpha\phi}g_{\mu\nu}dX^\mu dX^\nu + e^{2\beta\phi}(dX^d)^2 \tag{3.11}$$

with $\alpha^2 = \frac{1}{2(d-2)(d-1)}$ and $\beta = -(d-2)\alpha$ in order to get the Einstein frame. Now, from

[43] Some of them will be presented in subsection 3.5.

[44] For a complete reference regarding to Kaluza-Klein gravity and the evolution of the field we recommend [42].

$$2\pi R = \int_0^1 \sqrt{G_{dd}}\, dX^d = e^{\beta\phi} \tag{3.12}$$

we can start to visualize the exponential behaviour on the moduli space distance. Here the moduli space is parametrized by the d-dimensional vev of the dilaton/radion ϕ and is just $\mathscr{M} = \mathbb{R}$, so the distance in the moduli space is just the value of ϕ itself. Besides, it can be checked that when introducing the ansatz (3.11) in the D-dimensional Einstein-Hilbert action one obtains d-dimensional Einstein-Hilbert action plus a canonically normalized term for the dynamical dilaton (as mandatory in the Einstein frame).

We can now take a massless D-dimensional scalar field Ψ and, using periodicity of the compact dimension, expand it in a Fourier series:

$$\Psi(X^M) = \sum_{n=-\infty}^{\infty} \varphi_n(X^\mu) e^{2\pi i n X^d} \tag{3.13}$$

where φ_n constitute a tower of d-dimensional scalar fields and n is the so-called momentum number coming from the fact that the momentum is quantized along the compact direction.

Taking, for simplicity, the Minkowski metric $\eta_{\mu\nu}$, one obtains from the massless Klein-Gordon (KG) equation of motion for Ψ the equation of motion for the φ_n modes giving us a massive KG equation with the following masses for the KK modes:

$$M_n^2 = \left(\frac{n}{R}\right)^2 \left(\frac{1}{2\pi R}\right)^{\frac{2}{d-2}} . \tag{3.14}$$

So we find in the effective d-dimensional theory a massive tower of states with a KK mass scale given by the first massive mode ($n = 1$). It clearly experiences the SDC behaviour (3.9) when introducing (3.12) into (3.14)! This can be considered the first encounter with the SDC and, historically, its starting point.

In [11] one can also find a nice extension of the same compactification for strings (String theory) were, as expected, the winding modes enter the mass expression with the opposite dependence on the moduli (here the radius R) and so on the distance. One finally finds in the Einstein frame:

$$M_{(n,w)}^2 = \left(\frac{n}{R}\right)^2 \left(\frac{1}{2\pi R}\right)^{\frac{2}{d-2}} + (2\pi R)^{\frac{2}{d-2}} \left(\frac{wR}{\alpha_0'}\right)^2 \tag{3.15}$$

with w the winding number and α_0' coming from taking out the R scaling in α'.

That is just a reflection of *T-duality*, which exchanges the KK and winding towers such that the product of the mass scale of both towers remains constant. Therefore, when there is a great displacement in any direction, it causes that one of the towers must tend to become massless in order to compensate the other getting more and more massive.

In fact, the SDC behaviour (3.9) is purely stringy. In the sense that the presence of the winding sector guarantees that, independently of the sign of the dilaton displacement, there is always one tower (the KK or the winding) becoming massless exponentially on such displacement. From (3.15) and (3.12) we can easily observe that the tower of states becoming light is the KK tower when

$\triangle\phi \leq 0$ whereas the winding tower if $\triangle\phi \geq 0$. Thus, we can affirm that the SDC is not a QFT but a QG conjecture.

Remarkably, the above compactification was performed at a classical level, such that we expect quantum corrections causing instability of the towers, as well as possible decay to vacuum by energetical reasons and the fact of not being charged under a symmetry. At this point is where comes the role of the $S^1 \simeq U(1)$ isometry. It suggests that a more general KK ansatz (3.16) is possible, so that we obtain a KK gauge field A_μ[45] in the non-compact space, under which the tower of φ_n states is charged with $q_n(A) = 2\pi n$ and so protected from (most of) instabilities and vacuum decay.

Modifying the metric (3.11) to include the diagonal fluctuations, we find that the ansatz preserving the Einstein frame condition is:

$$ds^2 = G_{MN}dX^M dX^N = e^{2\alpha\phi}g_{\mu\nu}dX^\mu dX^\nu + e^{2\beta\phi}(dX^d + A_\mu dX^\mu)^2 \tag{3.16}$$

and dimensionally reducing the D-dimensional Einstein-Hilbert action gives us also a gauge field kinetic term in the non-compact space for the aforementioned $U(1)$ with gauge coupling:

$$g_{(A)} = e^{(d-1)\alpha\phi} = \frac{1}{2\pi R}\left(\frac{1}{2\pi R}\right)^{\frac{1}{d-2}} \tag{3.17}$$

Therefore, from $q_n^{(A)} = 2\pi n$, (3.17) and (3.15) we obtain a familiar relation between the charge and mass of the KK states:

$$g_{(A)}q_n^{(A)} = M_{n,0} \tag{3.18}$$

That's pretty like the lower bound of the eWGC (3.2). Actually, we know that we can trust the effective theory, at most, up to energies near to the mass scale where the higher order states become relevant and we really have the D-dimensional UV complete theory, that is when $n = 1$, and so we have a higher bound for the cutoff of the theory:

$$\Lambda \lesssim M_{1,0} \sim g_{(A)} \tag{3.19}$$

that is like the mWGC (3.3). Please note, as an asside, that in these cases we actually encounter a lattice of charges populated and so the Lattice WGC discussed before.

Coming back to the stringy example, clearly we will find the same gauge field coming from the gravitational sector but also a second gauge field V_μ coming from the B-field with one leg in the compact dimension. A similar analysis (look at [11]) leads us again to:

$$g_{(V)}q_w^{(V)} = M_{0,w} \tag{3.20}$$

where the winding modes have charge $q_w^{(V)}$ under V_μ, whose gauge coupling is given by $g_{(V)}$.

Both gauge fields A_μ and V_μ, as well the KK and winding modes charged respectively under them, are exchanged in the theory by the aforementioned T-duality. Let us be a bit more explicit. We have

[45] Being the gauge transformation $A_\mu \to A_\mu - \partial_\mu\gamma(X^\nu)$, $X^d \to X^d + \gamma(X^\nu)$.

already seen that $M_{KK} \equiv M_{1,0} \sim g_{(A)} \sim e^{\alpha\phi}$ and one can check that $M_w \equiv M_{0,1} \sim g_{(V)} \sim e^{-\alpha\phi}$. A T-duality transformation exchanges momentum and winding number at the same time as the radius R and its inverse $\sqrt{\alpha'}/R$. Thus, both KK and winding sectors are exchanged by T-duality. In fact, these relations permit us to discern an identification between the towers of states linked to the mWGC cutoff Λ and the SDC for this example. From the work of [4],[5] on, multiple of papers on the topic look for such identification (also commonly called "duality") within their settings and/or implicitly assume it must be there.

We observed that the mass depended on the charge number, so it is protected by $U(1)$ gauge symmetry. It is actually here where the relation between the WGC and SDC, and a germ of the BPS condition can be firstly visualized through stability. This point will be essential in our project and so will deserve the full subsection 3.8 for it. We would just like to advance that *stability* played apparent no role so far in the definitions, even if noted in the examples, but since the work of [5] seems that it is a kind of new implicit requirement and we will see that it might be too strong.

A final remark is that while the WGC has a clear physical motivation from arguments regarding to avoidance of BH remnants and global symmetries in QG, the fundamental origin of the SDC is yet unclear, even though, we have seen already (and will see later in an $N = 2$ highly non-trivial example) that both conjectures, in some contexts, may be not exactly independent. Precisely, both are supposed to be connected by a recent proposal called Emergence, whose details we are going to expose in the next subsection.

3.4 Emergence Proposal

3.4.1 Statement

Firstly, we proceed to present the statement of the proposal such that it serves as a reference for the reader along the project. The following formulation comes from [11] and represents a compromise between the original ideas proposed in [5], [43], [44] and [45].

Emergence Proposal. *The dynamics (kinetic terms) for all fields are emergent in the infrared by integrating out towers of states down from an ultraviolet scale Λ_s, which is below the Planck scale.*

At a practical level this means that, at the (unknown a priori) UV scale Λ_s, the renormalization group (RG) flow has a boundary condition on all the fields forcing them to have vanishing kinetic terms. That is the viewpoint of [5] and [43]. Nevertheless, assuming IR coupling is weak, this effectively leads to the same results as just taking the classical contribution to the kinetic terms in the UV to be subdominant to the one-loop RG running down from the UV. This second picture was defended by [44] and [45]. As we observe, there exist some subtle differences between them, even though they have no real consequences at a practical level and play no role in this project.

Let us just mention, that in [11] an extension to *Emergent potentials* is firstly discussed and also a check for Emergence, in the previously discussed KK example, regarding the 1-loop contribution for the scalar kinetic term, observing that the Emergence proposal is consistent under dimensional reduction. Remarkably, taking the D-dimensional and the non-compact effective d-dimensional towers, [11] finds that they cannot be decoupled! Applied to the important case of $N = 2$ Type IIB CY3[46],

[46] This point will become clear after reading subsection 3.6.2.

we would expect that already the kinetic term for the RR C_4-form would actually be emergent from integration out of D3-branes in ten dimensions.

3.4.2 Motivation

Emergence is a proposal whose main objective is to explain from microscopic physics the Swampland conjectures. There is some evidence for it (based on arguments coming from CP^{N-1} toy model [11] and a detailed $N=2$ Type IIB CY3 analysis [5]) and a number of outstanding relations which follow from it, linking together the WGC and the SDC (and also the species scale Λ_s). Nevertheless, one should keep in mind that these analysis are, so far, approximations (one-loop usually) and no exact computation has been performed to check it factor by factor (mainly because the UV theories are usually unknown and so a rigurous “integrating out" procedure cannot be performed yet). Moreover, a certain behaviour for the towers of states is always assumed.

These kind of ideas really start with the analysis of Seiberg-Witten [23], where massless monopoles were responsible for the singularities in moduli space, and was followed later by [7], taking care of global symmetries when approaching the conifold (one state); but applied instead to infinite distance loci (infinitely many states). We are going to discuss in subsection 3.7 the latter, because it can be regarded as the germ of the Emergence current formulation.

Let us just present the argument given in [11], in order to obtain the WGC and SDC from Emergence. The aim is to show that the the kinetic terms for the emergent fields are induced at one-loop in the IR from integrating out states, recovering by the way the species scale, the mWGC and the SDC, such that can all be though of as statements relating the kinetic terms of fields in the IR to towers of states which couple to those fields. This kind of relation already exists in QFT because integrating out the towers of states will lead to running for the kinetic terms into the IR. The proposal is then that the two concepts, Swampland constraints and QFT renormalisation, are in fact the same. In order for this to be true, we will see that the UV boundary conditions must vanish (or be sub-dominant enough to the running). A strong way to argue for such boundary conditions is to assume that at this UV scale the fields are actually not dynamical.

Consider a tower of particles with an associated mass scale m, such that the masses scale like $m_n = mn$. We will also consider that the particles are charged and its quantized charge goes like $q_n = n$. Let us now impose emergence of gravity (i.e. $M_P|_{\Lambda_s} \to 0$), keeping just track of the RG flow one-loop order magnitude, such that

$$M_P^2|_{IR} \sim \cancelto{0}{M_P^2|_{\Lambda_s}} + N\Lambda_s^2 \tag{3.21}$$

and then we find $\Lambda_s = M_P/\sqrt{N}$, that is, the UV scale for emergent gravity is the species scale [47], being N the number of species below such scale. The relation between one-loop corrections and the species scale is well-known, and essentially tells us that Λ_s is a scale where gravity gets strongly coupled. Moreover, the kinetic terms for the fields involve spatial derivatives and so seems pretty natural that from this scale dynamics may emerge.

Now, it is simple to relate N and m because we have $Nm \sim \Lambda_s$ and then

$$N \sim \left(\frac{M_P}{m}\right)^2 . \tag{3.22}$$

[47] Energy scale at which gravity becomes strongly coupled. The idea of the species scale is quite old and it played a major role in a well-known proposal for solving the Hierarchy problem [47].

Next, let us impose emergence for a gauge field (i.e. $g|_{\Lambda_s} \to \infty$ for the canonically normalized gauge kinetic term)

$$\frac{1}{g^2}|_{IR} \sim \cancelto{0}{\frac{1}{g^2}|_{\Lambda_s}} + \sum_i^N q_i^2 \ln\left(\frac{\Lambda_s}{m_i}\right) \tag{3.23}$$

obtaining the mWGC for the IR gauge coupling:

$$\frac{1}{g^2} \sim N^3 \sim \left(\frac{M_P}{m}\right)^2 \tag{3.24}$$

such that the scale gM_P is associated with an infinite tower.

Finally, let's impose emergence for a scalar field x (i.e. its kinetic term prefactor [48] verifies $g_{xx}|_{\Lambda_s} \to 0$) using (see e.g. [3]) that the coupling of a scalar to a particle is given through the derivative of its mass respect to the scalar

$$g_{xx}|_{IR} \sim \cancelto{0}{g_{xx}|_{\Lambda_s}} + \sum_i^N (\partial_x m_i)^2 \ln\left(\frac{\Lambda_s}{m_i}\right) \tag{3.25}$$

getting the IR value

$$g_{xx} \sim N^3 (\partial_x m)^2 \sim \left(\frac{M_P \partial_x m}{m}\right)^2 . \tag{3.26}$$

Next, if we compute the proper distance in field space measured by the emergent metric

$$d_\gamma(x) = \int_{x_i}^{x_f} \sqrt{g_{xx}} dx \sim M_P \int_{x_i}^{x_f} |\partial_x(\ln(m))| dx \sim M_P \ln\left(\frac{m(x_i)}{m(x_f)}\right) \Rightarrow m(x_f) \sim m(x_i) e^{-\alpha \frac{d_\gamma(x)}{M_P}} \tag{3.27}$$

we already obtain the SDC with $\alpha \sim \mathcal{O}(1)$!

Lastly, let us note that one can also find a relationship between SDC and De Sitter (dS) Swampland Conjecture (next conjecture to be defined), such that ultimately also this last conjecture might arise from Emergence [46]. We are not going to go into details because in this project the dS conjecture will play no role.

3.5 Other Swampland Conjectures and extensions

For completion, we would like to introduce some important conjectures to the reader, even though many of them will not be studied in our setting. Firstly, those which are directly Swampland conjectures, dividing between the ones which are contained in (sections 6 and 7 of) [11] and two very recent ones not included there, and finally one gravitational conjecture which could be ultimately related really to QG.

[48] Representing the moduli/field space metric.

Permit us start with those conjectures reviewed in [11]. The most important one is the so-called De Sitter Swampland Conjecture [34], and we are going to present here its most well-known refinement [46], which takes care of the counterexamples found for the original version.

The Refined de Sitter Conjecture.
The scalar potential of a theory coupled to gravity must satisfy either

$$|\nabla V| \geq \frac{c}{M_P} V \tag{3.28}$$

or

$$min(\nabla_i \nabla_j V) \leq -\frac{c'}{M_P^2} V \ . \tag{3.29}$$

Here $c, c' > 0$ are constants of order one, and the left hand side of (3.29) is the minimum eigenvalue of the Hessian in an orthonormal frame.

The main motivation for the conjecture is the fact that it is very difficult (if not impossible) to construct de Sitter vacua in String theory. Alternatives have been provided by scenarios building de Sitter vacuum in a rather stringy way but without requiring explicitly a proof that the solution belongs to String theory. The major example of such is the KKLT scenario [51]. Nevertheless, one can view the technical difficulties as fundamental and consider that they hint at a deep obstruction to construct de Sitter vacua in String theory, this is what motivates (3.28) which effectively means that de Sitter vacua are in the Swampland. The reason for the refinement (3.29) was to include a more general argument (not only stringy) for the conjecture based on the entropy of de Sitter space and the (R)SDC. In the phenomenological side, we recommend to the interested reader the following reference [67], where the SDC and the dS Conjectures are employed in the context of Dark Matter.

Next, there are two conjectures which arise directly from applying the WGC to the Stückelberg fields, associated to the extra propagating degrees of freedom for massive fields, with respect to their massless analogue, in the decomposition where they explicitly constitute the mass term of the higher spin field. This actually constraints some properties of the higher spin fields.

We will present the **Spin-1 Conjecture** [52] because is somehow more related to our project, even if it will not be studied in this thesis. It just takes the kinetic term for the Stückelberg field (longitudinal mode in the decomposition $A_\mu = A_\mu^\perp + \partial_\mu \theta$) contained in Proca mass term for a massive gauge field $m^2 A_\mu A^\mu \supset m^2 (\partial_\mu \theta)(\partial^\mu \theta)$ and proposes that the mass term m is an axion decay constant f. Therefore, applying the axion version of the mWGC one obtains automatically:

$$\Lambda < \left(\frac{m M_P}{g}\right)^{\frac{1}{2}} \tag{3.30}$$

with g the massive gauge field coupling. As usual, there are many subtleties which we are not going to discuss here and one can find in [11]. However, let us just say that the conjecture only applies to gauge fields gaining a mass thorugh stringy Stückelberg mechanism (not just simple Higgs mechanism) and so that the $m \to 0$ limit is at infinite distance in θ field space. Then it can be considered more like an application of the SDC than an alternative conjecture.

Although the **Spin-2 Conjecture** [53] is quite unrelated to our work, it has important implications and it comes actually from applying the mWGC to the gauge coupling of the massive gauge mode (helicity-1mode) coming from the massive spin-2 field decomposition (à la Stückelberg decomposition for the case of Spin 2). The procedure is similar to the previous conjecture and one obtains again a cutoff scale

$$\Lambda_m \sim \frac{mM_P}{M_w} \tag{3.31}$$

where m is the mass of the massive spin-2 field coupled to gravity and M_w is a mass scale specifying the coupling of the gauge field to matter current. Remarkably, there exists an extension to it called the **Strong Spin-2 Conjecture**, coming from an analogy to the SDC-WGC relation and the Emergence proposal [53], which asserts that in the case of massive gravity the cutoff scale is given by its mass $\Lambda_m \sim m_{\text{grav}}$, which in case of being correct basically implies that the mass of the graviton must be zero!, because currently m_{grav} has an experimental upper bound which is too tiny so in case of being massive we should observe in the measurable energy scales a huge tower of states associated with the coupling. As this is not the case, the graviton should be exactly massless such that the Spin-2 conjecture does not apply to it. That's an amazing proposal which deserves further studies, we recommend to the interested reader [53] and [11].

Now, we would like to comment on **two more geometrical conjectures**, which are straightforwardly verified in our $N = 1$ examples (section 5) exactly in the same way as in the $N = 2$ ones (e.g. [5]), basically stating that moduli spaces are conjectured to: 1) have finite volume [54] and 2) to be non-compact [2] (therefore it must be that near infinite distance loci some orthogonal direction has to shrink to zero size so as to maintain a finite volume).

Let us just comment that the resulting cone-like structure is related to *two other conjectures* [2]: 1) The scalar curvature of the moduli space approaching infinite distance loci is always negative and 2) there are no finite minimal length one-cycles in the moduli space within a given homotopy class. (This means that if there is an axion in the moduli space, then one can always make its decay constant vanish at infinite distance). With more than 8 supercharges ($N > 2$), these properties can be related to properties of gauge symmetries, such as the *Completeness Conjecture*, and after [5] (and similar works motivated by it) one may include also safely the case with exact $N = 2$.

Of course, one can find in [11] much more details on the previous conjectures and even some more conjectures. However, the ones which follow are not covered in [11].

We would like to start with an extension to the SDC (based on an study for warped KKLT ([51]) in the conifold) by [55] because its very tied to our examples (and to the Emergence proposal) in section 5.

Extension to SDC.
There exist points at finite or infinite distance in moduli space with singular tree-level metric $g^{(0)}_{xx}$ at which towers of modes become lighter than the species scale $\Lambda_s = \Lambda/\sqrt{N}$. Adding these states to the action, they induce a one-loop $\tilde{g}^{(1)}_{xx}$ to the field space metric whose functional form is always proportional to the former tree-level metric $g^{(0)}_{xx}$.

The next conjecture is just a generalization of the SDC to AdS and dS (although in the previous formulations it was not assumed that the non-compact space should be Minkowski). In [48], the authors firstly discuss the well-known case of $AdS_5 \times S^5$ compactification, where we again, as expected, observe a KK tower with analogue properties as discussed before (and later). The non-trivial part is that they relate the massless limit at infinite for the towers to the vanishing cosmological costant (which happens at infinite distance in field space) such that they can extend the treatment to dS by taking the absolute value of it and using the rest of the interconnected conjectures. As a consequence they propose:

Generalized Distance Conjecture.
Consider the non-compact space to be an Einstein space, i.e. AdS, Minkowski, or dS (if it exists). Then for large distance variation in fields, we get a light tower of states in the Einstein frame of the external effective theory, whose mass scale in Planck units is given by

$$m \sim e^{-\alpha \triangle} \tag{3.32}$$

where $\alpha \sim \mathcal{O}(1)$

The following conjecture/proposition has no name yet so permit us call it, temporarily, the K-charge Cancellation Conjecture[49]. In [49], the authors realized that dS Swampland Conjecture would be falsified if a setting with a single non-BPS D7-brane in classical fluxed Type IIA orientifold vacua would form part of the Landscape. The logical consequence of assuming that dS conjecture is correct is that K-theory charges cancel on the compact space. Extending the reasoning to any setting, one would find

K-charge Cancellation Conjecture.
If the (refined) de Sitter swampland conjecture is correct, then the K-theory charge on a compact space has to be trivial.

Let us just remark that the charges of the D-branes are non completely determined by Homology/Cohomology (that is RR charges) but there may also exist branes, called non-BPS branes, which do not carry any RR charge but are charged under discrete $\mathbb{Z}_2$ symmetries, such that they are no subject to tadpole cancellation conditions. Both cases are described by the so-called K-theory. It turns out that there can be single stable non-BPS branes but a stack of two is unstable and decays[50]. The natural question we should ask ourselves now is if the total K-theory charge on a compact space must cancel (i.e. it is even) in order to generalize the tadpole cancellation condition. That is the motivation for the above proposal. Another paper deepening on the topic is [66].

Finally, it is the turn of the non-Swampland conjecture. That is the famous Penrose's (Weak) Cosmic Censorship!, stated in [59]. The idea is that singularities arising in Einstein's equations are usually hidden by event horizons, such that cannot be observed in the outside region and so classical causality is protected. However, there exist Black Hole (BH) solutions like Reissner-Nordström (charged non-rotating BH) where the singularities are hidden for subextremal and extremal solution ($M \leq |Q|$) but, formally, nothing forbids us to take the superextremal case ($M > Q$) where the $r = 0$ singularity

[49]Similarly to the Spin-1 Conjecture, it is actually not a Swampland Conjecture but a proposed consequence of it.
[50]Making them useless for our future purposes as we will comment in subsection 6.1.

is naked (i.e. is not surrounded by a horizon). Mathematically nothing is wrong with these solutions but physically seem to be rather non-sensical, in general one would expect a classical theory like GR to be causal (and deterministic). Penrose's ideas are very difficult to state in a completely formal way (it is hard for example to formalize the notion of singularity) and, precisely the lack of formality gives enough space to have (at least) two independent formulations as follows:

Weak Cosmic Censorship Conjecture.

Mathematics: For generic initial data, the maximal Cauchy development posseses a complete null infinity.

Physics: There can be no singularity visible from future null infinite. No "naked" singularities can exist in nature, other than the Big Bang singularity.

Strong Cosmic Censorship Conjecture.

Mathematics: The maximal Cauchy development of generic compact or asymptotically flat initial data is locally inextendible as a regular Lorentzian manifold.

Physics: GR is a measurable and deterministic theory, and is predictable from the initial data with zero radius ($r = 0$) and would continue to be predictable with all other radius values, including final data at radius equals to infinity ($0 \leq r \leq \infty$).

Remarkably, these two conjectures are independent at a mathematical level such that one can find space-time solutions where one is valid and the other is violated and viceversa. A very recent interesting case where (a strong version of the) Strong version is violated is [65].

There is evidence that the Weak version does not hold in more than four dimensions, because there are unstable black holes in higher dimensions, and numerical evolution shows that the horizons break down in finite time. This produces regions of arbitrarily large curvature generically from smooth initial data. A promising class of counterexamples to cosmic censorship in four dimensions with asymptotically AdS boundary conditions has been conjectured in [56] with ample numerical evidence provided in [57]. They are based on coupling gravity to a Maxwell field and a negative cosmological constant $\Lambda < 0$.

During several decades there was intense investigation trying to find the way to "look beyond the horizon" and these counterexamples opened the door to such a possibility. However, Cumrun Vafa, in a private communication, suggested, to the authors of [58], that the counterexamples might be removed if the WGC holds. In fact, [58] explores the connection between both "Weak" conjectures, finding that in fact WGC avoids such counterexamples. Rather than dealing with the difficult QFT in curved spacetime problem, the charged matter was modeled by a classical charged scalar field. It was found that if the charge to mass ratio of the scalar field was large enough, the original Einstein-Maxwell solutions became unstable to turning on the scalar, and the electric field did not diverge. Remarkably, the minimum value of this ratio (sufficient and necessary) to preserve the cosmic censorship was proven to be precisely the weak gravity bound. Further evidence was found in [50].

Suddenly the picture changed, it seems that QG and not really GR conspires against "naked singularities" in nature. This connection between both conjectures is intriguing, deserves further study and gives a new strong argument in favor of the WGC, turning it into the most important and solid conjecture, by far.

