Moduli Stabilization and Entropy Maximization in Type II Calabi-Yau compactifications

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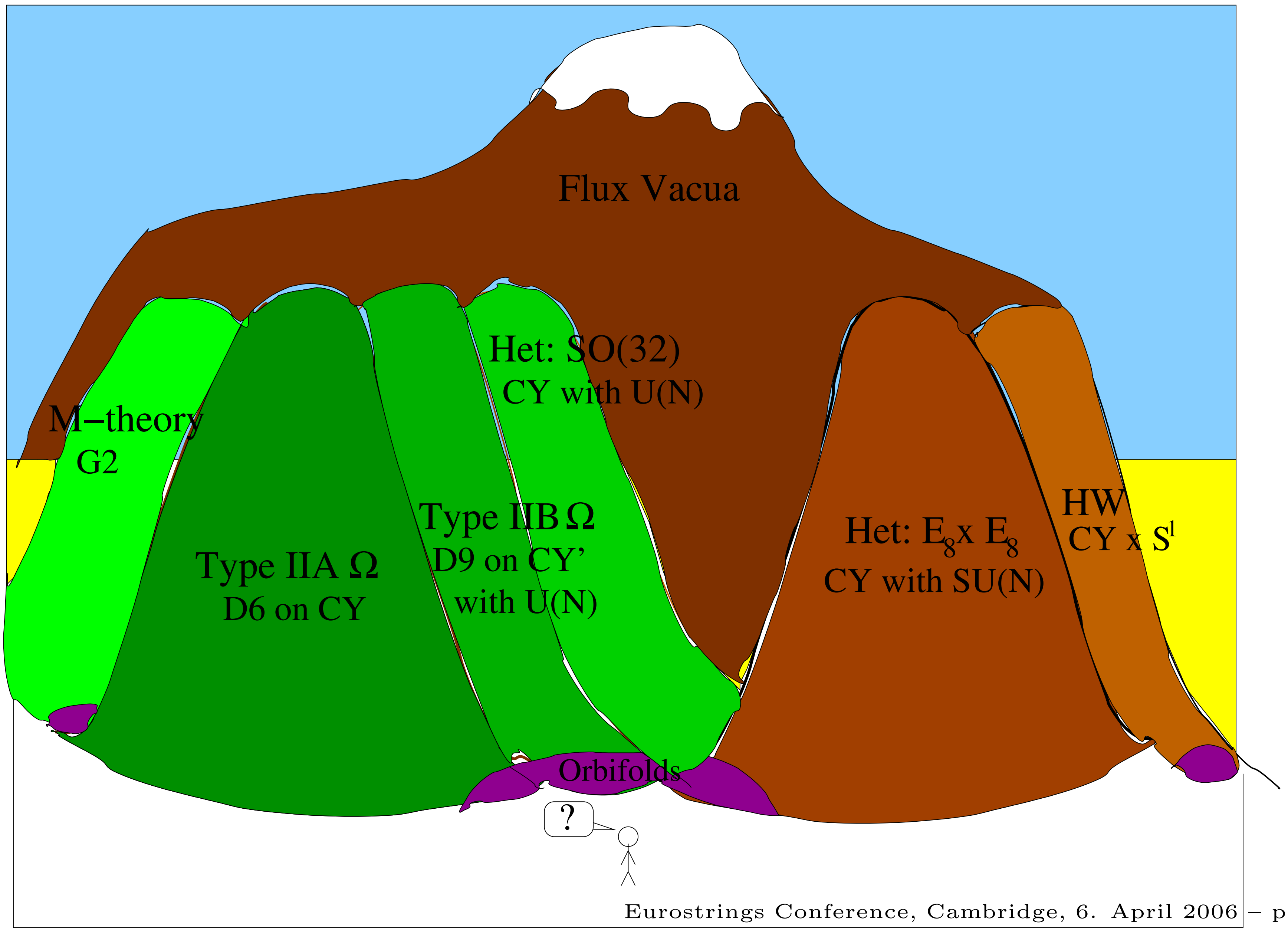
Introduction - String Model Building



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- New forces?
- Uncalculable couplings?
- How gets supersymmetry broken?
- How to get inflation?
- Dark energy?

Introduction - Moduli stabilization

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- Tree level: background fluxes
- Non-perturbatively: D-brane instantons, gaugino condensates.

