Intersecting Brane World Models

Dieter Lüst

Work in collaboration with K. Behrndt, N. Bernhardt, R. Blumenhagen, V. Braun, G. Dall'Agata, F. Epple, L. Görlich, B. Körs, S. Mahapatra, P. Mayr, T. Ott, R. Richter, S. Stieberger, T. Taylor

Review: hep-th/0401156

Chios, April 2004

I) Introduction

Goal of superstring theory:

Embedding of the Standard Model into a unified description of gravitational and gauge forces.

Obstacles on the way:

- How to derive the precise SM spectrum?
- How to determine the precise SM couplings?
- How to understand the hierarchy of scales: $M_W << M_{\text{Planck}}$?
- How to select the groundstate from an (apparent) huge vacuum degeneracy?
- How to describe the cosmological evolution of the universe?
- What is the structure of space and time at short distances?

Further plan of the talk:

II) Intersecting brane world models:

- Local intersecting D-brane constructions
- The question of space-time supersymmetry
- Compactifications – embedding of intersecting branes into a compact CY-manifold

III) The effective low-energy action

- Effective gauge couplings
- Effective matter couplings
- Gauge coupling unification

IV) Conclusions

II) Intersecting Brane World Models

The progress in type II string physics was made possible due the discovery of D-branes.

(Polchinski)

D(p)-branes are higher(p)-dimensional topological defects, i.e. hypersurfaces, on which open strings are free to move.

They have led to several new insights:

- Non-Abelian gauge bosons as open strings on the world volumes π of the D-branes $\rightarrow$ Brane world models
- Chiral fermions are open strings living on the intersections of two D-branes

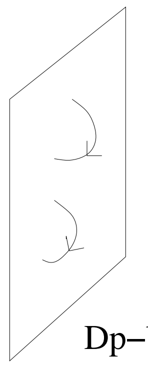
$$N_F = I_{ab} \equiv \#(\pi_a \cap \pi_b) \equiv \pi_a \circ \pi_b$$

- Correspond to non-trivial gravitational backgrounds $\rightarrow$ AdS/CFT correspondence

II) Intersecting Brane World Models (flat branes)

Consider first flat D-branes in Minkowski space $\mathbb{R}^{1,9}$.

Simplest D-brane configuration: 1 single Dp-brane:



Massless open string spectrum: $U(1)$ gauge boson $\longrightarrow$ supersymmetric $U(1)$ gauge theory in $p+1$ dimensions

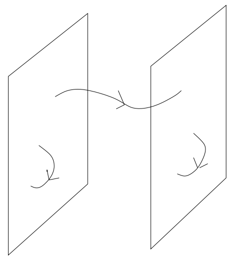
$$\mathcal{S}_{\text{eff}} = \int_\pi dx^{p+1} \; (\underbrace{\mathcal{L}_{\text{DBI}}(g, F, \phi)}_{\text{Tension}} + \underbrace{\mathcal{L}_{\text{CS}}(F, C_{p+1})}_{\text{Charge}})$$

Effective gauge interactions due to the exchange of open strings:

$$\begin{aligned} S_{DBI} &= \tau_p \int \mathrm{d}^{p+1}x \sqrt{\det(g_{\mu\nu} + \tau^{-1} F_{\mu\nu})} \\ &= (\frac{M_{\text{string}}^{p-3}}{g_{\text{string}}}) \int \mathrm{d}^{p+1}x \, F_{\mu\nu}^2 + \dots \end{aligned}$$

Other D-brane configurations (in flat space-time):

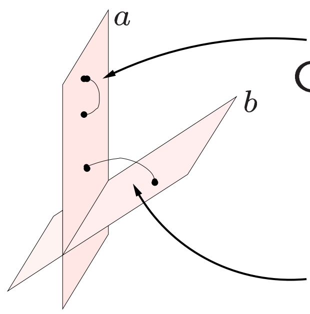
- N parallel Dp-branes



$\mathcal{N}=4$ supersymmetric $U(N)$ gauge theory in $p+1$ dimensions

- Intersecting D-branes

(Berkooz, Douglas, Leigh)



Gauge bosons in adj.

Chiral matter in $(N,\bar{M})$

Open string spectrum:

(i) $\mathcal{N}=4$ gauge bos. A^a_μ in adj. repr. of $U(N)\times U(M)$

(ii) Massless fermions ψ_i in chiral $(N,\bar{M})$ repres.

