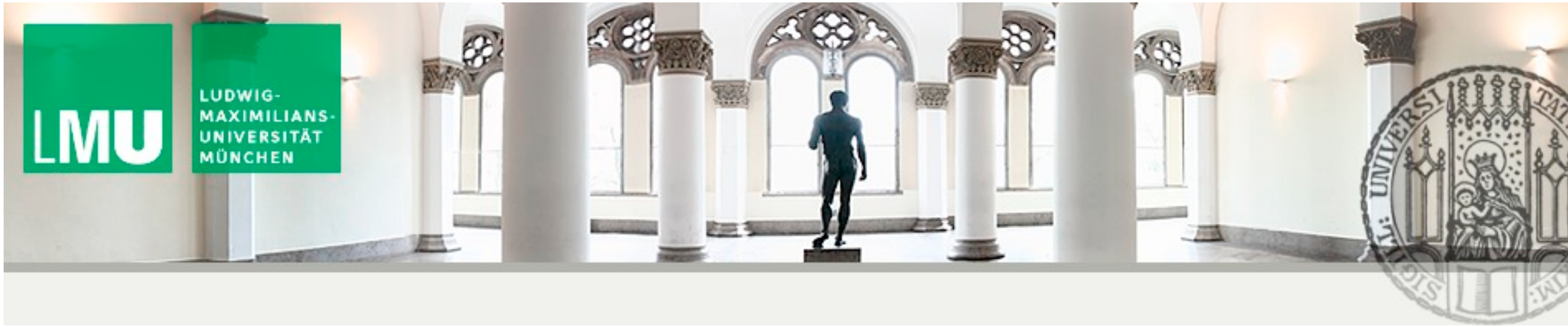

Strings and non-commutative/ non-associative geometry

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MAX-PLANCK-GESELLSCHAFT

EISA
European Institute for Sciences and Their Applications

Outline:

I) Introduction

II) Non-geometric flux compactifications

III) Effective action of non-geometric string backgrounds

D. Andriot, M. Larfors, D.L. P. Patalong, arXiv:1106.4015

D. Andriot, O. Hohm, M. Larfors, D.L. P. Patalong, arXiv:1202.3060, arXiv:1204.1979

IV) Non-commutative/non-associative geometries from non-geometric string backgrounds

R. Blumenhagen, E. Plauschinn, J. Phys A44 (2011), 015401, arXiv:1010.1263
D. Lüst, JEHP 1012 (2011) 063, arXiv:1010.1361; arXiv:1205.0100
R. Blumenhagen, A. Deser, D.Lüst, E. Plauschinn, F. Rennecke, J. Phys A44 (2011), 385401, arXiv:1106.0316
C. Condeescu, I. Florakis, D. Lüst, JHEP 1204 (2012) 121, arXiv:1202.6366
D. Andriot, M. Larfors, D. Lüst, P. Patalong, to appear.

V) Outlook & open problems

I) Introduction

Geometry in general depends on, with what kind of objects you test it.

Point particles in classical Einstein gravity see smooth & continuous manifolds.

➢ Classical & quantum singularities !

Strings may see space-time in a different way.

Shortest possible scale in string theory: L_s

Ways for describing stringy geometry:

- Quantum CY geometry, mirror symmetry (world sheet instantons).

- Non-geometric backgrounds:
  - Asymmetric orbifolds
  - Covariant lattices
  - Fermionic constructions
  - T-folds
  - ......

Here new observation:

Ways for describing stringy geometry:

- Quantum CY geometry, mirror symmetry (world sheet instantons).

- Non-geometric backgrounds: Asymmetric orbifolds
  Covariant lattices
  Fermionic constructions
  T-folds
  ……

Here new observation:

Stringy (non)- geometry: deformed geometry:

- Non-commutative geometry: $[X_i, X_j] \simeq \mathcal{O}(L_s)$
- Non-associative geometry: $[[X_i, X_j], X_k] \simeq \mathcal{O}(L_s)$

