Minimal Black Holes and Species Thermodynamics

DIETER LÜST (LMU, MPP)

Joint work with
Niccolo Cribiori and Carmine Montella, arXiv:2305.10489,
and with Ivano Basile, N. Cribiori, C. Montella, to appear

Harvard University, 2nd. November 2023

Outline :

I) Introduction: species scale and entropy

II) Minimal BHs as species bound states

III) Minimal charged BHs

IV) String constructions

V) Summary

Introduction:

Apparently Quantum Gravity is of holographic nature and also possesses thermodynamic and entropic properties.

Universal relations for Schwarzschild black holes:

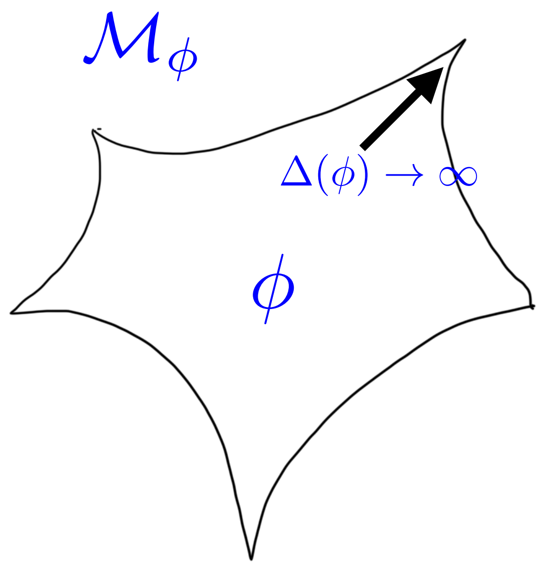
- BH entropy of black hole geometries ⟶ area law:

$$\mathcal{S}_{BH} \simeq \left(R_{BH}\right)^{d-2} M_P^{d-2}$$

- Thermal black hole decay ⟶ Hawking temperature:

$$T_{BH} \simeq \left(R_{BH}\right)^{-1}$$

Quantum Gravity moduli space:

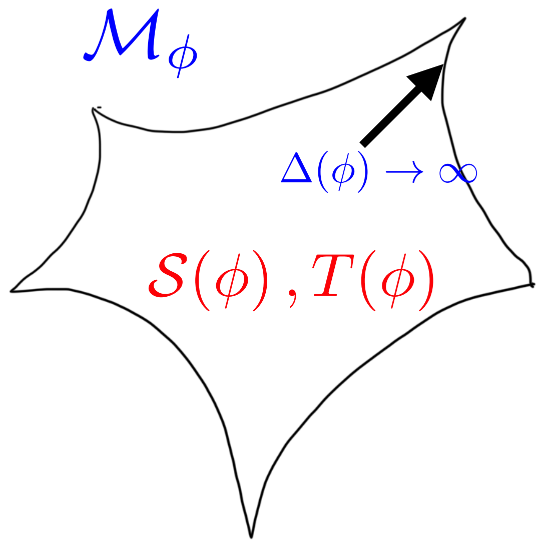


Tower of light states:

$$m_n(\phi) \simeq M_P e^{-\alpha\Delta(\phi)} \to 0$$

[H. Ooguri, C. Vafa (2006)]

Quantum Gravity moduli space:



Tower of light states:

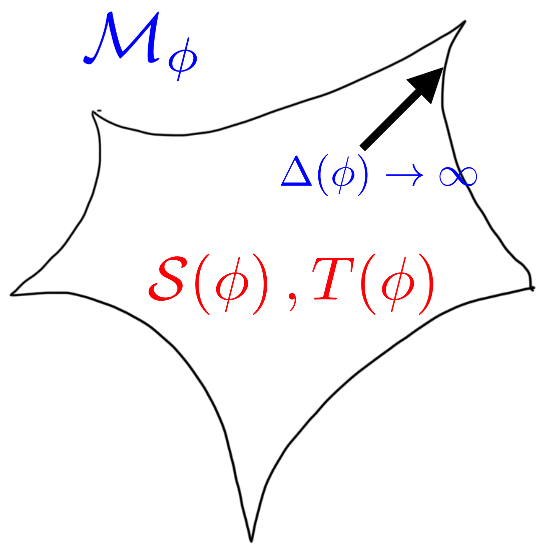
$$m_n(\phi) \simeq M_P e^{-\alpha\Delta(\phi)} \to 0$$

[H. Ooguri, C. Vafa (2006)]

$$\mathcal{S}(\phi), T(\phi) \to ?$$

Can we associate an entropy and a temperature to each point in moduli space?

Quantum Gravity moduli space:



Tower of light states:

$$m_n(\phi) \simeq M_P e^{-\alpha \Delta(\phi)} \to 0$$

[H. Ooguri, C. Vafa (2006)]

$$\mathcal{S}(\phi), T(\phi) \to ?$$

Can we associate an entropy and a temperature to each point in moduli space?

Our proposal: The entropy is given in terms of the entropy of certain particle species.

What is the entropy of particles species in quantum gravity ?

Is it extensive (volume law) or does it follow an area law ?

Does it follow universal thermodynamic relations ?

Species Scale :

UV cut-off in an EFT coupled to (quantum) gravity:

$$\Lambda_{UV} \longleftrightarrow \text{„shortest" possible length in EFT}$$

$$\longleftrightarrow \text{Scale where gravity becomes strongly coupled.}$$

This scale is related to the formation of black holes.