3.6 Non-trivial tests of the conjectures

Up to now, we have shown the two main conjectures, the Emergence proposal (and some other conjectures whose relevance in our setting will be less important). We have motivated them with several arguments and some “trivial" examples. Nevertheless, in order to take them seriously and sharpen the statements, more evidence through more complicated models is clearly demanded. Let us give an overview on the evolution of research around the WGC, SDC and Emergence, surely biased by our project interests.

3.6.1 State-of-art

Many tests where performed along the last decade regarding to the WGC, mainly because it was stated during the first years of the “Swampland program" and was already clear from the beginning that its implications would be incredibly powerful. We are not going to focus on the following tests, but just provide the reader with a general overlook and some references treating several explored guidelines. It is fair to begin with the most prominent area of research around the WGC, that is INFLATION, where some examples are [35], trying to get an answer on why several inflation models seem to resist an embedding in String theory by means of the contraints that arise from the WGC, and [60],[61] both investigating Large Field Inflation [51] constraints from the WGC. Another line of research was that attempting to DEEPEN INTO THE FUNDAMENTAL KNOWLEDGE ABOUT THE CONJECTURE AND TO GENERALIZE IT. In this case the most influencial paper is [3], where E. Palti develops a exhaustive bound state analysis of the conjecture [52] and also discuss important generalizations, including the scalar version aforementioned. Also, we would like to highlight [43], where the origin of Emergence, regarding the WGC, can be found. Finally, the aforementioned relation between WGC and Weak Cosmic Censorship [58] is taking center stage in recent years. Lastly, there is a branch using ALGEBRAIC GEOMETRY WITHIN CALABI-YAUS, where a very recent example regarding to the axionic WGC and Type IIB orientifolds is [68]. These can be considered as the main lines that focus only on the WGC.

The (R)SDC was also proposed rather early but one could say that it has really been subject of intense study in the last three-four years. That is because it is in general technically hard to test and the Physics around it was not clear. Here, REFINEMENT and TESTING can be regarded as the main research lines. One could say that the refinement by [4] renewed the interest of the community, playing a major role. For a case by case testing reference we would like to cite [62], where explicit tests in CY moduli spaces were performed. Nevertheless, the main testing reference is [5] (contains a single-divisor analysis, look at [63] for a multi-divisor discussion), where the authors introduced useful and strong mathematical tools towards generalize the testing process to any Calabi-Yau.

The idea that this two (and more) conjectures could be linked can be attributed to [44]-[45] and [5] (clearly influenced by the previous ideas of [43]). Thanks to them and the highly non-trivial test of

[51]The research binding WGC and Large Field Inflation (that is, a potential, generated by a large inflaton field displacement, carries out Inflation), experienced a sudden surge of interest with the BICEP2 Collaboration [76] pointing to a tensor to scalar ratio according to Large Field Inflation. The experimental results were shown to be false later but Large Field Inflation is not discarded.

[52]Which has a clear influence in this thesis and some of his ideas form part of the realm of our subsection 3.8.

[5], arose the Emergence proposal. From [5] on, the main research current within both conjectures was to study diverse settings where both conjectures and Emergence could be checked.

One can divide such (String theory motivated) settings into two classes: $N \geq 2$ and $N \leq 1$ (bulk) theories. Within the former, the principal one is Type IIB CY CS moduli space [5], which will be described with more detail in subsection 3.6.2. A generalization of the previous setting to Type IIA CY3 Kähler moduli space was performed in [19], where the expectancies from mirror symmetry mapping were checked. Also in [19] interesting links to M-theory and F-theory were treated. In [9] the role of instantons in Type IIB hypermultiplet moduli space is discussed and matched, by means of the c-map, to [5] results. Regarding to the case of $N > 2$, [11] states that, using [64], the SDC holds by means of group theoretical arguments, and, due to the higher number of supercharges, we will continue having BPS states, so we expect the whole WGC-SDC-Emergence picture introduced in [5] holds trivially.

With respect to $N = 1$, a first study within the "Swampland program" was performed in Heterotic by [12] using modular symmetries and focussing on the dS Conjecture. Later, [55] discussed both conjectures and a bit of emergence in warped KKLT (with fluxes).

Our project can be framed in this last line of investigation, so that we face the non-trivial task of testing both conjectures and their connection through Emergence for unfluxed $N = 1$ Orientifolds. As we will discuss later, our main goal is to check if one can generalize the picture of [5] to orientifold (SUGRA) background, where many of the bulk fields are projected out and the resulting SUGRA multiplets have drastically changed. Let us remark that during the period of this project another work related to $N = 1$ orientifolds was performed by [10], where the picture of [5] was tested for BPS D-domain walls and the SDC behaviour was sketched for D-strings and D-particles in $N = 2$ and/or $N = 1$ untwisted sector. A comparison between our project and their paper will be presented in subsection 5.2.4.

As a final remark, just add that the previous studies were realized taking into account only the bulk sector. Nevertheless, we discussed before that in the open string sector we can also have global symmetries associated to the worldvolume Dp-brane gauge theory. This case is worked in [28], finding that the open string global symmetry limit corresponds to the bulk infinite volume limit (Kähler moduli space) and so finding again agreement with [5] picture, where SDC and Sub-Lattice WGC towers are dual.

3.6.2 $N = 2$ Type IIB Calabi-Yau threefold complex structure moduli space

In this subsection we will summarize the important parts of the $N = 2$ Type IIB CY3 CS moduli space case, which is the starting point for our project, and provide the reader with some comments on delicate aspects. The original results come from [23] (WGC) and [5] (Monodromy Orbit, SDC and Emergence). A complete review can be found in [11]. More details, especially those concerning computations, will be provided in the examples of section 5.

Overall Summary.

Beginning with the eWGC with Scalar Fields (**eWGCSF**) (also called sometimes Gauge-Scalar WGC), we clearly observe from (2.26) and (3.4) that the D3-BPS particles (2.42) verify the lower bound of the conjecture and clearly imply the usual WGC (as the second term in the right hand side is clearly positive definite) $m^2 \leq \mathcal{Q}^2$. An explicit check in the LCS for $q_I = (0, q_i)$, $p^I = 0$ can be found in [23] [53] and, due to the symplectic structure/Electric-Magnetic duality, it is expected to hold in general for arbitrary values of q_I and p^I. Furthermore, from (2.42) we observe that it may be possible to

[53] Also in [23] one can find an analogous computation for the axionic WGC in the case of E2-instantons in Type IIA CY3.

build stable towers with these BPS branes verifying both the Sub-Lattice WGC and the Completeness Conjecture. In [5] such towers were found as we will discuss soon.

Let us sketch the explicit proof for $q_I = (0, q_i)$ and $p^I = 0$, which is a short and straightforward computation. In such a setting and with our conventions we obtain $\mathcal{Q}^2 = -\frac{1}{2}q_i(\mathscr{I}^{-1})^{ij}q_j = e^{K_{cs}}(q_i G^{ij} q_j + 4(q_i u^i)^2)$ (using (2.16), (2.20) and (2.29)) and $m^2 = e^{K_{cs}}|q_i x^i|^2$ (by means of (2.16), (2.20) and (2.42)). Now, we choose $v^i >> u^i$ in $x^i = u^i + i v^i$ for simplicity and we just need to plug in and compute, finding that the RHS and LHS of (2.26) agree.

Next, permit us focus on the **SDC**. In the context of Type IIB CY3 CS moduli space, [5] realized that a clear candidate for the towers of states, whose mass decreases exponentially in the geodesic distance, are the D3-BPS particles at infinite geodesic distance loci. Their study focuses on the LCS points where just one of the v^i goes to infinity (i.e. a single divisor treatment) because the multi-divisor analysis is much harder (look at [63] for more information). In these cases, one can check [54] that the exponential factor in (2.42) can be written as e^{-ad_γ}, with a an $\mathcal{O}(1)$ constant which, in general, depends on the path and will be determined later in some cases. Therefore, we clearly observe that the mass fits into the SDC behaviour (3.9), concretely one finds (4.1), observing that the cases with $p^I = 0$, i.e. electrically charged D3-BPS particles, generally experience the (decreasing) exponential behaviour.

At first sight, one would say that we are done because we can just take whichever among the electrically charged cycles and just superpose a tower of them wrapping n-times the electric cycles, obtaining the SDC towers. Notwithstanding, [5] introduced for first time a requirement, which from that paper on is usually implicit, in the definition of the SDC conjecture: *stability*. It will be the topic of subsection 3.8, but we will treat it a bit now because it is a crucial condition in the work of [5]. There, they just focus on the *BPS stability* because treat with BPS particles. The point is that, even though we know the mass of the so-called would-be BPS states of a given charge, we do not really know if such states are already populated. BPS states may only decay to BPS states such that the aforementioned superposition way to build the tower is not guaranteed to be stable and so populated. The reason for this, is that for a BPS state with, for instance, charge $q = 2$ is energetically (marginally) possible to decay to two states with charge $q = 1$ as the mass after and before is the same (energy conservation) and charge is clearly conserved. Thus, if we want to be sure that our tower is actually populated [55] we need to refine our analysis and search for other candidates.

There is a physical intuitive way to guess one stable tower for each single-divisor LCS limit, that is, to make bound states using the basis of D3-particles given by each term of (4.1). It is clear that in the LCS the SDC behaviour is guaranteed always for the the D3-branes wrapping A^0 and for the ones wrapping A^i, as long as the $x^i \neq 0$. Then for $v^i \to \infty$, the towers of bound states formed by D3 wrapping n-times A^0 and D3 wrapping once A^i clearly verify the (stable version of the) SDC.

On the other hand, in order to find the stable tower of states, there is a precise mathematical formalism introduced in [5], called *Monodromy Orbit formalism*. The idea is to use the BPS structure such that the D3-particles mass depends on the charges forming a symplectic structure. Next, [5] proposes to use monodromy actions on the period vector, which have a natural action on the charges of the would-be BPS states. Starting from a given charge, the monodromy action will transform it, and acting once and again with the monodromy determines an orbit through the possible charges called the monodromy orbit. At infinite distance, the monodromy is of infinite order, so that the monodromy orbit contains either one element, if the action of the monodromy on a charge is trivial, or an infinite number of charges. As a consequence they, roughly, propose that a candidate for an

[54] We will do it explicitly in section 5.

[55] BPS to BPS decay is not completely understood, in fact, it is not clear if it happens or not. It is just energetically possible so one should not consider this option when searching for stable states.

infinite tower of states becoming massless at infinite distance is generated by an infinite monodromy orbit starting from a single BPS state. The formal statement is more involved and includes several caveats. By means of it, they find several different singularities clasiffied by an index $d = 0, 1, 2, 3$. Where the $d = 0$ are finite distance singular loci (containing the conifold loci) and the rest are located at infinite distance. We are not going to get into technical details and just take as a result that their analysis allows us to identify an infinite stable tower of massless particles formed by bound states of D3-branes wrapping A^0 n-times and D3-branes wrapping the A^i cycles ([10]), as was argued before from a physics point of view. For the complete treatment, please follow [5] and [63], as well as [11] for a clear short-cut treatment which gets into more technical details than here but less than the previous two references.

The **link between both conjectures in** $N = 2$ is stablished by the two following facts:

1. *SDC-mWGC nexus*: [5] identifies the SDC tower with bound states of the aforementioned charged D3-BPS particle states and realizes that, as $d(P, Q) \to \infty$, the SDC tells us that infinite distances cannot be described by an EFT with finite cutoff. Similarly, the mWGC (3.3) states that global symmetries ($g \to 0$) cannot be described by an EFT with finite cutoff.
2. *SDC-eWGC nexus*: the D3-BPS particles are charged and verify, also the eWGC with Scalar fields (concretely its Completeness-tower version, the Sub-Lattice WGC), establishing a "duality" between both conjectures. [56]

The authors relate both statements and towers such that, similarly to the WGC, the SDC can be understood as a QG obstruction to fundamental global symmetries in this setting. Thus, following this line of thought, it is tempting to propose that they can emerge after integrating out the infinite towers of states, similarly to what happened in the conifold singularity resolution by [7] [57], leading to the Emergence proposal in [5].

The **Emergence proposal** is formulated and checked in this setting also by [5] [58]. The origin of the idea is that the String theory setting is perfect for testing the Emergence proposal because it has been known since [7] that String theory automatically integrates out wrapped D3 branes. This is true both for the CS moduli but also for the gauge couplings of the closed-string $U(1)$ gauge fields in the vector multiplets. The prime example is the conifold singularity [7]. It is pretty analogous to the understanding of Seiberg-Witten theory [33] in terms of integrating out monopoles. So when studying the moduli space of String theory, it is known that already the tower of states has been integrated out. The test they perform is then to compare the integrating out by hand analysis, with the properties of the bulk moduli space. The Emergence proposal would predict that they should give the same answer. In this precise setting, the authors observed that if one adds a tower of massive states to (2.21) (charged D3-BPS particles) and integrates them out, one obtains the correct behaviour (no exact matching of prefactors just approximately) of the gauge kinetic function at energies below the lightest state of the tower and also recovers the distance behaviour. The argument and computations are analogous to those provided in subsection 3.4.2 and rely on the fact that the towers have a very concrete structure with constant mass gap. For more information, look at sections 5 and 6 of [5].

Finally, we provide the 1-loop corrected gauge kinetic function, which will be important later:

[56] Up to possible caveats which will be explained in comments of next pages.

[57] In that case there was just one singular state instead of an infinite number of them. See subsection 3.7 for more information.

[58] For a concise but detailed summary, read section 5.3 of [11].

$$\mathscr{I}_{IJ} \simeq \cancelto{0}{\mathscr{I}_{IJ}^{UV}} - \frac{8}{3\pi^2}\sum_{k}^{S}(q_{k,I}q_{k,J}\ln\frac{\Lambda_{UV}}{m_k}) \tag{3.33}$$

where m_k and $q_{k,I}$ are the mass and charge of the k-th particle in the tower, $\mathscr{I}_{IJ}^{UV} = 0$ is the Emergence proposal ansatz, $\Lambda_{UV} \sim \Lambda_{\text{species}} = \frac{M_P}{\sqrt{S}}$ and S is the number of species (below the species scale).

Comments and details.

The following remarks and ideas are not indispensable for understanding the $N = 2$ picture of [5], so that the reader can safely continue without reading this part. Nevertheless, we include them because might help to increase the general understanding on the topic and some of them will appear later, with a renewed strength, in our setting.

- **No potential for the CS moduli.**

  Let us note that for $N = 2$ Type IIB CY3 there is no potential and so all the moduli remain massless, such that we can test the original conjectures and there is no caveats regarding masses/potentials for the moduli. For the $N = 1$ unfluxed orientifold case we are going to discuss later, that is not true and we have potential for the moduli generated by the space-time filling D3/D7 branes. However, that will not be relevant for our study because the CS moduli will remain massless without fluxes [59] and, as a consequence, we can test the SDC and WGC without potential.

- **Do the non-BPS D3-particles verify the eWGCSF.**

  We would like to clarify that, even if for the BPS we found the bound of the eWGC with Scalar Fields, it is not so obvious that the D3-particles verify the correct inequality ((3.4), (2.26)) when displacing a bit from the BPS condition. On the one side, it is clear that the LHS remains in such a case because $\mathscr{Q}^2$ just depends on the bulk, which remains unchanged. On the other hand, the first term of the RHS will grow because the BPS condition was the lower bound on the mass (for non-BPS states $m > |Z|$) and in the second $g^{i\bar{j}}$ and $\partial_i K$ remain because $g^{i\bar{j}}$ and K depend on the unchanged bulk and again the mass grows. The only contribution we do not know if grows or decreases is the associated with $\partial_i m$ because it would require the knowledge of the analytic dependence on the CS moduli for the non-BPS mass, that is unknown. Therefore, we observe that it is not obvious that the non-BPS D3-particles verify the eWGC. Although in [28] the authors checked for six dimensional F-theory compactifications that indeed there was towers of non-BPS objects verifying the WGC, we are not aware of any study testing explicitly that the same happens for the non-BPS D3-particles in this $N = 2$ setting.

- **Apparent tension between bound states and eWGCSF.**

  The aforementioned bound states, consisting of D3 wrapping A^0 n-times - D3 wrapping A^i once, are supposed to be stable because there would exist a gravitational binding energy such that their mass is lower than the sum of the isolated masses. In order to form such bound states they need to attract more than repel and so the LHS in (2.27) must be overcome by the RHS. Therefore, if one assumes that in the non-BPS case the WGCSF (2.26) is verified then seems pretty difficult to imagine how could (2.27) have the opposite inequality in order to make the bound states (i.e. gravitational and scalar interaction greater than gauge interaction). A priori, from (2.26) for both D3 A^0 and D3 A^i, and (2.27) for the interaction between both, one would

[59] Recall subsection 2.3.

say that in the non-BPS case both are most likely to experience the same inequality. Again, the derivative term on the mass prevents us from making a more precise statement.

In adittion, BPS states verify the equality in (2.27) and so it is difficult to imagine how to make bound states with them if they do not feel mutual net force.

- **Are superextremal particles and tower versions of eWGCSF compatible?**

  For BPS states we saw that stability is guaranteed by building certain bound states; however, how can one build bound states out of superextremal particles? They would always repel such that seems not possible to find a gravitational binding potential V_{bound} in order to make a stable bound state. We will discuss it later in subsection 3.8, but seems that for non-BPS states we can have three possibilities:

  1. One superextremal eWGC particle (so no Lattice/tower).
  2. Towers of bound states made out of subextremal (so non-eWGC) particles.
  3. Towers of stable superextremal eWGC such that an state with $q = k > 1$ would not decay to k-states with $q = 1$.

  It appears hard to distinguish between 1) and 3) because one would need to know the exact dependence of the mass on the charges (wrapping numbers) to decide if it is energetically favorable to decay or not. Furthermore, perturbing the BPS condition implies adding instability issues, making very difficult to reach a conclusive answer.

- **We do not delve into the Monodromy Orbit formalism.**

  We will not profundize more because such a tool will not be useful for us, due to the fact that, in our case, gauge fields and CS moduli will decouple and we will not find the BPS mass-change relation which was required for this mathematical treatment. Therefore, being a difficult topic and not useful for our purposes, we consider it beyond the scope of this thesis.

- **The link between WGC and SDC is not completely new.**

  Somehow, we have seen before the connection described in this section between both conjectures in $N = 2$. Such a behaviour was already present in the KK-winding examples of subsection 3.3.2. There, we noted that the towers verified both SDC and Sub-Lattice eWGC behaviour. Nonetheless, no stability analysis was performed because of the lack of non-renormalization theorems. The behaviour found in that example, $M_w \equiv M_{0,1} \sim g_{(V)} \sim e^{-\alpha\phi}$, is absolutely analogous to which one finds here (look at section 5 for explicit computations) binding WGC and SDC to avoidance of global symmetries in the infinite distance limit. In some sense, one can regard the setting in this section as the $N = 2$ supersymmetric framework with the chracteristics of the KK-winding example.

- **Emergence is approximate so far.**

  Take always into account that [5] is comparing just the approximate behaviour in the LCS regime without matching prefactors. In fact, [10] argued that SDC towers made of extended objects (D-domain walls and D-strings), whose masses were introduced in subsection 2.2.2, might contribute to the matching (even though it is unknown how to integrate them out precisely). In any case, what actually [5] does is not integrating out, in the sense that they do not know the UV theory and they just simulate the effect of integrating out scalars (or fermions) with the mass of the D-particles, in the IR theory, trying to match it with the bulk quantities, and then we include them hoping that the divergences arising get canceled. It is not clear that this process is correct due to the alteration and mixing of scales. The lesson to be learned from here is that we cannot expect to find an exact agreement, and one can only regard this as an intriguing and ambitious proposal, which probably cannot be exactly checked without the UV theory (that is, requires QG!).

- $N = 2$ **picture from [5] is based on a delicate structure.**

  Seemingly, the picture presented in this section relies fundamentally in the structure of the towers, analogous to the KK-winding example, and on the BPS condition which guarantees stability through a protected relation between mass (CS moduli) and charge (gauge fields), which in $N = 2$ fit in the same multiplet, as visible in Table 1, such that the D3 can couple to both simultaneously. It turns out that in our $N = 1$ setting (subsection 5.2.2), there will be no BPS particle coupling to both simultaneously, as can be guessed from Table 9 and we will show later. One can start to imagine that a new understanding will be demanded to address this crucial issue, that is a prominent piece of the “Puzzle".

Finally, we encourage the reader to continue through the next sections, where many explicit computations (masses, couplings, diverse divergencies) and arguments in our setting will be presented (in a language which makes them extrapolable to this $N = 2$ case), and then come back to this section. The reader will understand trivially those parts which might not be completely clear at this point, mainly because the intuitive understanding of the conjectures and the Emergence proposal in this setting is much simpler than in ours.

3.6.3 $N = 1$ subtleties

We have already noted some important differences between our $N = 1$ orientifold setting and subsection 3.6.2 one. For $N = 1$ Orientifolds there exist several subtleties which bring doubts about the validity of the previous [5] picture, where both the WGC (Sub-Lattice version) and the SDC were regarded as “dual" and seem to come from the Emergence proposal. Permit us to concretize and condense here the crucial differences.

- **Decoupling of gauge fields and moduli fields.**

  In the case of $N = 2$, the picture can be summarized as follows: The appropriate bound states of D3-BPS particles constitute infinite towers of states satisfying simultaneously the Sub-Lattice WGC and the SDC. The “integration out by hand" of the basis states (treated as scalar/fermion fields), forming the bound states, reproduces approximately the logarithmic divergence and the global symmetry behaviour found in the IR bulk theory, once Emergence (UV kinetic prefactor vanish or are irrelevant compared to the IR ones) is assumed.

  In the previous section we mentioned several subtleties and ignored details regarding the picture in $N = 2$. However, the BPS condition somehow provides a way of escape. This condition for the D3 branes wrapping (sLag) 3-cycles was possible because they coupled simultaneously to the gauge fields and the CS scalar in the same multiplet, as shown in Table 1. Notwithstanding, for $N = 1$ orientifolds the $N = 2$ vector multiplet decouples into $N = 1$ vector multiplet (contains the gauge fields) and $N = 1$ chiral (contains the CS moduli) as observed from Table 9 and Table 12. Moreover, the graviphoton V^0_μ got projected, and it is a well-known fact that the central charge $|Z|$ corresponds to the graviphoton charge at infinity. What this is telling us is that, as the bulk is $N = 1$ now, it makes no sense to consider central charges because there is no extended symmetry and so it loses its real meaning. In this way we observe that the $N = 2$ connection between the gravity multiplet and the $N = 2$ vector multiplet gets lost in the $N = 1$ case, which will be manifest in the fact that the D3-particles coupling to the bulk theory can be massive or charged but not both simultaneously. However, when we truncate from $N = 2 \to N = 1$ some part of the previous relations remain such that we do not get an arbitrary $N = 1$ theory but a constrained one. In fact the classical $N = 1$ mass is obtained by (2.42) truncation such that we will find (classically) massless charged states or massive uncharged states [60], which by obvious reasons (breaking of the “no force" condition) are no more BPS protected.

As a consequence, we observe that this $N = 1$ case is notably different to the $N = 2$ one, and that the subtleties considered in the previous subsection join our discussion with renewed strenght. Questions like: are the massive uncharged states stable? or, can we make bound states with charged massless particles?, are in the heart of the emerging “Puzzle" and our efforts in order to answer them entail a central point in this thesis. The aforementioned tension between stable towers and WGC states cannot be avoided now by a BPS condition.

Just as a prelude for experts in the field, who can already surely visualize it, there is ONE D-particle verifying the WGC and an unstable tower of D-particle bound states which seem to verify the SDC (without the “implicit" [5] stability requirement). Therefore, the relation between both conjectures, and so also Emergence, seems in danger due to this decoupling. We observe that $N = 2 \to N = 1$ transition may break, for particles (the story is different for D-domain walls, see [10] and later discussion), the "duality" between both conjectures, and now they may go on their own. This would also be expected from the SUGRA analysis described above! These points will become a primary part of the thesis and will be accordingly treated in the rest of this project.

Lastly, let us mention, that these features seem to be general within orientifolds, including also Type IIA. We just need to look at Table 2, Table 13 and the masses given in subsection 2.2.2. It is clear that the characteristics we are describing go beyond our concrete setting.

- **Monodromy Orbit treatment of [5] is not valid.**

  From the discussion in the previous bullet point we observe that the Monodromy Orbit treatment of [5], based on the $M = |Z|$ BPS relation, which bound the masses of the BPS D3s and their charges through the central charge, is not applicable to our case. Now charges and masses decouple (see later) and so this mechanism is not useful/reliable anymore.

  As a consequence, we lose the main mathematical tool evolved for detecting the (charged) towers becoming massless à la SDC (3.9) and we are left almost only with pure Physics intuition to solve an even more difficult problem than before.

- **Does general argument for Emergence remain valid?**

  Observe from the discussion in subsection 3.4.2 that the Emergence proposal giving us (from the respective runnings) the species scale, mWGC and SDC was based on the integration out of a tower (with a concrete structure) with massive and charged states. In our setting we will find massive-uncharged and massless-charged states, so that general argument does not hold anymore. We will later explore from this perspective what potentially changes, but observe already a new complication regarding $N = 2$.

- **Is one-loop analysis not enough?**

  One of the strongest properties of the $N = 2$ setting and the BPS condition is that both together protect against quantum corrections and instabilities in the LCS regime. Type IIB CY3 bulk theory enjoys a 4D $N = 2$ SUGRA and so, as a consequence, it is protected by *non-renormalization* theorems such that at the perturbative level receives just one-loop corrections (look at subsection 3.7 for more details). In addition, the D3-particles considered in [5] are

[60]The attentive reader would be surely wondering about the stability issues regarding these states. This kind of stability will be discussed in more detail in subsection 3.8.