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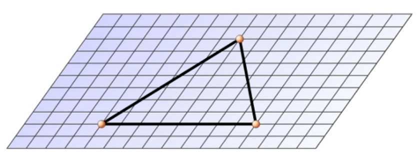
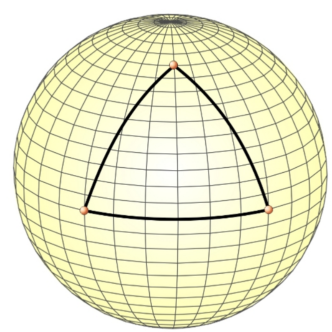
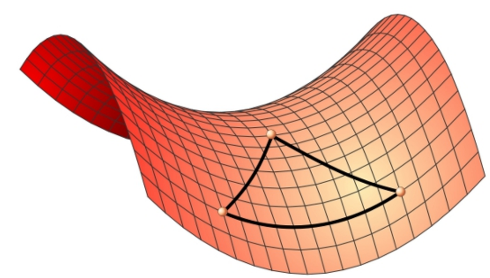
- Flat space: Triangle: $\alpha + \beta + \gamma = \pi$
- Curved space: Triangle: $\alpha + \beta + \gamma > \pi (< \pi)$

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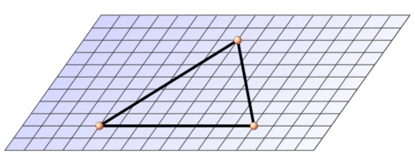

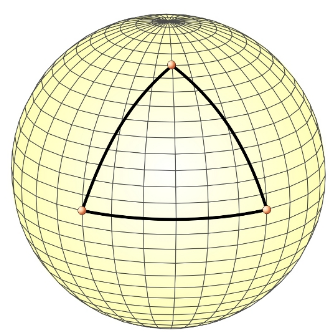

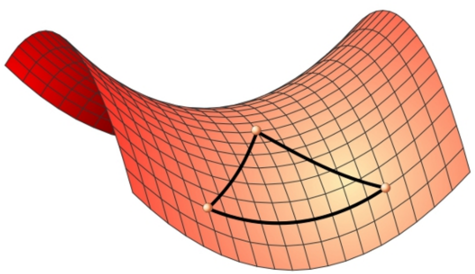

Manifold: need different coordinate charts, which are patched together by coordinates transformations, i.e. group of diffeomorphisms: $\mathrm{Diff}(M): \quad f: U \rightarrow U'$

Properties of Riemannian manifolds:

- distances between two points can be arbitrarily short.
- coordinates commute with each other:

$$[X_i, X_j] = 0$$

This is the situation, if one is using point particles to probe distance and the geometry of space.

Now we want to understand, how extended closed strings may possibly see the (non)-geometry of space.

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- Non-geometric Q-fluxes: spaces that are locally still Riemannian manifolds but not anymore globally.

Transition functions between two coordinate patches are not only diffeomorphisms but also T-duality transformations:

$$\mathrm{Diff}(M) \quad \rightarrow \quad \mathrm{Diff}(M) \times SO(d,d)$$

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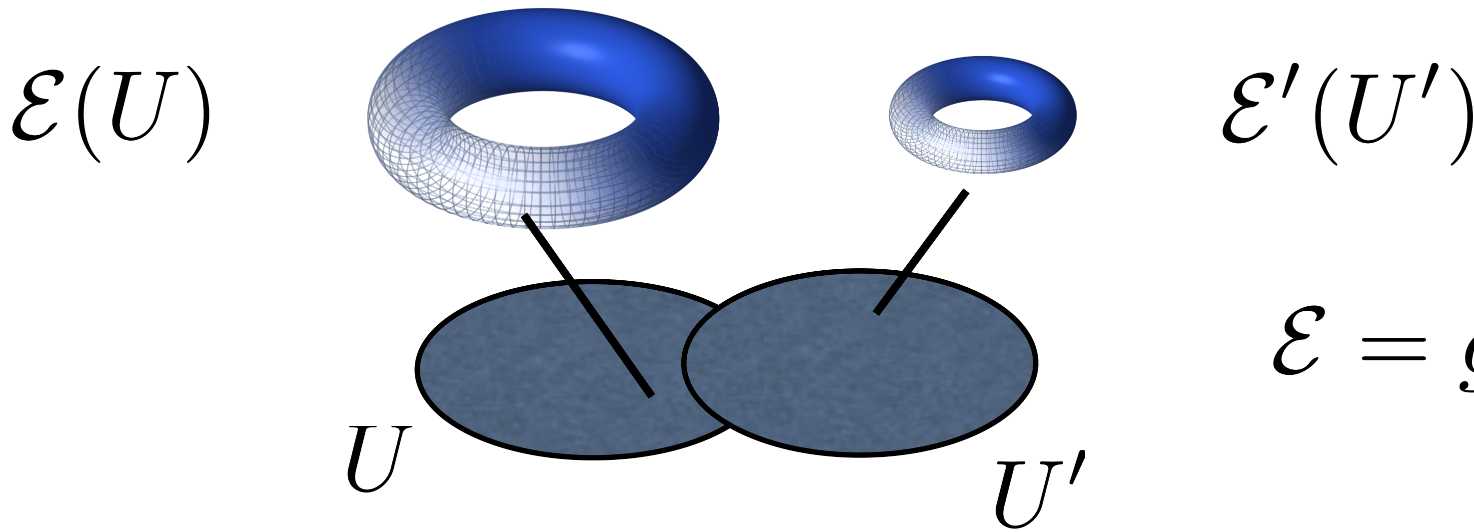
Physics is nevertheless smooth and well-defined!