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In this way one can obtain a discrete set of vacua with either

- negative cosmological constant, $\Lambda < 0$, (AdS vacua),
- zero cosmological constant, $\Lambda = 0$, (Minkowski vacua),
- positive cosmological constant, $\Lambda > 0$, (dS vacua)

and with various possibilities for the low-low energy matter fields (gauge bosons, quarks, leptons, ...).

Introduction - Landscape problem

2nd. problem: Count the number of consistent string vacua $\Longrightarrow$ Vast landscape with $N_{sol} = 10^{500-1500}$ discrete vacua! (Lerche, Lüst, Schellekens (1986); Douglas (2003))

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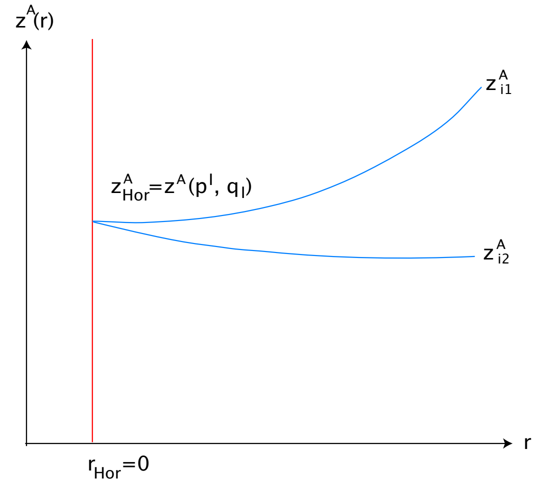
- Entropy of string vacua (Entropic answer): determine a probability wave function in moduli space,

$$|\psi(\phi)|^2 = e^{\mathcal{S}(\phi)} \, ,$$

and see if $|\psi|^2$ is peaked, i.e. has maxima, at vacua with good phenomenological properties.

Entropy of string (flux) vacua

Use correspondence between flux vacua and attractor mechanism for supersymmetric black holes: moduli are fixed at the horizon of supersymmetric black holes:



(H. Ooguri, C. Vafa, E. Verlinde (2005)):
Probability distribution for (5-form) AdS flux vacua (in 2 dimensions):

$$\text{HH wave function}: \quad |\Psi_{p,q}|^2 \sim \exp(\mathcal{S}), \quad \mathcal{S} = \frac{A_{horizon}}{4}$$

Further outline of the talk

- Moduli stabilization in type IIB $Z_N \times Z_M$ orientifolds

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- Entropy maximization of flux compactifications

  (G. L. Cardoso, D. Lüst, J. Perz (2006))

Moduli stabilization in orientifolds

KKLT-Proposal:

(Kachru, Kallosh, Linde, Trivedi (2003))

Step 1: Fix all moduli (preserving SUSY)
Dilaton (S) and complex structure moduli (U) are stabiized by 3-form fluxes, Kähler moduli are fixed by non-perturbative effects $\rightarrow$ SUSY AdS vacuum.

