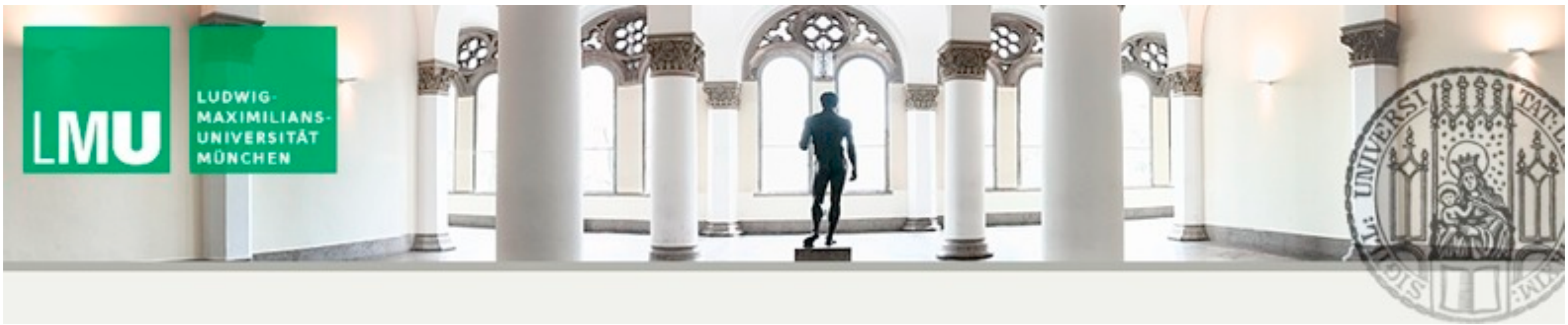
Classical and Quantum Black Hole Hair

DIETER LÜST (LMU-München, MPI)



Latin-American Conference, Habana, July 20th, 2016

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Work in collaboration with

Gia Dvali, Cesar Gomez, arXiv:1509.02114

Artem Averin, Gia Dvali, Cesar Gomez, arXiv:1601.03725, 1606.06260

Latin-American Conference, Habana, July 20th, 2016

Outline:

I) Introduction:

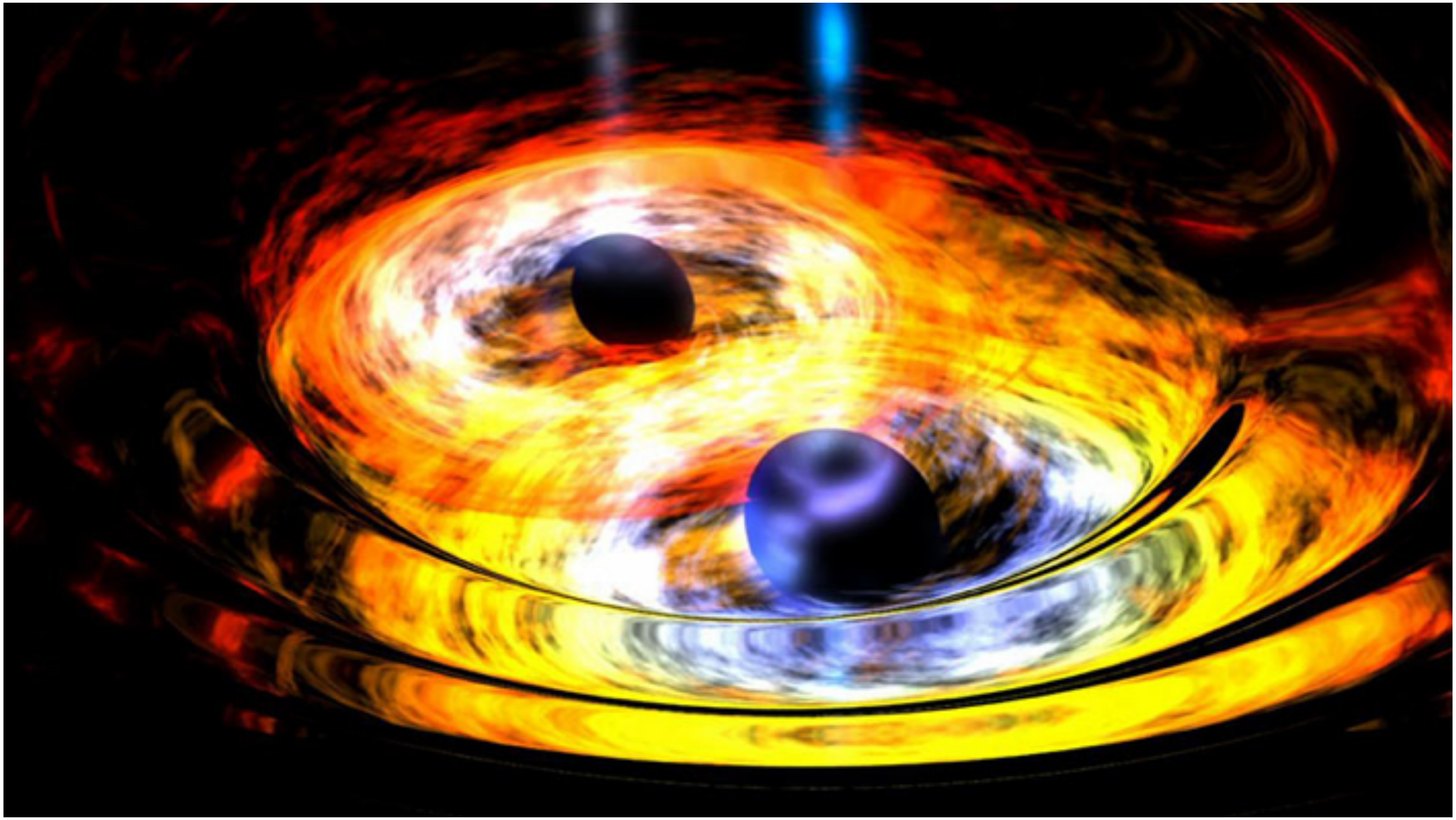
II) Classical Gravitational Black Hole Hair from Event Horizon Supertranslations