BPS supersymmetric, and so preserve one linear combination of gravitini, such that where the D3-particles are located $N = 1$ still remains [61] protecting their mass and charge in the LCS regime. As a consequence the one-loop analysis of [5] is exact and so the Emergence results can be trusted (recall that the argument in subsection 3.4.2 was at one-loop).

In our setting (subsection 5.2.2), we will start with $N = 2$ bulk and then take the orientifold projection, forcing the presence of space-time filling D3/D7 branes. If these are supersymmetric (i.e. verify (2.33)) can be located such that preserve one linear combination of both gravitini giving us bulk $N = 1$ [13]. In principle $N = 1$ is also partially constrained by the non-renormalization theorems (see e.g. [69]), such that still the gauge coupling receives just one loop corrections, but the Kähler potential receive all order loop corrections (unknown functions of Einsenstein series in the CS moduli). Nevertheless, now we need to add the D3-particles, clearly not BPS, which due to Dirichlet boundary conditions seem to not preserve now any supercharge, giving us $N = 0$ along the D3-particle. It is very difficult now to quantify their contribution when integrated out in Emergence. On the one side, the bulk coupling is one loop exact and the Eisenstein series vanish fastly in the LCS. On the other hand, the particles we are integrating in do not preserve any supercharge and the bulk we find is supposed to have already integrated out these particles so in the Emergence process of integrating them in and then simulate their integration out seems that one should take into account more loop corrections.

We will see later that we will find quantitavely exactly the same bulk distance divergence for $N = 1$ than for $N = 2$ (in the LCS). One could also worry about this, because now the Kähler potential is not exact. Nevertheless, as hinted in the previous paragraph, in the LCS the Eisenstein series tend to vanish fastly and so the classical contribution is the leading term. As a consequence, when we compute the distance using it, it is logical to find a similar divergence in such a regime. Thus, this is an argument in favor to disregard the higher loop corrections in the LCS when using Emergence, because as we will compute, the first loop order (up to stability issues) again would give us the correct matching.

It is not completely clear to us if one should consider higher loop order correction when integrating out by hand in Emergence and we will see later that this might have some relevance. In any case, these become irrelevant in the LCS.

- **Are there more contributions to Emergence?**

  This one is actually not an specific $N = 1$ subtlety, but plays a different role in such a context by truncation of the towers.

  The argument used in subsection 3.4.2 and in[5] for the Emergence matching just takes towers of particles into account. Nevertheless, [10] proposed that one might also take into account towers of extended objects (D-domain walls and D-strings) into the integration out matching. It is not clear at all how to integrate out extended objects, and so one cannot make any quantitative analysis which, added to the fact that already the Emergence matching for particles was verified just in behaviour (without any prefactor), just increases the speculation regarding the proposal.

  In the present, nobody can check if extended objects contribute and how they would contribute in behaviour (which in the LCS limit means we have three options: irrelevant, relevant or marginal) and relative prefactor (in the marginal case (same analytic behaviour in the LCS) the prefactor would determine the relative contribution regarding to, for instance, the towers of particles). Furthermore, the integration out of extended objects received a lot of effort in

[61] Recall that Dirichlet boundary conditions reduce the SUSY by 1/2. That is the reason why these objects are called 1/2-BPS.

the past without success, being technically difficult to tackle even today. Sincerely, it does not seem that in the near/medium term these computations can be made with rigor.

Anyway, one should bear in mind the possibility that these extended towers actually contribute. In such a case, there would be again issues concerning $N = 2 \to N = 1$ because many BPS towers in $N = 2$ will be no more stable in $N = 1$ (look at subsection 3.8). Notwithstanding, in the LCS the distance divergence and the global symmetry behaviour seem (look at subsection 5) pretty identical to the parent $N = 2$. Thus, our current understanding of Emergence seems to be in tension again with our setting. Another piece that adds to the “Puzzle".

3.7 Conifold vs ∞ distance singularities

Regarding to our examples and analysis, these two singularities are crucial and that is the reason why we dedicate to them the next pages. Concerning this section, we use as the main references [7], [70] and [71], appart from hinting some of our findings, which might seem obvious for a part of the community, but it took us a lot of time to realize of them and they will be crucial for an improved understanding of our models. Moreover, we realized that [9] also treats this topic but with a different objective. We recommend to everyone interested to also read it, in order to compare our situation with the Type IIA CY3 hypermultiplet case (a thorough reading would also require [8]). Besides, [6] and [22] were also really helpful in this section.

In this subsection the concept of *stability of the towers* will become crucial. It will be treated in detail in subsection 3.8 but in deference to the reader we advance [62] an essential argument for this section, which is quite extended also in the literature. If we consider the tower of D3-BPS particles that arises from wrapping the same brane n-times along the same cycle, clearly the mass of each of these states is given by n-times the mass of the once-wrapping state (just look at (2.42)), and it might then be unstable against decay to its constituents (as BPS states can only decay to other BPS states) because the "triangle inequality of masses" is saturated in this case, implying that one cannot ensure that the tower is populated by physical states. Hence, if we want to consider infinite towers composed by branes wrapping cycles several times, we must make sure that they consist of BPS states that are bound states (and not just superpositions) of branes, such that now the mass of the bound states is lower than the mass of the isolated BPS components, and so they are stable and therefore populated by physical states. This argument is a refined version of that provided by [7] to justify the resolution of the conifold singularity in Type IIB CY3 CS moduli space but we still continue without having a better argument that really ensures us that the superpositions really decay and not just “might not be stable". Why do we care about conifold singularities here? We are going to see that the orientifold loci observed from the perspective of the parent $N = 2$ are precisely located on certain conifold singularities making sense of unusual massless towers of D3-branes wrapping even (respectively odd for O5/O9) sLag 3-cycles, that as we will realize do not verify the SDC.

3.7.1 Conifold singularities

We must remark that there exist several kinds of conifold singularities and their resolution is by no means the same. Notwithstanding, for us the relevant is the one related to Type IIB CS moduli space, which was the first staged [7] and, in some sense, the simplest.

[62] We already sketched a bit this argument in subsection 3.6.2.

Conifold singularity in Type IIB CS moduli space

The original source is [7] and useful references are [70] and [71]. Before starting, just remark that in [7] the author explains that the consistency of his picture requires that there is only one supermultiplet which becomes massless. However, that is not a problem for our setting because that will be exactly the case. Although, in principle this can be extended (with modifications) to > 1 case [70]-[71].

From the expression (2.42), a very interesting case is that of all charges equal to zero except, for example, $q_1 = 1$. In such a case, we see that when $X_1 \to 0$ the mass vanishes linearly on X^1 (not exponentially on the distance, as we will check later). According to (2.15) we see that this corresponds to the case where the cycle schrinks to zero size and is what we call a conifold singularity. The word conifold is clear due to its cone structure, and the fact that there is a (physical) singularity can be easily deduced from the CS kinetic term in the effective action (2.21) and (3.35).

Circling the singularity, there is an usual $Sp(b_3; Z)$ monodromy (firstly encountered in the quintic and appears to be generic following the SUGRA bulk perspective): $X^1 \to X^1$, $F_1 \to F_1 + X^1$ which implies that near $X^1 = 0$:

$$F_1(X^1) \sim \text{constant} + \frac{1}{2\pi i} X^1 \ln X^1 \tag{3.34}$$

while all the other periods are smooth and non zero. At this point is where the singularity emerges, precisely we observe that replacing in (2.16) the metric diverges as:

$$G_{1,\bar{1}} \sim \ln(X^1 \bar{X}^1) + \text{single-valued} \tag{3.35}$$

It will be verified explicitly in our examples that this is precisely the case for our setting and it can be checked (quite straightforward but tediously) that the distance to $X^1 = 0$ measured by G is finite and that the scalar curvature diverges there (as expected by one of the *geometric conjectures* presented in subsection 3.5). What about the gauge coupling in the direction in which we approach the conifold singularity? It vanishes (global symmetry limit), as we will explicitly test in subsection 5.2.1.

The resolution of this singularity comes à la Seiberg-Witten [33] (integration out of BPS monopoles solves the singularity there), the low-energy bulk effective action involving only moduli fields must break down near $X^1 = 0$, because this state becomes light and can be excited. It should then be replaced by an effective field theory in which the kinetic term of the X^1 modulus does not diverge and there is an additional charged $N = 2$ hypermultiplet coming from the “integrated in" D3 wrapping once A^1. From this situation, we can take into account the correction induced by integrating out this state and that leads us to (3.34). Thus, the conifold singularity in the metric is just the usual type of singularity produced by integrating out massless charged fields in QFT.

Finally, a good argument given in [7] is that one regarding to the number of states which become light near the conifold. From (2.42) we already see that the n-times wrapping D3-particle would also vanish. If we interpret it as a charge-n single-particle state, there would be an extra contribution to the β-function ruining the integrating out agreement with (3.34). Therefore, we assume that this does not correspond to a single particle state. Rather it is regarded a n-particle state consisting of n $q_1 = 1$ particles. From our point of view, this argument is strong but incomplete and, even together with the previous BPS decay argumentation, it is not really a proof. Furthermore, one can check already this agreement by means of Emergence (actually, as stated before, the idea of Emergence comes from this resolution of the conifold).

Conifold singularity in Type IIB hypermultiplet

Based on Appendix B of [9]. As this case will not be relevant for us, just mention the principal features such that one can compare and observe that the resolution of this singularity is a bit different from the previous one.

The main difference is due to the fact that now we have both perturbative and non-perturbative α' and g_s corrections (look at figure in page 8) to the Kähler potential. These actually modify the Kähler potential (now called contact potential), especially the contribution from the worldsheet instantons gives the expected logarithmic divergent term as dominant in the metric when we approach the conifold locus (here the classical contribution to the Kähler potential, as proportional to the volume, vanishes). In this case, the resolution of the singularity is due to D1/D(-1)-brane instantons which, when integrated out, give a quantum corrected metric, such that now the logarithmic divergence caused by the worldsheet instantons gets cured by the Euclidean D1-brane instantons.

Another difference is that a large (but finite) number of states (D-instantons) is required to cure the singularity instead of just one (D3-particle).

A similar reasoning and resolution is applicable to the mirror Type IIA Hypermultiplet [8] where the relevant instantons are the Euclidean D2-instantons.

3.7.2 ∞ distance singularities

This is precisely the case discussed within the SDC context. As it is widely discussed along the text, we will just explain their relation to their conifold partners.

In this case, as its name hints, the uncorrected distance becomes infinite, instead of remaining finite. Firstly [5] (by means of emergent distance and couplings) and then [9] (through explicit computation) argue and check that this signals that an infinite tower of states has been integrated out, rather than a finite number (like in [7]). Interestingly, the same underlying principle that allows to resolve the conifold singularity in [7] was invoked in [5] to propose the emergence of infinite distances and weakly coupled gauge interactions in CY vector moduli spaces.

The main difference with respect to the case in [7] is that, instead of one, an infinite number of D3-particle states becomes light when reaching an infinite distance point, in agreement with the SDC.

The work of [9] is helpful to compare the ∞ distance and conifold cases within the Type IIB hypermultiplet framework. To take care of the former, an infinite number of instantons (becoming exponentially massless in the geodesic distance) is required, whereas a large (but finite) number (not exponentially massless in the geodesic distance) is necessary to cure the conifold singularity, following the general trend in the difference between the resolution of these singularities.

In fact, the $\mathcal{M}_{VM}^{IIB}(Y)$ conifold singularity resolution has a counterpart via the c-map (see figure in page 8) in $\mathcal{M}_{HM}^{IIA}(Y)$, in which the singularity is removed by the quantum corrections to the metric due to D2-instantons. Via mirror symmetry, the same effect appears in $\mathcal{M}_{HM}^{IIB}(\tilde{Y})$, where one can use the contact potential to see how D(-1)/D1-instanton effects resolve the singularity. If we extend this way of thinking to the case of infinite distance trajectories, the work of [9] can then be understood as a systematic manner to take into account the one-loop effects of D3-particles in [5].

3.8 Stability, BPS condition and bound states

In this subsection we are going to discuss those aspects related to a key concept, **stability**, as well as its intrinsic relation with the BPS condition (in supersymmetric theories) and the possibility of bound state formation [3], [11]. As commented before, we will consider two different kinds of stability. On the one hand, the *non-BPS stability*. On the other hand, we will briefly recall the arguments regarding to the aforementioned *stability under BPS to BPS decay*. Useful references regarding the former are [25] and [26], and, concerning the latter, we recommend [5], [11], [7] and [10]. Nevertheless, the several parts along this section are not contained in any of the previous references.

3.8.1 Bound states and stability role

As a warming up, let us briefly show why bound states (and their stability) are relevant.

Firstly, we can obtain from them, by means of a simple argument [11], the original eWGC (3.2). By demanding the absence of stable gravitacionally bound states in Einstein-Maxwell theory with charged particles, there must exist a particle on which gravity acts as the weakest force, independently of the RG running.

Consider the particle with the largest charge-to-mass ratio in the theory. Both F_{gravity} and F_{EM} are long range forces, the former attractive and the latter repulsive. When the particles are separated by r greater than their Compton wavelength, the magnitudes of the forces are:

$$F_{\text{gravity}} = \frac{m^2}{8\pi M_P^2 r^2}\ , \quad F_{\text{EM}} = \frac{(gq)^2}{4\pi r^2}\ . \tag{3.36}$$

Then the eWGC (3.2) is equivalent to state $F_{\text{EM}} \geq F_{\text{gravity}}$ for the particle with largest charge-to-mass ratio!

If the above formulation is violated, then two such (stationary) particles would attract instead of repel. Then the energy of the bound state would be smaller than $2m$ due to the gravitational binding potential, but its charge would be exactly $2q$. As a consequence, this bound state would possess a larger charge-to-mass ratio than the particle with the largest ratio in the theory. Therefore, the bound state would be stable under decay by means of charge and energy conservation. Thus, we can always add more and more of such particles and, since they are mutually attractive, there would always exist a stable bound state with arbitrarily large charge which cannot decay.

These stable bound states are pretty similar to the aforementioned remnants, and the main difference is that they can be weakly coupled, while the remnants are usually considered as strongly coupled. Again, it is not fundamentally clear what goes wrong in the microscopic level if such stable gravitationally bound states exist.

Remarkably, we can extend this argument to the case in which the particle experiences also scalar force:

$$F_{\text{scalar}} = \frac{\langle \partial_\phi m \rangle^2}{4\pi r^2} = \frac{\mu^2}{4\pi r^2} \tag{3.37}$$

obtaining the WGC with Scalar Forces (3.4) as an equivalent statement to $F_{\text{EM}} \geq F_{\text{gravity}} + F_{\text{scalar}}$ for the particle with largest charge-to-mass+scalar charge ratio.

Secondly, we have observed in subsection 3.6.2 that the SDC requires stable bound states and not superposition. These bound states are pretty much like the above ones in the sense that gravitational and scalar forces fight against the gauge interaction. Then they look like gravitationally stable bound states again. $N = 2$ overcomes this paradoxical situation because of the BPS condition which is precisely the intersection point where both phenomena are mutually consistent. However, we already noted in subsections 3.6.2 and 3.6.3 that, when the BPS condition is abandoned and/or there are no BPS states, some subtleties emerged.

Once we realize that bound states and their stability are in the core of the conjectures, let us focus on the two kinds of stability.

3.8.2 Non-BPS stability

It is usually implicitly assumed as a fact that in Type IIB (IIA) there are just Dp-branes with odd (even) p coupling to the bulk theory. That is motivated from the fact that in Type IIB (IIA) we dispose only of even (odd) bulk RR gauge fields to couple to the Chern-Simons part (2.32) of the Dp-brane action. As a consequence, these states are charged under the RR fields, enjoying stability under vacuum decay. We can have any of these “allowed Dp-branes" with a mass provided by the DBI term (2.31) and their charges can avoid them to decay to vacuum, although their mass would decay because it is not protected, in general, under perturbative and non-perturbative corrections, and there might be states energetically favoured. These, in turn, sometimes exist and are called BPS states. Such states preserve (for our case) 1/2 of the original supersymmetry and have the special feature that are protected against perturbative corrections, so that the BPS condition is preserved making the states stable. The main feature of these states, is that they feel “no net force" such that the repulsive force (gauge through charge) and the attractive force (gravity through mass and, in our setting, scalar force through the CS moduli) cancel. Adding together the “no net force" and the 1/2-SUSY protection against corrections seems obvious that these states will be stable (appart from BPS to BPS decay, see next subsection).

It is instructive to comment two different cases of how both conditions can be obtained.

Firstly, the D3-branes wrapping 3-cycles, in $N = 2$ bulk, have both their charge (2.39) and their mass (2.42) bound through the number of times they wrap the cycle, and the BPS condition (2.40) precisely minimizes their mass such that these are energetically favorable making them stable because of enjoying “no net force" (reflected in (2.26)) and are BPS SUSY protected under perturbative corrections (which might modify these strict equalities making them unstable).

Another different case is that of the space-time filling D3 [13], which we add to the $N = 2$ setting to obtain $N = 1$ O3/O7 orientifold bulk. These do not wrap any cycle and so the only required condition (calibration condition) we require (appart from the tadpole cancellation) for them to be stable is (2.33). When expanding in the open string modes, it makes that the mass term coming from (2.31) and the charge term coming from (2.32) are exactly equal, such that again “no net force" is verified, and makes them stably BPS SUSY protected under corrections [13]. There exists also an analogous calibration condition for the space time filling D7 but is a bit more complicated and depends on the fluxes because now the D7 wraps a 4-cycle which is required to be holomorphic to preserve supersymmetry (look at [13]).

Nevertheless, we can have in general other Dp-branes in the theories, as well as we had Dp-branes in the Bosonic String theory, which are just neither BPS nor stable. The feature all of them share is that they are not charged under any bulk RR field and so there is a “net force" due to their mass. As a consequence, nothing protects them from decay to vacuum. This happens by the so-called “Tachyon condensation" as described by A. Sen two decades ago [25]. Tachyons are not projected from the

D-brane spectrum making them unstable (similarly to the Mexican-hat potential and tachyon condensate within the Higgs mechanism) and generating a potential whose stable minimum is displaced such that through the tachyonic condensation the potential reaches the “real" minimum where usually, due to energetical reasons, the D-brane decays to vacuum.

The above paragraph picture is accurate for the case of Dp-branes in ten dimensions with trivial topology. Nevertheless, it is not clear to us how to generalize it to the case of compactifications over a non trivial (here CY3) internal manifold. Without going further, in our case the branes wrap (homologically non-trivial) cycles such that even if they are not charged under RR fields, the wrapping number is topologically protected. It is clear that the uncharged massive D3-branes wrapping odd 3-cycles ($N = 1$ orientifolds, subsection 5) are not BPS and so they should not be protected under perturbative corrections which makes them unstable in general. However, it is tremendously difficult to address any scale or magnitude to such instability because, in this case, it is absolutely mandatory to take into account the backreaction on the geometry, meaning we must left the probe limit. In the BPS case, the equilibrium between repulsive and attractive forces permitted to safely depreciate the backreaction on the geometry, but now there is a “fight" between the D3-particle tension, trying to shrink the cycle, and the geometric cycle resisting it. What is clear is that, when approaching the LCS regime they become asymptotically massless and, therefore, such backreaction becomes less and less important, as well as they become more and more stable under any kind of instability. These objects are complicated because they experience tachyonic instabilities and every kind of possible corrections but, at the same time, they fit in a topologically protected sector.

Using K-theory [63] (look at [25] and [26]), one can sometimes find non-BPS stable Dp-branes by means of a discrete symmetry protecting them against decay. However, these cannot form towers because then the symmetry gets broken when considering more than one non-BPS stable brane and the whole system becomes again unstable. As a consequence, they will not play a serious role for our purposes.

An even different case is the one where the Dp-branes are charged but lose the mass, e.g. in the even D3-particles we will obtain in section 5. These are stable under decaying to vacuum, because are charged, but are not BPS protected.

Next, let us touch a delicate issue. In the formulation of SDC and Lattice WGC it is not specified that the towers need to be necessarily stable. Let us consider the first example of KK-winding towers of states without gauge field, firstly considered in subsection 3.3.2 in order to motivate the SDC (for Kähler volume moduli). These towers are in general not stable because the states forming them do not have any symmetry protecting them when the gauge field is turned off. However, the SDC divergence in the moduli metric is exactly the same as when turning on the gauge sector. Then it might seem that the SDC behaviour is independent of the gauge field, and so of the stability of the towers themselves!

These towers have exactly the same “asymptotic stability" property which we are going to discuss for our “bound states" of odd D3-particles in the $N = 1$ examples of subsection 5. As a consequence, the same question that arises here will appear later in our setting: Is it really necessary to require stability to get the SDC behaviour? The possible answer to this question will be tied to the Emergence proposal, as easily expected.

Interestingly, stability started to become an implicit requirement after the work of [5], because in the case of $N = 2$ it is possible to find BPS stable objects (concretely D3-particles) whose charge and mass are linked such that these states feel no net force and there is no other energetically favorable state to decay, appart from the BPS to BPS decay.

[63] Recall subsection 3.5.

Finally, let us consider general BH formation by superposing a large N number of D3-particles. In the BPS case, we fit in the limit of being able to add more and more of this D3-particles to build the BH. However, in the non-BPS case if the repulsive forces overcome the attractive forces we cannot add more than one particle, which is in fact an eWGC particle allowing any BH to decay to it, avoiding remnants. If the attractive forces are stronger than the repulsive ones, then we can make, as explained in the very beginning of this section, arbitrarily charged gravitationally bound states forming, for example, a BH. We then observe that the eWGC and the bound state behaviour appear incompatible for the same particles in the non-BPS case. The former case seems to be more related to the avoidance of naked singularities (Weak Cosmic Censorship, subsection 3.5), and the second to the towers appearing in the $N = 2$ conjectures and to microscopical BH building and microstate counting.

3.8.3 Stability under BPS to BPS decay

This is the kind of stability where the Monodromy Orbit treatment of [5] plays a fundamental role, as discussed in subsection 3.6.2. BPS objects are stable under perturbations, because they are protected by a (gauge) symmetry and their mass takes a minimum value $m = |Z|$ which is energetically favourable and protected by a narrow link between mass and charge. In other words, they are protected under vacuum decay because are charged under a symmetry of the theory. Moreover, their mass is related to their charge such that they preserve a fraction of supersymmetry, protecting them from arbitrary corrections. Besides, their mass takes a lower bound such that they are favoured energetically against any other state with $m \geq |Z|$, which ultimately should decay to BPS. Finally, they cannot decay to an state with $m \leq |Z|$ because supersymmetry protects them.

Hence, the only plausible way in which a BPS state can decay is by means of a BPS to BPS transition. For example, take a look at (2.42). It is clear that one D3-BPS particle with $q^1 = 2$ as unique non-zero charge, has exactly the same mass as two D3-BPS particles with only $q^1 = 1$. Then, by means of energy and charge conservation, the decay channel is open and a transition may in principle take place, although it is not completely clear if and when such a process would actually happen. In order to be sure that this is not an issue, the superpositions are considered unstable and avoided due to a possible BPS to BPS decay.

As a consequence, we search for bound states of the previous D3-BPS particles, whose formation is subtle as commented in subsection 3.6.2, which are marginally possible because of the kinetic mixing present in the gauge coupling. Then, by means of the gravitational binding potential, these bound states (which in principle are no more BPS, because now a part of the mass goes to build the bond) cannot decay to their BPS constituents by a simple triangle inequality. The non-BPS stability issues (subsection 3.8.2) with respect to this (non-BPS) bound states are a mystery to us from the intuitive point of view, but it seems that the Monodromy Orbit treatment somehow guarantees they are stable [5].

4 General Case

Even though in the previous sections we started to include several original ideas, most of those will be recycled in what follows, such that the reader can consider that the original part of this research project starts here and goes to the end. As we have already remarked, some of our results intersect with those of [10], but there are crucial differences, which also modify our interpretation of the phenomena.

Unfortunately, we were not able to get too much information from the general case, appart from that given in [5] for $N = 2$ Type IIB CY3 CS moduli space in LCS regime (taking into account that our ultimate focus are the $N = 1$ orientifolds). That is why we will need to concentrate on concrete examples, where we can really get a complete picture of what is happening.

Nevertheless, in order to make more understandable our treatment for the reader, we will introduce the general case and from it reveal surprising behaviour, which could be firstly thought of as non-sense, but we will see that it fits in a higher picture, which is, to our understanding, pretty beautiful although difficult to interpret.

Let's firstly work on the parent $N = 2$ as a warm up.

From (2.42), it is not completely obvious which are the states realizing the SDC for $N = 2$. Naively, we just observe states giving us zero mass linearly, quadratically or in a cubic way for special configurations when we approache limits where some of the X^i [64] went to zero, but nothing related to an exponential behaviour at infinite distance. However, concerning the SDC, the essential term is precisely the exponential in (2.42), which in the LCS is equal to: $e^{\frac{1}{2}(-\ln(\frac{4}{3}d))} = (\frac{4}{3}d)^{-1/2}$ with $d = d_{ijk}v^i v^j v^k$. This actually hints to the (exponential) decay of the mass in the LCS ,when the coordinates are sent to infinity. Besides, in LCS, $F_0 = \frac{1}{6}d_{ijk}x^i x^j x^k$ and $F_i = -\frac{1}{2}d_{ijk}x^i x^j$. Therefore, what we find is precisely that in the LCS (recall we are using special coordinates):

$$M_{D3}^{BPS} = \frac{1}{\sqrt{\frac{4}{3}d}}|q_0 + q_i x^i + p^i \frac{1}{2}d_{ijk}x^i x^j - p^0 \frac{1}{6}d_{ijk}x^i x^j x^k| \, . \tag{4.1}$$

From this crucial but simple computation we already note that, depending on the v^i we decide to send to infinity, some states or others go to zero, and this also depends on the d_{ijk} of the mirror CY. Notwithstanding, it is quite clear that the D3 wrapping A^0 and, at least, one kind of D3 wrapping A^i become massless at all infinite distance points. Bearing this in mind, we can motivate the *Monodromy Orbit* [5] result, finding that one infinite stable tower becoming massless at infinite distance is formed by bound states of D3-branes wrapping n-times A^0 and D3-branes wrapping the A^i cycles [65]. Following the aim of [10] we could consider each of the D3-branes wrapping once each cycle as a basis and perform a detailed analysis, but this was similarly worked in the reference and would be rather analogous. Besides, it will not be really important for us. Before continue, just mention that we will primarily focus on the electric side ($p^I = 0$), however an in-depth analysis concerning the magnetic and dyonic cases would be also interesting.