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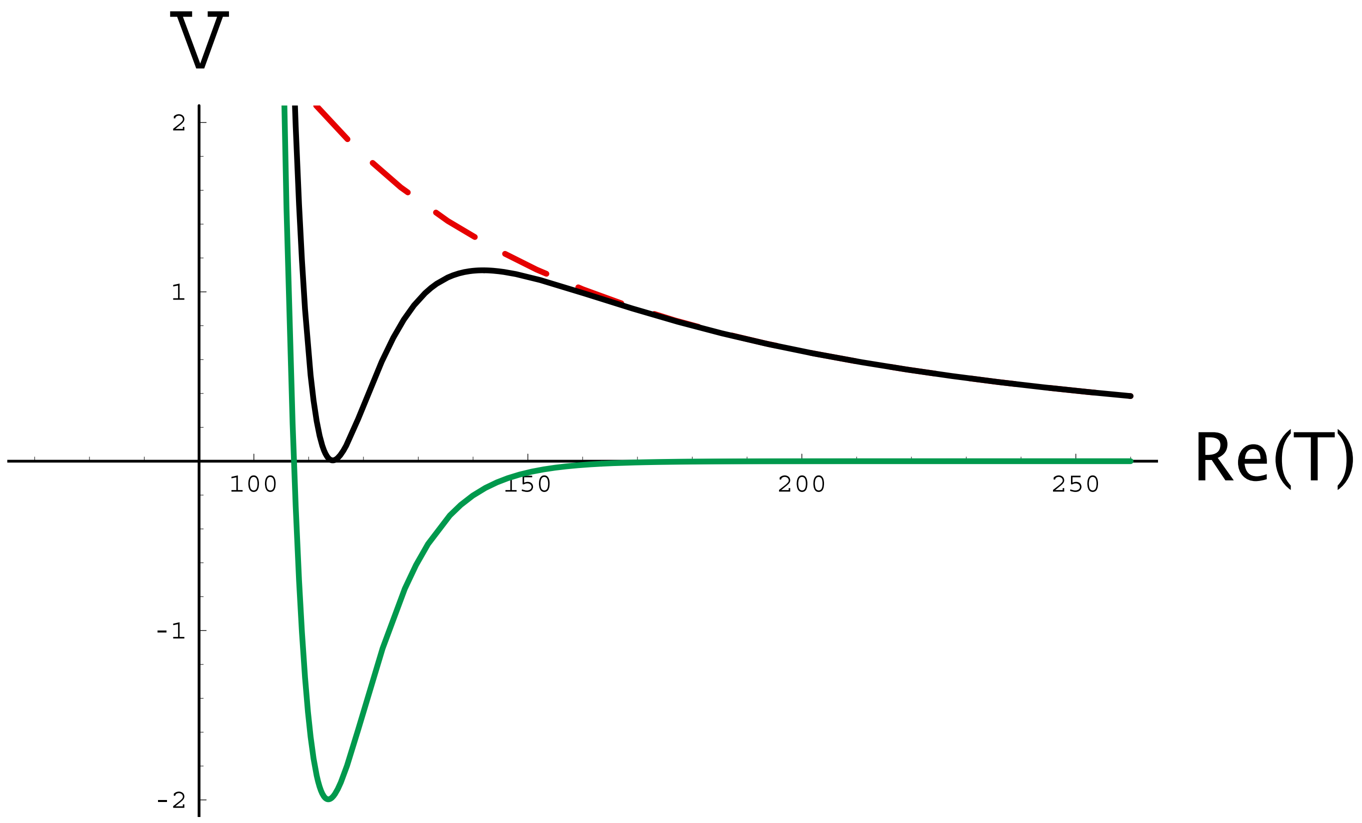
Step 2: Uplift the minimum of the potential to a positive, non-SUSY (metastable) dS vacuum (by $\bar{D}3$-branes).

This form of the up-lift requires that the AdS vacuum is "stable" in the sense that all scalar masses are positive definite (stronger requirement that Breitenlohner/Freedman bound):

$$\frac{\partial V}{\partial \phi_i \partial \bar{\phi}_j} > 0, \quad (\phi_i, \phi_j = S, T, U)$$

Moduli stabilization in orientifolds



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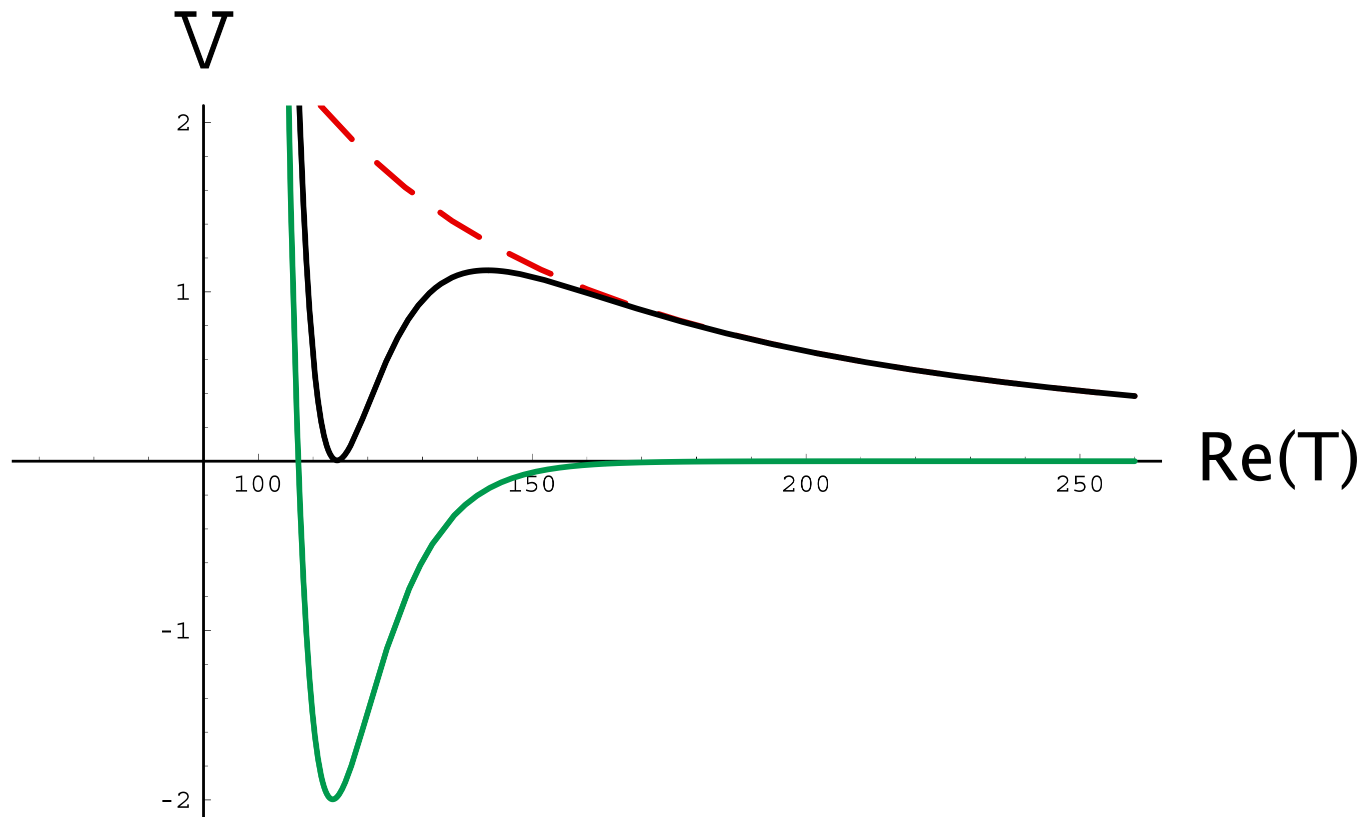