Q-space T-fold: Patching uses T-duality.

e.g. torus fibrations



$$\mathcal{E} = g + b$$

Geometric background: $\mathcal{E}' = a\mathcal{E}a^t$ in $U \cap U'$, $a \in GL(d, Z)$

Non-geometric background:

$$\mathcal{E}' = \frac{a\mathcal{E} + b}{c\mathcal{E} + d} \text{ in } U \cap U'$$

III) Effective action of non-geometric string backgrounds

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Standard effective action is in general not well-defined for non-geometric backgrounds:

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D. Andriot, M. Larfors, D.L. P. Patalong, arXiv:1106.4015

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Mathematical framework:

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Mathematical framework:

- Doubled field theory: T-duality (field redefinition).

T-duality: $$T: \quad R \longleftrightarrow \frac{\alpha'}{R}, \quad M \longleftrightarrow N$$

$$T: \quad p \longleftrightarrow \tilde{p}\,, \quad p_L \longleftrightarrow p_L\,, \quad p_R \longleftrightarrow -p_R\,.$$

- Dual space coordinates: $\tilde{X}(\tau, \sigma) = X_L - X_R$

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Doubled field theory:

(O. Hohm, C. Hull, B. Zwiebach (2009/10))

- Manifestly O(D,D) invariant string action.
- Coordinates: use O(D,D) vector $X^M = (\tilde{X}_i, X^i)$

- Background: $$\mathcal{H}^{MN} = \begin{pmatrix} g_{ij} - b_{ik}g^{kl}b_{lj} & b_{ik}g^{kj} \\ -g^{ik}b_{kj} & g^{ij} \end{pmatrix}$$

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- O(D,D) invariant DFT action:

$$S_{\rm DFT} \;=\; \int dx d\tilde{x}\, e^{-2d} \,\Big(\, \frac{1}{8}\,\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\,\partial_L\mathcal{H}_{MK} \\ - 2\,\partial_M d\,\partial_N\mathcal{H}^{MN} + 4\mathcal{H}^{MN}\,\partial_M d\,\partial_N d\,\Big)\,.$$

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- O(D,D) transformation (field redefinition):

bi-vector

$$\mathcal{E} = g + b \quad \rightarrow \quad \tilde{\mathcal{E}}^{-1} = \tilde{\mathcal{E}} = g^{-1} + \beta$$

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- O(D,D) transformation (field redefinition):

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- Introduce the following objects (non-geometric fluxes):

Connection: $\boxed{Q_m{}^{nk} = \partial_m \beta^{nk}}$

Field strength (tensor):

$$\boxed{R^{ijk} = 3 \tilde{D}^{[i} \beta^{jk]} , \quad \tilde{D}^i \equiv \tilde{\partial}^i - \beta^{ij} \partial_j}$$

- Bianchi-identities:

$$4\,\tilde{\partial}^{[i} R^{jkl]} + 4\,\beta^{p[i} \partial_p R^{jkl]} + 6\, Q_p{}^{[ij} R^{kl]p} \;=\; 0$$

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- Rewrite DFT action

$$\begin{aligned} S_{\rm DFT}(\tilde{g},\beta,\tilde{\phi}) \;&=\; \int dx d\tilde{x}\,\sqrt{|\tilde{g}|}\,e^{-2\tilde{\phi}}\Big[\mathcal{R}(\tilde{g},\partial) + \mathcal{R}(\tilde{g}^{-1},\tilde{\partial}) \\ &- \;\frac{1}{4}Q^2 - \frac{1}{12}R^{ijk}R_{ijk} + 4\Big((\partial\tilde{\phi})^2 + (\tilde{\partial}\tilde{\phi})^2\Big) + \ldots\Big] \end{aligned}$$

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- Final action („supergravity limit“): $\tilde\partial = 0$

$$\boxed{e^{2d}\mathcal{L}_{\rm final}(\tilde g,\beta,d)(x)=\mathcal{R}(\tilde g)+4(\partial\tilde\phi)^2-\frac{1}{12}R^{ijk}R_{ijk}-\frac{1}{4}\tilde g_{ik}\tilde g_{jl}\tilde g^{rs}\,Q_r{}^{kl}\,Q_s{}^{ij}+\ldots}$$

This action is indeed well-defined for non-geometric fluxes!