The exponential behaviour for these towers comes from analyzing the expression for the distance in the CS moduli space:

[64] Recall from subsection 2.1.3 that $X^i = x^i X^0 = x^i$ when taking special coordinates, which will be always the case.

[65] Remark, that a very similar but maybe more complete argument was found in [10] but we introduce this also as an original result because we got it before it appeared published.

$$d_\gamma(P,Q) = \int_\gamma \sqrt{2G_{ij}\dot{x}^i\dot{x}^j}ds \ . \tag{4.2}$$

From this expression and (2.18), taking the infinite limit in one modulus and following that trajectory, is quite easy to get the exponential behaviour on the distance. Nevertheless, it must be geodesic and so depends on the trajectory chosen. The difficulty on taking intermediate trajectories increases a lot and we will not deepen into this. Concerning our examples and goals, it is not indispensable, as the number of dimensions we choose is low. Nevertheless, we refer to [5] and [10] for more information about many LCS moduli infinite limits, even though it is not known yet how to check stability in such cases.

Now let's briefly visualize what changes for the $N = 1$ orientifold configurations, from the parent $N = 2$ perspective. Basically, what we use is the fact that, in this abstract setting, we can already choose the parent $N = 2$ basis to be the even-odd one required for the orientifold (look at [24]). This opens the door to the possibility that the structure related to the SDC might be mainly inherited from $N = 2$. Notwithstanding, the condition $F_{k_+}|_{z^{k_+}=0} = 0$ (O3/O7) (identically for O5/O9) forces several conditions. In the LCS, the mirror CY3 (and so by mirror symmetry the Kähler part of our CY3) must verify certain restrictions on its triple intersection numbers d_{ijk}. Furthermore, as we have seen, the spectrum changes a lot due to the orientifold projection.

By now, everything points to the intuitive naive idea that the SDC in $N = 1$ would be verified as a particular case of the $N = 2$ towers. The WGC with Scalar Fields (WGCSF) seems more tricky as in the orientifold case one gets a *potential* from the spacetime filling D3 and/or D7 (also from fluxes but we turn them off). Regarding to the Emergence proposal, up to now everything seems fine, but we will see that the even (odd) property, under the orientifold projection, of the $U(1)$ vector coming from C_4, will make it really much more complicated. Finally, we have those even infinite towers of states whose mass becomes zero also when x^{i_+} goes to zero (as can be seen from (4.1) with $x^{i_+} = 0$ and $p^0 = p^i = q_0 = q^{i_-} = 0$), where is precisely located the orientifold moduli subspace!

In the general case, it seems quite difficult to make this more precise, this is why we will search for concrete examples to unreveal the real nature behind $N = 1$ orientifolds.

Finally, let us just recall again that we are always taking the probe limit for the Dp-branes and so neglecting their backreaction.

5 Examples

It is clear from the previous section, that if we want to adress the singular behaviour coming from the orientifold projection, in order to get a better understanding, we will need to explore concrete models.

5.1 Toroidal Orientifold compactifications

As it's usually the case, when one searches for concrete String theory realizations and not just SUGRA backgrounds, it is worthy to firstly look at toroidal compactifications because they were extensively studied. In [20] we found a survey of toroidal orbifolds and their orientifolds but there is a problem: even (odd) 3-cycles, under the orientifold projection, fit into the O3/O7 (O5/O9) twisted sector! As far as we know, just the topology has been worked in these cases but not the resolved geometry, making these examples not really useful for our testing purposes [66]. This is why we focus better on the following concrete abstract SUGRA models, hoping they are actually realized within String theory. Finally, we anticipate that this "twisted issue" is actually not a problem but a hint of the real nature of the D3-particles wrapping those cycles.

5.2 Abstract Models

Our first choice did not seem to be the appropriate to shed some light on our problems. Therefore, we just decided to go to the simplest but also most illuminating models one can imagine, which are no more than concrete abstract SUGRA models, whose CS moduli space dimensions are determined by just taking low even and odd Hodge numbers in order to compute analytically couplings and distances in the appropriate loci. We are aware about the fact that might be the case that no CY3 has this numbers, but what we are going to learn from this seems clearly extendable to any higher Hodge numbers in even and odd sectors. The tools we are going to utilize were already introduced in subsection 2.

Our perspective is always the following: we work in the parent $N = 2$ Type IIB CY3 CS moduli space (subsection 2.1) and approach the conifold/$N = 1$ orientifold limits (subsection 2.3), that is basically to require the conditions $X^{k_+} = 0$, $F_{k_+}|_{X^{k_+}=0} = 0$ for O3/O7 (subsection 2.3.1) and $X^{k_-} = 0$, $F_{k_-}|_{X^{k_-}=0} = 0$ for O5/O9 (subsection 2.3.1), taking the appropriate field configurations. The conditions on F_k, in the regime where the CS, within the orientifold locus, are greater enough than zero, basically give constraints on the allowed triple intersection numbers of the mirror CY which make possible to have an orientifold projection and, in the intermediate regime, these conditions fix the quadratic and linear coefficients a_{ij} and b_i in the prepotential (2.17) to get concrete values, which then basically cancel their contribution in the mass and distance calculation (of course, in the LCS just the cubic term is relevant).

We will note that for our cases the differences between the conifold loci in $N = 2$ moduli space and orientifold loci in $N = 1$ moduli space are just the conditions on the F_a and, of course, the field configuration corresponding to the orientifold projection, where the most relevant fact is that C_4 is even (odd) for O3/O7 (O5/O9) projecting out the odd gauge vector terms. In fact, one can obtain [67] the $N = 1$ CS moduli space from the orientifold abstract SUGRA model in the following way:

[66] It will become clear later that the new features distinguishing $N = 1$ case from the parent $N = 2$ are not suitable to be tested in this settings.

[67] This is not completely correct at a technical level, but is schematically valid for our purposes.

1. Begin with the parent $N = 2$ CS moduli space in the even-odd basis.
2. Approach a certain conifold locus $x^{i_+} = 0$ (O3/O7) or $x^{i_-} = 0$ (O5/O9).
3. We will note in subsection 5.2.1 that precisely the above limits correspond to [7] conifold singularities. Then it must be resolved by incorporating the (ONE) D3-BPS particle wrapping the vanishing cycle, so that it cures the metric divergence and the global symmetry limit for the gauge coupling (recall subsection 3.7.1).
4. Once resolved the conifold, by means of "integrating in" the massless D3-BPS particle, take the orientifold projection in the spectrum.
5. As as result one finds the $N = 1$ CS moduli space, as long as the conditions (especially on the triple intersection numbers), coming from $F_{k_+}|_{x^{k_+}=0} = 0$ (O3/O7) or $F_{k_-}|_{x^{k_-}=0} = 0$ (O5/O9), are verified by the mirror CY3.

5.2.1 $h^{2,1}_+ = 1$, $h^{2,1}_- = 0$

Before starting, let us advance that the situation is analogous when $h^{2,1}_+ = 0$, $h^{2,1}_- = 1$ changing the parity of the cycle wrapped by the D3 brane and the orientifold projection to O5/O9. At the end of this subsection, this will become obvious for the reader.

This is the simplest non-trivial example and appears to have no relation with the orientifold case, because we just have one even dimension in moduli space and precisely when we go to the conifold/orientifold locus $x^+ \to 0$ the CS moduli space is just a point. Nevertheless, this example is perfect to address some of the previous issues: 1) The role of the (infinite tower?) of even D3-particles when going to $x^+ \to 0$ and, specially, 2) Which is the distance to the orientifold locus? Is it finite or infinite? and 3)What is the nature of the orientifold locus? Does it really fit as a conifold?

To answer these questions, we develop a treatment from the parent $N = 2$ as described above.

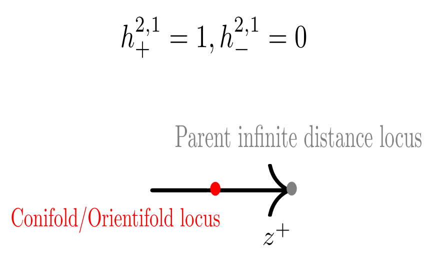

Figure 3: Example $h^{2,1}_+ = 1$, $h^{2,1}_- = 0$ setting.

Let us check here the SDC exponential behaviour for $N = 2$ for the sake of concreteness, as there is just one direction, so that the distance is actually geodesic. Going to the LCS regime we find $G_{++} = \frac{3}{4(v^+)^2}$. Then, we obtain that:

$$d_\gamma(P,Q) = \int_\gamma \sqrt{2G_{++}}dv^+ = \sqrt{\frac{3}{2}}\ln(\frac{v^+_Q}{v^+_P}) \, . \tag{5.1}$$

Thus, we obtain the exponential behaviour in the mass:

$$M_{D3}^{BPS} = \frac{1}{\sqrt{\frac{4}{3}d}}|...| = \frac{1}{\sqrt{\frac{4}{3}d_{+++}}}(\nu^+)^{-3/2}|...| = \frac{1}{\sqrt{\frac{4}{3}d_{+++}}}e^{-\sqrt{\frac{3}{2}}d_\gamma}|...| \,, \tag{5.2}$$

where the dots represent the terms associated to the wrapping numbers in between the absolute value in (4.1). This is a behaviour we can always expect in our settings when travelling to the LCS regime in one of the basis moduli directions.

Now, going to the $N = 1$ orientifold locus in the O3/O7 sense means $X^+ \to 0$ and $F_+|_{X^+=0} \to 0$. This might seem trivial but it is not, because then we are no more in the LCS regime forcing us to take the complete prepotential (2.17). Firstly, we missed this part a bit, because here the D3-particles wrapping the even 3-cycle seem to vanish linearly (4.1) and in a rather different way than expected for the SDC (the distance does not seem to diverge). Furthermore, it seems that we have an infinite tower of states that become massless (D3-branes wrapping n-times the even 3-cycle) and not in the SDC exponential way. As the distance (will be computed soon) is finite, these states would not be in contradiction with the SDC but are puzzling. Does make sense a theory with infinitely many massless charged states? A priori not too much.

The resolution of this rare situation comes somehow from more than 20 years ago [7]. $X^+ \to 0$ is precisely a conifold singularity in $N = 2$, whose resolution was explained in section 3.7. These states that we found become massless à la Seiberg-Witten [33] and the logarithmic singularity on the metric is cured by taking them into account in the bulk theory. Also the supossed tower is argued to be unstable avoiding the main conceptual problem. Although, in this example the orientifold limit does not make too much sense, we see that precisely this conifold singularity is linked to the orientifold locus corresponding to the fractional D3-branes in the twisted sector we noted in the subsection 5.1!

From the parent $N = 2$ viewpoint everything makes sense now and it is known how to resolve the conifold/orientifold singularity. Nevertheless, when we impose the orientifold projection, $X^+ = 0$ strictly. So the fractional D-branes we find in the twisted sectors of the $N = 1$ orientifold really come from the parent ones at the conifold locus! This is a CY3 result and one would expect that, the fact that in the conifold we have usually to introduce fluxes, making the geometry warped and not anymore CY near to these singularities ([21]), might permit somehow take care of these fractional D3-branes in the $N = 1$ orientifold (even though, the mass formula for the D3 wrapping sLag 3-cycles seemed to be independ on fluxes, so we do not know how these could get any mass and we assume they continue to be massless up to unknown QG effects [68], keeping in mind that this is not a common behaviour).

Anyway, the answers to the previous questions are respectively: 1) These states are precisely a concrete case of the conifold singularity D3-branes of [7], 2) The distance (see below) is finite in concordance with the features of the conifold singularity and 3) From the next computation we match exactly the conifold behaviour, so for this model the O3/O7 orientifold locus is just the (resolved) conifold locus in the parent $N = 2$ plus the field configuration required by the orientifold projection (even-, oddness).

Just add, that from an O5/O9 perspective the O5/O9 orientifold is the whole parent $N = 2$ plus the orientifold field configuration and that, following exactly the same reasoning, there the D3 wrapping

[68] In [72] our suspicions about that (2.42) is quantum-mechanically exact were confirmed. The only change is, of course, in the prepotential, which now should include all the terms as in (2.17). However, for electrically charged cycles, this makes no modification because K_{CS} never receives the instantonic IR contribution $\ln x^+$ without a factor x^+ accompanying it, and so making it vanishing. Two derivatives with respect to the vanishing CS moduli are mandatory to get the divergent $\ln x^+$ alone. The other contributions are finite except in the LCS regime, where the first term in (2.17) domains, and so nothing avoids the massless limit.

even cycles seem to satisfy the SDC but are not charged, due to the orientifold projection, leading to the stability issues discussed in subsection 3.8. We will profundize more on this behaviour in the next example.

For completion, we will show explicitly that the conifold behaviour (3.35) effectively fits, by explicitly computing the metric through (2.17) and (2.16). This computations are quite tedious and long, involving properties of the polylogarithms. Then we will check (2.42) and look at (4.2), in order to check that, in fact, this is not the SDC behaviour and the distance is finite. It would be also interesting to compute here the gauge coupling to check that, unlike in SDC, does not decay exponentially on the distance. Just mention, that we do not need to do it, because [7] arguments directly apply to our case, but it is instructive to see how powerful are those arguments by checking explicitly the conifold behaviour.

From (2.16), using the holomorphicity of the prepotential (and so the periods) we arrive for the general case at:

$$G_{ij} = -\frac{[(F_j - X^I(\partial_{\bar{x}^j}\bar{F}_I))(\bar{F}_i - \bar{X}^I(\partial_{x^i}F_I)) + (\partial_{x^i}F_j - \partial_{\bar{x}^j}\bar{F}_i)(\bar{X}^I F_I - X^I \bar{F}_I)]}{(\bar{X}^I F_I - X^I \bar{F}_I)^2} \tag{5.3}$$

which for our case reduces (in special coordinates) to:

$$G_{++} = -\frac{(F_+ - \partial_{\bar{x}^+}\bar{F}_0 - x^+\partial_{\bar{x}^+}\bar{F}_+)(\bar{F}_+ - \partial_{x^+}F_0 - \bar{x}^+\partial_{x^+}F_+) + (\partial_{x^+}F_+ - \partial_{\bar{x}^+}\bar{F}_+)(F_0 - \bar{F}_0 + \bar{x}^+F_+ - x^+\bar{F}_+)}{(F_0 - \bar{F}_0 + \bar{x}^+F_+ - x^+\bar{F}_+)^2} \tag{5.4}$$

Now, taking into account (2.17), we use the identity (B.10) in [22] which permits us to express the polylogarithms in a more manageable form:

$$\mathrm{Li}_3(e^{-x}) = -\frac{1}{2}x^2\ln(x) + p(x) \;\;,\; p(x) = \zeta(3) - \zeta(2)x + \frac{3}{4}x^2 + \frac{1}{12}x^3 + \mathcal{O}(x^4) \tag{5.5}$$

which applied to our case gives (take into account that we will examine this in the limit $X^+ \to 0$):

$$\mathrm{Li}_3(e^{2\pi i k_+ X^+/X^0}) = -\frac{1}{2}(-2\pi i k_+\frac{X^+}{X^0})^2\ln(-2\pi i k_+\frac{X^+}{X^0}) + \zeta(3) + \mathcal{O}(x) \tag{5.6}$$

From it, we automatically see that:

$$F_+(x^+)|_{x^+\to 0} \sim \text{constant} + \sum_{k_+} n_{k_+} 2\pi^2 k_+^2 x^+ \ln x^+ \tag{5.7}$$

in agreement (up to factors which surely depend on the normalization) with the expected (3.34).

Finally, taking into account that $F_0 - \bar{F}_0 = 2i\gamma = \text{const}$, $F_{++} = \text{constant} + i\sum_{k^+} n_{k^+} 2\pi^2 k_+^2 \ln x^+ + ...$ and that in the conifold limit, the leading term in (5.4) is the second, we automatically find the expected behaviour (3.35)):

$$G_{++} \sim \ln(|x^+|^2) + \text{non-singular} \,. \tag{5.8}$$

From this metric we observe that the distance is clearly finite and there is no exponential massless SDC behaviour on the distance for the mass.

Permit us now examine the behaviour of the gauge coupling constant.

From the expression (2.24), we clearly observe that for our case:

$$\mathscr{I}_{++} = \mathrm{Im}(\mathscr{N}_{++}) = Im[\bar{F}_{++} + 2i\frac{\mathrm{Im}(F_{+K})X^K \mathrm{Im}(F_{+L})X^L}{\mathrm{Im}(F_{00}) + 2\mathrm{Im}(F_{+0})X^+ + \mathrm{Im}(F_{++})(X^+)^2}] \ . \tag{5.9}$$

When $X^+ \to 0$ clearly:

$$\mathscr{I}_{++} \to \mathrm{Im}[\bar{F}_{++} + \frac{2i}{\mathrm{Im}(F_{00})}(\mathrm{Im}F_{+0})^2] \tag{5.10}$$

Here, we see that the leading term is the first one (two derivatives with respect to X^+ give us the free divergent logarithm without any vanishing linear or quadratic term accompanying it) and using $F_{++} = \text{constant} + i\sum_{k^+} n_{k^+} 2\pi^2 k_+^2 \ln x^+$ we get the expected logarithmic divergence for the (inverse) gauge couplings:

$$\mathscr{I}_{++} \sim \sum_{k^+} n_{k^+} 2\pi^2 k_+^2 \ln \bar{x}^+ \ . \tag{5.11}$$

From (2.21) we know that (5.11) corresponds to the inverse of the gauge coupling [69]. Therefore, we note that this leads us to the divergence of the gauge kinetic term corresponding to a global symmetry. Moreover, again we see that there is no exponential SDC behaviour on the distance but approximately a logarithm dependence on the vanishing mass.

Before continue, just four remarks:

- As explained before, our analysis in these models is based on the A^I electrically charged cycles (recall that the A^0 cycle is always odd for O3/O7 and even for O5/O9).
- Please take into account that, although we do not mention it, regarding to the SDC, we are also including the A^0 wrapping D3-branes [70] and assuming the bound state such that (schematically) $M_{\text{bd state}} \leq e^{K_{cs}/2}|n + x^+| \sim e^{-d_\gamma}|n + x^+|$, $d_\gamma \sim \ln((v^+)^{3/2})$.
- This conifold behaviour will be encountered also in the next example. When we approach through a one-dimensional basis path the conifold/orientifold loci, by obvious reasons, exactly the same happens as for the SDC one-dimensional analysis we performed before.
- When we talk about conifold singularities we always refer to the the $N = 2$ parent ones where the $N = 1$ Orientifold "lives", never to the conifold singularities inside the orientifold loci, which also appear when considering higher Hodge numbers. The latter are not relevant for our purposes.

[69]When higher Hodge numbers are involved, one needs to diagonalize in order to disentangle the gauge kinetic mixing.

[70]Bear in mind that this was the cycle corresponding to the $X^0 = 1$, which due to the projective nature of CS moduli space is not a coordinate itself.

5.2.2 A symmetric picture - $h_+^{2,1} = h_-^{2,1} = 1$

The in-depth analysis of this toy model is probably the main part of this thesis, because it gives, in the simplest possible way, a very precise and complete setting, which includes all the ingredients involved in the relation between the $N=1$ orientifold CS moduli space and its parent $N=2$. Concretely, it highlights the special role of its O3/O7 and O5/O9 $N=1$ "orientifold loci", and provides us with a completely different picture to that in [5], leading to possible modifications in our understanding of the conjectures for particles in $N=1$ unfluxed orientifold.

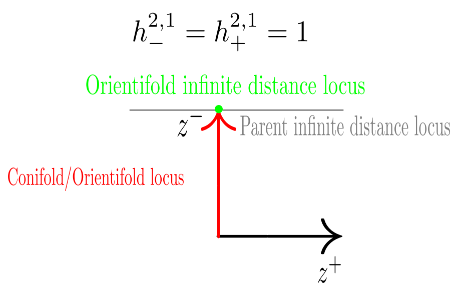

Figure 4: Example $h_+^{2,1} = h_-^{2,1} = 1$ setting.

Fortunately, this simple picture contains all the indispensable elements together, and it will not be really necessary to explore higher Hodge numbers. Firstly, we have a parent $N=2$ CS moduli parametrized by two complex coordinates x^+ and x^-, whose conifold limit (with the appropriate orientifold field configuration) correspond [71] respectively to the O3/O7 and O5/O9 orientifold CS moduli loci! We will focus in this text on the O3/O7 side, whereas the O5/O9 is analogous, just exchanging $+ \longleftrightarrow -$ and even $\longleftrightarrow$ odd everywhere.

As an asside, let us note that complementing both orientifolds and undoing each projection we recover a great deal of the parent $N=2$ (the LCS limit with both moduli going to infinite would be uncovered and also the double conifold would correspond to the internal conifold of both orientifolds). We are not going to deepen into this point but can be interesting to further study it.

Before going into the SDC, WGCSF and Emergence proposal, let us compute the gauge couplings in the Orientifold LCS regime ($v^- >> 1$) (the rest of the necessary elements are computed in a similar fashion as in subsection 5.2.1, taking into account that the orientifold locus is precisely 1-dimensional). Just comment that for this analysis we will always move within the orientifold locus $X^+ = 0$ and that $F_+|_{x^+=0} = 0$, in the LCS regime ($v^- \to \infty$), just imposes $d_{+--} = 0$ as the necessary condition that the mirror CY must verify in order the orientifold projection to be possible. From this condition (c.f. [14]) the gauge coupling cross terms (even-odd) decouple (are zero) so we just need to take care of $\mathcal{I}_{00}$ and $\mathcal{I}_{--}$ (also the mixed odd $\mathcal{I}_{0-} = \mathcal{I}_{-0}$ can be computed with the expected mixed behaviour) and $\mathcal{I}_{++}$ (similar results arise regarding the CS moduli space metric in (2.18) when sending $v^- \to \infty$[72]). Then we compute from (2.29), (2.18), (2.19) and using $d^{ij}d_{jk} = \delta^i_k$:

$$\mathcal{I}_{00}^{-1} = -\frac{6}{d_{---}(v^-)^3} \sim e^{-\sqrt{6}d_{\gamma_-}} \to 0 \;,\; \mathcal{I}_{--}^{-1} = -\frac{2}{d_{---}v^-} \sim e^{-\sqrt{\frac{2}{3}}d_{\gamma_-}} \to 0 \;,\; \mathcal{I}_{++}^{-1} = \frac{1}{d_{++-}v^-} \sim e^{-\sqrt{\frac{2}{3}}d_{\gamma_-}} \to 0 \tag{5.12}$$

Note that, as the orientifold locus is (complex) one dimensional, $d_{---} \neq 0$, otherwise the mirror

[71] In the sense precised in the beginning of subsection 5.2.

[72] For the skeptical reader, one can also take (5.3) and compute the dominating terms, obtaining the same result.

orientifolded CY would have zero volume. Nonetheless, nothing a priori can be said regarding d_{++-}, what will be an essential point in subsection 6.3.3.

It is also really important to note, that (5.12) is an LCS expression, which is not completely correct and gives us only the most relevant term [73]. In general, the mismatch is negligible, except in two cases: 1) when we have two derivatives with respect of x^+ of the prepotential (2.17) [74] and 2) when the most relevant term vanishes, for example if $d_{++-}=0$, one needs to compute (2.24), using (2.17). Taking both into account one obtains a fundamental difference for $\mathscr{I}_{++}^{-1}$ in (5.12):

$$\mathscr{I}_{++} \sim \text{Im}\bar{F}_{++} \sim d_{++-}v^- + \sum_{k^+} n_{k^+} 2\pi^2 k_+^2 \ln\bar{x}^+ + ... \tag{5.13}$$

As one can observe, there are two main contributions: the UV, already noticed in (5.12), and the IR divergence, coming from the parent $N=2$ conifold (5.11). The UV just dominates the IR in the exact LCS point, because the orientifold is exactly located at $x^+=0$, such a situation reminds us to the mass of the D3 wrapping A^+, where $M \sim e^{-d_{\gamma^-}}|x^+|$, the first term is the UV contribution and the second seems IR. The main difference is that the coupling receives one-loop corrections, while the mass is quantum mechanically exact [72]. Therefore, we note that $\mathscr{I}_{++}^{-1}$ conifold like diverges in the whole orientifold locus and moreover diverges exponentially on the LCS distance in an SDC way. This is hinting us that the SDC still has some remnants from the $N=2$ and is now associated to the avoidance of global symmetries at infinite distance in the parent $N=2$ which now are overlapped by the conifold effect (except in the LCS point). This *direction suppressing* interpretation is rather impressive and happens exactly in the same way for the locus $X^-=0$ (O5/O9) exchanging $+ \longleftrightarrow -$ everywhere, but, concerning the coupling, fits just as an artifact of the still unresolved conifold singularity.