(iii) Massive scalars ϕ_i in $(N,\bar{M})$ repres.

II) Intersecting Brane World Models (flat branes)

Intersecting D-branes break space-time supersymmetry!
This supersymmetry breaking manifests itself as the a massive/tachyonic scalar groundstate:

$$M^2_{ab} = \frac{1}{2}\sum_I \Delta\theta^I_{ab} - \max\{\Delta\Phi^I_{ab}\}$$

Massless scalars $\Leftrightarrow$ open string sector is supersymmetric.

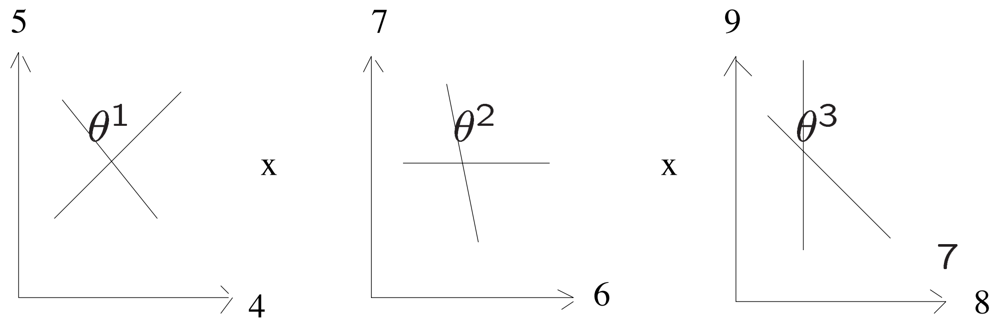
Two flat supersymmetric D6-brane configurations:

- 2 intersecting D6-branes, common in 123-directions, intersect in 4-5 and 6-7 planes, parallel in 8-9 plane:

  1/4 BPS ($\mathcal{N}=2$ SUSY): $\theta^1+\theta^2=0$

- 2 intersecting D6-branes, common in 123-directions, intersect in 4-5, 6-7 and 8-9 planes:

  1/8 BPS ($\mathcal{N}=1$ SUSY): $\theta^1+\theta^2+\theta^3=0$

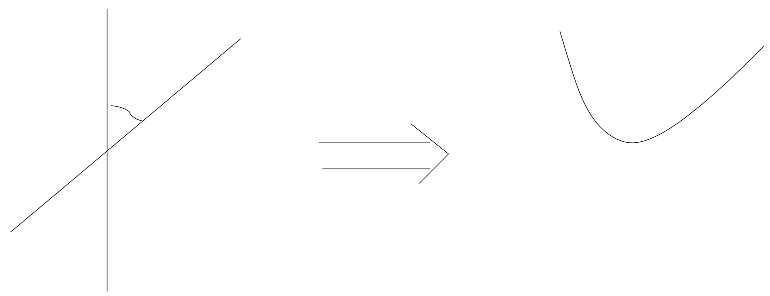


II) Intersecting Brane World Models (flat branes)

In case the open string scalar is tachyonic ($M_{ab}^2 < 0$) $\longrightarrow$ the 2 different branes will recombine into a single brane.

Brane recombination $\longleftrightarrow$ Tachyonic Higgs effect

(F. Epple, D.L., hep-th/0311182)



Spectrum : $M^2 = (-\frac{1}{2} - n)\frac{|\theta|}{\pi\alpha'} + n\,, \quad n = 0, 1, 2, \ldots\,.$

- Small intersection angle: one tachyonic mode $\rightarrow$ field theory Higgs effect. (The vacuum strucure is determined by the D- and F-flatness conditions in the world volume field theory.)

  (Erdmenger, Guralnik, Helling, Kirsch, hep-th/0309043)

- Large intersection angle: many tachyonic modes $\rightarrow$ string field theory