IV) Non-commutative/non-associative geometries from non-geometric string backgrounds

Now we want to derive the stringy quantum geometry of non-geometric backgrounds .

$\Rightarrow$ Deformed (NC/NA) string geometry with

Q- reps. R-flux as deformation parameters.

i) Elliptic monodromy: symmetric $\leftrightarrow$ asymmetric orbifold

D. Lüst, JEHP 1012 (2011) 063, arXiv:1010.1361; arXiv:1205.0100

C. Condeescu, I. Florakis, D. Lüst, JHEP 1204 (2012) 121, arXiv:1202.6366.

ii) Parabolic monodromy: T-duality as canonical transformation

A. Andriot, M. Larfors, D. Lüst, P. Patalong, to appear; I. Bakas, D. Lüst, work in progress

iii) CFT amplitude computation

R. Blumenhagen, A. Deser, D.Lüst, E. Plauschinn, F. Rennecke, J. Phys A44 (2011), 385401, arXiv:1106.0316

i) Elliptic = finite order monodromy

ω - background, geometric space

Symmetric (freely acting orbifold): commutative

$\updownarrow$ T-duality

Q-background, non-geometric space

Asymmetric (freely acting orbifold): non-commutative

Reacll: three-dimensional flux backgrounds:

Fibrations: 2-dim. torus that varies over a circle:

$$T^2_{x^1,x^2} \hookrightarrow M^3 \hookrightarrow S^1_{x^3}$$

The fibration is specified by its monodromy properties.

Two T-dual cases:

(i) Geometric spaces (manifolds): geometric ω - flux

complex structure is non-constant:

$$x^3 \to x^3 + 2\pi \quad \Rightarrow \quad \tau(x^3 + 2\pi) = -1/\tau(x^3)$$

(ii) Non-geometric spaces (T-folds): non-geometric Q-flux

size + B-field is non-constant:

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R

Fibrat

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The fibra … es.