This point is very subtle and will motivate subsection 6.3.3. On the one hand, we expect that, resolving the conifold à la Strominger [7], the IR contribution would dissapear when "integrating in" the massless D3-particle, obtaining the behaviour (5.12) in the LCS plus finite corrections in the rest. On the other hand, it is not clear to us that the truncation of [14] is completely correct (apparently is based on several SUGRA studies which are far beyond the scope of this text), in the sense that the proper way to derive the $N=1$ quantities would be to develop the $N=1$ action, but this would require the knowledge of $\int_Y \alpha \wedge \star\alpha$ and, for $N=1$, contrary to $N=2$, we care of the special geometry to obtain them without truncating. In addition, the meaning of the orientifold projection is that $x^+=0$ and $F_+|_{x^+=0}=0$ imply that this two periods are no more dynamical degrees of freedom, such that the second derivative $F_{++}|_{x^+=0} = \partial_{X^+}F_+|_{x^+=0}$ seems a kind of IR convective term which should not be present, as now both $F_+|_{x^+=0}$ and x^+ are neither coordinates nor degrees of freedom.

Let us remark that, even if it seems reasonable to follow the interpretation of [14] added to the view of the orientifold as the $N=2$ parent (resolved) conifold (once added the orientifold field configuration), leading to (5.12) plus finite non-LCS corrections; one should bear in mind the possibility that F_{++} should not be present in the truncated theory. In such a case, a different analysis to ours should be performed.

Furthermore, recall that the electric gauge couplings correspond to the inverse of $\mathscr{I}$. In fact, due to the gauge coupling kinetic mixing, one should diagonalize $\mathscr{I}$. Notwithstanding, the actual couplings in the odd sector would be a combination of the previous, and so experience a similar behaviour. Besides, as noted above, the orientifold projection eliminates the even-odd components [14], making

[73] In analogy, the same situation we describe for the coupling in this paragraph happens concerning the metric.

[74] In such a case, we obtain a "free" $\ln x^+ = -\infty$ coming from the $N=2$ IR conifold behaviour inherited by $N=1$ orientifold in the truncation [14].

already the even ++ part diagonal.

From now on, permit us concentrate on the concrete details concerning both conjectures and the Emergence proposal in our setting.

SDC

At a first sight, it seems obvious from (2.42), (4.1), the previous toy model and subsection 3.6.2 that the infinite stable tower verifying the conjecture for the $N = 1$ orientifold is precisely the one formed by bound states: D3 wrapping n-times A^0 - D3 wrapping A^- (for O5/O9 would be D3 wrapping n-times A^0 - D3 wrapping A^+). It comes just from the application of the previous model (subsection 5.2.1) to the Orientifold $N = 1$ locus of this model.

Nevertheless, the current situation is not so simple. From the O3/O7 spectrum in Table 9 and (2.39), we observe that the D3-particles wrapping the odd cycles are not charged under the RR gauge field V^+ coming from the compactification of C_4 and so they are unstable following the discussion in subsection 3.8 [75]. In such a section we already commented that the KK-winding example without gauge fields experienced a similar situation and, like here, the moduli space distance diverged exactly in the same way as with the gauge fields turned on (providing stability).

These “pseudo"-bound states involving only odd D3-particles would be highly susceptible to instabilities, due to their gravitational (and scalar) interaction, which cannot be contrarrested by any repulsive gauge force. However, the gravity and scalar interaction goes (exponentially on the distance) to zero as long as we approach the LCS point. This hints to a kind of condensate where these bound states would become “asymptotically stable", preventing us from reaching such a point.

We are aware that this interpretation would imply to relax the implicit assumption of stability for the SDC towers. Notwithstanding, we will argue when analyzing the Emergence proposal that quantitatively would fit.

It is oustanding that our unstable tower is not directly associated with the avoidance of the odd sector global limit of a $U(1)$ symmetry in the orientifold theory. Even though, in the parent $N = 2$, (5.12) would be associated to avoidance of global symmetry in the odd sector, after orientifold projecting (so with the orientifold projection configuration turned on) the C_4, giving us the $V^{\pm}_{\mu}$, is even and thus V^-_{μ} is projected (as well as the graviphoton V^0) making the bulk theory (and the D3 wrapping these odd 3-cycles) uncharged under the U(1). What we notice is a remnant of the parent $N = 2$ structure inherited by the $N = 1$ orientifold in the conifold locus. In other words, the SDC for $N = 2$ was related with the avoidance of a global symmetry, but, after the $N = 1$ projection, the tower satisfying the SDC remains (unstable), while the global symmetry is just projected out by the orientifold. This means that somehow the fundamental physics of the SDC, which is unclear from a pure $N = 1$ setting [76], can be partially understood in terms of its parent $N = 2$ as inheritance.

Even more subtle is the global symmetry limit in the even sector (I^{-1}_{++} in (5.12)). As we discussed before, it is not completely clear if one can trust the LCS computation and how to interpret it. This will become a crucial piece in the “Puzzle", because, from an Emergence point of view, the global symmetry $I^{-1}_{++} \to 0$ should be addressed to a tower of (maybe extended) objects charged under V^+. We will thoroughly argue that these do not exist in our setting!

[75] In fact, they are not BPS.

[76] To be honest, this is not completely true. We will see soon that the integration out of these states still can generate the infinite distance. As a consequence, the unstable SDC would be still fundamentally geometrical, but denoting the appearance of a condensate of D3-branes at infinity in $N = 1$, and the stable SDC in $N \geq 2$ gets related to global symmetries just because the towers becoming massless are also charged and, appart from generating the infinite distance, also cause the global symmetry limit.

Let us then propose a relaxation for the SDC in orientifolds:

SDC for particles in unfluxed Type II orientifold vector multiplet.
The SDC, concerning particles, is verified in Type II orientifold vector multiplet moduli space without fluxes, as long as the stability condition is relaxed.

The main arguments to propose this modified version of the SDC for particles in unfluxed Type II orientifold vector multiplet are:

1. Stability is guaranteed as long as the particles are charged under a gauge symmetry. We observe from Table 1, Table 2, Table 9, Table 12 and Table 13 that the $N = 2$ vector multiplet splits into $N = 1$ vector and $N = 1$ chiral multiplets. Therefore, the gauge fields coupling to the particles and the CS scalars providing them with mass decouple. As a consequence, the particles coupling to the $U(1)$ are massless in the whole orientifold moduli space and those which are massive (and verify the SDC behaviour at infinity) cannot couple to it. Thus, the SDC D-particle towers do not have a gauge symmetry protecting them, being unstable.

2. Emergence quantitative matching for the infinite distance divergence, which will be discussed later on this subsection.

Lastly, let us also advance that in [10], the authors found, for Type IIB $N = 1$ Orientifolds, BPS stable towers of D5-branes wrapping 3-cycles (D5-BPS domain walls) verifying the SDC and (Sub-Lattice) WGC for 3-forms (adjusting to the Emergence interpretation of [5]). This is possible because the D5 couple to C_6 (2.32), which is odd under the orientifold projection, as well as the CS moduli. Then, their mass (Table 8) is determined by the wrapping number of the 3-cycles and these objects are protected as they are “charged" (charge corresponding to the wrapping number) under a 3-form (topological?) bulk gauge field arising from the democratic compactification of the 10D action. Nonetheless, these states were already present in $N = 2$, such that quantitatively they will not change anything.

WGCSF

If the reader found the SDC rather subtle, the WGCSF for D-particles in orientifolds is considerably more involved. Along this section one should always keep in mind (2.26), (2.27), $\mathcal{Q}^2 = -\frac{1}{2}q_i(\mathcal{I}^{-1})^{ij}q_j$ and (5.12).

Looking again to the mass of the D3-BPS particles in $N = 2$ for any regime (2.42), we observe that at $x^+ = 0$ the D3-branes wrapping A^+ (“even D3-particles") are exactly massless. Moreover, when imposing the orientifold projection, also $F_+|_{x^+=0} = 0$, such that the D3-branes wrapping B_+ (“even D3-monopoles") and the D3-branes wrapping both A^+ and B_+ (“even D3-dyons") also become exactly massless, whereas in $N = 2$ they had a mass (if $d_{+--} \neq 0$ they get exponentially massive in the LCS, and if $d_{+--} = 0$ they become exponentially massless in the LCS, but outside the LCS point they are massive due to the corrections in (2.17)). Remarkably, the condition $F_+|_{x^+=0} = 0$ must be verified for any x^-, telling us that, if the parent $N = 2$ admits an O3/O7 orientifold projection, already must verify $d_{+--} = 0$ before the projection. Therefore, in the parent $N = 2$ the second case in the previous parentheses follows. Let us recall that this result is quantum-mechanically exact [72]!

Then, we observe that in the orientifold theory the even sector, i.e. the charged sector, is completely massless. Notwithstading, the gauge coupling (I^{-1}_{++}) coming from (5.13) is perfectly finite (5.12) after “integrating in" the even D3-particle and only leads to a global symmetry in the LCS limit. This already tells us that these even D3-particles are no more BPS because they feel a gauge interaction but they cannot feel a gravitational force due to the fact they are massless and also no scalar force because they do not couple to the x^- [77]. As a consequence, these objects are no more BPS and, due to the fact that they are massless and just interact through the $U(1)$, they cannot make any kind of bound state with another even particle because they just repel (annihilate) if the members of the state have same (opposite) charges. Moreover, as we noted before, the even components of I_{+J} in the orientifolds vanish [14] ($I_{+0} = I_{+-}$) and decouple, such that they do not interact with the odd D3-particles through charge or mass.

One can then claim that the even sector is isolated and we have found no way in which a tower (even just one state) of bound states verifying the SDC can arise from it (see section 6 for more details). Even more, we have only one even D3-particle! By the superposition argument of Strominger [7] (and subsections 3.6.2, 3.8) there was just one state, the even D3-particle, becoming massless when $x^+ \to 0$ in order to solve the conifold singularity. Once we “integrate it in" we resolve the conifold singularity and then project to get the O3/O7 theory. It was crucial, in order to match the one-loop correction for the gauge coupling, that just one state contributed ($q^+ = 1$), while the rest just decay to it [7]. From another perspective, now we have charged massless particles, the particles with $q^+ > 1$ are marginally stable and the decay channel to $q^+ = 1$ is open, making them unstable under decay to $q^+ = 1$ [78]. In addition, this particle is stable, in the sense of subsection 3.8.2, because it is charged, so cannot decay to vacuum, and is massless, so there is no energetically favoured decay.

Following this logic, we just find the $q^+ = 1$ D3-particle (instead of a tower of states), which clearly verifies (3.2) and, of course, does not verify the SDC behaviour (3.9) due to the aforementioned direction suppressing by $x^+ = 0$.

Thus, we observe that our example is in clear tension with the (Sub-Lattice) WGC (subsection 3.2) and also with the Completeness Conjecture (subsection 3.2.2) if one requires stability in their statements. In the other hand, such a particle fits perfectly within the realm of the Weak Cosmic Censorship (subsection 3.5), in the sense that any charged Black Hole would have an open decay channel to such a particle and it avoids superextremal macroscopic Black Hole solutions, because any such configuration would decay by means of this massless charged particle much before reaching a macroscopic regime. Somehow this D-particle is a kind of microscopic (in non-compact 4D) naked Black Hole.

An open question, which constitutes the heart of the “Puzzle" is to find what is wrong with the global symmetry limit $I^{-1}_{++} \to 0$ in the LCS. Following [5] line, one should find a (stable) tower of (bound) states charged under it which become (exponentially) massless at the LCS. In some sense, the aforementioned “asymptotically stable" odd “pseudo" bound states prevent us from reaching the $I^{-1}_{++} \to 0$ in the LCS, avoiding that global symmetry. Nevertheless, in terms of Emergence they are not satisfactory (as uncharged under V^+) and their “integration out" cannot reproduce the $I^{-1}_{++} \to 0$ behaviour. This question will play a major role in what follows of this section and the main part of section 6 will be devoted to this topic.

A similar story takes place regarding the massless D3-monopole and D3-dyon. By the same reasoning, we expect to have just the D3-monopole with $p_+ = 1$ and the D3-dyon with $q^+ = p_+ = 1$. In both cases, we observe that the magnetic (dyonic) coupling is constant and diverges in the LCS, as opposite to the electric case but also expected due to the usual $g_{\text{mag}} \sim 1/g_{\text{el}}$. In both cases the original version [1] of the mWGC would be verified by far due to the fact that their mass is exactly

[77]This can be noticed from (2.26), (2.27) and the fact that $D\_m = 0$, because $m = 0$ and $\partial\_m \sim |x^+|... = 0$. From a SUGRA perspective was expected.

[78]This latter argument goes in the line of the superposition argument described before.

zero:

$$m_{\text{mag}} \lesssim g_{\text{mag}} M_P \ , \quad m_{\text{dyon}} \lesssim g_{\text{dyon}} M_P \ . \tag{5.14}$$

To be precise, the couplings now are associated to the factor which accompanies the wrapping numbers in $\mathcal{Q}^2$ in (2.25). For completion, we will sketch how the magnetic coupling diverges in the LCS. The dyonic computation is analogous.

Let us take a D3 branes wrapping p^+ times B^+, then the interaction between two such states is given by:

$$\mathcal{Q}^2 = -\frac{1}{2}(p^+)^2(\mathcal{I} + \mathcal{R}\mathcal{I}^{-1}\mathcal{R})_{++} \ . \tag{5.15}$$

We already observed the predicted inverse behaviour from (5.13). The second term can be tediously computed taking all the corrections (2.17) into account and starting from (2.24). Nevertheless, as we discussed before, the unique term which is non-finite, within the non-LCS regime, is the $\ln(x^+)$ (coming from the ++ contraction), which would disappear once we “integrate in" [7]. Then we can safely focus on the LCS, which makes the computation much shorter by using (2.28) and (2.29). In such a regime we find:

$$(\mathcal{R}\mathcal{I}^{-1}\mathcal{R})_{++} = -\frac{6}{d_{---}(v^-)^3}[d_{++-}u^- \frac{1}{4}G^{++}d_{++-}u^-] \sim \frac{(u^-)^2}{v^-} \ . \tag{5.16}$$

Thus, considering (5.13) and (5.16), and taking into account the correction to the conifold singularity, we find the predicted LCS behaviour.

From what we observe in this example, we can propose the following modification for the WGC in our orientifold setting.

WGC for particles in unfluxed Type II orientifold vector multiplet.
The eWGC, mWGC and dWGC regarding to particles are verified in Type II orientifold vector multiplet moduli space without fluxes by the parent conifold inherited states. On the contrary, their (Sub-Lattice) tower versions are not verified, as long as stability is required.

We are aware that this $U(1)$, with just three stable objects charged under it and all being massless, is absolutely strange, as well as the tension existent between a setting which comes from String theory and so it is supposed to be UV complete with several conjectures and/or versions of them. Without any doubt, this setting deserves further studies in order to clarify all the mentioned issues around it. For example, we needed to test the initial version of the dual conjecture (5.14), because the usual version of the mWGC (3.3), which comes from $m_{\text{mag}} \lesssim g_{\text{mag}} M_P$ as shown in [1] and in subsection 3.2.2, is related to an UV cutoff Λ_{UV} usually associated to the scale of the towers verifying the eWGC. In that argument the expression (3.7) was used, but here we observe that the monopole is massless, even if the electric coupling goes to infinite in the LCS. It is not clear to us whether the behaviour (3.7) is restricted to the massive case or it is valid also for massless particles and monopoles. The first option seems to be the right one, because in the second case, the UV cutoff for the theory would have to be exactly zero, which looks rather inconsistent.

Relation between both for $N = 1$

We have already discuss a bit about the change in the relationship between both conjectures (e.g. footnote 76), and we advanced before that the Emergence interpretation of these results will become absolutely puzzling. In $N = 2$ we had charged massive towers and in $N = 1$ we apparently have uncharged massive "asymptotically stable" towers and three charged massless states, each of those serves to respectively verify the simplest versions of the eWGC, mWGC and dWGC. From this point of view, we expect a challenging Emergence analysis. In $N = 2$ the SUGRA structure with gauge fields and CS moduli in the same multiplet, allows for BPS states coupling to both (and to the Gravity multiplet with metric and graviphoton) simultaneously, providing us with a (subtle) "duality" and an Emergence proposal regarded as the fundamental microscopic origin for both.

From our studies, we noted that this duality fits more as an artifact of the $N = 2$ SUGRA, bound to the BPS condition, than as general statement. In $N = 1$ we do not have such an structure, we lack of D-BPS particles and so we observed that [5] duality for particles appears to break down.

Let us summarize in a proposal the relationship that we have found so far between both conjectures (for particles) in our unfluxed orientifold setting:

Proposal for relation between SDC and WGCSF in $N = 1$ Orientifolds for D-particles. *The $N = 2$ "duality" SDC≡WGCSF for D3-particles in Type IIB CY3 breaks down in $N = 1$ Type IIB CY3 Orientifolds exactly as the $N = 2$ vector multiplet spectrum (gauge vector and CS moduli) splits into $N = 1$ vector (gauge field) and chiral multiplets (CS moduli). This is because the D3 couples to both, through its mass to the CS and through its charge to the gauge vector. To sum up, regarding the D3-BPS particles, when $N = 2 \to N = 1$ the conjectures decouple just as the SUGRA spectrum.*

The evidence for this proposal has been already provided and we also think that it extends to similar cases, where the Dp-branes couple to the bulk spectrum splitting under the projection into different multiplets when orientifolding. For example, we expect that in Kähler moduli space the same happens regarding to the D-branes wrapping 2-cycles (e.g. D-strings of subsection 2.2.2, which are SDC candidates for Kähler moduli in Type IIB) coupling to both J (through DBI mass) and C_2 (through Chern-Simons charge), up to quantum or flux effects, which could modify the exactness of the expressions, although we strongly believe that the fundamental grounds remain. It is worth to mention that precisely in O3/O7 case the Kähler twisted sector is given by the $h^{1,1}_-$ cycles, so that we find again the same story, at least at a classical level.

What is clear from this proposal is that an analysis of the conjectures and their relation for different SUGRA charges $N = 0, 1, 2, 4, 8$ would be really interesting and, from our viewpoint, actually mandatory. One expects that the behaviour found for $N = 2$ straightforwardly extends to $N \geq 2$, but the cases of $N = 1$ (partially studied in this project) and specially $N = 0$ are not only the most challenging ones, but also the most interesting at a phenomenological level.

Emergence proposal

The emergence proposal analysis is challenging. The differences with respect to subsection 3.6.2 are evident and the picture of [5] gets modified.

First of all, naively one would expect more than one-loop corrections playing a role. We discussed before that, from a stringy point of view, the gauge coupling is still one-loop exact in $N = 1$. Besides, K_{cs} receives infinite corrections. The first-loop correction comes from Eisenstein series, but these become negligible when reaching infinite distance in the LCS regime [69], and so it will not play a role in the Emergence analysis. There is no information in the literature about the higher order corrections to K_{cs} so we are not going to take them into account, assuming they vanish at the LCS regime. This is the expenctancy from String theory and so we are going to adopt it in the following analysis, considering the one-loop correction as "asymptotically exact" in the LCS regime.

The analysis of [5] was analogous to that one presented in subsection 3.4.2. This is because the towers of $N = 2$ verified that all the states were stable, charged and massive, the full lattice was assumed to be populated and had a constant mass gap.

The situation in our framework is absolutely different. On the one hand, we found the "asymptotically stable" towers consisting of D3 wrapping n-times A^0-D3 wrapping once A^+ massive uncharged bound states, whose mass goes exponentially to zero at infinite distance. We already discussed the stability issues regarding to such states and argued that in the LCS point they may form a kind of condensate (precisely avoiding we reach that point). Taking them into account, the Emergence derivations (3.21), and (3.27) providing us, respectively, with the species scale $\Lambda_s = M_P/\sqrt{N}$ and the SDC behaviour remain untouched.

Moreover, was observed in [10] that might be necessary to consider the "integration out" of, not only particles, but also extended objects like D-domain walls and D-strings. Many of such objects would be projected out by the orientifold unless one takes into account our "asymptotic stability" interpretation for the uncharged "pseudo"-bound states. At the same time, we observe exactly the same bulk distance divergence as in $N = 2$ [79], which suggests that in order to trust Emergence quantitatively the same contribution from the towers must be present at the LCS. For unfluxed orientifolds that seems only possible taking into account these unstable states, which become more and more stable as we approach the LCS point. Let us summarize this line of thought in the following concrete statement.

Emergence for massive sector in unfluxed orientifolds. *The Emergence proposal for the massive sector of unfluxed orientifolds is inherited from $N = 2$ if, and only if, the "asymptotic stability" interpretation is correct.*

Next, let us turn our focus to the charged sector, where we just have the massless D3-brane wrapping once ($q^+ = 1$) A^+. From the very beginning, it seems quite difficult that the integration out of just one particle could cause exactly the same effect in the gauge coupling I^{-1}_{++} (5.12) as in $N = 2$, where the global symmetries were addressed by the integration out of whole towers of states [80].

Following [5] Emergence proposal, this global symmetry should be cured by "integrating in" towers of states charged under V^+_μ and becoming SDC (3.9) massless at the LCS point, where the global symmetry arises. Searching for them will be the topic of subsection 6.1 and we advance that, unfortunately, such towers do not seem to exist. As a consequence, we will analyze Emergence considering just this even D-particle.

The main equation in [5], concerning the charged sector, was (3.33). It comes from the self-energy loop diagram involving the coupling of the $U(1)$ gauge boson ("photon") with the tower of charged D3-BPS particles in the theory. In fact, this kind of computation comes from the conifold analysis of [7] and was extended to N particles in [5] by adding the contributions of all the particles below the species scale (the cutoff for the theory in [5]). Let us briefly write the one particle contribution [73]:

[79] Appart from the higher order loop corrections which are thought to be negligible in the LCS regime.

[80] Bear in mind this last sentence. We will later show that even from an $N = 2$ perspective I^{-1}_{++} in (5.12) behaves strangely.

$$\frac{1}{g_{\rm el}^2} \sim \int d^4p \frac{1}{(p^2+m^2)^2}\frac{\Lambda_{UV}^2}{p^2+\Lambda_{UV}^2} \sim \ln\left(\frac{\Lambda_{UV}}{m}\right) \tag{5.17}$$

where the second factor in the integrand guarantees a smooth high momentum cutoff and can be really omitted for our purposes. As a consequence, we obtain the logarithmic behaviour from (5.17); the sum and prefactors come from a detailed computation taking into account the N charged particles.

In [5] $\Lambda_{UV}=\Lambda_s$, because the charged particles were also massive with the constant gap structure. In our setting the situation changes dramatically. First of all, we just have one charged particle, which is actually massless. Secondly, the Λ_{UV} for the charged side of the theory should not be the species scale or, at least should not include the uncharged towers and so $N=1$ in $\Lambda_s = M_P/\sqrt{N}$, corresponding to the unique massless even D3. In fact, we discussed before that was difficult for us to establish any candidate for the cutoff for a theory with just one particle, one monopole and one dyon [81].

Firstly, let us naively consider (5.17) and take the massless limit. Then, we obtain:

$$\frac{1}{g^2}|_{IR} \sim \cancelto{0}{\frac{1}{g^2}|_{\Lambda_{UV}}} + \ln\left(\frac{\Lambda_{UV}}{M_{D3-A^+}}\right) = \ln\left(\frac{\sqrt{\frac{4}{3}d_{---}}\Lambda_{UV}}{|x^+|e^{-\sqrt{\frac{3}{2}}d_{\gamma_-}}}\right) \sim -\ln|x^+| + \ln\left(\frac{\Lambda_{UV}\sqrt{d_{---}}}{e^{-\alpha d_{\gamma_-}}}\right) \tag{5.18}$$

where the first term corresponds to the conifold behaviour observed in subsection 5.2.1. Once we "integrate in" the even D3-particle in the theory this logarithmic term gets corrected and so we obtain only the LCS contribution [82]:

$$\frac{1}{g^2}|_{IR} \sim \left(\frac{\Lambda_{UV}\sqrt{d_{---}}}{e^{-\alpha d_{\gamma_-}}}\right) + \ldots \tag{5.19}$$

It seems absolutely unlikely that this Λ_{UV} would take a kind of $e^{e^{\cdots}}$ behaviour in order to match the I^{-1}_{++} (5.12). Let us note that if one takes the UV scale as the species scale with N=1 in (5.19) one obtains a kind of first order result $1/g^2 \sim -\alpha d_{\gamma_-}$ of I^{-1}_{++} (5.12). In any case, we argued before that the one-loop approximation should be also valid for $N=1$, so seems that this caveat would not arrange the situation. Besides, the crucial factor d_{++-} in (5.12) cannot be obtained in this way.

Now, instead of approaching the massless limit, we directly take the massless propagator and integrate, obtaining:

$$\int_{0_{IR}}^{\Lambda_{UV}} d^4p \frac{1}{p^4} \sim \ln(\Lambda_{UV}) - \ln(0_{IR}) \tag{5.20}$$

[81]Recall that the D3-branes wrapping 3-cycles are not just one particle. In fact, we commented before that for $N=2$ an $N=2$ hypermultiplet (4 real scalars/2 complex scalars + 2 Weyl fermions) arose in 4D from the conifold even D3-BPS particle. For $N=1$ it is not completely clear to us exactly how many particles would arise from the even D3-particle in non-compact 4D.

[82]The authors of [44] explore the ultralight charged particle case, proposing a renormalized version of mWGC-Emergence by means using the renormalized coupling instead of the IR. In some sense, the "integrating in" of the even D3-particle, and so supression of the logarithm, that we perform is equivalent to that statement. Nevertheless, again they are in trouble for the exactly massless case which they do not explore in detail. We believe that this is due to the fact that they realized of a similarly strange behaviour like our.

In this case we find a rather non-sensical result with two cutoffs, an IR coming from the fact that the massless charged particles move at speed of light and must have a non-zero energy/momentum (in order to be detectable through any kind of collision), and the UV about which we have little information. In any case, it does not appear possible that this Λ_{UV} would take the required behaviour in order to match I_{++}^{-1} (5.12).

We observe that, in both cases, there is no agreement with I_{++}^{-1} (5.12), such that the Emergence proposal for the charged sector seems in clear tension with orientifolds. We will resume it in the following sharp statement.