Species Scale:

Species scale: $\Lambda_{UV} \equiv \Lambda_{sp} = L_{sp}^{-1}$

$$\Lambda_{sp} \simeq \frac{M_P}{(N_{sp})^{\frac{1}{d-2}}} \iff N_{sp} = (L_{sp})^{d-2} M_P^{d-2}$$

[G. Dvali (2007)]

(Up to log-corrections)

Species Scale:

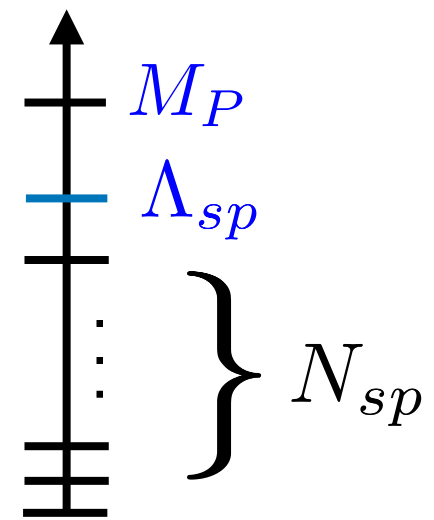
Species scale: $$\Lambda_{UV} \equiv \Lambda_{sp} = L_{sp}^{-1}$$

$$\Lambda_{sp} \simeq \frac{M_P}{(N_{sp})^{\frac{1}{d-2}}} \quad \Longleftrightarrow \quad N_{sp} = (L_{sp})^{d-2} M_P^{d-2}$$

[G. Dvali (2007)]

(Up to log-corrections)

N_{sp} : Number of particles below Λ_{sp} .



.... it depends on moduli fields: $$N_{sp} = N_{sp}(\phi)$$

Species Scale - three possible definitions:

- Perturbative: scale where the inverse one-loop gravity propagator becomes small.

$$G^{-1}(p^2) \simeq p^2 \left(1 - N_{sp}^2 \left(\frac{p^2}{M_p^2} \right)^{\frac{d-2}{2}} \log\left(-\frac{p^2}{\mu^2} \right) \right) \Rightarrow \Lambda_{sp} = \frac{M_P}{N_{sp}^{\frac{1}{d-2}}}$$

Species Scale - three possible definitions:

- Perturbative: scale where the inverse one-loop gravity propagator becomes small.

$$G^{-1}(p^2) \simeq p^2 \left(1 - N_{sp}^2 \left(\frac{p^2}{M_p^2} \right)^{\frac{d-2}{2}} \log\left(-\frac{p^2}{\mu^2} \right) \right) \Rightarrow \Lambda_{sp} = \frac{M_P}{N_{sp}^{\frac{1}{d-2}}}$$

- Scale where higher curvature operators in EFT become relevant.

$$S_{eff} \simeq M_P^{d-2} \int d^d x \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 + \sum_{n=2}^{\infty} a_n \frac{\mathcal{O}_{2n}(R)}{\Lambda_{sp}^{2n-2}} \right)$$

[D. van de Heisteeg, C.Vafa, M.Wiesner, D.Wu (2022)]

Species Scale - three possible definitions:

- Perturbative: scale where the inverse one-loop gravity propagator becomes small.

$$G^{-1}(p^2) \simeq p^2 \left(1 - N_{sp}^2 \left(\frac{p^2}{M_p^2}\right)^{\frac{d-2}{2}} \log\left(-\frac{p^2}{\mu^2}\right)\right) \Rightarrow \Lambda_{sp} = \frac{M_P}{N_{sp}^{\frac{1}{d-2}}}$$

- Scale where higher curvature operators in EFT become relevant.

$$S_{eff} \simeq M_P^{d-2} \int d^d x \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 + \sum_{n=2}^{\infty} a_n \frac{\mathcal{O}_{2n}(R)}{\Lambda_{sp}^{2n-2}} \right)$$

[D. van de Heisteeg, C.Vafa, M.Wiesner, D.Wu (2022)]

- Schwarzschild radius of minimal black hole.

[G. Dvali, M. Redi (2009)]

[N. Cribiori, D.L., G. Staudt (2022)]

$$\Lambda_{sp} \simeq R_{BH,min}^{-1}$$

Species Thermodynamics:

Here we propose the following generalisation of the definition of the species scale:

Species Thermodynamics:

Here we propose the following generalisation of the definition of the species scale:

- Replace the number of species N_{sp} by the species entropy $\mathcal{S}_{sp}$:

$$\Lambda_{sp} = \frac{M_P}{\mathcal{S}_{sp}^{\frac{1}{d-2}}}$$

Species Thermodynamics:

Here we propose the following generalisation of the definition of the species scale:

- Replace the number of species N_{sp} by the species entropy $\mathcal{S}_{sp}$:

$$\Lambda_{sp} = \frac{M_P}{\mathcal{S}_{sp}^{\frac{1}{d-2}}}$$

- In other words: definition of species entropy:

$$\boxed{\mathcal{S}_{sp} \simeq (L_{sp})^{d-2} M_P^{d-2}}$$

Species Thermodynamics:

Here we propose the following generalisation of the definition of the species scale:

- Replace the number of species N_{sp} by the species entropy $\mathcal{S}_{sp}$:

$$\Lambda_{sp} = \frac{M_P}{\mathcal{S}_{sp}^{\frac{1}{d-2}}}$$

- In other words: definition of species entropy:

$$\boxed{\mathcal{S}_{sp} \simeq (L_{sp})^{d-2} M_P^{d-2}}$$

- We will also introduce a species temperature T_{sp}

Species Thermodynamics:

Here we propose the following generalisation of the definition of the species scale:

- Replace the number of species N_{sp} by the species entropy $\mathcal{S}_{sp}$:

$$\Lambda_{sp} = \frac{M_P}{\mathcal{S}_{sp}^{\frac{1}{d-2}}}$$

- In other words: definition of species entropy:

$$\boxed{\mathcal{S}_{sp} \simeq (L_{sp})^{d-2} M_P^{d-2}}$$

- We will also introduce a species temperature T_{sp}

We will require that species entropy and species temperature follow the rules of black hole thermodynamics.

Properties of species scale and species entropy:

Discussed recently by many authors: L. Anchordoqui, I. Antoniadis, R. Blumenhagen, J. Calderon-Infante, A. Castellano, Cribiori, M. Delgado, A. Gligovic, A. Herraez, L. Ibanez, C. Montella, D.L., S. Lüst, A. Paraskevopoulo, G. Staudt, A. Uranga, C. Vafa, M. Wiesner, D. Wu, ..

Properties of species scale and species entropy:

Discussed recently by many authors: L. Anchordoqui, I. Antoniadis, R. Blumenhagen, J. Calderon-Infante, A. Castellano, Cribiori, M. Delgado, A. Gligovic, A. Herraez, L. Ibanez, C. Montella, D.L., S. Lüst, A. Paraskevopoulo, G. Staudt, A. Uranga, C. Vafa, M. Wiesner, D. Wu, ..