Two T-du

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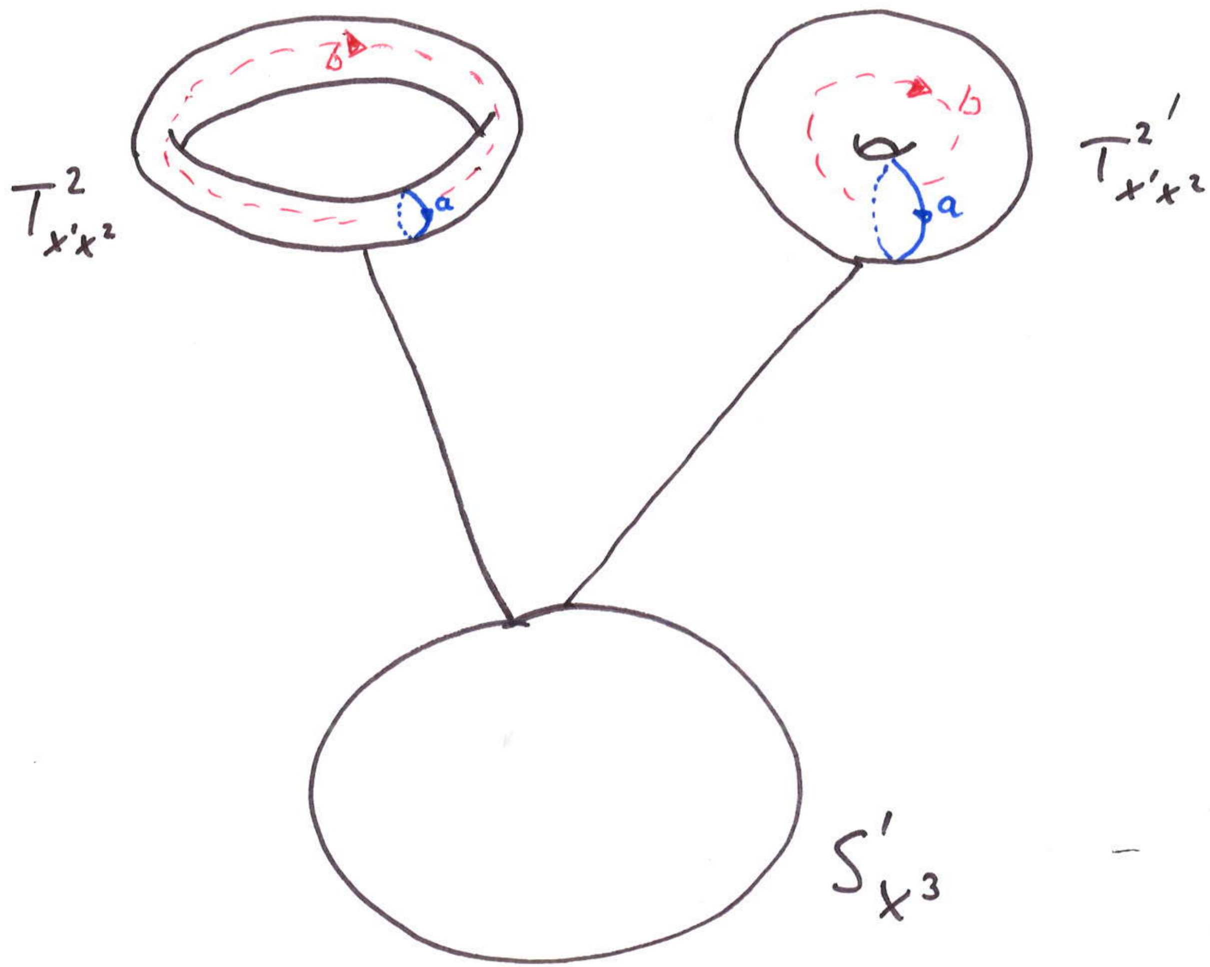
com

$x^3 -$ … (x^3)

(ii) Non- … Q-flux

size

$x^3 \rightarrow$ … (x^3)



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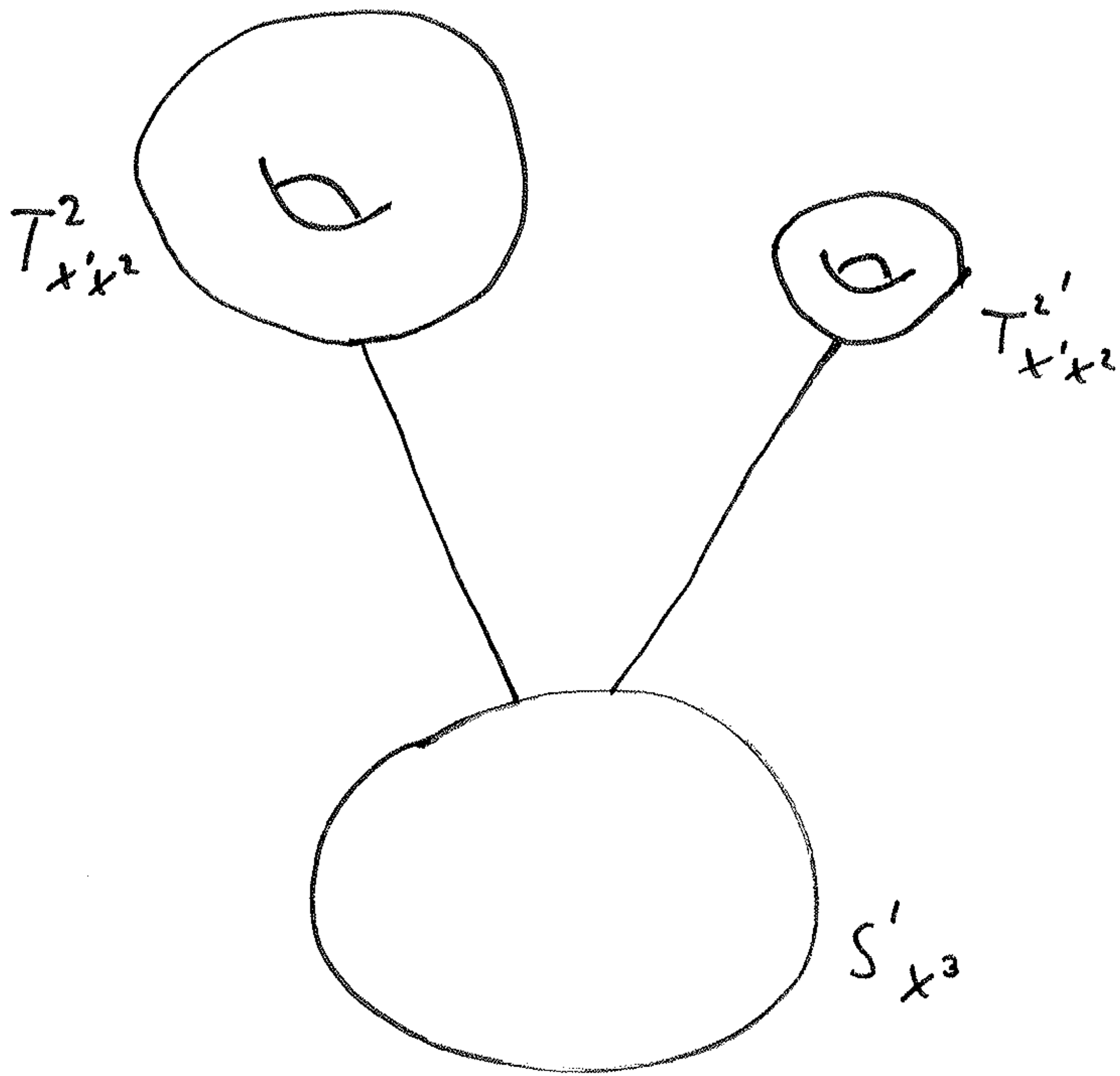