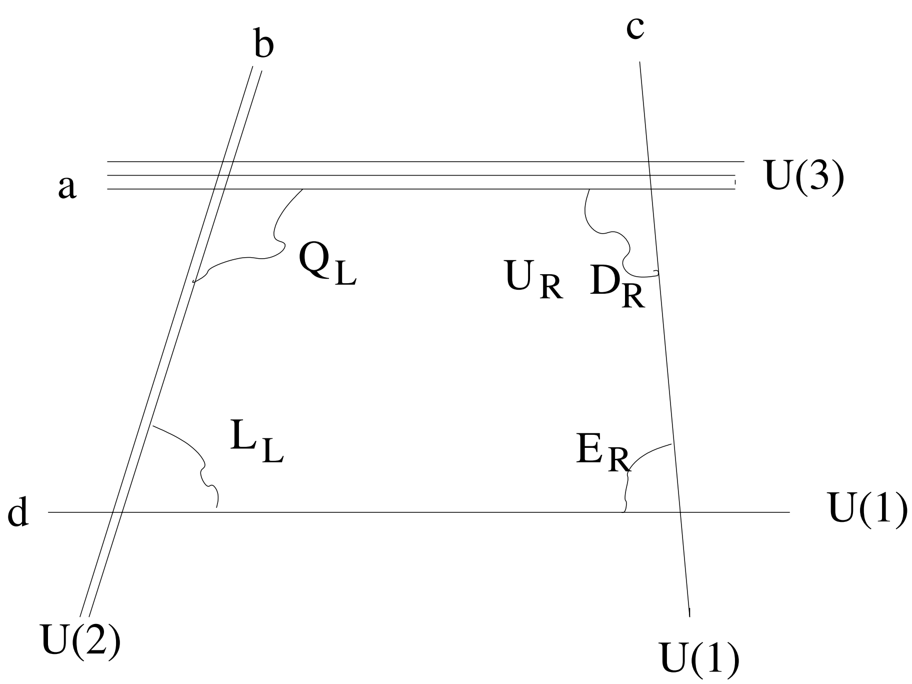
  (Sen)

II) Intersecting Brane World Models (flat branes)

Local picture of the Standard Model:

Minimal realization by four stack of D-branes:

Stack a:	$N_a = 3$	$SU(3)_a \times U(1)_a$	QCD branes
Stack b:	$N_b = 2$	$SU(2)_b \times U(1)_b$	weak branes
Stack c:	$N_c = 1$	$U(1)_c$	right brane
Stack d:	$N_d = 1$	$U(1)_d$	leptonic brane



Space-time supersymmetry is preserved all four stacks of D-branes preserve the angle conditions among each other.

II) Intersecting Brane World Models (compact space)

Now we have to embed these D-branes into a compact space $\mathcal{M}^6$:

Space-time: $\mathbb{R}^{1,3} \times \mathcal{M}^6$.

- $D6_a$-branes are wrapped around 3-cycle π_a inside $\mathcal{M}_6$.

  Brane world volume: $\mathbb{R}^{1,3} \times \pi_a$.

- Multiple intersections $\Longrightarrow$ Family number

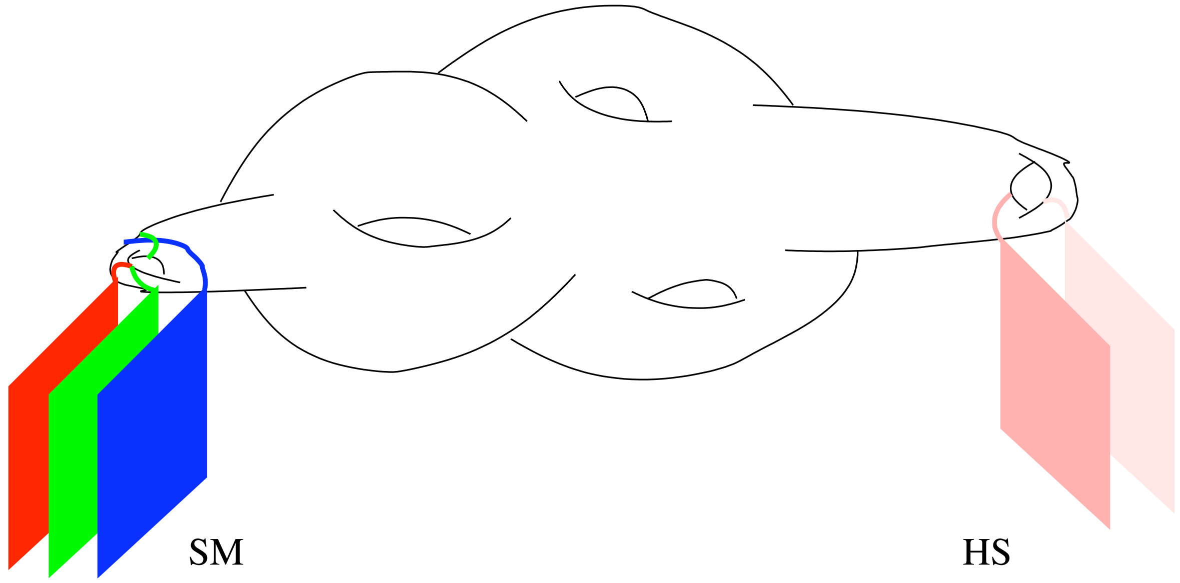
- Cancellation of internal Ramond charges on compact space (Gauss law)

- Cancellation of internal brane tensions/forces (stability problem)

$\Longrightarrow$ Need orientifold plane(s) (branes of negative RR-charge and tension).