III) Quantum Hair & Charges from Black Holes as Graviton Boundstate (Coherent State Picture)

IV) Summary & Outlook

aLIGO / GW150914:

- Merging of binary black holes
- Discovery of gravitational waves



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Quantum mechanically: one expects quantum hair.

Today: we will consider purely gravitational hair, as it is relevant for the Schwarzschild black hole.

Infinite classical hair: degeneracy of the black hole metrics:

Non-uniqueness of the classical black hole metric under supertranslations:

$\mathcal{A}$ - supertranslations: $\delta_{\mathcal{A}}\, g_{\mu\nu}^{BH} = \tilde{g}_{\mu\nu}^{BH}$

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The $\mathcal{A}$ - supertranslation group is spontaneously broken.

Infinitely many massless Goldstone modes

$\Rightarrow$ infinite classical black hole entropy:

$\mathcal{S}_{class} = \infty$ $\qquad$ $\mathcal{Q}_{class}^{\mathcal{A}} = 0$

Finite quantum hair: non-uniqueness of finitely many almost degenerate black hole vacua:

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- Finitely many charges which correspond to finitely many pseudo Goldstone modes:

$\mathcal{Q}^{\mathcal{A}}_{q.m.} \neq 0 \ , \quad \mathcal{S}_{q.m.} = N \quad \Rightarrow$ finite quantum hair.

II) Gravitational Black Hole Hair from Event Horizon Supertranslations

[G. Dvali, C. Gomez, D.L. arXiv.1509.02114;
A. Averin, G. Dvali, C. Gomez, D.L., arXiv:1601.03725;
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Schwarzschild radius: $r_S \equiv 2G_N M$

Planck length: $L_P^2 \equiv \hbar G_N$

Entropy: $\mathcal{S} \sim N = \frac{r_S^2}{L_P^2}$

Classical limit:

$$\hbar \to 0\,, M = \text{finite},\;\; r_S = \text{finite}$$

Semiclassical limit:

$$\hbar = \text{finite},\;\; r_S = \text{finite},\;\; G_N \to 0,\;\; M \to \infty$$