- $$\Lambda_{sp} \leq 1 \quad \longleftrightarrow \quad \mathcal{S}_{sp} \geq 1$$

Properties of species scale and species entropy:

Discussed recently by many authors: L. Anchordoqui, I. Antoniadis, R. Blumenhagen, J. Calderon-Infante, A. Castellano, Cribiori, M. Delgado, A. Gligovic, A. Herraez, L. Ibanez, C. Montella, D.L., S. Lüst, A. Paraskevopoulo, G. Staudt, A. Uranga, C. Vafa, M. Wiesner, D. Wu, ..

- $$\Lambda_{sp} \leq 1 \quad \longleftrightarrow \quad \mathcal{S}_{sp} \geq 1$$
- Asymptotic behaviour at the boundary of moduli space:

[D. van de Heisteeg, C.Vafa, M.Wiesner (2023)]

$$\Lambda_{sp} = e^{-\gamma\phi}\,, \;\; \gamma_{KK} = \sqrt{\frac{D-d}{(D-2)(d-2)}}\,, \;\; \gamma_{\rm string} = \sqrt{\frac{1}{d-2}}$$

$$\mathcal{S}_{sp} = e^{\delta\phi}\,, \;\; \delta_{KK} = \sqrt{\frac{(d-2)(D-d)}{D-2}}\,, \;\; \delta_{\rm string} = \sqrt{d-2}$$

Properties of species scale and species entropy:

Discussed recently by many authors: L. Anchordoqui, I. Antoniadis, R. Blumenhagen, J. Calderon-Infante, A. Castellano, Cribiori, M. Delgado, A. Gligovic, A. Herraez, L. Ibanez, C. Montella, D.L., S. Lüst, A. Paraskevopoulo, G. Staudt, A. Uranga, C. Vafa, M. Wiesner, D. Wu, ..

- $$\Lambda_{sp} \leq 1 \quad \longleftrightarrow \quad \mathcal{S}_{sp} \geq 1$$

- Asymptotic behaviour at the boundary of moduli space:

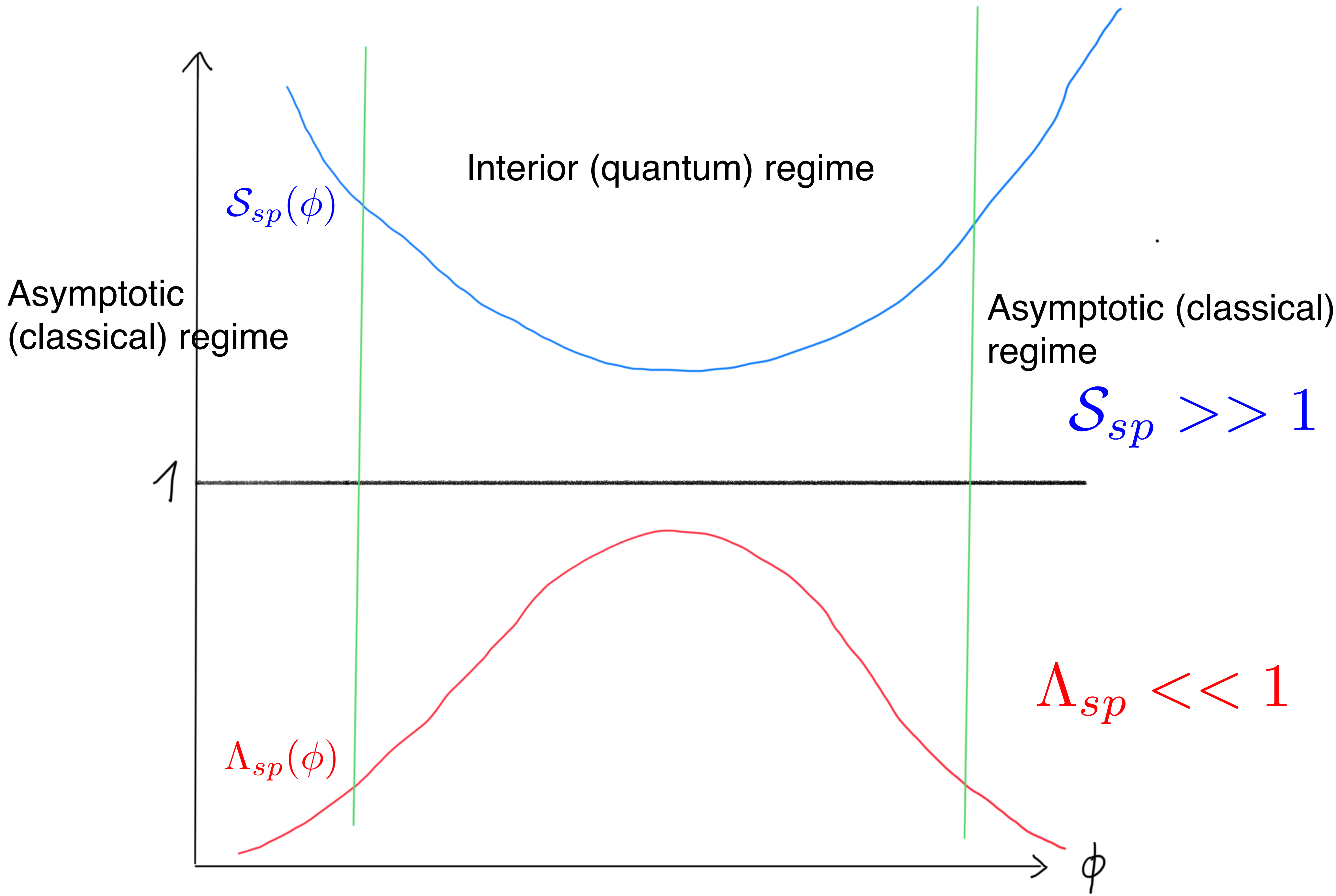
[D. van de Heisteeg, C.Vafa, M.Wiesner (2023)]

$$\Lambda_{sp} = e^{-\gamma\phi}\,, \; \gamma_{KK} = \sqrt{\frac{D-d}{(D-2)(d-2)}}\,, \; \gamma_{\rm string} = \sqrt{\frac{1}{d-2}}$$

$$\mathcal{S}_{sp} = e^{\delta\phi}\,, \; \delta_{KK} = \sqrt{\frac{(d-2)(D-d)}{D-2}}\,, \; \delta_{\rm string} = \sqrt{d-2}$$

- They are globally defined over the entire moduli space $\longleftrightarrow$ modular functions.