$\Longrightarrow$ Strong restrictions on brane configurations

View on the internal Calabi-Yau space $\mathcal{M}^6$:



II) Intersecting Brane World Models (compact space)

Model building: Two simple ways to embed the SM!
(Blumenhagen, Körs, D.L., hep-th/0012156; Cremades, Ibanez, Marchesano, Rabadan, hep-th/0105155, hep-th/0302105)

Both of them use four stacks of $D6$-branes:

$$A \quad : \quad U(3)_a \times SP(2)_b \times U(1)_c \times U(1)_d$$
$$B \quad : \quad U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d.$$

The chiral spectrum of the intersecting brane world model should be identical to the chiral spectrum of the standard model particles.

This fixes uniquely the intersection numbers of the four 3-cycles, $(\pi_a, \pi_b, \pi_c, \pi_d)$.

field	**sector**	**I**	$SU(3) \times SU(2) \times U(1)^3$
q_L	(ab)	3	$(3, 2; 1, 0, 0)$
u_R	(ac)	3	$(\overline{3}, 1; -1, 1, 0)$
d_R	(ac')	3	$(\overline{3}, 1; -1, -1, 0)$
e_L	(db)	3	$(1, 2; 0, 0, 1)$
e_R	(dc')	3	$(1, 1; 0, -1, -1)$
ν_R	(dc)	3	$(1, 1; 0, 1, -1)$

The hypercharge Q_Y is given as the following linear combination of the three $U(1)$s

$$Q_Y = \frac{1}{3}Q_a - Q_c - Q_d.$$

Then an ISB model is constructed by the following six steps:

(i) chose a compact Calabi-Yau manifold $\mathcal{M}^6$,

(ii) determine the orientifold 6-plane π_{O6}

(iii) chose four 3-cycles $\pi_{U(3)_a}$, $\pi_{U(2)_b}$, $\pi_{U(1)_c}$, $\pi_{U(1)_d}$ for the four stacks of D6-branes, as well as their orientifold mirrors

(iv) compute their intersection numbers

(v) ensure that the RR tadpole conditions vanish (possibly by adding hidden D6-branes)

(vi) ensure that the linear combination $U(1)_Y$ remains massless.

II) Intersecting Brane Worlds (compact spaces)

Many intersecting brane world models on tori, orbifolds, or the quintic Calabi-Yau manifold with gauge group

$$G = SU(3)_c \times SU(2)_L \times U(1)_Y$$

and 3 families of quark and leptons can be explicitly constructed ($\mathcal{N} = 1$ supersymmetric models are more restricted).

(Blumenhagen, Braun, Görlich, Ott, Körs, D.L. (2000/01/02); Aldazabal, Cremades, Franco, Ibanez, Marchesano, Rabadan, Uranga; Cvetic, Shiu, Uranga; Bailin, Kraniotis, Love; Kokorelis; Förste, Honecker, Schreyer; Ellis, Kanti, Nanopoulos)

(Some of the authors also use more than four stack of D6-branes or different types of D-branes.)

III) Effective low-energy action

In $\mathcal{N}=1$ supersymmetric models the low-energy super gravity action of the massless gauge A^a_μ and matter fields Φ_i is determined by three moduli-dependent functions:

Gauge kinetic function: $f_a(M)\ W^a_\alpha W^a_\alpha$

$\rightarrow$ Gauge couplings $1/g_a^2(M) = Re\, f_a$.

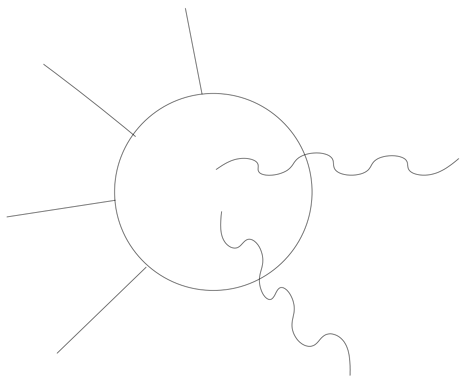
Superpotential: $W = W_{ijk}(M)\Phi_i\Phi_j\Phi_k + \ldots$.