Can the KKLT scenario be realized in concrete orientifold models?

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- A consistent orientifold projection has to be performed. This yields $O3$- and $O7$-planes. The associated tadpoles must be cancelled by $D3$- and $D7$-branes and/or background fluxes. The space-time supersymmetry is reduced to $\mathcal{N} = 1$.

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- The Kähler potential for the moduli fields has to be computed.

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- (The uplift to dS-vacua has to be performed.)

Moduli stabilization in orientifolds

Closed string moduli space:

- $h^{(1,1)}$ Kähler moduli T^i (size),
- $h^{(2,1)}$ complex structure moduli U^i (shape),
- dilaton $S = e^{-\phi_{10}} + iC_0$.

Moduli stabilization: Dilaton and complex structure moduli are stabilized via background fluxes (Giddings, Kachru, Polchinski):

$$G_3 = F_3 + iSH_3\,, \quad F_3 = dC_2\,, \quad H_3 = dB_2\,.$$

Flux superpotential (Gukov, Vafa, Witten; Taylor, Vafa):

$$W_{flux} = \kappa_{10}^{-2} \int_X G_3 \wedge \Omega\,.$$

Kähler moduli are stabilized via non-perturbative effects.

Moduli stabilization in orientifolds

Two possible origins for the non-perturbative superpotential:

- Euclidean D3-brane instantons wrapping internal 4-cycles (divisors)

$$W_{np} = g_i e^{-2\pi V_i}$$

- Gaugino condensation in the world volume of D7-branes wrapped on internal 4-cycles

$$W_{np} = g_i e^{-\frac{2\pi V_i}{b_i}}$$

Condition for existence of non-vansishing non-perturbative superpotential (F/M-theory) (Witten):

$$\chi(\text{wrapped divisor}) = h^{(0,0)} - h^{(0,1)} + h^{(0,2)} - h^{(0,3)} = 1$$

Zero modes may change in the presence of 3-form fluxes and/or O-planes!

Moduli stabilization in orientifolds

Effective $\mathcal{N} = 1$ superpotential:

$$W = W_{flux}(S, U^i) + W_{np}(T^i) = \kappa_{10}^{-2} \int_X G_3 \wedge \Omega + \sum_i g_i e^{-h_i T^i} .$$

Scalar potential:

$$V = e^{\kappa_4^2 K} \left(|D_S W|^2 + \sum_i |D_{T^i} W|^2 + \sum_i |D_{U^i} W|^2 - 3|W|^2 \right)$$

Impose SUSY condition:

$$D_i W = \partial_i W + \kappa_4^2 W \partial_i K = 0, \quad \Rightarrow \quad S_0,\ T_0^i,\ U_0^i$$

Generically the SUSY vacua with fixed moduli are AdS: $V_0 = -3 e^{\kappa_4^2 K_0} |W_0|^2$.