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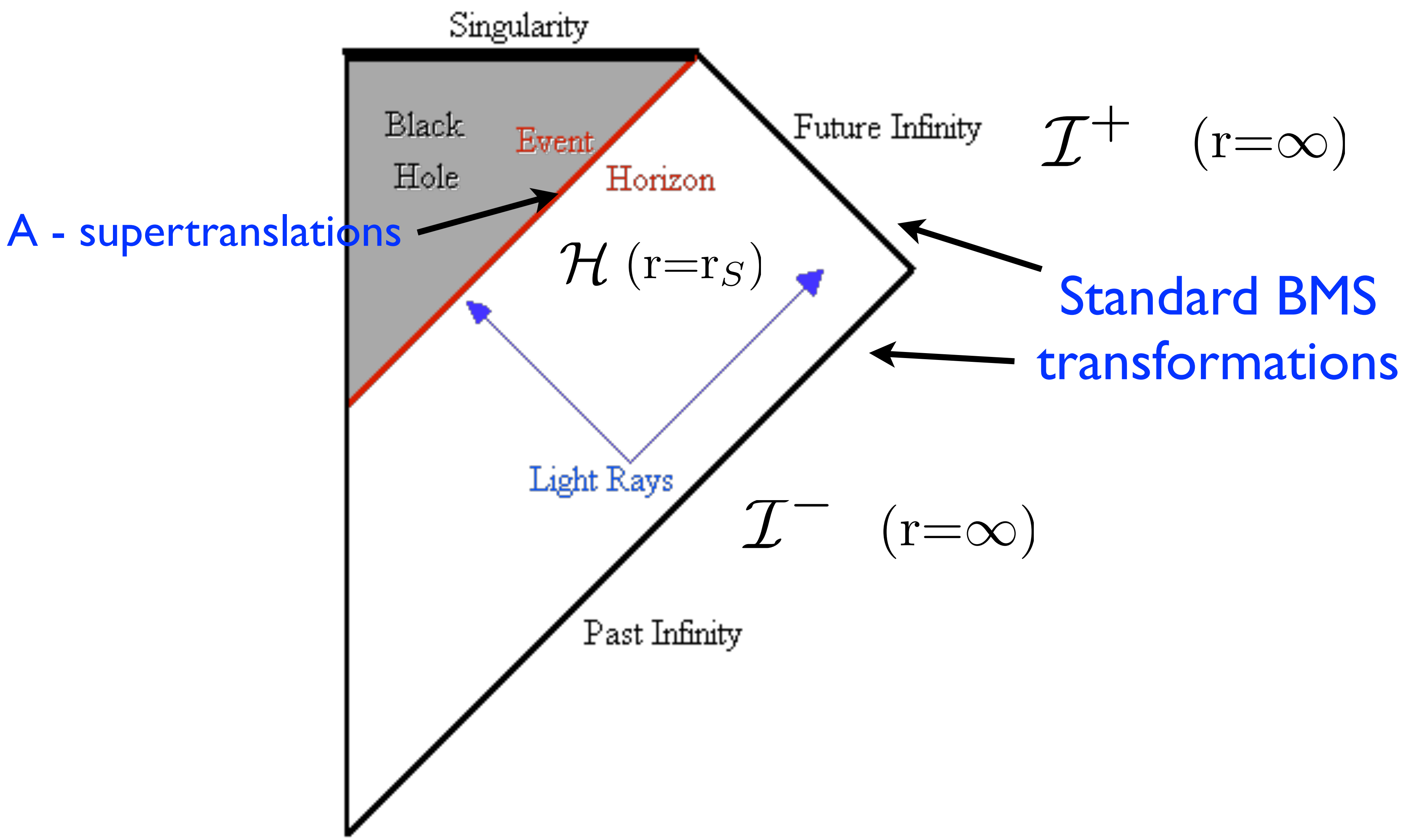
$\Big\}\ N \to \infty$

Consider Penrose diagramme for Schwarzschild metric:



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Diffeomorphisms that keep the boundaries (= null surfaces) of certain space-time metrics invariant.

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Bulk: gauge redundancies, no physical charges

Boundary: not gauge redundancies, but correspond to asymptotic symmetries, like large gauge transformations at infinity, which provide physical charges.

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These are the standard BMS - transformations.

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They even act non-trivially on Minkowski space-time:

Infinite family of flat Minkowski metrics.