…(x^3)

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Specific example: Z_4-monodromy

D. L., JEHP 1012 (2011) 063, arXiv:1010.1361,
C. Condeescu, I. Florakis, D. L., arXiv:1202.6366

$$X^3(\tau,\sigma+2\pi) = X^3(\tau,\sigma) + 2\pi N_3$$

winding number

$$\omega : \quad \tau(x^3+2\pi) = -1/\tau(x^3)$$

$$Q : \quad \rho(x^3+2\pi) = -1/\rho(x^3)$$

$$X_L(\tau,\sigma+2\pi) \;=\; e^{i\theta} X_L(\tau,\sigma)\,, \quad \theta = -2\pi N_3 H$$

(Complex coordinates: $X_{L,R} = X^1_{L,R} + iX^2_{L,R}$)

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Corresponding closed string mode expansion $\Rightarrow$

$$X_L(\tau+\sigma) = i\sqrt{\frac{\alpha'}{2}} \sum_{n\in\mathbb{Z}} \frac{1}{n-\nu} \alpha_{n-\nu} e^{-i(n-\nu)(\tau+\sigma)}\,, \qquad \nu = \frac{\theta}{2\pi} = -N_3 H$$

(shifted oscillators!)

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Then one obtains:

$$[X_L(\tau,\sigma), \bar{X}_L(\tau,\sigma)] = \Theta\ , \qquad \Theta = \alpha' \sum_{n\in\mathbb{Z}} \frac{1}{n-\nu} = -\alpha'\pi\cot(\pi N_3 H)$$

Right moving torus coordinates:

$$\omega: \quad \tau(x^3+2\pi) = -1/\tau(x^3) \Rightarrow X_R(\tau,\sigma+2\pi) = e^{i\theta} X_R(\tau,\sigma)$$

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ω-flux	$[X^1, X^2] = 0$
Q-flux:	$[X^1, X^2] \simeq i L_s^3\, F^{(3)}\, \tilde{p}^3$

dual momentum (winding) in third direction

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dual momentum (winding) in third direction

Corresponding uncertainty relation:

$$(\Delta X^1)^2 (\Delta X^2)^2 \geq L_s^6 (F^{(3)})^2 \langle \tilde{p}^3 \rangle^2$$

The spatial uncertainty in the X_1, X_2 - directions grows with the dual momentum in the third direction: non-local strings with winding in third direction.

These non-geometric Q-backgrounds with rotated closed string boundary conditions can be realized as freely acting asymmetric orbifolds.