Emergence for charged sector in unfluxed orientifolds. *The Emergence proposal for the charged sector of unfluxed orientifolds quantitatively fails, if our analysis is correct and we assume these settings are not in the Swampland.*

We will come to this point in more detail in section 6, where we will refine the above statement, seeking to reconcile our setting with Emergence and a physical explanation for the I_{++}^{-1} (5.12) global symmetry and presenting two alternatives in subsection 6.3.

We are aware that our is a rather rudimentary treatment for this unusual theory with just massless charged perturbative and non-perturbative states. The above sketches were performed with the intention of showing to the reader that the requirement of a tower seems mandatory in order to obtain the global symmetry behaviour I_{++}^{-1} (5.12). Might be that a more evolved treatment for such a theory would false this analysis and make sense of the situation. Notwithstanding, we have not found such a treatment in the literature and suspect that it has not been already developed due to its technical difficulty, and also because phenomenologically massless charged particles and, especially, the massless charged monopoles and dyons are not experimentally observed and even less a theory containing all them together.

5.2.3 Expected behaviour for higher Hodge numbers

It is rather clear from the above analysis, that we will find an identical behaviour where the O3/O7 and O5/O9 pictures again complement themselves, obtaining exactly an straightforward (but technically hard to develop) generalization of the above subsection 5.2.2 example. The main difference is that now, of course, not every conifold in parent $N = 2$ can be related to an orientifold locus. The latter are precisely located in interesections of some of the former and, regarding to these cases, [7] already tells us that its treatment is not trivially generalized. References regarding to multiple shrinking cycles are [70] and [71], where topological transitions and homological relations between the vanishing cycles are explored. However, we are not going to delve into such situations in this thesis.

Finally, just add that might be interesting, following this view of the relation between parent $N = 2$ conifold singularities and $N = 1$ orientifold loci, to find consistent generalizations of the orientifold projections which could live in these conifolds for the higher dimensional CS moduli space case, corresponding to higher $h_{\pm}^{2,1}$.

5.2.4 Comparison with [10]

This master's thesis research project was performed between October 2018 and August 2019, and will be presented in September 2019. Along this period of almost one year, the field was very active and many papers were published. Among these, there was a publication [10] by L. E. Ibáñez et al., whose central topic was similar to our, in the sense that they examined the SDC and WGC for D-domain walls, and the SDC for charged D-strings and D-particles. Their point of view was different to ours, focusing on those similarities with respect to $N = 2$ [5] and establishing an analogy to their Emergence picture. Nevertheless, some of their results/methodology are shared by our analysis, although others are in tension.

When [10] was published, we had already found some of their results. Therefore, we note these as points in common.

Their paper was published quite before the expected end of our project, thus we decided to take profit of those aspects which, we consider, were formulated in a better way and those ideas which we had not contemplated at that time, as well as those statements in which we differ.

We consider that the similarities between our work and [10] are mainly superficial and inherited from [5], while the differences are much more stronger and fundamental, deepening into the core of the “Swampland Program" and the Emergence proposal.

Similarities

Along this thesis, we have always cited [10], whenever our and their ideas/treatment were similar and also when we borrowed original ideas or results that we did not find before the publication date.

Due to the fact that we have already noted several times the similarities along the text and they can be checked reading [10], we are not going to spend more space on this and only focus on the differences. As a consequence, we will just enumerate the main similarities:

1. The prior knowledge coming from the literature and especially [5].
2. The approach of using basis of D-branes wrapping once each cycle.
3. The superposition argument motivating the need of bound states in order to satisfy the SDC.
4. The mass formulae regarding to D-particles and D-strings in Type IIB. [83]

Results/Ideas borrowed

Next, we mention again (we already did in the text) those ideas and results we took from their work and we had not found before the publication date.

1. The mass formulae regarding to all D-domain walls and the D-particles and D-strings in Type IIA.
2. The whole D-domain wall analysis.
3. The proposal of taking into account the “integration out" of extended objects in the Emergence proposal.

[83] In the text (subsection 2.2.2) we adopted their result and notation in order to unify criteria as they found more masses than us, and so in their notation we can present the masses in an uniform way.

4. The concrete mathematical check that the tower of bound states of D3-branes wrapping n-times A^0 - D3-branes wrapping once A^i, satisfying the SDC when $v^i \to \infty$, can be rigurously obtained from the Monodromy Orbit formalism.

Differences

Much more relevant and fundamental are the differences than the similarities. Therefore, permit us focus on them.

- **Only charged Dp-branes were considered in [10].**

  They only consider charged Dp-branes, and so, for our previous models, they would not find any D-particle tower of states regarding the SDC. It is stated that the uncharged ones are projected, but we did not find arguments for such a conclusion within their paper. We think it might be due to the following reasons:

  1) They surely expected to find the [5] $N = 2$ interpretation, so that they may not have noticed about these states which are precisely in conflict with it. (Quoted page 42 of [10]: "Finally, it is remarkable that in the orientifold case the towers of particles and several towers of strings are projected out by the orientifold action. This is always the case when the 1-forms or 2-forms that couple to the particle or string in question are projected out by the orientifold, as in the toroidal example. It is, moreover, consistent with the relation between the towers coming from the WGC and those from the SDC and the idea that these towers prevent the appearance of a global symmetry. This is due to the fact that when the corresponding q-form fields are projected out there is no gauge symmetry giving rise to a global one at the infinite distance point, hence not requiring the presence of the corresponding tower.")

  2) As they already found D-domain walls satisfying the SDC for the orientifold case and matching the [5] $N = 2$ interpretation, everything seemed to fit perfectly and it was not mandatory that any particle satisfied the SDC because the domain walls already did it.

  3) This is also related to the fact that they do not delve into (quoted page 47 [10], "the volume of the corresponding 3-cycles could not be computed integrating the holomorphic 3-form which is odd under the orientifold involution") the even D3-particles (verifying the eWGC for particles), such that to match the above quoted argumentation it was not required for any D3-particle to satisfy the SDC.

  4) Again, as they checked the (p-form version (3.5) of the) eWGC for D-Domain Walls, they did not need to focus on the role of D-particles concerning it.

  5) The uncharged Dp-branes are supposed to be unstable (subsection 3.8) because no gauge symmetry protects them. This kind of D-branes usually contain tachyons in the spectrum and so with "project" [10] might mean they are GSO projected or unstable. In any case, we considered before the possibility of an "asymptotic stability" interpretation when reaching infinite distance.

- **They consider that $M_{D3}^{BPS} \propto \int_{\Gamma^+} \Omega = 0$ might be wrong.**

  Following the quote from [10] in 3) of the previous difference, we do not observe anything wrong with taking $M_{D3}^{BPS} \propto \int_{\Gamma^+} \Omega = 0$ with Ω odd and Γ^+ even. It is consistent with the fact that we inherit these twisted even D3-particles, charged and massless, in the $N = 1$ (identically for the odd case in O5/O9) orientifold setting from the parent $N = 2$ in the conifold. These states were essential for resolving the conifold singularity and seem to remain with the role of being the eWGCSF (really eWGC because they also decouple from the scalars x^-) particles. Another point would be that, due to fluxes and/or space-time filling D3/D7, the geometry became warped ([21]) causing changes in the CY analysis near the conifold (and so the orientifold loci). Nevertheless, such effects would be relevant in the $N = 2$ framework but once we take the unfluxed $N = 1$ orientifold they would not come into our setting.

From (2.34), as well as the fact that the orientifold imposes rigorously $X^+ = 0$ and many other unsuccessful trials, we have not already found a way to make precise any quantum effect into the mass formula, so two options remain: 1) Actually these states are quantum mechanically exactly massless in the orientifold limit ([72]) and simply this is not a phenomenological model or 2) The mass receives some Quantum Gravity corrections whose origin is unknown to us. In any case, if such corrections already existed it is unlikely that they modify our picture in the LCS regime and/or become big enough to permit the even D3-particles to form bound states.

- **Their analysis is more conservative regarding [5] and our is more critical.**

  [10] followed a continuist approach, using the techniques present in the literature and in [5]. Concretely they analyzed D-BPS domain walls, which, as discussed before, are a very particular case where the BPS structure seems to remain, so that the picture of [5] was verified. Once they observed that the picture remained for that case, we believe they considered it would still be correct for all the objects in the theory.

  Our point of view was more incisive, always trying to get the complete physical picture. It was motivated because we focussed from the very beginning on the D-particle case, where there is no apparent BPS structure, and all the subtleties described in this thesis arose at an early stage. Unfortunately, many of the questions and subtle points required of a deeper knowledge than our and may be that also of the whole community, judging in base of the literature that we have been able to read and find.

  It is fair to affirm that our study of particles goes deeper in the physical sense and seems clearly extendable to more D-particles, D-strings and, some features, also to the D-Domain Walls, in different moduli spaces, by taking [10] analysis and then including our contribution regarding to all the “forgotten" uncharged Dp-branes. Even though the branes are not charged under the projected $U(1)$ gauge vectors, they can wrap cycles, even if these cycles do not give them any charge under the remaining $U(1)$s after the orientifold projection. The price to pay is stability, but, appart from that, we do not find any strong reason to not contemplate such states.

  In addition, we consider that our picture concerning particles, although not so simple and possibly incomplete, fits much better with the expectancies of having bulk $N = 1$ SUGRA (without BPS D-particles) and the viewpoint of the orientifold locus as a corrected parent conifold adding the orientifold field configuration.

  As a consequence of taking all states into account, the pictures for $N = 1$ Orientifold SDC and the WGCSF concerning the particles are more logical to us, corresponding to what one would expect from a pure SUGRA spectrum analysis.

  Finally, the crucial “Puzzle" of subsection 6 is not noticed within their analysis. In our treatment, it arises naturally.

- **Their treatment is more technical and our is more intuitive.**

  They used more evolved mathematical techniques and are more precise, with more concrete results. That was possible, because the mathematical machinery required for their objectives was already available in the literature, whereas we gave priority to those subtle points for which, unfortunately, technical tools are lacking in the literature (up to our knowledge). As a consequence, we needed to be more intuitive, take more risks, be less precise (sometimes even rudimentary) and barely conclusive in order to get some insights.

6 Analysis of the “Puzzle"

The present section is devoted to state more concretely and try to solve the aforementioned “Puzzle".

We call **“Puzzle"** to the situation encountered in subsection 5.2.2, where we do not have a fundamental physical origin for the global symmetry $I^{-1}_{++} \to 0$ (5.12). More concretely, there is apparent tension between the global symmetry behaviour in the bulk $U(1)$ charged sector of the theory and the Emergence proposal. That is, in order to match Emergence, one should find some tower coupling simultaneously to V^+_μ and x^-, in order to justify the LCS global symmetry behaviour $I^{-1}_{++} \to 0$ (5.12).

As remarked in the subsection 5.2.2, we “inherit" from the parent $N = 2$ one stable electrically charged D3-particle (massless) and appears to be incompatible with the exponential behaviour experienced by the coupling, which is typical from the integration out of infinite towers of states becoming exponentially massless on the geodesic CS moduli space distance.

The most natural attempt to solve this apparent inconsistency would be to search for another states coupling simultaneously to both V^+_μ and x^- and being able to form an infinite tower. Thus, we are going to exhaust all possibilities, that we were able to conceive, within String theory and the result will be that there is no more states within Type IIB O3/O7 (same for O5/O9) satisfying the required conditions appart from the familiar D3-branes wrapping 3-cycles (up to our knowledge).

Later, we will explore the possibility that the bulk $U(1)$ gauge theory might be actually sick. The result will be that we cannot claim such an strong statement.

Finally, we will comment on the possible solutions which are compatible with Emergence and an intuitive Physics explanation for $I^{-1}_{++} \to 0$ (5.12). Firstly, we will explore the option with less sense, that is to consider multiparticle states/resonances of D3-branes wrapping even 3-cycles and/or (towers of) D3-branes wrapping even and odd cycles simultaneously. We will quickly discard this option due to several reasons, among which the definitive one is that they cannot provide the d_{++-} present in $I^{-1}_{++} \to 0$ (5.12). Next, we will present a bold statement, sending the orientifold twisted sector to the Swampland. Lastly, a geometric alternative against the interpretation of $I^{-1}_{++} \to 0$ (5.12) being really a global symmetry at the LCS, will be proposed. Both will be motivated pointing out benefits, disadvantages and possible ways to test them.

In this section it is really important to keep in mind the previous discussion and arguments developed, especially, in subsections 3.6.2, 3.6.3, 3.7, 3.8, 5.2.1 and 5.2.2.

6.1 Searching for towers of states coupling to V^+_μ and x^-.

We have already computed the relevant $\mathscr{I}^{-1}$, D3-brane masses and their relation to the CS moduli space distance previously in (5.12), (4.1), (5.2), (5.1). We must add to them the O3/O7 projection of the spectrum given by Table 9 and the information provided in subsection 2.3 in order to follow better this subsection.

From previous discussions, we realize that we require states coupling to the V^+_μ and becoming also SDC massless when $v^- \to \infty$, in order to explain, by means of integration out, using (may be an addapted version of) (3.33), the $I^{-1}_{++} \to 0$ global symmetry limit from a physical perspective.

The initial direct proposal (also motivated by the splitting of the SUGRA vector multiplet) is that the odd (coupled to CS and so massive) bound states n D3 A^0 - D3 A^- satisfy the SDC and explain the distance divergence, while the even (coupled to C_4 vector and so charged) D3 A^+ satisfies the eWGC and takes care of the even global symmetry. Nevertheless, when we add the stability issues discussed before and try to find how this massless charged D3-particle could take care of the global symmetry limit at the LCS (they even do not couple to the CS), we observe that the situation is far from trivial and rather “puzzling".

Another alternative would be to find some states that still in the projected theory could couple to both x^- and V_μ^+ simultaneously. A priori, these do not need to be D-particles, might be D-strings, D-domain walls or come from the open string sector.

Following this last guideline, we will analyze right now different cases and observe that just one expected kind of D-particles fit. Firstly, we begin with the easiest alternative, that is to explore the D3-branes wrapping simultaneously even (charged) and odd (massive) cycles. Next, we take a closer look to other possible states and check if they could couple to both CS moduli and bulk RR gauge vector. The possible options are probe Dp-branes (also NS5) wrapping q-cycles and the open string sector.

- **Even-odd D3-particles couple simultaneously to C_4 and CS moduli.**

  We have already observed that the even D3-particles couple to the V_μ^+ vector (coming from C_4) while the odd ones couple to x^- CS moduli. Of course, if we wrap D3-branes in even and odd cycles simultaneously we can get mass (CS coupling) through the odd ones and charge (gauge vector coupling) through the even ones. In such a case, might be that we could make bound states with these, because we have both repulsive and attractive forces. The main problem is again the stability. These states are not BPS because the mass is not bound to the charge (the even wrapping numbers are associated to the charge and the odd ones to the mass) such that energetical arguments point to instability and possible decay.

  Besides, it is rather plausible that they decay ultimately to odd and even D3-particles, finding the cases described along the text, because as both sectors (even and odd) clearly decouple the mass and charge of the final and initial states are the same [84]. In other words, there is a decay channel open and nothing prevents them from decay.

- **Probe Dp/NS-branes are not the solution.**

  Coupling to the CS moduli necessarily requires for them to wrap a 3-cycle, and coupling to the bulk RR gauge vector requires them to be Dp-branes with $p \geq 3$. Following (2.32), we see that the three terms consist of an even number of form and so, a Dp-brane with even p can never couple to the Type IIB RR gauge fields. As a consequence, we can discard those because We already have enough unstable material. The case of D3-particles has already been explained and what is left are the NS5 and D5 wrapping 3-cycles [85].

  The case of NS5- and D5-domain walls [86] was studied in [10], where the authors found that these verify the p-form eWGC (3.5) and can form SDC stable towers in the LCS regime. Notwithstading, these couple to C_6 but not to C_4 and so cannot take care (even qualitatively) of the $I_{++}^{-1} \to 0$ global symmetry when $v^- \to \infty$.

  The fact that D5-branes do not couple to C_4 can be seen from expanding the Chern-Simons action (2.32):

$$S_{CS}^{D5} \sim \int_{M^3\times\Gamma_3} C_6 + \int_{M^3\times\Gamma_3} C_4 \wedge \mathscr{F} + \dots \tag{6.1}$$

[84] Recall that this is not a bound state of branes, but just one brane wrapping both cycles. We already discussed that due to the total decoupling between the even and odd interactions there is no plausible way to build such “mixed" bound states between charged-massless and uncharged-massive states.

[85] D7 or D9 cannot wrap only a 3-cycle in the CY.

[86] Both in practice just differ by a factor of $g_s = e^\phi$, so by strong-weak regime, and the fact that the NS branes couple to the NS-NS B-field instead of the RR gauge fields.

The relevant term for our purposes is the second. First of all, it requires we allow fluxes and/or a B-field, so it would not be general enough to provide us with a satisfactory explanation [87]. However, it does not really matter because we can just have three legs in the non-compact 4D space-time and one must come from the vector inside C_4 expansion (2.12), so that just two legs are left for the B and/or F fields. The B field is odd under the projection (2.49), which eliminates the two-form over the non-compact space $B_2(x)$. Concerning to F, it is also odd under the projection [27] and just fluxes might be even but, in order to preserve four-dimensional Lorentz invariance, we consider gauge flux only in the internal (CY3) space (look at [27]).

Remarkably, we observe that in $N = 2$ these D5-domain walls would couple to the V_μ contained in C_4 and the projection breaks this connection, again as expected from a SUGRA point of view.

Finally, the NS5-branes couple electrically to B_2 and magnetically to B_6, and not to RR fields, so do not help to solve the "Puzzle".

- **Probe (p,q)-branes are not an alternative.**

  S-duality mixes axion C_0 and dilaton $e^{-\phi}$, as well as C_2 and B_2 leaving C_4 invariant [18]. The fact that B_2 and C_2 have both the same (odd) parity causes that such a mixing cannot help us in (6.1).

  By means of S-duality D7-branes are "mapped" to (p,q) 7-branes [18] (for D5-branes a similar situation might occur). As a consequence, these can be safely discarded.

- **Open string sector has no relation to the "Puzzle".**

  Permit us to consider the space-time filling D3 and D7, as well as their moduli fields and the possible open strings attached to the D-branes. In this section becomes relevant the general analysis sketched for the O3/O7 case in section 2.3 (c.f. [13]) where we included the open string sector and its coupling to the bulk.

  It is quite clear that the space-time filling D3 do not couple to V_μ^+ (2.32). This can be ultimately understood as the fact that they do not wrap a 3-cycle. For the concrete expressions and dependence regarding to the stack of N space time filling D3 we refer respectively to equations (4.24) and (4.25) in [13]. It is also interesting to compare them to note that these feel "no net force" because the first terms of the equations cancel each other. The remaining terms are solitonic contributions to the 4D theory introducing couplings between the previous bulk theory and the open string sector coming from the brane. Just recall that this D3 is exactly superposed over the 4D Minkowski and so acts as a background modification rather than a dynamical object from the perspective of a non-compact 4D observer (in the strong coupling limit might also play that role).

  For the general action we refer to [13], equation (4.36). One observes there that nothing couples to the bulk gauge vector V_μ^+ and the CS moduli in this new background, including the stack of N space time filling D3, are no more isolated but couple to the other moduli. Concretely, they contribute (third line of [13], equation (4.36)) to the open string scalar moduli ϕ^i mass but through derivatives $\partial_\mu z^{k-}$ such that there seems to be no exponential supression in the LCS limit.

  It might be interesting to examine this new coupling to the other moduli, which makes no more diagonal the CS moduli contribution to the Kähler potential (2.51). As a consequence, strictly speaking there is no direct product $\mathcal{M} = \mathcal{M}^{cs} \times \mathcal{M}^k$ and we should not talk about CS moduli space alone without taking into account the coupling effects from the Kähler moduli

[87] Recall, we can avoid fluxes including only space-time filling D-branes to cancel the tadpole and the (v.e.v. of the) B-field is not required to be turned on. Nevertheless there is no inconvenient with including open string fluxes, which provide with a richer "T-dual" picture.

side. Nevertheless, in the LCS the coupling term is negligible with respect to the K_{cs} such that the logarithmic divergence in the distance (as long as we do not take also at the same time the other moduli to infinite) and the I_{++}^{-1}, when freezing the rest of the moduli, are not modified.

Remarkably, including the open string sector, κ (notation of [13] for $d = d_{ijk}v^i v^j v^k$) depends implicitly on the CS moduli. In the Einstein frame, the mass of the space-time filling D3 and many fermions depend exponentially on κ ([13]) and so are likely to verify the SDC behaviour (3.9) in the LCS due to the introduction of the open sector and its coupling to the bulk [13]. In any case, no open string moduli, nor the space-time filling D3 couple to the V_μ^+, so that they are not useful to explain the global symmetry limit.

Regarding to the space-time filling D7, the derivation is much more complicated so we refer to [13]. In the reference, one can obtain the DBI and Chern-Simons couplings as the equations (4.47) and (4.49). In both we see that there is no coupling to the x^-, nor the V_μ^+. The quick explanation is that these D7 wrap holomorphic (to preserve SUSY) four cycles, and so they couple to Kähler moduli but not to CS nor the vector multiplet [88]. Again from a 4D perspective these solutions (in the weak coupling limit) can be regarded more as a superposed background over 4D Minkowski.

In the D7 case (look at (4.51) in [13]) the CS moduli is completely isolated and remains exactly as in the bulk theory without any extra coupling to the open string sector and the same happens to the V_μ^+. As a consequence, no open string moduli can help us to solve the “Puzzle" and the CS moduli remains factorized (recall section 2.3) making completely valid our previous bulk analysis of Kähler potential, distances, global symmetry limit ... for this moduli space.

Next, we can also consider the strings attached to D3/D7 but we do not see how they could be charged under $V_\mu^{k_+}$ and their mass be dependent on the CS moduli (in an exponential SDC behaviour (3.9)).

Finally, let us note that even if the appropriate states were present in the open string sector, for a general setting there is not an infinite tower of them but a finite number. The number of space-time filling D3/D7 is constrained by the tadpole cancellation conditions and is finite for an specific setting. In addition, the number of open string moduli associated to each of these space-time filling D-branes is finite. Therefore, there would be in practice a finite number of states instead of the required infinite number.

Open string gauge field is not the answer.

Concerning the fact that the odd D3-particles are unstable in their coupling to the bulk, we thought that maybe the open string sector by means of the D-brane worldvolume gauge field could give a charge to the these unstable branes making them stable. In [28], the authors, in a completely different context (F-theory, 6D compactifications) found that for a global open string symmetry limit there exists a tower of critical bulk heterotic strings becoming tensionless at the LV limit. It is a beautiful description and, in our case, it may be that something similar happens (recall that the Kähler moduli in general is much more related to mass of the space-time filling branes and couples completely to the open string moduli unlike the CS moduli), even though seems hard to explore this point. In any case, these would not address our issues because we need objects charged under the C_4 in order to explain the bulk global symmetry so both problems appear to be decoupled.

Let us focus on the possibility that the odd D3-particles could somehow be charged under the open string gauge field. What makes them unstable is that they have a mass that is not bound to any charge (in $N = 2$ by means of wrapping cycles both were necessarily related, that's not the case now). Even if an open string gauge field actually coupled to these D3-particles, their

[88] If 4-cycle=1-cycle × 3-cycle, the D7 wrapping a 4-cycle might couple to CS moduli. However, such an splitting depends on rather special homology constraints and so leads to lack of generality.

mass would still be unstable (because it is unrelated to the symmetry), so that it does not solve our bulk C_4 problem nor give an SDC stable tower.

- **K-theory does not help.**

  We discussed before that the charges for Dp-branes were actually classified by K-theory and not only by (co)homology. Appart from the RR charges, K-theory includes non-BPS stable solutions by means of a discrete symmetry.

  Unfortunately, the new symmetry is not related to the V_μ^+, coming from the C_4, and these "extra K-theoretical" states cannot make towers, but just one state as discussed in [49]. Therefore, this alternative is not valid.

After having analyzed all the previous possibilities, we are pretty convinced that no state contained in String theory other than the D3-branes wrapping 3-cycles can provide us with a solution, if any, to the "Puzzle".

6.2 Is the bulk $U(1)$ theory well defined?

After the failure of the previous subsection, the most natural guideline to follow is to investigate if the $U(1)$ theory is sick.

The starting point would be to analyze *anomalies*. Although, this model is unlikely to be anomalous due to its simplicity. Checking, for example, chiral anomalies [89], we observe that, without fluxes and satisfying the tadpole cancellation conditions (2.47), (2.48), we avoid these anomalies. In any case, anomalies should not be the answer because the "Swampland program" constraints are related to QG UV completion, not to QFT consistency in the IR.

The next point would be to investigate if the $U(1)$ *symmetry is somehow broken* but without fluxes it does not seem to be broken at all. From the SUGRA analysis of [14] and [15], the bulk $U(1)$ symmetry is not broken without fluxes.

Lastly, we should recall the fact that our charged sector consisted of just three states, a massless D3-particle, a massless D3-monopole and a massless D3-dyon. A priori, nothing is wrong with massless charged particles, even if they are uncommon. Notwithstanding, having at the same time massless D3-monopoles (and dyons) is rather strange. It is also not clear to us which one should be the cutoff of this theory. Unfortunately, we are not experts on this branch of Physics and we have not found any related literature.

From this subsection, we cannot conclude that the $U(1)$ sector is sick. As a consequence, we will continue **assuming that the theory is not ill-defined, only rare**. Nevertheless, one should keep in mind the previous paragraph, so that further studies in order to determine if such a theory is consistent are highly recommended.

6.3 Possible solutions

After a considerable investment of time and effort, we have been able to get a decent understanding on the "Puzzle" but we have not found a concrete solution yet.