We will instead consider supertranslations at $\mathcal{I}^-$ and at the black hole horizon $\mathcal{H}$.

Schwarzschild metric in (infalling) Edington - Finkelstein coordinates:

$$ds^2 = -(1 - \frac{r_S}{r})dv^2 + 2dvdr + r^2 d\Omega^2$$

$$v = t + r^* \qquad dr^* = (1 - \frac{r_S}{r})^{-1} dr$$

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Supertranslations:

Diffeomorphisms that correspond to vector fields

$$\zeta^\mu = (f, A, B, C)$$

with $\quad \delta_f g_{\mu\nu} = \tilde{g}_{\mu\nu}$

Supertranslations at the horizon $\mathcal{H}$

[see also: S. Hawking, arXiv:1509.01147; M. Perry, talk 2015;
S. Hawking, M. Perry, A. Strominger, arXiv:1601.00921;
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Boundary conditions:

The supertranslations should preserve the structure of the metric at the horizon:

$$f|_{r=r_S} = f(\theta,\phi)\,,$$

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Furthermore we require that $\delta_f g_{r\mu} = 0$

$\mathcal{A}$ - supertranslations: microstates of black hole:

Compare supertranslations on $\mathcal{I}^-$ and on $\mathcal{H}$:

Horizon supertranslations contain a part which cannot be compensated by standard BMS-supertranslations.

Belong to the factor space: $\mathcal{A} \equiv BMS^{\mathcal{H}} / BMS^{-}$:

$$\chi^{\mu}_{f} = (f, 0, -\frac{1}{r_S}\frac{\partial f}{\partial \vartheta}, -\frac{1}{r_S \sin^2\vartheta}\frac{\partial f}{\partial \varphi})$$

These transformations are intrinsically due to the presence of the horizon.

Infinitely many classical black hole metrics:

Massless modes of the black hole:

$$\mathcal{A}: \quad \delta_{\chi_f} g_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2r^2 \frac{1}{r_S} \frac{\partial^2 f}{\partial \vartheta^2} & -2r^2 \frac{1}{r_S} (\frac{\partial^2 f}{\partial \vartheta \partial \varphi} - \cot \vartheta \frac{\partial f}{\partial \varphi}) \\ 0 & 0 & * & -2r^2 \frac{1}{r_S} (\frac{\partial^2 f}{\partial \varphi^2} + \sin \theta \cos \vartheta \frac{\partial f}{\partial \vartheta}) \end{pmatrix}$$

We call them gapless Bogoliubov $\mathcal{A}$ - modes.

They correspond to gravitational waves with finite wave length of order r_S , but zero energy (gap).

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- They keep the ADM mass of the black hole invariant .
- They are solutions of the full non-linear Einstein equations on the horizon.

(A. Gußmann)

Hence the $\mathcal{A}$ - modes should be identified with the microstates of the black hole $\Rightarrow$ Classical Hair

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$$\mathcal{Q}_{class}^{\mathcal{A}} \sim \int d^3x \; g^{\mu\nu} \partial_v (\delta_{\chi_f} g_{\mu\nu}) = 0$$

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Infinite entropy in (semi) classical limit: $\mathcal{S}_{class} = \infty$

However it needs infinite time to resolve the information. Information is not accessible.

III) Quantum Hair & Charges from Black Holes as Graviton Boundstate (Coherent State Picture)

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How does the quantum hair become finite?

How many angular Bogoliubov modes must be counted as information carriers?

Need microscopic quantum picture for black hole:

Black hole: bound state (Bose Einstein Condensate) of N soft gravitons at a quantum critical point.