[N. Cribiori, D.L. (2023); D. van de Heisteeg, C.Vafa, M.Wiesner, D.Wu (2022/23)]



Interior (quantum) regime
$\mathcal{S}_{sp}(\phi)$
Asymptotic (classical) regime
Asymptotic (classical) regime
$\mathcal{S}_{sp} >> 1$
1
$\Lambda_{sp} << 1$
$\Lambda_{sp}(\phi)$
ϕ

This picture is closely related to the BH entropy conjecture:

In the limit $\mathcal{S}_{BH} \to \infty$ there is a tower of light states with masses

$$m = \left(\frac{1}{\mathcal{S}_{BH}}\right)^{\gamma}, \quad \gamma > 0$$

[Q. Bonnefoy, L. Ciambelli, S. Lüst, D.L. (2019);
N. Cribiori, M. Dierigl, A. Gnecchi, M. Scalisi, D.L. (2022)]

This picture is closely related to the BH entropy conjecture:

In the limit $\mathcal{S}_{BH} \to \infty$ there is a tower of light states with masses

$$m = \left(\frac{1}{\mathcal{S}_{BH}}\right)^{\gamma}, \quad \gamma > 0$$

[Q. Bonnefoy, L. Ciambelli, S. Lüst, D.L. (2019);
N. Cribiori, M. Dierigl, A. Gnecchi, M. Scalisi, D.L. (2022)]

Smallest possible BH with minimal entropy → tower is given in terms of species and the BH entropy becomes the species entropy $\mathcal{S}_{sp}$.

This picture is closely related to the BH entropy conjecture:

In the limit $\mathcal{S}_{BH} \to \infty$ there is a tower of light states with masses

$$m = \left(\frac{1}{\mathcal{S}_{BH}}\right)^{\gamma}, \quad \gamma > 0$$

[Q. Bonnefoy, L. Ciambelli, S. Lüst, D.L. (2019);
N. Cribiori, M. Dierigl, A. Gnecchi, M. Scalisi, D.L. (2022)]

Smallest possible BH with minimal entropy → tower is given in terms of species and the BH entropy becomes the species entropy $\mathcal{S}_{sp}$.

Large species entropy limit:

$$\mathcal{S}_{sp} \to \infty \quad \Rightarrow \quad m_{sp} = \left(\frac{1}{\mathcal{S}_{sp}}\right)^{\gamma} \to 0$$

Outline :

I) Introduction: species scale and entropy

II) Minimal BHs as species bound states

III) Minimal charged BHs

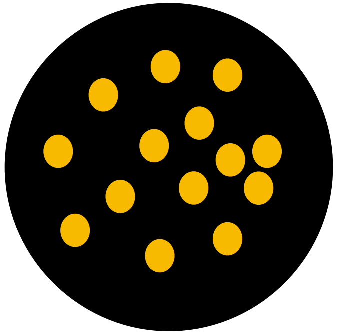
IV) String constructions

V) Summary

Minimal black holes as species bound states:

N-portrait picture for black holes: bound state of N_g gravitons: ●

$$\alpha_g = \frac{L_P^2}{R^2} = N_g^{-1}$$



[G. Dvali, C. Gomez (2011)]

- BH Entropy: $\mathcal{S}_{BH} \simeq N_g$

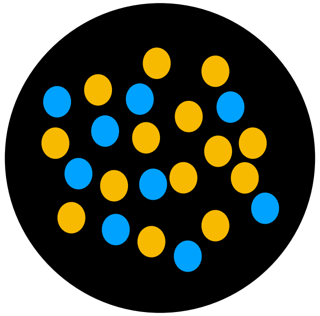
- BH Radius: $R_{BH} \simeq \sqrt{N_g} L_P$ (Wave length of gravitons)

- BH Mass: $M_{BH} \simeq E_g \simeq \sqrt{N_g}/L_P$ (Energy of gravitons)

Now introduce in addition N_{sp} massive species: ●

Assume that they are couple like gravitons (e.g. spin-2 tower of KK modes)

Black hole also contains the species particles as constituents:



[G. Dvali, C. Gomez, D.L. (2012)]

$$\alpha_g = \frac{L_P^2}{R^2} = (N_g N_{sp})^{-1}$$

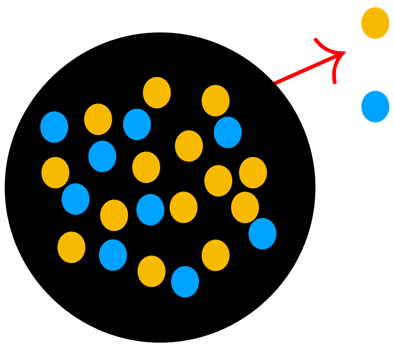
So effectively $N_g \rightarrow N_{tot} = N_g N_{sp}$

- BH Entropy: $\mathcal{S}_{BH} \simeq N_g N_{sp}$

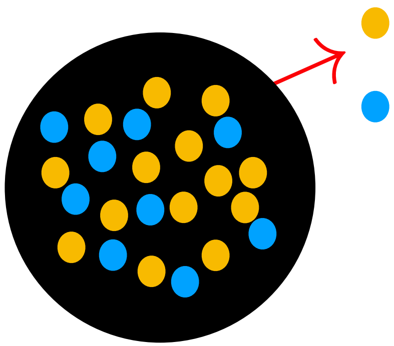
- BH Radius: $R_{BH} \simeq \sqrt{N_g N_{sp}} L_P = \sqrt{N_g} L_{sp}$

- BH mass: $M_{BH} \simeq \sqrt{N_g N_{sp}} / L_P$

BH decay: leakage of constituents:



BH decay: leakage of constituents:

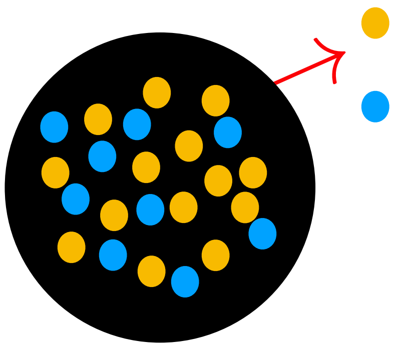


- BH Temperature: $T_{BH} \simeq 1/(\sqrt{N_g N_{sp}} L_P) = 1/(\sqrt{N_g} L_{sp})$

Semiclassical (Hawking) BH Decay Rate for large entropy:

$$\Gamma^{th}_{BH} \simeq T_{BH}$$

BH decay: leakage of constituents:



- BH Temperature: $T_{BH} \simeq 1/(\sqrt{N_g N_{sp}} L_P) = 1/(\sqrt{N_g} L_{sp})$

Semiclassical (Hawking) BH Decay Rate for large entropy:

$$\Gamma_{BH}^{th} \simeq T_{BH}$$

- However quantum decay rate for small entropy is suppressed:

$$\Gamma_{BH}^{qm} \simeq \frac{T_{BH}}{\mathcal{S}_{BH}^{n}} \simeq \mathcal{S}_{BH}^{\frac{1-n(2-d)}{2-d}}$$

Mass, temperature and entropy of Schwarzschild-like BHs satisfy the following general thermodynamic relations:

$$\mathcal{S}_{BH} \simeq (R_{BH})^{d-2} M_P^{d-2} \simeq (M_{BH})^{\frac{d-2}{d-3}} M_P^{\frac{2-d}{d-3}}$$

$$T_{BH} = \left(R_{BH}\right)^{-1} \quad \Longrightarrow \quad \mathcal{S}_{BH} T_{BH}^{d-2} = M_P^{d-2}$$

Mass, temperature and entropy of Schwarzschild-like BHs satisfy the following general thermodynamic relations:

$$\mathcal{S}_{BH} \simeq (R_{BH})^{d-2} M_P^{d-2} \simeq (M_{BH})^{\frac{d-2}{d-3}} M_P^{\frac{2-d}{d-3}}$$

$$T_{BH} = \left(R_{BH}\right)^{-1} \quad \Longrightarrow \quad \mathcal{S}_{BH} T_{BH}^{d-2} = M_P^{d-2}$$

These relations are consistent with the first law of thermodynamics:

$$dM_{BH} = T_{BH} d\mathcal{S}_{BH}$$

$$\frac{1}{T_{BH}} = \frac{\partial S_{BH}}{\partial M_{BH}}$$

Minimal Black Holes:

Set $N_g = 1 \quad \Leftrightarrow \quad N_{tot} = N_{sp}$

Minimal Black Holes:

Set $N_g = 1 \quad \Leftrightarrow \quad N_{tot} = N_{sp}$

This defines all relevant species properties:

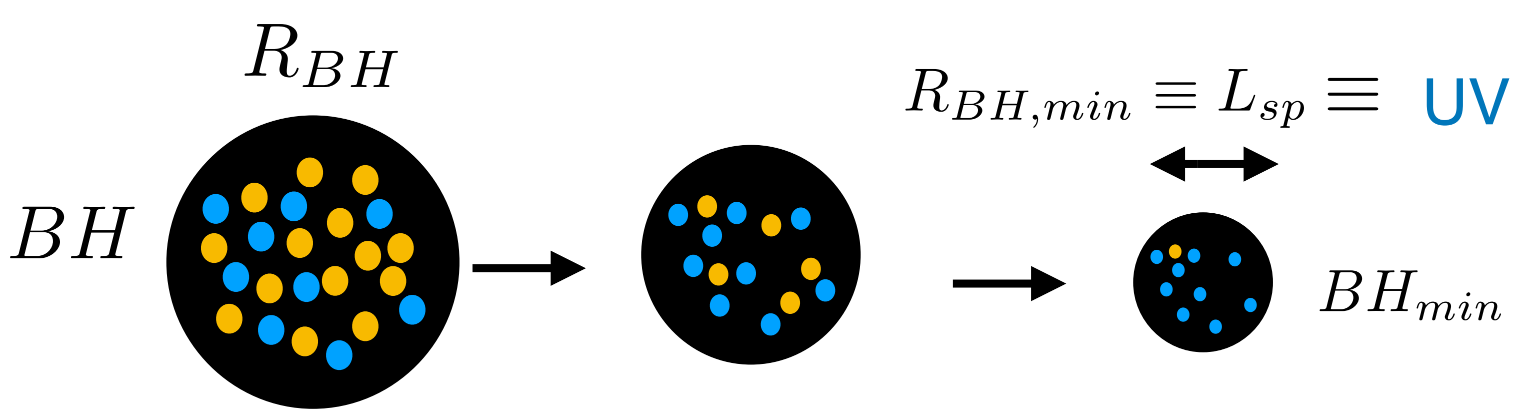
- Species Entropy: $\mathcal{S}_{sp} := \mathcal{S}_{BH,min} = N_{sp}$
- Species Scale: $L_{sp} := R_{BH,min}$
- Species Mass: $M_{sp} := M_{BH,min}$
- Species Temperature: $T_{sp} := T_{BH,min} = L_{sp}^{-1} = \Lambda_{sp}$

Minimal Black Holes:

Set $N_g = 1 \quad \Leftrightarrow \quad N_{tot} = N_{sp}$

This defines all relevant species properties:

- Species Entropy: $\mathcal{S}_{sp} := \mathcal{S}_{BH,min} = N_{sp}$
- Species Scale: $L_{sp} := R_{BH,min}$
- Species Mass: $M_{sp} := M_{BH,min}$
- Species Temperature: $T_{sp} := T_{BH,min} = L_{sp}^{-1} = \Lambda_{sp}$



Black Hole - Species Correspondence

Dual description:	Particles	-	Geometry
	Species	-	Minimal BH as species bound state

Black Hole - Species Correspondence

Dual description:	Particles	-	Geometry
	Species	-	Minimal BH as species bound state

There is a transition between the particle and the BH description:

At strong coupling species form a black hole.

Large entropy → Collective BH description

Small entropy → Individual description as qm particles

Black Hole - Species Correspondence

Dual description: Particles - Geometry

Species - Minimal BH as species bound state

There is a transition between the particle and the BH description:

At strong coupling species form a black hole.