C. Condeescu, I. Florakis, D. L., arXiv:1202.6366

- The model is an exactly solvable CFT
- Partition function:

$$\boxed{Z = \frac{1}{\eta^{12}\,\bar{\eta}^{12}}\; R \sum_{\tilde{m},n\in\mathbb{Z}} e^{-\frac{\pi R^2}{\tau_2}|\tilde{m}+\tau n|^2}\;\; Z_L\big[{}^h_g\big](\tau)\,\tilde{Z}_R(\bar{\tau})\,\Gamma_{(5,5)}\big[{}^h_g\big](\tau,\bar{\tau})}$$

T-duality in x^3-direction $\Rightarrow$ R-flux tau boundary

$$\tilde{p}^3 \longrightarrow p^3$$

$\Rightarrow$ For the case of non-geometric R-fluxes one gets:

$$\boxed{[X^1, X^2] \simeq i L_s^3 \, F^{(3)} \, p^3}$$

Use $[p^3, X^3] = -i$

$$\boxed{\Longrightarrow \quad [[X^1, X^2], X^3] + \text{perm.} \simeq F^{(3)} \, L_s^3}$$

Non-associative algebra!

This nicely agrees with the non-associative closed string structure found by Blumenhagen, Plauschinn in the SU(2) WZW model: arXiv:1010.1263

Twisted Poisson structure (same as for point particle in the field of a magnetic monopole, being related to co-cycles)

ii) Parabolic = infinite order monodromy

Four different 3-dimensional closed string flux backgrounds, which are related by T-duality: (Shelton, Raylor, Wecht, 2005; Dabholkar, Hull, 2005)

Chain of 3 T-dualities:

$$F^{(3)}: \qquad H \underset{T_{x_1}}{\longleftrightarrow} \omega \underset{T_{x_2}}{\longleftrightarrow} Q \underset{T_{x_3}}{\longleftrightarrow} R$$

(not isometry)

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NS H-flux

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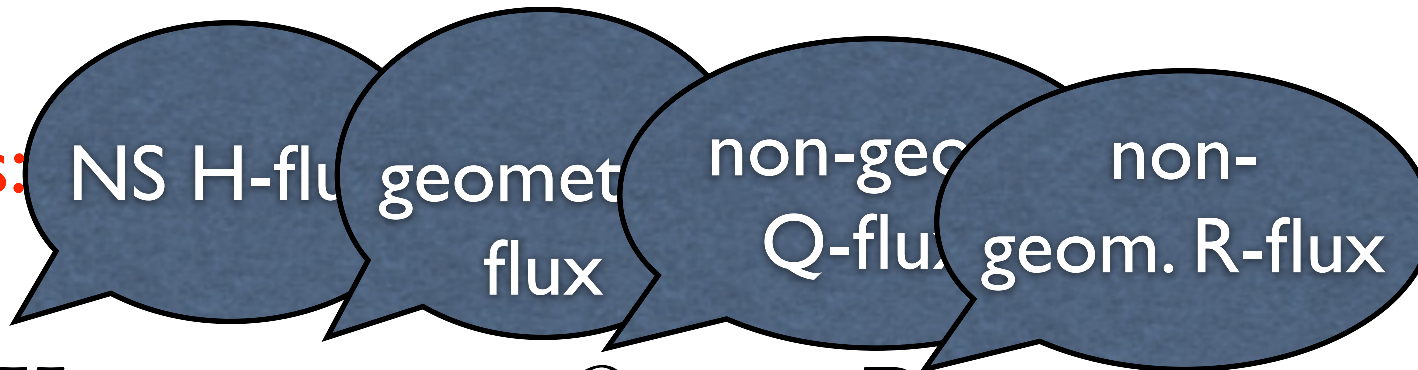

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Non-associative „Space“ with R-flux

$$[[X_i, X_j], X_k] \neq 0$$

Procedure for the quantization of these backgrounds:

1. step: Standard canonical quantization of H and ω - backgrounds

$$[\mathcal{X}^\mu(\tau,\sigma),\mathcal{X}^\nu(\tau,\sigma')] = 0$$

$$[\mathcal{P}_\mu(\tau,\sigma),\mathcal{P}_\nu(\tau,\sigma')] = 0$$

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- Obeying the following closed string boundary (SO(2,2)-monodromy) conditions:

$$\begin{aligned}
Y^1(\tau,\sigma+2\pi) &= Y^1(\tau,\sigma)+2\pi N^3_Y H Y^2(\tau,\sigma)\,,\\
Y^2(\tau,\sigma+2\pi) &= Y^2(\tau,\sigma)\,,\\
\tilde{Y}^1(\tau,\sigma+2\pi) &= \tilde{Y}^1(\tau,\sigma)\,,\\
\tilde{Y}^2(\tau,\sigma+2\pi) &= \tilde{Y}^2(\tau,\sigma)+H N^3_Y \tilde{Y}^1(\tau,\sigma)\,;\\
Y^3(\tau,\sigma+2\pi) &= Y^3(\tau,\sigma)+2\pi N^3_Y\ .
\end{aligned}$$