Kähler potential: $K = \hat{K}(M,\bar{M}) + K(M,\bar{M})_{i\bar{j}}\ \Phi_i\bar{\Phi}_j + \ldots$.

$\rightarrow$ Yukawa couplings: $Y_{ijk} = e^{\sqrt{\hat{K}/2}}\sqrt{K^{-1}_{i\bar{l}}K^{-1}_{i\bar{m}}K^{-1}_{i\bar{n}}}\ W_{lmn}$

String tree level scattering amplitudes on the disk:
(N. Bernhardt, D.L., P. Mayr, R. Richter, S. Stieberger)

Amplitude between N_o open and N_c closed strings:



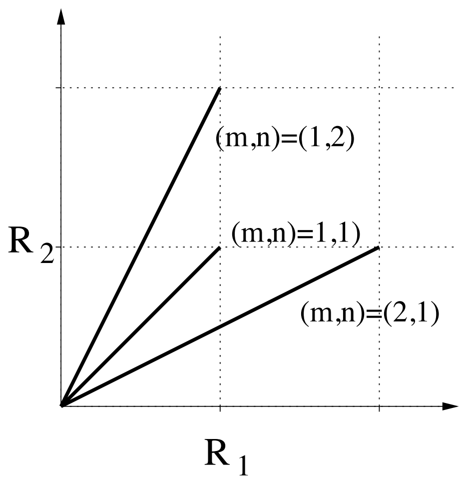
$N_0 = 4,\ \ N_c = 2$

III) Effective low-energy action

Explicit string calculation for $\mathcal{M}_6 = \prod_{j=1}^{3} T_j^2$:

Closed string moduli M: $U_j = R_j^2 / R_j^1$.

D6-branes are wrapped around the product of three 1-cycles with wrapping numbers (m_j, n_j):



$$\tan(\pi\theta_j) = \frac{m_j}{n_j} U_j \, .$$

(i) 3-point amplitude $< A_\mu^a \; A_\nu^a \; U_j >$ $(N_o = 2, \; N_c = 1)$:

Gauge coupling constant:

$$g_a^{-2} = e^{-\phi_4} \prod_{j=1}^{3} \frac{|n_j^a(1 + \tan\theta_j^a)|}{\sqrt{U_j}} \sim e^{-\phi_4} \; \mathrm{Vol}(\pi_a) \, .$$

III) Effective low-energy action

(ii) 3-point amplitude $< C_\theta \; \bar{C}_\theta \; U_j >$ $(N_o = 2,\; N_c = 1)$:

Matter field Kähler metric

$$\frac{\partial K_{C_\theta \bar{C}_\theta}}{\partial U_j} \sim \frac{1}{4U_j} e^{-\pi i \theta_j} \sin(\pi\theta_j)[2\gamma_E + \psi(\theta_j) + \psi(1-\theta_j)] K_{C_\theta \bar{C}_\theta}$$

$$K_{C_\theta \bar{C}_\theta} \sim \prod_{j=1}^{3} \sqrt{\frac{\Gamma(\theta_j)}{\Gamma(1-\theta_j)}} \,.$$

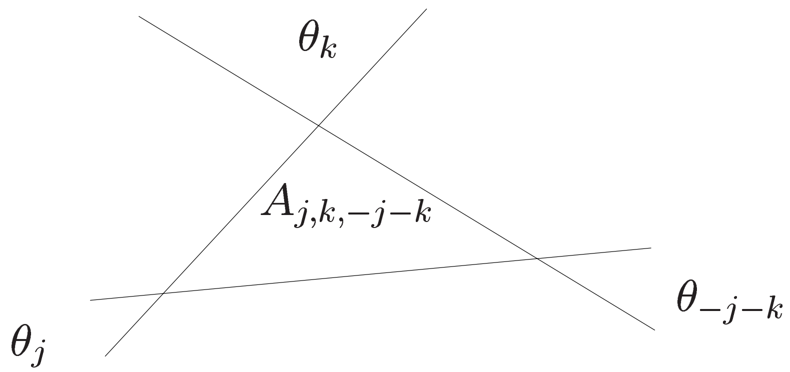
(iii) 4-point amplitude $< C_\theta \; \bar{C}_\theta \; C_\theta \; \bar{C}_\theta >$ $(N_o = 4,\; N_c = 0)$:

Yukawa couplings:

(See also: Cvetic, Papadimitriou; Cremades, Ibanez, Marchesano)

$$Y_{j,k,-j-k} \sim \prod_{i=1}^{3} \left[\frac{\Gamma(1-\theta_j)\Gamma(1-\theta_k)\Gamma(\theta_j+\theta_k)}{\Gamma(\theta_j)\Gamma(\theta_k)\Gamma(1-\theta_j-\theta_k)} \right]^{1/4} W_{j,k,-j-k} \,.$$

W is the exponential superpotential describing the classical world sheet instantons: $W_{j,k,-j-k} \sim e^{-A_{j,k,-j-k}}$, A being the area between the 3 D6-branes:



Gauge coupling unification

(Blumenhagen, Stieberger, D.L., hep-th/0305146;
cfr. Antoniadis, Kiritsis, Tomaras, hep-th/0004214)

In D-brane models with gauge group $SU(3) \times SU(2) \times U(1)_Y$ each gauge factor comes with its own gauge coupling:

$$\frac{4\pi}{g_a^2} = \frac{M_s^3 V_a}{(2\pi)^3 g_{st} \kappa_a}, \qquad V_a = (2\pi)^3 R_a^3 .$$

with $\kappa_a = 1$ for $U(N_a)$ and $\kappa_a = 2$ for $SP(2N_a)/SO(2N_a)$.

By dimensionally reducing the type IIA gravitational action one can express the Planck mass in terms of stringy parameters ($M_{pl} = (G_N)^{-\frac{1}{2}}$)

$$M_{pl}^2 = \frac{8 M_s^8 V_6}{(2\pi)^6 g_{st}^2}, \qquad V_6 = (2\pi)^6 R^6 .$$

Eliminating the unknown string coupling g_{st} gives

$$\frac{1}{\alpha_a} = \frac{M_{pl}}{2\sqrt{2}\,\kappa_a M_s} \frac{V_a}{\sqrt{V_6}}.$$

The gauge coupling only depends on the complex structure moduli.

III) Gauge coupling unification

Consider the gauge coupling unification in a model indepedent bottom up approach.

3 phenomenological requirements:

- The SM branes mutually preserve $\mathcal{N} = 1$ supersymmetry.
- The intersecting numbers realize a 3 generation MSSM
- The $U(1)_Y$ gauge boson is massless

One can show that these restrictions provide one relation between the 3 gauge coupling constants:

$$\frac{1}{\alpha_Y} = \frac{2}{3}\frac{1}{\alpha_s} + \frac{1}{\alpha_w}.$$

This relation will allow for natural gauge coupling unification!

III) Gauge coupling unification

In the absence of threshold corrections, the one-loop running of the three gauge couplings is described by the well known formulas

$$\begin{aligned} \frac{1}{\alpha_s(\mu)} &= \frac{1}{\alpha_s} + \frac{b_3}{2\pi}\ln\left(\frac{\mu}{M_s}\right) \\ \frac{\sin^2\theta_w(\mu)}{\alpha(\mu)} &= \frac{1}{\alpha_w} + \frac{b_2}{2\pi}\ln\left(\frac{\mu}{M_s}\right) \\ \frac{\cos^2\theta_w(\mu)}{\alpha(\mu)} &= \frac{1}{\alpha_Y} + \frac{b_1}{2\pi}\ln\left(\frac{\mu}{M_s}\right), \end{aligned}$$

where (b_3, b_2, b_1) are the one-loop beta-function coefficients for $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$.

Using the tree level relation at the string scale yields

$$\frac{2}{3}\frac{1}{\alpha_s(\mu)} + \frac{2\sin^2\theta_w(\mu) - 1}{\alpha(\mu)} = \frac{B}{2\pi}\ln\left(\frac{\mu}{M_s}\right)$$

with

$$B = \frac{2}{3}b_3 + b_2 - b_1.$$

III) Gauge coupling unification

Employing the measured Standard Model parameters

$$\begin{aligned} M_Z &= 91.1876 \text{ GeV}, \quad \alpha_s(M_Z) = 0.1172, \\ \alpha(M_Z) &= \frac{1}{127.934}, \quad \sin^2\theta_w(M_Z) = 0.23113 \end{aligned}$$

the resulting value of the unification scale only depends on the combination B of the beta-function coefficients.