Large entropy → Collective BH description

Small entropy → Individual description as qm particles

Certain consistency conditions must be fulfilled for the BH - Species Correspondence !

Species as tower of particles - emergent string conjecture

Only KK towers or string excitations can be light at infinite distance.

[S. Lee, W. Lerche, T. Weigand (2019)]

Species as tower of particles - emergent string conjecture

Only KK towers or string excitations can be light at infinite distance.

[S. Lee, W. Lerche, T. Weigand (2019)]

General ansatz for species tower:

Mass spectrum: $m_n = f(n) m_0 \, , \quad n = 1, \ldots, N_s$

Degeneracy at each level: d_n

Species as tower of particles - emergent string conjecture

Only KK towers or string excitations can be light at infinite distance.

[S. Lee, W. Lerche, T. Weigand (2019)]

General ansatz for species tower:

Mass spectrum: $m_n = f(n) m_0 \,, \quad n = 1, \ldots, N_s$

Degeneracy at each level: d_n

From that we derive:

Species entropy: $$S_{sp} \equiv N_{sp} = \sum_{n=1}^{N_s} d_n$$

Species as tower of particles - emergent string conjecture

Only KK towers or string excitations can be light at infinite distance.

[S. Lee, W. Lerche, T. Weigand (2019)]

General ansatz for species tower:

Mass spectrum: $m_n = f(n) m_0 \,, \quad n = 1, \ldots, N_s$

Degeneracy at each level: d_n

From that we derive:

Species entropy: $S_{sp} \equiv N_{sp} = \sum_{n=1}^{N_s} d_n$

Total species mass: $M_{sp} = \sum_{n=1}^{N_s} f(n) d_n m_0$

Species as tower of particles - emergent string conjecture

Only KK towers or string excitations can be light at infinite distance.

[S. Lee, W. Lerche, T. Weigand (2019)]

General ansatz for species tower:

Mass spectrum: $m_n = f(n) m_0 \,, \quad n = 1, \ldots, N_s$

Degeneracy at each level: d_n

From that we derive:

Species entropy: $$S_{sp} \equiv N_{sp} = \sum_{n=1}^{N_s} d_n$$

Total species mass: $$M_{sp} = \sum_{n=1}^{N_s} f(n) d_n m_0$$

Species scale, i.e. species temperature: $$\Lambda_{sp} \equiv T_{sp} = f(N_s) m_0$$

Consistency with thermodynamics laws requires:

1st. law of species thermodynamics: $$dM_{sp} = T_{sp} d\mathcal{S}_{sp}$$

$$\mathcal{S}_{sp} \simeq \left(M_{sp}\right)^{\frac{d-2}{d-3}} \,, \quad \mathcal{S}_{sp} \simeq \left(T_{sp}\right)^{2-d}$$

$\Longrightarrow$ Bounds on $f(n)\,, \quad d_n$

Consistency with thermodynamics laws requires:

1st. law of species thermodynamics: $$dM_{sp} = T_{sp} d\mathcal{S}_{sp}$$

$$\mathcal{S}_{sp} \simeq \left(M_{sp}\right)^{\frac{d-2}{d-3}} , \quad \mathcal{S}_{sp} \simeq \left(T_{sp}\right)^{2-d}$$

$\Longrightarrow$ Bounds on $f(n) , \quad d_n$

2nd. law of species thermodynamics:

The species entropy does not decrease when moving towards the boundary of the moduli space:

$$\delta_\phi \mathcal{S}_{sp} \geq 0 \quad \text{for} \qquad \phi \to \infty$$

(This is satisfied for the asymptotic behaviour shown before.)

Consistency with thermodynamics laws requires:

1st. law of species thermodynamics: $$dM_{sp} = T_{sp} d\mathcal{S}_{sp}$$

$$\mathcal{S}_{sp} \simeq (M_{sp})^{\frac{d-2}{d-3}} \,, \quad \mathcal{S}_{sp} \simeq (T_{sp})^{2-d}$$

$\Longrightarrow$ Bounds on $f(n)\,, \quad d_n$

2nd. law of species thermodynamics:

The species entropy does not decrease when moving towards the boundary of the moduli space:

$$\delta_\phi \mathcal{S}_{sp} \geq 0 \quad \text{for} \qquad \phi \to \infty$$

(This is satisfied for the asymptotic behaviour shown before.)

So let us see which kind of towers satisfy the thermodynamic laws.

1st Example:

Choose $d_n = 1 \quad \Rightarrow \quad N_s = N_{sp} = \mathcal{S}_{sp}$

Then we obtain from the consistency requirement (1st. law) that

1st Example:

Choose $d_n = 1 \quad \Rightarrow \quad N_s = N_{sp} = \mathcal{S}_{sp}$

Then we obtain from the consistency requirement (1st. law) that

$$f(n) = n^{1/p} \quad \text{with} \quad p \geq 1$$

i.e. $m_n \simeq n^{1/p} m_{KK}$

1st Example:

Choose $d_n = 1 \quad \Rightarrow \quad N_s = N_{sp} = \mathcal{S}_{sp}$

Then we obtain from the consistency requirement (1st. law) that

$$f(n) = n^{1/p} \quad \text{with} \quad p \geq 1$$

i.e. $m_n \simeq n^{1/p} m_{KK}$

This is precisely the spectrum of a KK tower from p extra dimensions !

So there is a KK - black hole correspondence.

2nd Example: string tower

Minimal BH is bound state of tower of massive string modes

Choose $$f(n) = n^{1/2}\,, \qquad d_n \sim e^{\sqrt{n}}$$

$\Longrightarrow$ String spectrum: $$m_n = \sqrt{n} M_s$$

2nd Example: string tower

Minimal BH is bound state of tower of massive string modes

Choose $f(n) = n^{1/2}\,, \qquad d_n \sim e^{\sqrt{n}}$

$\Longrightarrow$ String spectrum: $m_n = \sqrt{n} M_s$

$$\Lambda_{sp} = m_{N_s} \simeq \sqrt{N_s} M_s$$

$$N_{sp} = \sum_{n=0}^{N_s} d_n \sim \sum_{n=0}^{N_s} e^{\sqrt{n}} \simeq \sqrt{N_s} e^{\sqrt{N_s}}$$

$$\Lambda_{sp} \simeq M_s \log \frac{M_P}{M_s}$$

Total species mass for string tower:

$$M_{sp} = M_s \sum_{n=1}^{N_s} e^{\sqrt{n}} \sqrt{n} = \mathcal{S}_{sp}^{\frac{3-d}{2-d}}$$ ✓

This is in agreement with the 1st law of species thermodynamics.