Moduli stabilization in orientifolds

Type IIB orientifolds that allow for tadpole cancellation:

Z_N-orbifolds

Twist Γ	$h^{untw.}_{(1,1)}$	$h^{untw.}_{(2,1)}$	$h^{twist.}_{(1,1)}$	$h^{twist.}_{(2,1)}$
Z_3	9	0	27	0
$Z_{6-I}^{(1)}$	5	0	24	5
$Z_{6-I}^{(2)}$	5	0	20	1
$Z_{6-II}^{(1)}$	3	1	32	10
$Z_{6-II}^{(2)}$	3	1	26	4
$Z_{6-II}^{(3)}$	3	1	28	6
$Z_{6-II}^{(4)}$	3	1	22	0
Z_7	3	0	21	0
$Z_{12}^{(1)}$	3	0	26	5
$Z_{12}^{(2)}$	3	0	22	1

Moduli stabilization in orientifolds

$Z_N \times Z_M$-orbifolds

Twist Γ	$h^{untw.}_{(1,1)}$	$h^{untw.}_{(2,1)}$	$h^{twist.}_{(1,1)}$	$h^{twist.}_{(2,1)}$
$Z_2 \times Z_2$	3	3	48	0
$Z_3 \times Z_3$	3	0	81	0
$Z_6 \times Z_6$	3	0	81	0
$Z_3 \times Z_6$	3	0	70	1
$Z_2 \times Z_3$	3	1	32	10
$Z_2 \times Z_6$	3	1	48	2
$Z_2 \times Z_6$	3	0	33	0

Kählerpotential for untwisted moduli at the orbifold point:

$$K = -\log(S + \bar{S}) - \sum_i \log(T^i + \bar{T}^i) - \sum_i \log(U^i + \bar{U}^i)\,.$$

Superpotential for untwisted moduli at the orbifold point:

$$W = (a_0 - ia_1 S)(b_0 + \sum_i b_i U^i + \sum_i c_i \prod U^k/U^i + d_0 \prod U^k) + \sum_i g_i e^{-h_i T^i}\,.$$

Moduli stabilization in orientifolds

Given this K and W, are there stable AdS vacua at the orbifold point?

(D. Lüst, S. Reffert, W. Schulgin, S. Stieberger, hep-th/0506090; see also Choi, Falkowski, Nilles, Olechowski, Pokorski, hep-th/0411066):

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- Dilaton and 3 Kähler moduli and one complex structure modulus: YES!
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Moduli stabilization in orientifolds

Problems at the orbifold points:

- In many orientifold models, the $D7$-branes wrap divisors of the form $D_i = T^2 \times T^2$ in the covering space of orbifold. These divisors have $\chi = 0$ and do not contribute to W_{np}.
  2 possible ways out: lift zero modes by fluxes or use fractional D7-branes which wrap shorter 4-cycles $\tilde{D}_i = P^1 \times P^1$.
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Go to smooth Calabi-Yau with resolved singularites!

($Z_2 \times Z_2$: F. Denef, M. Douglas, B. Florea, A. Grassi, S. Kachru, hep-th/0503124; all orbifolds: S. Reffert, E. Scheidegger, hep-th/0512287; D. Lüst, S. Reffert, E. Scheidegger, W. Schulgin, S. Stieberger, to appear)

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- Perform the orientifold projection on the smooth CY $\Rightarrow$ O-planes and D-branes.
- Determine the divisor topologies to decide whether a non-pert. superpotential is generated.
- Determine the Kähler potential from the triple intersection ring of X:

$$
\begin{aligned}
K &= -\log(S+\bar{S}) - \log\int \Omega\wedge\bar{\Omega} - 2\log V \\
V &= \frac{1}{6}\int J\wedge J\wedge J = \frac{1}{6}D_{ijk}T^iT^jT^k\,.
\end{aligned}
$$

Moduli stabilization in orientifolds

Results for the blown-up orientifolds:

- The Kähler potential for the Kähler moduli can be computed for all models. E.g. Z_{6-II}-orientifold ($h^{(1,1)}_{un.tw.} = 3$, $h^{(1,1)}_{tw.} = 22$, $h^{(2,1)}_{un.tw.} = 1$, $h^{(2,1)}_{tw.} = 0$):

$$\begin{aligned} V &= 3r_1r_2r_3 + r_3\sum_{\beta=1}^{3} t_{2,\beta}t_{4,\beta} - \frac{1}{2}r_2\sum_{\gamma=1}^{4} t^2_{3,\gamma} - r_3\sum_{\beta}(2t^2_{2,\beta} + \frac{1}{2}t^2_{4,\beta}) \\ &+ \frac{1}{2}\sum_{\beta\gamma} t^3_{1,\beta\gamma} - 2\sum_{\beta} t^2_{2,\beta}t_{4,\beta} + \frac{16}{3}\sum_{\beta} t^3_{2,\beta} + \frac{2}{3}(\sum_{\beta} t^3_{4,\beta} + \sum_{\gamma} t^3_{3,\gamma}) \\ &- \sum_{\beta\gamma}(2t_{1,\beta\gamma}t^2_{2,\beta} + \frac{1}{2}t_{1,\beta\gamma}t^2_{3,\gamma}\frac{1}{2} + t_{1,\beta\gamma}t^2_{4,\gamma}) \end{aligned}$$

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- The Kähler potential for the complex moduli can be computed for models with $h^{(2,1)}_{tw.} = 0$.
- The non-pert. superpotential for the Kähler moduli can be computed for all models.

Moduli stabilization in orientifolds

Result from analyzing the scalar potential:

Candidate models with stable, supersymmetric AdS minima: Z_4, Z_{6-II}, $Z_2 \times Z_2$ $Z_2 \times Z_4$ orientifolds! (Stable minima are excluded for orientifolds with no complex structure moduli).

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Alternative schemes to KKLT:

- Uplift from AdS via D-terms $\Rightarrow$ Stability analysis has to be refined.

  (Burgess, Kallosh, Quevedo, hep-th/0309187; Villadoro, Zwirner, hep-th/0508167; Achucarro, de Carlos, Casas, Doplicher, hep-th/0601190)

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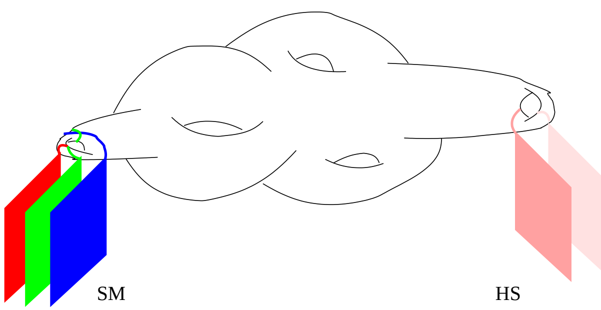
- Supersymmetric Minkowski vacua with $W_{vac} = 0$ and $D_W = 0 \Rightarrow$ These are automatically stable vacua; however one needs more complicate race-track superpotentials that may not lift all flat directions.

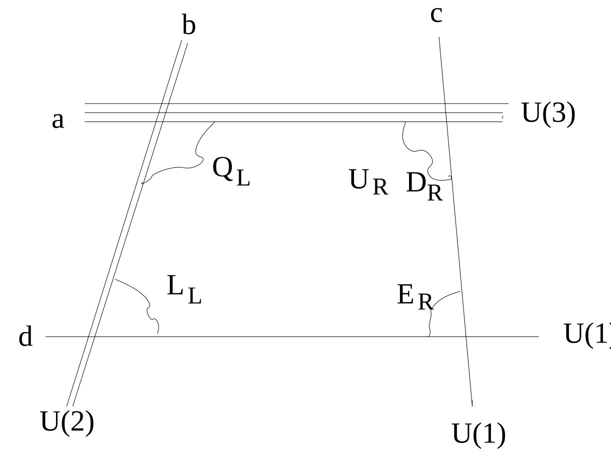
  J. Blanco-Pillado, R. Kallosh, A. Linde, hep-th/0511042; D. Krefl, D. Lüst, hep-th/0603211))

Statistics of D-brane vacua

Now consider the open strings on the D-branes (D7-branes with F-flux in IIB resp. D6-branes at angles in IIA) on top of flux vacua:

- Possibilty of getting the spectrum of the MSSM. Standard Model D-brane quiver:





- Computation of soft masses. (D. Lüst, S. Reffert, S. Stieberger)

Now one has to solve the supersymmetry condition for D-branes (vanishing D-terms) plus the constraints from RR tadpoles and K-theory!