[G. Dvali, C. Gomez, 2011, G. Dvali, C. Gomez, D.L. (2012)]

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$\Rightarrow$

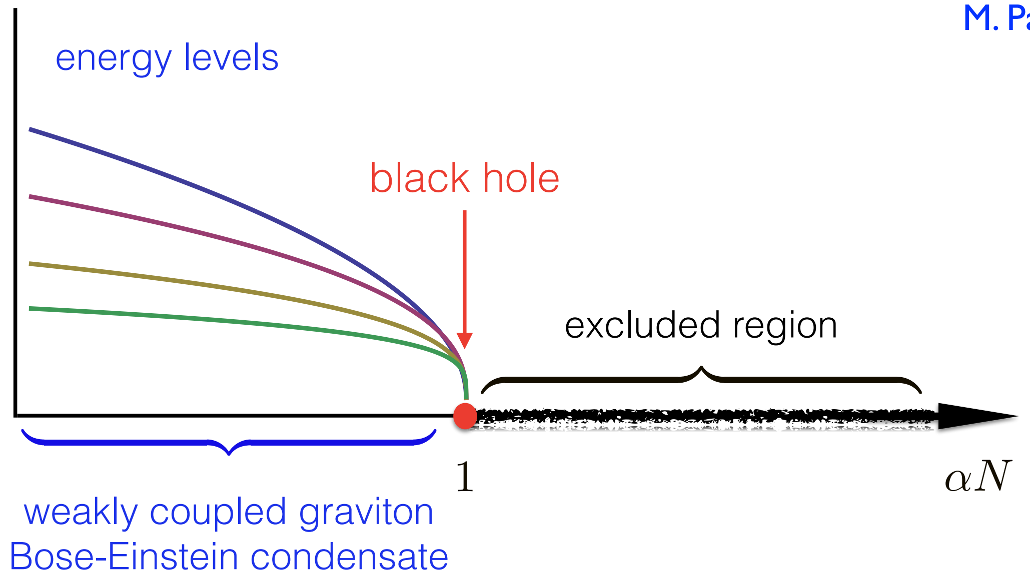
$\mathcal{A}$ - supertranslations are explicitly broken.

$\mathcal{A}$ - modes are collective, finite mass excitations of gravitons:

Bogoliubov excitations = pseudo Goldstone bosons.

Bogoliubov modes for black holes as BEC:

[D. Flassig, A. Pritzel, N. Wintergerst, arXiv:1212.3344; M. Panchenko, arXiv1510.04535]



Quantum critical point at leading order in N:

$$\epsilon = \sqrt{1 - \alpha N} \longrightarrow 0$$

$$\alpha \equiv \frac{L_P^2}{R^2} \rightarrow \frac{1}{N}$$

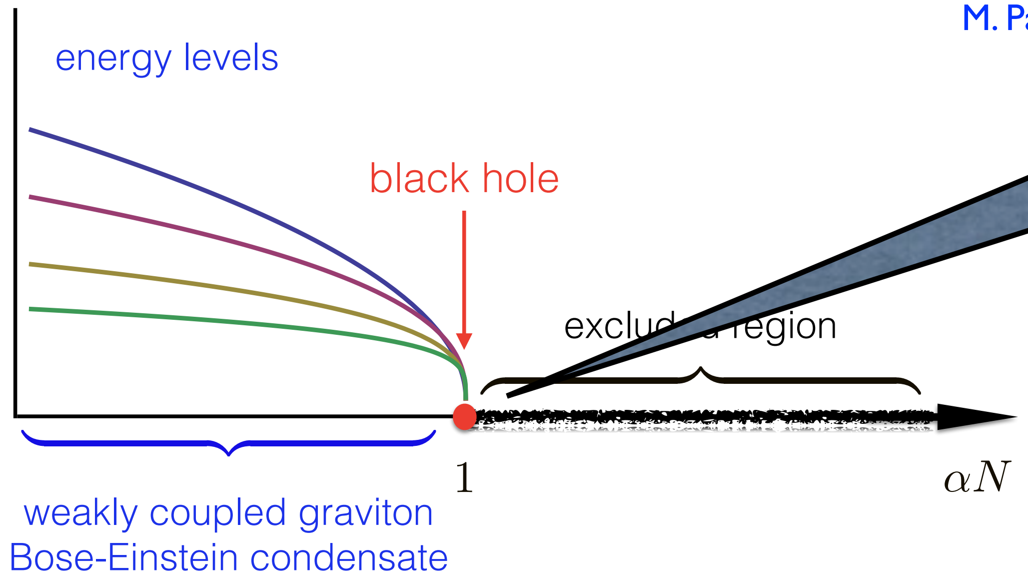
For finite N subleading corrections: finite energy gap:

$$\omega_{lm} = \Delta E = \frac{1}{N} \frac{\hbar}{r_S}$$

Bogoliubov modes for black holes as BEC:

[D. Flassig, A. Pritz ... 344;
M. Pan

Bogoliubov modes become (almost) gapless, degeneracy of states



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In quantum theory the $\mathcal{A}$ - modes correspond to operators of finite wave length gravitons:

$$\delta_{\chi_f}\hat{g}_{\mu\nu}(v,\vartheta,\varphi) = \sum_{l,m}\left(\hat{b}^{\mu\nu}_{lm} Y_{lm}(\vartheta,\varphi) e^{-iv\omega_{lm}} + \hat{b}^{\dagger,\mu\nu}_{lm} Y^{*}_{lm}(\vartheta,\varphi) e^{iv\omega_{lm}}\right)$$

Effective action on the horizon:

$$S_{eff} \sim \int dv d\vartheta d\varphi \left(\partial_v\left(\delta_{\chi_f}\hat{g}_{\mu\nu}(v,\vartheta,\varphi)\right)\right)^2$$

Charges:

$$\hat{Q}^{\mathcal{A}}_{lm} \sim \int g^{\mu\nu}\partial_v\left(\delta_{\chi_f}\hat{g}_{\mu\nu}(v,\vartheta,\varphi)\right) = -i\sqrt{\hbar\omega_{lm}}\left(e^{-i\omega_{lm}v}\hat{b}_{lm} - e^{i\omega_{lm}v}\hat{b}^{\dagger}_{lm}\right)$$

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Black hole vacua: coherent state of Bogoliubov modes:

$$|BH\rangle \equiv |N\rangle \sim e^{-\sum_{lm}\sqrt{n_{lm}}(\hat{b}_{lm}-\hat{b}^\dagger_{lm})}|0\rangle$$

Now it follows that

$$Q^A_{lm} = \langle N|\hat{Q}^A_{lm}|N\rangle \sim \sqrt{\hbar\omega_{lm} n_{lm}} \sim \frac{1}{N}$$

For finite N, the charges are non-vanishing, since the energy gap is non-zero:

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How many charges are there?

The energy gap can be estimated as $\Delta E \sim \frac{l^2}{N^2}\frac{\hbar}{r_S}$.

$$\Rightarrow \quad l_{max} \sim \sqrt{N}$$

$-l \leq m \leq l \quad \Rightarrow$ There exist $l^2 = N$ different charges.

These charges should be preserved during the Hawking radiation.

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There exist $l^2 = N$ different Boguliubov modes b_{lm} .

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$\Rightarrow$ The number of states is 2^N .

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Black hole qubits can be used as universal quantum computer. [G. Dvali, C. Gomez, D.L., Y. Omer, B. Richter, 2016]

IV) Summary & Outlook

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Our proposal:

BMS like $\mathcal{A}$ - supertranslations provide a gravitational hair for the black hole:

classical hair: gapless metric deformations provided by $\mathcal{A}$- supertranslations - gravitational waves of finite wave length along the horizon.

quantum hair: almost gapless Bogoliubov modes of BEC graviton condensate - gravitons of finite wave length and finite energy at the horizon.

- Note that Minkowski space can be regarded as the near horizon limit of the Schwarzschild geometry, obtained in the limit $r_S \to \infty$.

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In quantum language it means that the corresponding Minkowski vacua are infinitely degenerate and can be regarded as coherent state of infinitely many gravitons with zero momentum:

[G. Dvali, C. Gomez, D.L., arXiv:1509.02114]

$$\mathcal{T}^{\mathcal{BMS}^-} |Min\rangle = |\widetilde{Min}\rangle$$

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- How are the different black hole vacua and the different $\mathcal{A}$ - charges related to the black hole entropy counting in string theory?

[A. Strominger, C.Vafa (1996)]

Questions:

- How can one measure the $\mathcal{A}$ - charges?
  Is there a kind of gravitational memory effect?

- What is the algebra of the $\mathcal{A}$ - charges?
  - Area preserving diffeomorphisms - membrane?

- How are the different black hole vacua and the different $\mathcal{A}$ - charges related to the black hole entropy counting in string theory?

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Thank you very much !!