Question: does the 1st. law imply the string tower, i.e.

$$d_n = e^{\sqrt{n}} \quad \overset{?}{\Rightarrow} \quad f(n) = \sqrt{n}$$

Total species mass for string tower:

$$M_{sp} = M_s \sum_{n=1}^{N_s} e^{\sqrt{n}} \sqrt{n} = \mathcal{S}_{sp}^{\frac{3-d}{2-d}}$$ ✓

This is in agreement with the 1st law of species thermodynamics.

Question: does the 1st. law imply the string tower, i.e.

$$d_n = e^{\sqrt{n}} \quad \overset{?}{\Rightarrow} \quad f(n) = \sqrt{n}$$

Summary: KK tower - BH correspondence ✓

String - BH correspondence ✓

[See also: G. Horowitz, J.Polchinski (1996)]

Total species mass for string tower:

$$M_{sp} = M_s \sum_{n=1}^{N_s} e^{\sqrt{n}} \sqrt{n} = \mathcal{S}_{sp}^{\frac{3-d}{2-d}}$$ ✓

This is in agreement with the 1st law of species thermodynamics.

Question: does the 1st. law imply the string tower, i.e.

$$d_n = e^{\sqrt{n}} \quad \stackrel{?}{\Rightarrow} \quad f(n) = \sqrt{n}$$

Summary: KK tower - BH correspondence ✓

String - BH correspondence ✓

[See also: G. Horowitz, J.Polchinski (1996)]

Can we completely derive the emergent string conjecture from a bottom up black hole argument?

Outline :

I) Introduction: species scale and entropy

II) Minimal BHs as species bound states

III) Minimal charged BHs

IV) String constructions

V) Summary

Minimal charged BHs and species :

Multi parameter family of metrics: M, Q, P

Eg. Reissner-Nordström black hole: $M_{BH}^2 \geq Q_{BH}^2$

Non-extremality parameter: $c = \sqrt{M_{BH}^2 - Q_{BH}^2}$

$$dM_{BH} = T_{BH} d\mathcal{S}_{BH} + \Phi dQ_{BH}$$

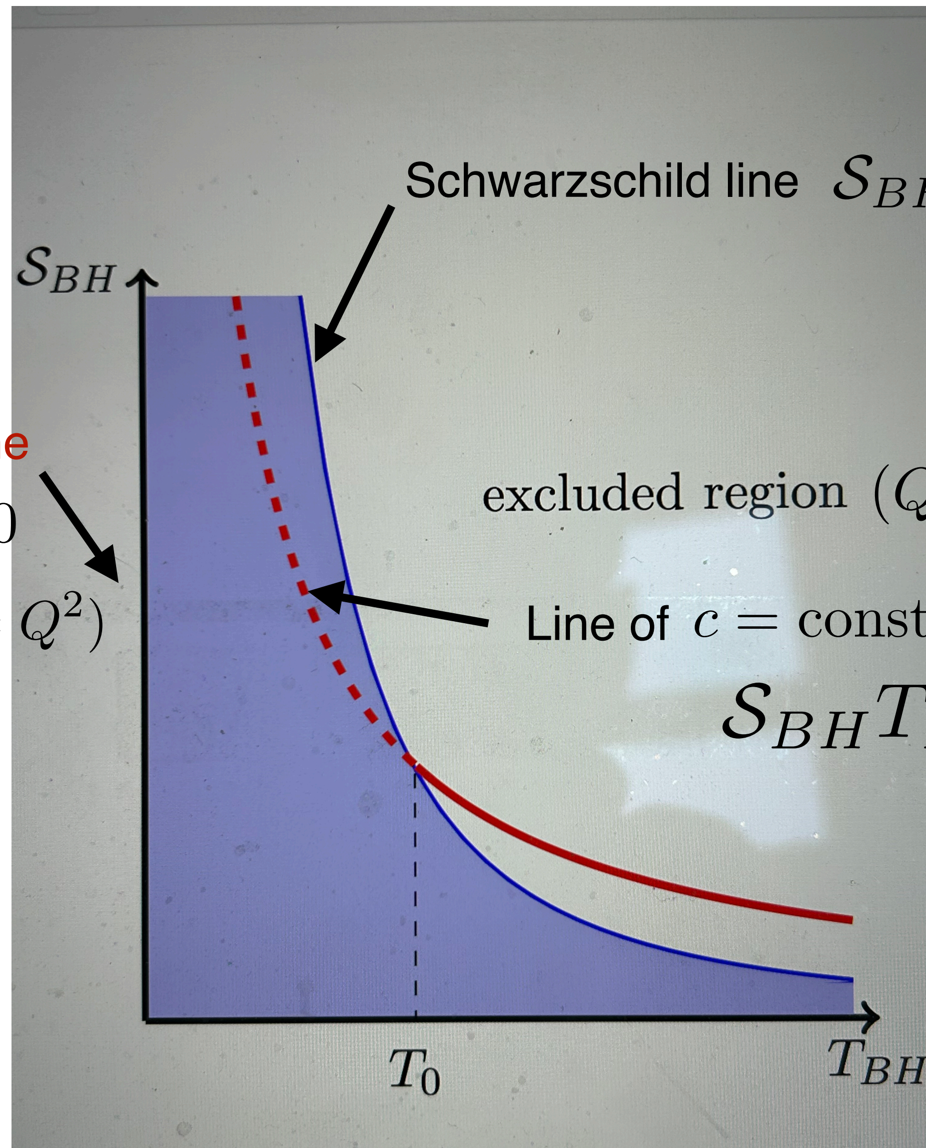
$$\Longrightarrow d(M_{BH} - \Phi Q_{BH}) = T_{BH} dS - Q_{BH} d\Phi = dc$$

$$T_{BH} = \frac{\partial c}{\partial \mathcal{S}_{BH}}\Big|_{\Phi}, \quad Q_{BH} = \frac{\partial c}{\partial \Phi}\Big|_{\mathcal{S}_{BH}}$$

$$T_{BH} \simeq \frac{c}{S_{BH}}$$

Phase diagram of RN black hole:

$$\mathcal{S}_{BH} \simeq \frac{R^2_{BH,+}}{L_P^2} \qquad T_{BH} \simeq \frac{c}{\mathcal{S}_{BH}}$$