Unfortunately, the lack of more time and mathematical tools lead us to an open problem. Nevertheless, we have highly constrained the number of possible alternatives to the point of arriving to three

[89] Look at section 7.2.1 of [17]

alternative situations, compatible with Emergence, which we will describe below. In each case, we will present the advantages, drawbacks and, whenever possible, hint a way in which they might be tested.

The reader will realize that from our perspective, the first option seems a bit preposterous, so we really consider the last two options as plausible. In turn, among these, the second one seems very drastic and has shocking consequences, but it is easier to discard using counterexamples, while the third is more continuistic, generates a better fit with respect to $N = 2$ situation and seems more logical, although it is mathematically more difficult to delve into.

6.3.1 A tricky getaway.

We have already pointed out numerous reasons of why the even D3-particles and the even-odd D3-particles could not solve the "Puzzle". Nevertheless, here we will provide a definitive argument discarding them, which is independent of all those provided in previous discussions.

Although we argued before that these states cannot make towers nor bound states in several ways, let us assume they do it. Then we can apply Emergence to them in a similar way that in [5] and [11] (or subsection 3.4.2).

Permit us firstly sketch how to obtain I_{00}^{-1} and I_{--}^{-1} global symmetries in (5.12) from the integration of towers. Before starting, we want to emphasize that to be rigorous one should integrate the stable towers of bound states found in [5] through the Monodromy orbit and match the properly diagonalized couplings. Nevertheless, here we will just integrate the basis towers of states to obtain the I_{II} couplings by means of Emergence, in order to show the reader a crucial point.

The first step is to use the expression (3.33), taking into account that the sums are dominated by the states with high charges which, at infinite distance, always satisfy $\Lambda_{UV}/m_k \to 1$, because $m_k \sim N\triangle m \sim \Lambda_{UV}$. As a consequence, the logarithm tends to a constant, leading to a power law result ($\sum_k^N k^2 \ln\frac{N}{k} \sim \frac{1}{9}N^3 + ...$).

Let us begin with the integration out of the D3-branes wrapping A^0:

$$I_{00} \simeq -\frac{8}{3\pi^2}\sum_k^N (q_{k,0}q_{k,0}\ln\frac{\Lambda_{UV}}{m_k}) \sim \sum_k^N k^2 \ln\frac{N}{k} \sim N^3 \sim \frac{1}{(\triangle M_{D3-A^0})^2} \sim \frac{1}{d_{---}(v^-)^3} \sim \frac{e^{\sqrt{6}d_{\gamma_-}}}{d_{---}} \quad (6.2)$$

and follow with the D3-branes wrapping A^-:

$$I_{--} \simeq -\frac{8}{3\pi^2}\sum_k^N (q_{k,-}q_{k,-}\ln\frac{\Lambda_{UV}}{m_k}) \sim \sum_k^N k^2 \ln\frac{N}{k} \sim N^3 \sim \frac{1}{(\triangle M_{D3-A^-})^2} \sim \frac{1}{d_{---}(v^-)} \sim \frac{e^{\sqrt{\frac{2}{3}}d_{\gamma_-}}}{d_{---}} \,. \quad (6.3)$$

We observe the agreement with (5.12) and one can also check in the same way the off-diagonal component I_{0-}.

The essential point concerning this section is the presence of d_{---}, which is a remnant of the fact that we are taking the limit $v^- \to \infty$ in the orientifold and so using the path: $X^+ = 0, v_0^- \in [\text{const}, \infty) \to v_f^- = \infty$.

Turning our focus to I_{++}, we note in (5.12) that the dependence on the distance only agrees with D3 wrapping A^- cycles, like (6.3), but the crucial factor d_{++-} cannot be obtained from D3 wrapping any cycle. In fact, it might only come from the term $d = d_{---}(v^-)^3 + d_{++-}v^-(v^+)^2 + ...$, but in the region we are exploring, $X^+ = 0 \to v^+ = 0$, it never shows up.

To sum up, there is no way in which d_{++-} can appear from D3-branes wrapping whichever cycle!, as can be straightforwardly deduced from (4.1) with $v^- \to \infty$ and $z^+ = 0$.

As a final remark, let us note that the even D3 multiparticle states/resonances are exactly massless in all the orientifold, so there is no way to obtain anything from them using this treatment, and in the case of the mixed D3-particle one can find the exponential behaviour on the distance for the coupling $I^{-1}_{++} \to 0$ (5.12). Nonetheless, in the latter case we cannot obtain the factor d_{++-}, and in the former, whatever the appropriate treatment for the massless case, such a factor has nowhere to come from.

6.3.2 A powerful speculation.

Along this section we tried to solve the “Puzzle" without success. Thus, it is worthy to consider the option that such a “Puzzle" would actually signal the incompatibility between QG and the twisted sector of, at least, unfluxed Type II orientifolds.

Up to now, we have just worked at a SUGRA level. Nevertheless, the proper way to test the conjecture is to examine a concrete String theory setting. Recalling subsection 5.1, we already noticed that in [20] the even sector under the orientifold projection corresponds precisely to the twisted sector (fixed point set under the discrete action) in the orientifolds of the toroidal orbifold compactifications.

Let us briefly explain this point. The toroidal orbifold compactifications consist of taking a $\mathbb{T}^6$, which is not a CY3, and quotient it by a discrete symmetry, so that we obtain a manifold with the appropriate Hodge numbers in order to be a CY3 but presenting some singularities corresponding to the fixed point set under the orbifold action. These are usually called singular CY3 because in order to be actual CY3 need to be smoothed out, which is technically called to resolve (sometimes blow-up) the singularities. The resolution of the singularity is rather abstract and the main idea is to “open" the singularities by means of introducing new moduli parametrizing correspondingly cycles/submanifolds, which in the “shrinking limit" correspond to the singular CY3. Therefore, in the different homologies for the singular CY3 we have normal cycles and shrinking cycles to which new moduli is assigned representing the smooth CY3. The latter represent the so-called twisted sector [90]. Now, when we take the toroidal orbifold and apply over it the orientifold projection, in the quotient, called the orientifold of the toroidal orbifold, we need to take care of a new fixed point set. It turns out that, regarding $h^{2,1}$, all the examples in [20] share a common feature: $h^{2,1}_+ \leq h^{2,1}_{\text{twisted}}$, with all the even 3-cycles coming from the twisted sector!

In some sense, this is not so surprising, because the orientifold projection requires of the condition $X^{k_+} = 0$, which effectively corresponds to a contracted cycle which cannot be opened.

Furthermore, the twisted sector is by no means restricted to $h^{2,1}$. In fact, an analogous situation is found for the Kähler moduli side: $h^{1,1}_- \leq h^{1,1}_{\text{twisted}}$ and again with all odd 2-cycles coming from the twisted sector, perfectly matching with the previous picture.

Summing up, these toroidal examples are not Calabi-Yau manifolds in general, because they are not smooth and this is due to the fact that they have singularities associated to the twisted cycles. In

[90] Note that we will use also such a name for the orientifold sector inherited from the orbifold twisted sector.

$N=2$, these twisted cycles can be mathematically resolved, which is equivalent to open the cycle. In order to do so, there must be a moduli associated to that cycle. However, when we project to $N=1$, the (O3/O7) conditions $X^{k_+}=0$ and $F_{k_+}|_{X^{k_+}=0}=0$ (for O5/O9 replace $+\leftrightarrow-$ and we observe the same situation) force the twisted cycles to remain singular and so the resultant manifold cannot be resolved, being consequently not smooth. Again for the Kähler moduli and associated Hodge numbers the situation is analogous.

Thus, naturally arises the possibility that the twisted sector of the SUGRA models would not be realized in String theory, which requires smooth CY3. Let us sharpen this intuition in a concise proposal:

Orientifold twisted sector is in the Swampland.
The twisted sector of unfluxed Type II orientifolds is incompatible with the Emergence proposal, is not realized in String theory and so belongs to the Swampland.

This is a bold statement [91] which deserves and requires deeper studies to be checked but seems, from our point of view, a plausible solution to the “Puzzle".

Among the advantages of this approach, we would like to highlight that it is based on a whole survey of examples [20], so there is evidence in favor, and that is easily falsifiable.

The main drawback is surely that usually in the models which fail to be UV compatible with gravity, at least, one of the conjectures WGC or SDC fails. Here, on the contrary, both are, strictly speaking, fulfilled in their weakest version (the SDC by “asymptotically" stable objects and the WGC by just one state). The unique point which fails is their connection tied to Emergence. Nevertheless, if this solution is the right one, our setting would constitute very strong evidence in favor of Emergence, as well as the stability requirement and the tower versions!

Furthermore, this alternative seems difficult to conciliate with the one in subsection 6.3.3 [92].

Coming back to falsifiability, one just needs to provide one example of an orientifold where, at least, one even 3-cycle is not coming from the twisted sector and/or can be smoothed out into an authentic CY.

6.3.3 Is it $d_{++-}=0$? A geometrical approach

At this point, we will describe the last alternative. The starting point is the following: the “Puzzle" is based on the premise that there is a global symmetry $I^{-1}_{++}\to 0$, which cannot be explained through the integration out of any combination of states. However, what if the global limit does not actually happen?

Being more precise, if $d_{++-}=0$ we observe from (5.13) that, once corrected the conifold behaviour in the way described before, the global symmetry limit disappears, as well as any dependence on the CS moduli x^- and what remains are, at most, constants coming from the coefficients of the corrections to the LCS (first) term in (2.17) after taking derivatives.

[91] We would like to thank Eran Palti for suggesting us this viewpoint.

[92] In case that both are correct, might be that a better statement for the previous proposal were: “The twisted sector of unfluxed Type II orientifolds is not realized in String theory and so belongs to the String Swampland".

Such a behaviour would fit perfectly not only with our setting, but also with the general $N = 2$ Emergence picture, where, independently of the path we take (that is, without restricting to $z^+ = 0$ along the way) to reach a region with $v^- \to \infty$ and finite z^+, again there is no object able to provide us with d_{++-} when integrated out. In this approach the "Puzzle" would be solved, because there would be actually no "Puzzle"!

From the intuitive point of view, the resulting picture would be what one expects. Starting from $N = 2$ (5.13), one approaches the conifold $X^+ = 0$ and resolves the conifold singularity by integrating in the even D3-brane wrapping once A^+ à la [7] (second term in (5.13) gets corrected becoming non-divergent). The first term is zero due to d_{++-}, causing that there is no global symmetry for V^+_μ and, therefore, no charged tower associated to it. There are the "asymptotically stable" towers of odd (neutral) D3 bound states, satisfying the SDC and corresponding to the infinite distance in the orientifold CS moduli space by Emergence, and the eWGC particle avoiding the conifold global symmetry.

Nevertheless, why should $d_{++-} = 0$? A priori, there is no reason (up to our knowledge) for demanding that the orientifolds cannot have $d_{++-} \neq 0$. Therefore, naively we observe two completely different situations when $d_{++-} = 0$ and when $d_{++-} \neq 0$. The first one seems perfectly fine and in the second one we have the "Puzzle".

One one might be tempted to propose that the problem is in the models with $d_{++-} \neq 0$ and send them directly to the Swampland. However, it seems rather arbitrary to do so, there is no physical mechanism to sustain such an option, contrary to the case of subsection 6.3.2, and concerning the parent $N = 2$, following the logic described above, one would also need to send such models to the Swampland for the same reason. We consider this too drastic and not so plausible [93].

Instead of sending models with $d_{++-} \neq 0$ to the Swampland, permit us think about the role and meaning of d_{++-} (and d_{+--}[94]). We distinguish the following two scenarios:

N=2

Let us consider the LCS regime in $N = 2$ where both coordinates satisfy $v^-, v^+ >> 1$. Here, we observe from (2.18) that the metric is diagonal in the odd-even basis if, and only if, $d_{++-} = d_{+--} = 0$. Thus, our even-odd basis is only orthogonal when $d_{++-} = d_{+--} = 0$.

In addition, we have, in general, gauge kinetic mixing in $N = 2$, that is, the gauge coupling matrix $\mathscr{I}$ is not diagonal and one should diagonalize it in order to obtain the three diagonal gauge couplings corresponding to the eigenbasis of gauge fields. After that, we should rewrite the new $\mathscr{I}_{II}$ in terms of the new CS moduli coordinates diagonalizing the metric matrix.

Once we have both matrices diagonal in the appropriate set of eigenbasis for the CS moduli and the gauge fields, we can send one of the new CS moduli to infinite (which is equivalent to pick a direction) and check if the new three gauge kinetic couplings tend to infinite (that points out a global symmetry limit).

The expected behaviour is that we find two couplings, coming from the direction we take and the mixing with X^0, approaching a global symmetry, whereas the one associated to the perpendicular direction (using the diagonal metric) is finite. Correspondingly there should be towers of states becoming massless approaching the new infinite limit locus along the space outside it, but not along itself.

[93] Although an interesting connection arises. It is remarkable that the O5/O9 projection would require precisely $d_{++-} \neq 0$ in the parent $N = 2$ in order to be possible.

[94] Recall that all the computations we performed in the model of subsection 5.2.2 took into account that $d_{+--} = 0$ impossed by the O3/O7 projection.

Said another way, what we believe is that when d_{++-} and/or $d_{+--} \neq 0$ and we send, e.g. $v^- \to \infty$, we are not really sending one CS moduli to infinite but a combination of the eigen-CS moduli. Once we diagonalize, we should obtain $\tilde{d}_{++-} = \tilde{d}_{+--} = 0$, recovering the normal behaviour without "Puzzle".

We can concisely propose this alternative as:

Geometric solution to the "Puzzle" in $N = 2$.
In the parent $N = 2$, the "Puzzle" is removed by a proper diagonalization.

N=1

Let us now approach the locus $X^+ = 0$ by means of a curve. Due to the conifold singularity, we expect that it is favorable (geodesic) for this curve to bend out near to the conifold locus. If $d_{++-} = 0$ (take into account that the O3/O7 already imposes $d_{+--} = 0$) the following metric is diagonal and once the conifold divergence is corrected we just find (at first order) a G_{--} as should be for the orientifold. In case that $d_{++-} = 0$, the metric is not diagonal until we reach exactly $X^+ = 0$ and there is also a G_{++} non-negligible component in the orientifold $v^- \to \infty$ limit which is rather unexpected.

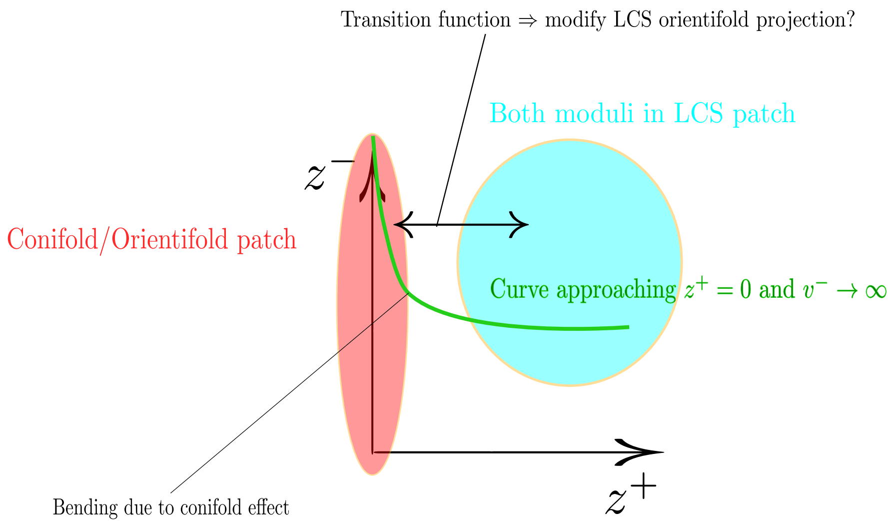

Figure 5: Geometric solution to the "Puzzle" in $N = 1$.

$$G \sim \begin{pmatrix} \frac{1}{(v^-)^2} & \frac{d_{++-}, d_{+--}}{(v^-)^2} \\ \frac{d_{++-}, d_{+--}}{(v^-)^2} & \frac{\ln(z^+)}{(v^-)^3} + \frac{d_{++-}}{(v^-)^2} \end{pmatrix} + \ldots \xrightarrow{\text{curing conifold singularity}} \begin{pmatrix} \frac{1}{(v^-)^2} & \frac{d_{++-}, d_{+--}}{(v^-)^2} \\ \frac{d_{++-}, d_{+--}}{(v^-)^2} & \frac{d_{++-}}{(v^-)^2} \end{pmatrix} + \ldots \tag{6.4}$$

Taking into account that the moduli space is not described by an unique patch, what we propose is that the orientifold projection of [14] should be modified because the perpendicular direction (with respect to the $N = 2$ diagonal metric) to x^- along the curve changes (bending) with the patches and is not x^+.

Geometric solution to the “Puzzle" in $N = 1$.
In the $N = 1$ orientifold, the “Puzzle" is removed by a transition function mapping the orientifold projection of [14] in the parent $N = 2$ LCS chart into the appropriate modified projection in the parent $N = 2$ conifold chart.

These formulations for the geometric alternative are no more than a vague idea, and so rather imprecise, because we had not time to develop them further. Nevertheless, we expect that the general idea for the geometric proposal is clear, that is:

Geometric solution to the “Puzzle".
A careful geometric analysis shows that the “Puzzle" is no more than a misinterpreted geometric effect.

Unfortunately, we realized about this possibility at the very end of my Master's thesis period and we had no time for testing it on a concrete example with given triple intersection numbers. In an abstract setting without specifying the triple intersection numbers seems hard to perform such analysis.

We would like to highlight three benefits. Firstly, it provides with a more natural picture from all the perspectives and fits better with the $I_{++} \to 0$ of $N = 2$ and links both “Puzzles". Secondly, contemplating the magnetic sector ((5.13), (5.15), (5.16)) a symmetric story appears to happen and, again, there is no infinite tower to justify the even coupling. Finally, from the orientifold point of view, we noticed before that the $d_{++-}v^-$ term comes directly from taking the imaginary part of $\bar{F}_{++}$, which is the derivative with respect to the non-dynamical coordinate $x^+ = 0$ of the non-dynamical period $F_+|_{x^+=0} = 0$. This leads to more doubts about the validity of the standard truncation developed in [14].

The main disadvantage is that is very speculative and difficult to formalize and check. Moreover, seems difficult to reconcile with the alternative given in subsection 6.3.2.

Concerning testing grounds, might be possible to pick a concrete example with given values for the triple intersection numbers and properly diagonalize, construct a transition map and check this picture. Nevertheless, checking it in general, for abstract intersection numbers, seems a formidable mathematical problem.

7 Summary of Results and Outlook

In this final section, we make firstly a brief summary of the setting, results we found and the conclusions which can be inferred from these. Then, we will enumerate several guidelines that would be interesting to explore, following this project.

Theoretical Introduction and Setting.

The first three sections were dedicated to motivate our object of study and, especially, to introduce very dense prerequisites in diverse areas. Much is standard, but some sections have been hard to a prepare due to the absence of literature on the subject and/or its difficulty. Nevertheless, these sections were precisely those that contain the most relevant details. Therefore, it is possible the presence of certain mistakes or inaccuracies but we believe that they are fundamentally correct and we strongly recommend reading them. We are referring mainly to sections 3.6.3, 3.7 and 3.8 as surely the reader has already noticed. Even in the standard sections, sometimes we included proper ideas (most of the times repeated later) and tried to offer an original style, that we hope will please the reader.

From section 4 on, we described our setting and performed the corresponding computations and analysis. Firstly, we searched for concrete String theory examples for complex structure moduli in Type IIB CY3 orientifold compactifications but we did not find any explicit example including the twisted (even) sector worked. Thus, we lowered our level of demand to the SUGRA settings. There we realized about a case, both illuminating and simple, in subsection 5.2.2, which cannot be understood without the previous 5.2.1. Along both sections, we noticed about the crucial role of the bulk SUGRA spectrum, to which the D3-particles couple to get their mass and charge, and the apparent paradox caused by the even sector under the orientifold projection regarding the Emergence proposal. We called this as the “Puzzle" and the whole section 6 was devoted to it.

Before we finished this Master thesis, a paper [10], whose object of study intersected partially with our, was published. Nevertheless, the approach and differences are fundamental and so the results. In subsection 5.2.4 we establish a comparison with them, highlighting the similarities and differences, as well as acknowledging (and rendering thanks, of course) those aspects that they obtained, were useful to us, and of which we had not realized before.

Summary of Results.

In this moment, we proceed to expose our original results, which basically start from the section 4 on. Even though in the sections prior to 4, we started to include some original ideas, these have been repeated in more detail in what followed. Therefore, the reader can consider that the original part of this project starts there and extends to the end.

- **$N = 1$ orientifolds seen as $N = 2$ conifolds.**

  We have introduced a helpful viewpoint in subsection 5.2, where we described the $N = 1$ orientifold loci in the following way: start with the parent $N = 2$, approach the conifold corresponding to $x^+ = 0$, resolve the conifold singularity by adding the even D3-particle and impose the orientifold projection. This is possible as long as the condition $F_+|_{x^+=0} = 0$ is verified by the parent $N = 2$, that in practice usually just means the vanishing of d_{++-}.

  Following this idea, the orientifolds can be regarded as the intersection of conifold loci in $N = 2$ with the orientifold truncated field configuration, and the O3/O7 and O5/O9 complement providing us with main features of the parent $N = 2$.

  We are aware that this construction might be not mathematically precise or correct. Nevertheless, it is conceptually useful, we use it with in order to get a better intuition and our computations are generally independent of such interpretation, such that this does not invalidate neither our results nor conclusions.

- **An enlightening model:** $h^{2,1}_{+} = h^{2,1}_{-} = 1$.

  This model, described in subsection 5.2.2 is ideal, in the sense that provide us with all the required ingredients (that is, even and odd cycles), while it is still possible to compute the required distances, having the control over them all the time, without getting lost in math and so being able to note every detail due to its simplicity (complex 2-dimensional). We would say that it is the minimal setting containing everything necessary to address the study of both conjectures and their relationship (through Emergence), as well as to connect the whole picture with its equivalent in $N = 2$ [5].

  We consider this model iself as a major result because it marked the way to follow and permitted us to maximally exploit intuition without getting too lost in the involved mathematical structure that surrounds orientifold moduli spaces. This model is outstading, because it is at the same time simple, complete and, almost in all probability, generalizable to higher hodge numbers. It basically permitted us to arrive to the following bullet points.

- **For $N = 1$ Type IIB orientifolds, concerning particles, the (e,m,d)WGC are verified by the D3-branes wrapping even 3-cycles in their simplest form.**

  In subsection 5.2.2 we showed that the D3-brane wrapping the even 3-cycle A^+ once (i.e. $q^+ = 1$), inherited in the $N = 1$ orientifold from the $N = 2$ conifold, satisfies the electric WGC. Similarly, we pointed out that the D3-brane wrapping the (magnetic) even 3-cycle B^+ once (i.e. $p_+ = 1$) and the D3-brane wrapping both even cycles once (dyonic) verify, respectively, the magnetic and dyonic WGC (5.14). Remarkably, if stability is required, the tower versions (3.2.1) of such conjectures are not realized in our setting.

  The first bullet point was essential to reach this result, because we understood from the conifold resolution of [7] the physical nature of the massless charged states wrapping even cycles, whose role has been so mysterious and strange to us, even raising the, finally discarded, cumbersome possibility of having infinite towers of charged and massless particles.

- **For $N = 1$ Type IIB orientifolds, concerning particles, the SDC is verified by "asymptotically stable" D3-branes wrapping odd 3-cycles.**

  In subsection 5.2.2 we noticed that concerning the SDC, the situation was rather analogous to that in $N = 2$ [5]. The distance divergence is exactly the same and also the infinite towers of (bound) states becoming exponentially massless on the distance (D3-branes wrapping A^0 n-times-D3 wrapping A^- once). Nevertheless, there is a crucial difference: these states are not charged in $N = 1$. As a consequence their stability must be discussed (subsections 3.8 and 5.2.2). We realized that in the LCS their gravitational and scalar interactions get exponentially supressed, such that one finds a condensate of "asymptotically stable" D3-branes wrapping odd 3-cycles in the infinite distance limit, which actually avoids us from reaching such a singular locus.

  Therefore, we proposed that if one disregards stability effects the SDC is verified by these neutral towers. To go beyond, backreaction effects and a more profound stability analysis are required. Notwithstading, in the next bullet point we find a powerful argument in favor of this interpretation.

- **For $N = 1$ Type IIB orientifolds, Emergence is verified in the odd sector but "puzzling" in the even sector.**

  The emergence analysis over our model was performed in subsection 5.2.2. There we observed that, considering the "asymptotically stable" D3-branes wrapping odd 3-cycles, the Emergence proposal, regarding the SDC infinite distance generated by integration out of such states, was quantitatively exactly matched as in $N = 2$.

Nonetheless, the even sector was much more problematic because the global symmetry $I_{++}^{-1} \to 0$ (5.12) survives the orientifold projection and there is no available tower which can generate it by integration out. To this paradox we refer by the name of "Puzzle" and we noticed that it was already present in $N = 2$ (in subsection 6).

- **The relationship between the WGC, SDC and Emergence seems determined by bulk SUGRA.**

  This result, accurately formulated in subsection 5.2.2, is probably the most important one. Along subsections 3.8 and 5.2.2 we emphasized the dependence of [5] picture (described in subsection 3.6.2) on the BPS condition and the bulk SUGRA spectrum to which the D3-branes couple to get their mass and their charge.

  We observed that the even D3-particles (wrapping even cycles) coupled to the even gauge field V_μ^+ but not to the CS moduli z^-, being therefore charged and massless, whereas the odd D3-particles coupled to z^- but not to V_μ^+ and, as a consequence, are uncharged but massive. The former satisfy the WGC and the latter the SDC. Thus, we observe an splitting which did not happened in $N = 2$ and which perfectly fits with the SUGRA spectrum according to $N = 2 \to N = 1$.