2. step: T-duality as canonical (Buscher) transformation:

(E. Alvarez, L. Alvarez-Gaume, Y. Lozano, 1994;
I. Bakas, K. Sfetsos, 1995)

$H \leftrightarrow \omega$: T-d. along $\iota = 1$

$$\left.\begin{aligned} \partial_\tau X^1 &= \partial_\sigma Y^1 - HY^3\partial_\sigma Y^2 \\ \partial_\sigma X^1 &= \partial_\tau Y^1 - HY^3\partial_\tau Y^2 \\ \partial_\tau X^{2,3} &= \partial_\tau Y^{2,3} \\ \partial_\sigma X^{2,3} &= \partial_\sigma Y^{2,3} \end{aligned}\right| \quad \underset{\text{(all orders)}}{\Longleftrightarrow} \quad \left|\begin{aligned} \partial_\tau Y^1 &= \partial_\sigma X^1 + HX^3\partial_\tau X^2 \\ \partial_\sigma Y^1 &= \partial_\tau X^1 + HX^3\partial_\sigma X^2 \\ \partial_\tau Y^{2,3} &= \partial_\tau X^{2,3} \\ \partial_\sigma Y^{2,3} &= \partial_\sigma X^{2,3} \end{aligned}\right.$$

$\omega \leftrightarrow Q$: T-d. along $\iota = 2$

$$\left.\begin{aligned} \partial_\tau Y^2 &= \partial_\sigma Z^2 + HZ^3\partial_\tau Z^1 \\ \partial_\sigma Y^2 &= \partial_\tau Z^2 + HZ^3\partial_\sigma Z^1 \\ \partial_\tau Y^{1,3} &= \partial_\tau Z^{1,3} \\ \partial_\sigma Y^{1,3} &= \partial_\sigma Z^{1,3} \end{aligned}\right| \quad \underset{\text{(up to } O(H^2))}{\Longleftrightarrow} \quad \left|\begin{aligned} \partial_\tau Z^2 &= \partial_\sigma Y^2 - HY^3\partial_\sigma Y^1 \\ \partial_\sigma Z^2 &= \partial_\tau Y^2 - HY^3\partial_\tau Y^1 \\ \partial_\tau Z^{1,3} &= \partial_\tau Y^{1,3} \\ \partial_\sigma Z^{1,3} &= \partial_\sigma Y^{1,3} \end{aligned}\right.$$

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T-dual SO(2,2)-monodromy conditions:

$$\begin{aligned}Z^1(\tau,\sigma+2\pi) &= Z^1(\tau,\sigma) - 2\pi N_Z^3 H\tilde{Z}^2(\tau,\sigma)\,, \\ Z^2(\tau,\sigma+2\pi) &= Z^2(\tau,\sigma) + 2\pi N_Z^3 H\tilde{Z}^1(\tau,\sigma)\,, \\ \tilde{Z}^1(\tau,\sigma+2\pi) &= \tilde{Z}^1(\tau,\sigma)\,, \\ \tilde{Z}^2(\tau,\sigma+2\pi) &= \tilde{Z}^2(\tau,\sigma)\,; \\ Z^3(\tau,\sigma+2\pi) &= Z^3(\tau,\sigma) + 2\pi N_Z^3\,.\end{aligned}$$

Mix coordinates with dual coordinates.

$\updownarrow$

Non-geometric background.

3. step: Derive (non-canonical) quantization for Q-background:

(consistent with the non-geometrical monodromy conditions)

$$[Z^1(\tau,\sigma), Z^2(\tau,\sigma)] = -\frac{1}{2}\frac{\pi^2}{3} H N^3$$

T-duality does not preserve the canonical commutation relations!

V) Outlook & open questions

- Mixed closed string bound conditions (in analogy to mixed D-N boundary conditions for D-branes) lead to closed string non-commutativity. This is a stringy, non-local effect - Wilson loop operator.

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- What is the generalization of quantum mechanics for this non-associative geometry? How to represent this algebra (octonians?)?