Non-extremal minimal BH as species bound state:

Consider a charged tower with $(m_n, q_n) \qquad n = 1, \ldots, N_{sp}$

Lattice weak gravity conjecture:
entire tower is super-extremal: $|q_n| \geq m_n \quad \forall \ n$

Non-extremal minimal BH as species bound state:

Consider a charged tower with (m_n, q_n) $n = 1, \ldots, N_{sp}$

Lattice weak gravity conjecture:
entire tower is super-extremal: $|q_n| \geq m_n \quad \forall\ n$

Minimal BH: [B. Heidenreich, M. Reece, T. Rudelius (2016)]

$$Q_{BH,min} := Q_{sp} = \sum_{n=1}^{N_{sp}} q_n \,, \quad M_{BH,min} := M_{sp} = \sum_{n=1}^{N_{sp}} m_n$$

Minimal BH is super-extramal: $|Q_{BH,min}| \geq M_{BH,min}$

Non-extremal minimal BH as species bound state:

Consider a charged tower with $(m_n, q_n) \qquad n = 1, \ldots, N_{sp}$

Lattice weak gravity conjecture:
entire tower is super-extremal: $|q_n| \geq m_n \quad \forall\, n$

Minimal BH: [B. Heidenreich, M. Reece, T. Rudelius (2016)]

$$Q_{BH,min} := Q_{sp} = \sum_{n=1}^{N_{sp}} q_n \,, \qquad M_{BH,min} := M_{sp} = \sum_{n=1}^{N_{sp}} m_n$$

Minimal BH is super-extramal: $|Q_{BH,min}| \geq M_{BH,min}$

Higher order corrections in EFT: $\alpha_1 (F_{\mu\nu} F^{\mu\nu})^2 \,, \qquad \alpha_3 F_{\mu\nu} F_{\rho\sigma} W^{\mu\nu\rho\sigma}$

Modified extremality condition: [N. Arkani-Hamed, L. Motl, A.Nicolsis, C. Vafa (2006); Y. Hamada, T. Noumi, G. Shiu (2019)]

$$c^2 = M^2_{BH,min} - Q^2_{BH,min} + 2\alpha \geq 0$$

$$\alpha = \frac{2}{5} (4\pi)^2 (2\alpha_1 - \alpha_3)$$

Example: charged KK tower:

Tower of KK masses: $m_n = n/R \qquad (k = 1 \ldots, N_{sp})$

Associated KK charges: $q_n = (n+\beta)/R \qquad \beta \geq 0$

Total species mass:

$$M_{sp} = \sum_{n=1}^{N_{sp}} m_n = (\mathcal{S}_{sp})^{\frac{d-3}{d-2}}(1 + 2(\mathcal{S}_{sp})^{-1}) = \Lambda_{sp}^{3-d} + 2\Lambda_{sp}$$

Total species charge:

$$Q_{sp} = \sum_{n=1}^{N_{sp}} q_n = (\mathcal{S}_{sp})^{\frac{d-3}{d-2}}(1 + (2+\beta)(\mathcal{S}_{sp})^{-1}) = \Lambda_{sp}^{3-d} + (2+\beta)\Lambda_{sp}$$

Extremality parameter: $c = \sqrt{2\alpha - \beta}\,(\mathcal{S}_{sp})^{\frac{d/2-2}{d-2}}$

$$2\alpha - \beta \geq 0$$

Species temperature:

$$T_{sp} \simeq \frac{\sqrt{2\alpha - \beta}}{\left(\mathcal{S}_{sp}\right)^{\frac{d}{2d-4}}} = \sqrt{2\alpha - \beta}\left(\Lambda_{sp}\right)^{d/2}$$

It agrees with thermodynamic relations for charged BHs in d dimensions.

Outline :

I) Introduction: species scale and entropy

II) Minimal BHs as species bound states

III) Minimal charged BHs

IV) String constructions

V) Summary

String constructions:

String constructions with broken susy what kind of towers are possible

Scherk-Schwarz like constructions

	Particle	Light	Additive shift	Parametrically small
F1 (heterotic)	✗	✓	✗	✗
D1 (orientifold)	✗	✗	✗	✗
NS5 (heterotic)	✗	✗	✗	✗
D1 (SS_{KK})	✗	✗	✓	✓
wrapped D1 (SS_{KK})	✓	✗	✓	✓
wrapped D1 (SS_{wind})	✓	✓	✓	✓

Table 1: Comparison of various candidates for near-extremal species black holes (or branes) in non-SUSY string constructions. Our ideal target are particle-like light species with a parametrically small additive shift in mass relative to the charge. The most promising candidate are wrapped D1-branes on small circles in Scherk-Schwarz settings, which for type IIB can be interpreted in terms of M-theory SUSY breaking. Scherk-Schwarz vacua have soft SUSY breaking, but also dynamical tadpoles.

Summary :

Summary :

- The laws of species thermodynamics and minimal black holes shed new light on swampland conjectures.

Summary :

- The laws of species thermodynamics and minimal black holes shed new light on swampland conjectures.

- There is a correspondence picture: species as particles - species as minimal black hole.

Strong constraints on the form of the species tower.

$$\overset{?}{\Longrightarrow}$$ Emergent string conjecture.

This correspondence might be also related to the emergence proposal:
species build space-time geometry

Summary :

- The laws of species thermodynamics and minimal black holes shed new light on swampland conjectures.

- There is a correspondence picture: species as particles - species as minimal black hole.

Strong constraints on the form of the species tower.

$$\overset{?}{\Longrightarrow}$$ Emergent string conjecture.

This correspondence might be also related to the emergence proposal: species build space-time geometry

- Cosmology

The production and the decay of species is determined by their temperature - nice agreement with dark matter in dark dimensions

Thank you !