  We propose that this is not a casual effect and that actually the relationship between both conjectures and also Emergence depends fundamentally on the bulk SUGRA spectrum.

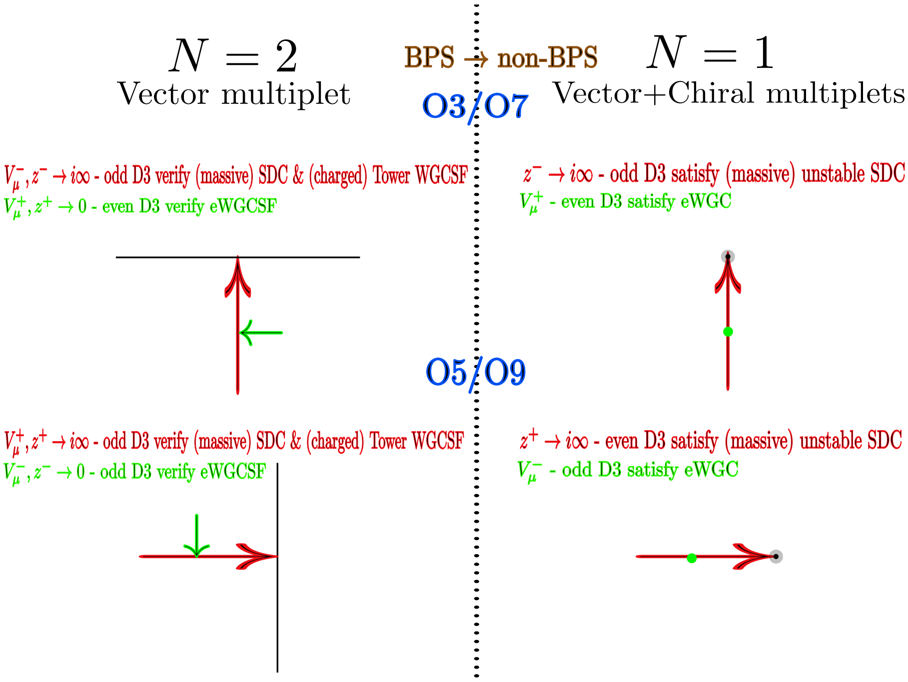

Figure 6: SUGRA splitting of the conjectures for D3-particles in Type IIB Orientifolds. In $N = 2$, the red and green arrows denote respectively the $v^\mp \to \infty$ infinite distance and $z^\pm \to 0$ conifold directions, whereas the black lines represent the infinite distance locus $v^\mp = \infty$. On the other hand, the $N = 1$ CS moduli space has one complex dimension less due to the O3/O7 (O5/O9) condition $z^+ = 0$ ($z^- = 0$), such that the parent green arrow and black line contract to (complex) points. The D3-particles couple to both moduli (BPS) in $N = 2$ and just to one (non-BPS) of them in $N = 1$.

- **The "Puzzle": an essential open problem and its current status.**

  Everything fits perfectly in the above interpretation, with the exception of the "Puzzle". We have dedicated to it the whole section 6, narrowing the possible solutions to two (if the charged sector of our theory is well-defined, look at subsection 6.2), assuming Emergence holds for our setting. The first alternative is more drastic but easier to test, and the second is more continuist and fits better with the $N = 2$ “Puzzle" but appears to be much harder to properly prove. Each one is described respectively in subsections 6.3.2 and 6.3.3. Intuition tells us that the latter makes much more sense, but we cannot discard the former, which is intriguing.

This is a deep problem, in the sense that whatever the solution it will be fundamental and possibly change or improve the current status/understanding of the “Swampland program". Due to the end of this Master thesis period we needed to stop and leave its complete resolution for future research. Nevetheless, we developed an intensive work restricting a lot the possible solutions.

Firstly, because it appears incompatible with Emergence, we needed to decide if being against or in favor of Emergence. A priori, following scientific rigor, one should not discard Emergence only because it would be easier. In addition, without Emergence we would lose a strong explanation for global symmetries and one should find some alternative to make sense of $I_{++}^{-1} \to 0$ (5.12), which we have not been able.

Next, we decided to continue and try to match it to Emergence. As a consequence, a requirement is to find towers of objects charged under V_μ^+ and coupling to z^- (look at beginning of section 6 and subsection 6.1). We dedicated the whole subsection 6.1 to such objective and we found that just the D3-branes wrapping even and odd cycles simultaneously verify such a behaviour but look unstable under decay to even and odd cases, and do not match the key factor d_{++-} in $I_{++}^{-1} \to 0$ (5.12) (subsection 6.3.1).

The following step was to ask ourselves about the validity of our strange even sector with an $U(1)$ and a massless charged particle, monopole and dyon under it. In subsection 6.2 we did not find any strong argument invalidating such a theory, even if it is rare.

Therefore, assuming a fundamental origin for the global symmetry (Emergence or other) and considering our theory well-defined, we found just two options left. That the even sector, contained in the twisted sector of the toroidal orbifold realizations of [20], is in the Swampland (subsection 6.3.2), or that a proper diagonalization of both moduli space metric and gauge coupling matrix ($N = 2$), as well as a modified version of [14] orientifold truncation analysis ($N = 1$), mapping between patches and so taking into account global effects and not only local, are indispensable and solve the $d_{++-} \neq 0$ (leading to the “Puzzle" apparent paradox) problem, such that there would be no “Puzzle" (once a rigurous diagonalization and geometric treatment are performed).

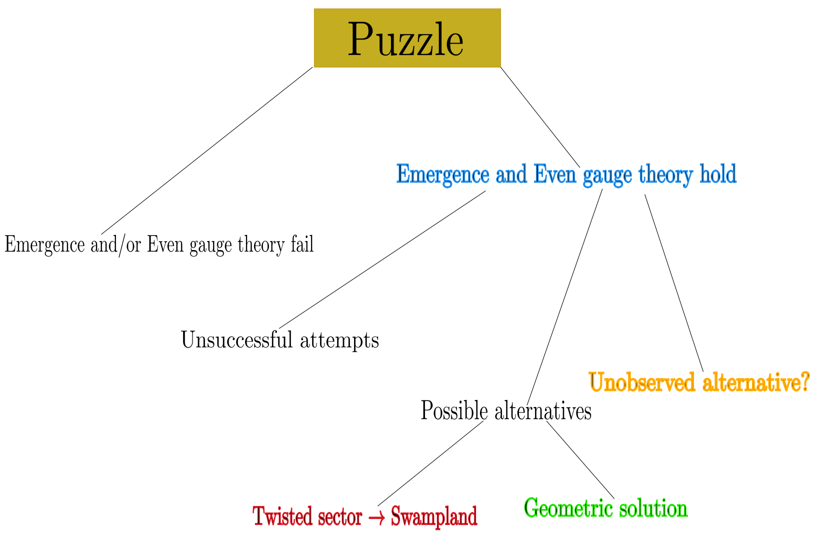

Figure 7: Current status of the “Puzzle".

Future Guidelines

As a final contribution within this thesis, we would like to hint several future lines to explore, some related to the subtleties we found, others opened by our analysis and the rest because are interesting by themselves.

- **Completely resolve the "Puzzle".**

  The most urgent research line is, without any doubt, to continue our study of the "Puzzle" in order to distinguish between the presented alternatives by means of, firstly, a concrete example and, later, a systematic formalism.

  We have profoundly narrowed the possibilities and carefully explained their potential implications. It is fair to claim that whatever the right resolution, our understanding of the Swampland program would really be increased. In a certain sense, the future of this conjectures network depends fundamentally on clarifying the subtleties and unknowns that we have pointed out and developed in this project.

  We have also remarked that it might be as simple as finding some counterexample or as difficult as developing a general geometric formalism mapping charts in moduli space. In any case, it seems obvious that every effort is justified in this direction.

- **Extension to the case with $h_+^{(2,1)}, h_-^{(2,1)} > 1$.**

  Even if naturally seems that our picture extends to frameworks with higher Hodge numbers, one should explicitly check it. Besides, it would be really interesting to find some generalization of the orientifold projections for the parent $N=2$ conifold loci and not only for the all odd/all even intersections, that is the case for the current orientifold projections.

- **Testing other conjectures in this setting.**

  Our model is particularly simple, such that for testing other conjectures and the Emergence ties might be a right choice. For example, Emergence has been proposed in [46] to also relate De Sitter Conjecture to the SDC (and WGC). It would be interesting to study if that binding survives in our setting, due to the modifications that the relation between SDC and WGC, as well as the Emergence picture of [5], suffer.

  Another intriguing object of study would be to further explore the relationship between our massless charged sector and the Weak Cosmic Censorship-WGC connection.

- **Extension to Type IIA orientifolds.**

  We focussed on Type IIB Orientifolds. It would be interesting to extend our treatment to the Type IIA Orientifold case, where there is also a similar splitting of the SUGRA multiplets in the sense that the Kähler moduli and the $U(1)$ gauge vector decouple. Taking into account that this is mirror symmetric to our setting, and that we noted in subsection 2.3.4 that mirror symmetry is preserved (up to subtleties) for this concrete case, we would expect that the mirror towers/D-branes satisfy a similar behaviour. For the $N=2$ analysis this was already the case, as showed in [19]. Furthermore, the E2-instantons of [23] present the characteristics to possibly experience the same behaviour in the CS moduli space of Type IIA.

- **Extension to D/NS-strings and D/NS-domain walls.**

  Following the the work of [10], it would be absolutely necessary to include the "forgotten" uncharged towers there (and all the discrepancies discussed in subsection 5.2.4) and analyze if once they are included a similar story to ours happens, at least for the D-strings. The case of the D-domain walls seems a bit different and we do not have strong arguments regarding it.

- **Analyze the subtleties regarding to the modifications which the $N = 1$ potential could induce in the results.**

  We have performed our analysis without Dp-brane backreaction and managed to avoid fluxes. This lead us to no have potential concerning the complex structure moduli, such that we can test the WGC as stated in subsection 3.2.1. Nevertheless, in general the backreaction effects and fluxes are relevant, in the line of [21], and we should aim to explore what changes if any. Some other subtleties concerning the mirror map have been mentioned and are worthy to investigate.

- **Further analysis of NS-branes and non-geometric/exotic -fluxes/branes.**

  Up to know these elements have just played a symbolic role in the Swampland Program (just some testing for NS5-brane domain walls in [10], where are also exotic branes start to play a role, and recently [75]). However, as hinted in the above guideline, they might play an essential role for a complete understanding of these conjectures and their links, as well as provide us with examples/counterexamples and modifications.

- **Approaching phenomenology.**

  The idea of these conjectures and the whole Swampland Program is precisely to join what we know and try to obtain restrictions over QG consistent theories. In order to make contact to nature, we should aim to relax Supersymmetry conditions as much as possible. The Orientifold models present $N = 1$ (which then can be broken to $N = 0$), and so fit in the potentially realistic models but require fluxes to stabilize the CS moduli [95]. Therefore, we consider that extensions to interesecting D-brane models, and, in general, models more realistic than our, are mandatory, and our findings concerning the relationship between the conjectures and the SUGRA spectrum should be taken into account in such analysis.

  Permit us remark the phenomenological relevance of our $N = 1$ proposal in subsection 5.2.2 (really linked to the bulk SUGRA splitting of the spectrum in $N = 2 \rightarrow N = 1$), because even if, of course, our models are not phenomenologically relevant, the models that are, also have $N = 1$ (broken to $N = 0$). These models present also a SUGRA spectrum and possibly the conjectures there suffer exactly the same decoupling as in this work. So our work might be rather important in what refers to nature.

- **SUGRA analysis of the conjectures.**

  The majority of the studies up to now have been based on $N = 2$, which is a really interesting and particularly symmetric and simple case. Nevertheless, from our work we observe that the connections between the conjectures, as well as their ties to the Emergence proposal essentially depend on the SUGRA we allow. Therefore, added to the above guideline, this justifies our aim for a SUGRA analysis of the conjectures and their relationship. From our results, we would expect that in (bulk) $N = 0$ both conjectures (for particles) had no relation (appart from that inherited from breaking another SUGRA (as was our case, where still remains some remnant relation because the $N = 1$ was broken from $N = 2$ parent)) and that in $N > 2$ the situation continues exactly as in $N = 2$, with the "duality" between the conjectures, matching perfectly the Emergence picture of [5]. Notwithstanding, this needs to be examined in detail.

  In [3] it was thought that the character of the conjectures was not tied to Supersymmetry but, from our findings, it seems that actually there is such nexus. Anyway, a more profound treatment should be performed, in the aim of demonstrating or rejecting our proposals/hypothesis.

[95] All moduli must be stabilized because have not been observed experimentally.

- **What about the magnetic and dyonic cases?**

  In addition, due to the electromagentic and self-duality (reflected in the symplectic BPS D3 mass formula) and the strong-weak duality of Type IIB, it would be interesting to analyze the magnetic case (e.g. $p^0 \neq 0$) in $N = 2$ (also for the $N = 1$ orientifold projection) and check if the inverse of the SDC (and the corresponding Emergence) holds for it. A mixed situation between electric and magnetic cases is expected for dyons just looking at (2.42).

  Concerning the WGC/WGCSF, we have pointed out the importance of the magnetic sector along the text and checked the electric-magnetic and dyonic WGC for out setting $N = 1$.

  We have commented a bit about these cases and their importance several times, but further studies are necessary.

- **Check if our even sector is well defined: an opportunity to delve into theories with massless charged particles, monopoles and dyons.**

  This last line of future research ties with the above one. We have noticed that our theory with one stable charged massless particle, monopole and dyon state is rather unconventional. We have not found any reason of why it should be ill-defined; however, the usual relations and the actual cutoff of the theory are a mistery for us, this is why we were forced to test the mWGC and dWGC in the most basic version (5.14), instead of the usual (3.3).

  We can not emphasize more that further studies are needed to address this point. By the way, it is an opportunity to improve the general knowledge about theories with charged massless states (particles, monopoles, dyons ...), about which we have found almost no information in the literature.

Surely many other lines are very interesting, we just mention the above ones because are related to our work and seem urgent to us to explore them.

References

[1] N. Arkani-Hamed et. al., *"The String landscape, black holes and gravity as the weakest force*", 2006, [hep-th/0601001].

[2] H. Ooguri and C. Vafa, *"On the Geometry of the String Landscape and the Swampland*", 2006, [hep-th/0605264].

[3] E. Palti, *"The Weak Gravity Conjecture and Scalar Fields*", 2017, [arXiv:1705.04328].

[4] D. Klaewer and E. Palti, *"Super-Planckian Spatial Field Variations and Quantum Gravity*", 2017, [arXiv:1610.00010].

[5] T. W. Grimm, E. Palti and I. Valenzuela, *"Infinite Distances in Field Space and Massless Towers of States"*, 2018, [arXiv:1802.08264].

[6] D. Robles-Llana, M. Rocek, F. Saueressig, U. Theis and S. Vandoren, *"Nonperturbative corrections to 4D string theory effective actions from SL(2,Z) duality and supersymmetry"*, 2007, [hep-th/0612027].

[7] A. Strominger, *"Massless black holes and conifolds in string theory"*, 1995, [hep-th/9504090].

[8] H. Ooguri and C. Vafa, *"Summing up D-Instantons"*, 1996, [hep-th/9608079].

[9] F. Marchesano and M. Wiesner, *"Instantons and infinite distances*", 2019, [arXiv:1904.04848].

[10] A. Font, A. Herráez and L. E. Ibáñez, *"The Swampland Distance Conjecture and Towers of Tensionless Branes*", 2019, [arXiv:1904.05379].

[11] E. Palti, *"The Swampland: Introduction and Review*", 2019, [arXiv:1903.06239].

[12] E. Gonzalo, L. E. Ibáñez and A. M. Uranga, *"Modular Symmetries and the Swampland Conjectures*", 2018, [arXiv:1812.06520].

[13] H. Jockers, *"The effective action of D-branes in Calabi-Yau orientifold compactifications*", 2005, [hep-th/0507042].

[14] T. W. Grimm and J. Louis, *"The effective action of $N = 1$ orientifolds*", 2004, [hep-th/0403067].

[15] T. W. Grimm and J. Louis, *"The effective action of Type IIA orientifolds*", 2005, [hep-th/0412277].

[16] S. Hosono, A. Klemm and S. Theisen, *"Lectures on mirror symmetry*", 1994, [hep-th/9403096].

[17] E. Plauschinn, *"Type IIB Orientifolds, D-Brane Instantons and the Large Volume Scenario*", 2009, Ph.D. thesis.

[18] R. Blumenhagen, D. Lüst and S. Theisen, *"Basic Concepts of String Theory*". Theoretical and Mathematical Physics. Springer, Heidelberg, Germany, 2013.

[19] P. Corvilain, T.W. Grimm, I. Valenzuela, *"The Swampland Distance Conjecture for Kähler moduli*", 2018, [arXiv:1812.07548].

[20] D. Lüst, S. Reffert, E. Scheidegger and S. Stieberger, *"Resolved Toroidal Orbifolds and their Orientifolds*", 2006, [hep-th/0609014].

[21] I. R. Klevanov and M. J. Strassler, *"Supergravity and a Confining Gauge Theory: Duality Cascades and χSB-Resolution of Naked Singularities*", 2000, [hep-th/0007191].

[22] F. Saueressig and S. Vandoren, *"Conifold singularities, resumming instantons and non-perturbative mirror symmetry*", 2007, [arXiv:0704.2229].

[23] C. Vittmann, *"The Axion-Instanton Weak Gravity Conjecture and Scalar Fields*", 2018, M.Sc. thesis.

[24] T. Grimm, *"The effective action of type II Calabi-Yau orientifolds*", 2005, Ph.D. thesis.

[25] A. Sen, *"Non-BPS States and Branes in String Theory*", 1999, [hep-th/9904207].

[26] K. Becker, M. Becker and J.H. Schwarz, *"String Theory and M-Theory*". Cambridge University Press, 2007.

[27] E. Plauschinn, *"The Generalized Green-Schwarz Mechanism for Type IIB Orientifolds with D3- and D7-Branes*, 2009, [arXiv:0811.2804].

[28] S.J. Lee, W. Lerche and T. Weigand, *"Tensionless Strings and the Weak Gravity Conjecture*", 2018, [arXiv:1808.05958].

[29] E. Plauschinn, *"Non-geometric backgrounds in string theory*", 2019, [arXiv:1811.11203].

[30] C. Vafa, *"The String landscape and the swampland*", 2005, [hep-th/0509212].

[31] L. Susskind, *"Trouble For Remnants*", 1995, [hep-th/9501106].

[32] E. Witten, *"Search for a Realistic Kaluza-Klein Theory*", Nucl. Phys. **B186** (1981) 412-428.

[33] N. Seiberg and E. Witten, 1994, [hep-th/9407087]; 1994, [hep-th/9408099].

[34] G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, *"De Sitter Space and the Swampland*", 2018, [arXiv:1806.08362].

[35] T. Rudelius, *"Constraints on Axion Inflation from the Weak Gravity Conjecture*", 2015, [arXiv:1503.00795].

[36] R. Blumenhagen, *"Large Field Inflation/Quintessence and the Refined Swampland Distance Conjecture*", 2018, [arXiv:1804.10504].

[37] B. Heidenreich, M. Reece and T. Rudelius, *"Sharpening the Weak Gravity Conjecture with Dimensional Reduction*", 2016, [arXiv:1509.06374].

[38] B. Heidenreich, M. Reece and T. Rudelius, *"Evidence for a sublattice weak gravity conjecture*", 2017, [arXiv:1606.08437].

[39] S. Andriolo, D. Junghans, T. Noumi and G. Shiu, *"A Tower Weak Gravity Conjecture from Infrarred Consistency*", 2018, [arXiv:1802.04287].

[40] J. Polchinski, *"Monopoles, duality and string theory*", 2004, [hep-th/0304042].

[41] D. Gaiotto, A. Kapustin, N. Seiberg and B. Willett, *"Generalized Global Symmetries*", 2015, [arXiv:1412.5148].

[42] J. M. Overduin and P. S. Wesson, *"Kaluza-Klein Gravity*", 1998, [arXiv:gr-qc/9805018].

[43] D. Harlow, *"Wormholes, Emergent Gauge Fields, and the Weak Gravity Conjecture*", 2016, [arXiv:1510.07911].

[44] B. Heidenreich, M. Reece and T. Rudelius, *"The Weak Gravity Conjecture and Emergence from an ultraviolet Cutoff*", 2018, [arXiv:1712.01868].

[45] B. Heidenreich, M. Reece and T. Rudelius, *"Emergence of Weak Coupling at Large Distance in Quantum Gravity*", 2018, [arXiv:1802.08698].

[46] H. Ooguri, E. Palti, G. Shiu and C. Vafa, *"Distance and de Sitter Conjectures on the Swampland*", 2019, [arXiv:1810.05506].

[47] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *"The Hierarchy problem and new dimensions at a millimeter*", 1998, [hep-th/9803315].

[48] D. Lüst, E. Palti and C. Vafa, *"AdS and the Swampland*", 2019, [arXiv:1906.05225].

[49] R. Blumenhagen, M. Brinkmann and A. Makridou, *"A Note on the dS Swampland Conjecture, Non-BPS branes and K-theory*", 2019, [arXiv:1906.06078].

[50] G. T. Horowitz and J. E. Santos, *"Further evidence for the weak gravity-cosmic censorship connection*", 2019, [arXiv:1901.11096].

[51] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, *"De Sitter vacua in string theory*", 2003, [hep-th/0301240].

[52] M. Reece, *"Photon Masses in the Landscape and the Swampland*", 2018, [arXiv:1808.09966].

[53] D. Klaewer, D. Lüst and E. Palti, *"A Spin-2 Conjecture on the Swampland*", 2019, [arXiv:1811.07908].

[54] M. Douglas, *"Is the number of string vacua finite?*" (Strings 2005)

[55] R. Blumenhagen, D. Klaewer and L. Schlechter, *"Swampland Variations on a Theme by KKLT*", 2019, [arXiv:1902.07724].

[56] G. T. Horowitz, J. E. Santos and B. Way, *"Evidence for an Electrifying Violation of Cosmic Censorship*", 2016, [arXiv:1604.06465].

[57] T. Crisford and J. E. Santos, *"Violating the Weak Cosmic Censorship Conjecture in* AdS_4", 2017, [arXiv:1702.05490].

[58] T. Crisford, G. T. Horowitz and J. E. Santos, *"Testing the Weak Gravity - Cosmic Censorship Connection*", 2018, [arXiv:1709.07880].

[59] R. Penrose, *"Gravitational collapse: The role of general relativity*", Riv. Nuovo Cim. **1** (1969) 252 [Gen. Rel. Grav. **34** (2002) 1141].

[60] M. Montero, A. M. Uranga and I. Valenzuela, *"Transplanckian axions!?*", 2015, [arXiv:1503.03886].

[61] J. Brown, W. Cottrell, G. Shiu and P. Soler, *"Fencing in the Swampland: Quantum Gravity Constraints on Large Field Inflation*", 2015, [arXiv:1503.04783].

[62] R. Blumenhagen, D. Klaewer, L. Schlechter and F. Wolf, *"The Refined Swampland Distance Conjecture in Calabi-Yau Moduli Spaces*", 2018, [arXiv:1804.10504].

[63] T. W. Grimm, C. Li, E. Palti, *"Infinite Distance Networks in Field Space and Charge Orbits*", 2018, [arXiv:1811.02571].

[64] S. Cecotti, *"Supersymmetric Field Theories*". Cambridge University Press, 2015.

[65] O. J. C. Dias, H. S. Reall and J. E. Santos, *"The BTZ black hole violates strong cosmic censorship*", 2019, [arXiv:1906.08265].

[66] C. Damian and O. L. Brito, *"Some remarks on the dS conjecture, fluxes and K-theory in IIB toroidal compactifications*", 2019, [arXiv:1906.08766].

[67] P. Agrawal, G. Obied and C. Vafa, *"H_0 Tension, Swampland Conjectures and the Epoch of Fading Dark Matter*", 2019, [arXiv:1906.08261].

[68] M. Demirtas, C. Long, L. McAllister and M. Stillman, *"Minimal Surfaces and Weak Gravity*", 2019, [arXiv:1906.08262].

[69] M. Haack, J. U. Kang, *"Towards the 1-loop effective action of type IIB orientifolds*", 2016, [PoS (CORFU2016) 099].

[70] E. Kiritsis, *"String Theory in a Nutshell*". Princeton University Press, 2019.

[71] B. R. Greene, *"String Theory on Calabi-Yau Manifolds*", 1997, [hep-th/9702155].

[72] B. R. Greene and C. I. Lazaroiu, *"Collapsing D-Branes in Calabi-Yau Moduli Space: I*", 2000, [hep-th/0001025].

[73] S. H. Shenker, *"Another Length Scale in String Theory?*", 1995, [hep-th/0001025].

[74] B. Heidenreich, M. Reece and T. Rudelius, *"Repulsive Forces and the Weak Gravity Conjecture*", 2019, [arXiv:1906.02206].

[75] N.C. Bizet, C. Damian, O. L. Brito and D. M. Peña, *"Leaving the Swampland: Non-geometric fluxes and the Distance Conjecture*", 2019, [arXiv:1904.11091].

[76] **BICEP2 Collaboration**, P. Ade et al., *"Detection of B-mode Polarization at Degree Angular Scales by BICEP2*", 2014, [arXiv:1403.3985].

Statement of Authorship

I hereby state that this Master's Thesis has been written by me and is the fruit of my own work. Every source more specific than generic statements or common knowledge has been acknowledged and included in the bibliografy. I further declare that I have not submitted this thesis at any other institution in order to obtain a degree.

München, August 9th, 2019

Martín Enríquez Rojo