Statistics of D-brane vacua

Statistical survey of $1.66 \cdot 10^8$ susy D-brane models on $Z_2 \times Z_2$ orientifold. (R. Blumenhagen, F. Gmeiner, G. Honecker, D. Lüst, T. Weigand, hep-th/0411173,0510170; see also T. Dijkstra, L. Huiszoon, A. Schellekens)

Restriction	Factor
gauge factor $U(3)$	0.0816
gauge factor $U(2)/Sp(2)$	0.992
No symmetric representations	0.839
Massless $U(1)_Y$	0.423
Three generations of quarks	2.92×10^{-5}
Three generations of leptons	1.62×10^{-3}
Total	1.3×10^{-9}

Only one in a billion models give rise to an MSSM like four stack D-brane vacuum.

Probability distribution of flux vacua

(G.L. Cardoso, D. Lüst, J. Perz, hep-th/0603211; related work: Gukov, Saraikin, Vafa, hep-th/0509109; Fiol, hep-th/0602103; Belluci, Ferrra, Marrani, hep-th/0602161)

Is it possible to assign a probability distribution $|\psi|^2$ to the string vacua to decide which of the many vacua is the most likely one?

Probability of the MSSM?

Consider $\mathcal{N} = 2$ closed string flux vacua without D-branes at particular points in their moduli spaces, where additional states become massless:

- Additional vector multiplets ($\beta > 0$): Asymptotic freedom!
- Additional hypermultiplets ($\beta < 0$): Infrared freedom!

Which of the two possibilties is more likely?

Probability distribution of flux vacua

Ooguri, Vafa, Verlinde (hep-th/0502211):

Instead of 3-form fluxes consider IIB on $S^2 \times CY$ with Ramond 5-form fluxes through $S^2 \times \Sigma_3$ $(\Sigma_3 \subset CY) \Longrightarrow$ Superpotential:

$$W = \int_{S^2 \times CY} (F_5 \wedge \Omega) \,, \qquad F_5 = \omega \wedge F_3 \,.$$

SUSY conditions: $D_A W = 0 \quad \Longrightarrow \quad AdS_2$ vacua!

$$\begin{aligned} p^I &= \mathrm{Re}(C X^I),\ X^I = \int_{A_I} \Omega,\ p^I = \int_{S^2 \times A_I} F_5,\ I = 0, \ldots, h^{(2,1)} \\ q_I &= \mathrm{Re}(C F_I),\ F_I = \int_{B^I} \Omega,\ q_I = \int_{S^2 \times B^I} F_5 \end{aligned}$$

Complex structure moduli: $\quad z^A = X^A / X^0$

$(A = 1, \ldots, h^{(2,1)})$.

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These are just the attractor equations of $\mathcal{N} = 2$ SUSY black holes!

$\mathcal{N} = 2$ black hole	$\mathcal{N} = 1$ landscape
D3-branes wrapped around Σ_3	F_5 through $S^2 \times \Sigma_3$
Black hole charges (q_I, p^I)	5-form fluxes (q_I, p^I)
Central charge $Z(z)$	Superpotential $W(z)$
Stabilization cond. $D_A Z = 0$	Supersymmetry cond. $D_A W = 0$
Entropy $\mathcal{S}$	Cosmological constant $\|V_0\|$
Near horizon geometry $AdS_2 \times S^2$	Vacuum space $AdS_2 \times S^2$

Probability wave function: $|\psi_{p,q}(z, \bar{z})|^2 = e^{\mathcal{S}_{p,q}(z,\bar{z})}$.

The attractor eqs. $p^I = \mathrm{Re}(Y^I)$ and $q_I = \mathrm{Re}(F_I(Y))$ provide a non-unique map between fluxes/charges and moduli $z^A = Y^A/Y^0$.

This non-uniqueness can be resolved by fixing one pair of (p, q)-charges, i.e. by fixing Y^0.

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Entropy at the two-derivative level:

$$\mathcal{S} = \pi\, i \left(\bar{Y}^I \, F_I^{(0)}(Y) - Y^I \, \bar{F}_I^{(0)}(\bar{Y}) \right) = \pi \, |Y^0|^2 \, e^{-G(z,\bar{z})} \, .$$

One modulus example with a singularity at $V \to 0$ (conifold):

$$F^{(0)}(V) = -i(Y^0)^2 (\frac{\beta}{2\pi} \, V^2 \log V + a) \, , \quad V = -iz^1 = -iY^1/Y^0$$

$$e^{-G(V,\bar{V})} = 4 \, \mathrm{Re}\, a - \frac{\beta}{2\pi} (V + \bar{V})^2 - \frac{2\beta}{\pi} |V|^2 \log |V| \, .$$

It follows that $\mathrm{Re}\, a > 0$ (large black holes).

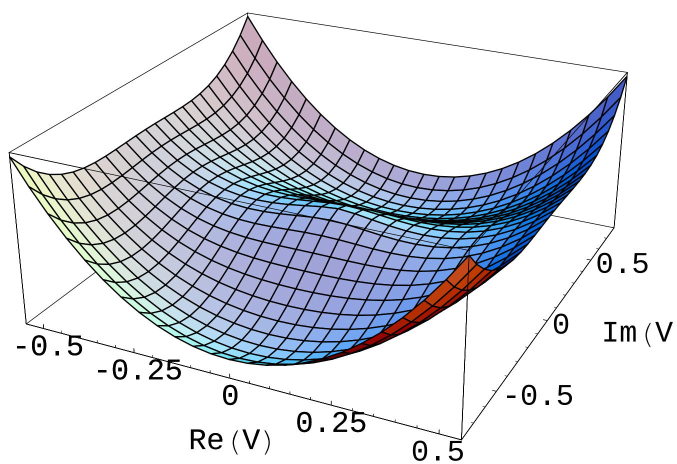
Probability distribution of flux vacua

Positivity of the Kähler metric:

$$g_{V\bar{V}} \approx \frac{\beta}{\pi} e^{G_0} \log|V|^2 > 0 \quad \beta < 0\,.$$

Together with the gauge coupling constant $g^{-2} \approx \frac{\beta}{4\pi} \log|V|^2$. we see that the corresponding gauge theory is infrared free $(\beta = (n_V - n_H)/2)$!

From this we finally get that the entropy exhibits a maximum at $V = 0$ (in agreement with Fiol):



Probability distribution of flux vacua

Entropy with higher curvature interactions:

(Cardoso, de Wit, Mohaupt (1998))

$$\mathcal{S} = \pi\Big(i\left(\bar{Y}^I\, F_I(Y,\Upsilon) - Y^I\, \bar{F}_I(\bar{Y},\bar{\Upsilon})\right) + 4\,\mathrm{Im}\,(\Upsilon\, F_\Upsilon)\Big),\ (\Upsilon = -64)$$

Perturbative expansion:

$$F(Y,\Upsilon) = \sum_{g=0}^{\infty} F^{(g)}(Y)\,\Upsilon^g\,.$$

$F^{(1)}$ for one modulus example near $V = 0$:

$$F^{(1)} \approx -\frac{i}{64\cdot 12\pi}\beta\,\log V$$

Maximum of entropy gets enhanced by Wald term!

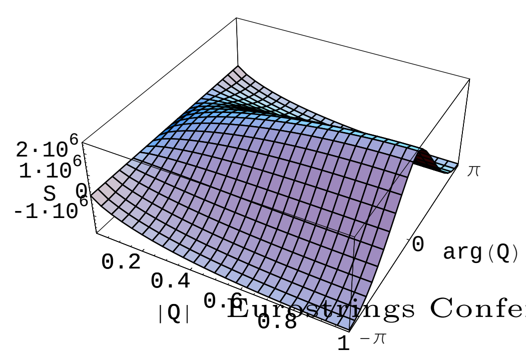
Probability distribution of flux vacua

The higher $F^{(g)}$ are divergent with alternating coefficients:
$F^{(g)}(Y) = i\frac{A_g}{(Y^0)^{2g-2}\, z^{2g-2}}$ $(g > 1)$.

However the full non-perturbative prepotential is known from the topological partition function: (Gopakumar, Vafa (1998))

$$F(Y, \Upsilon) = \frac{i\Upsilon}{128\pi} F_{\mathrm{top}} = \frac{i\Upsilon}{128\pi} \sum_{n=1}^{\infty} n \log\left(1 - q^n Q\right),$$

where $g_{\mathrm{top}}^2 = -\frac{\pi^2 \Upsilon}{16 (Y^0)^2}$, $q = e^{-g_{\mathrm{top}}}$, $Q = e^{-2\pi V}$.
For real g_{top} the entropy has again a maximum:



Probability distribution of flux vacua

OSV Free energy:

Entropy as a Legendre transformation:

$$\mathcal{S} = E - L\,,$$

Free energy

$$E = 4\pi \operatorname{Im} F, \quad Z_{\rm top} = e^{-F_{\rm top}}\,,$$

and where L is given by

$$L = \pi\, q_I\, \phi^I = 4\pi \operatorname{Im} F_I \operatorname{Re} Y^I$$

At the conifold point the entropy is maximized, however the free energy has a local minimum!

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In conclusion: following the entropic principle infrared free theories seem to be preferred!

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Thank You!