For the MSSM one has $(b_3, b_2, b_1) = (3, -1, -11)$, i.e $B = 12$ and the unification scale is the usual GUT scale

$$M_s = M_X = 2.04 \cdot 10^{16} \text{ GeV}.$$

For the individual gauge couplings at the string scale we get

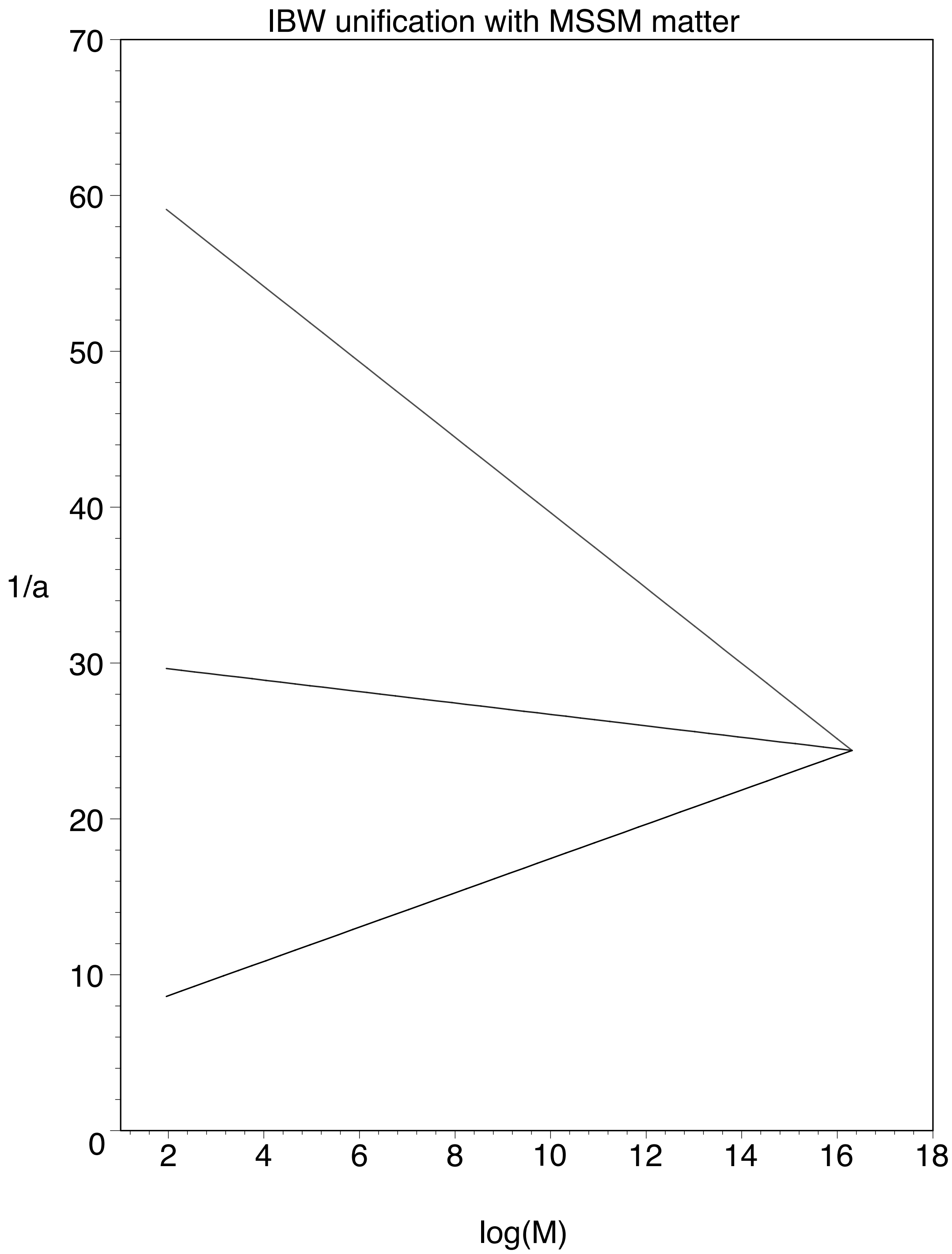
$$\alpha_s(M_s) = \alpha_w(M_s) = \frac{5}{3}\alpha_Y(M_s) = 0.041,$$

which are just the supersymmetric GUT scale values with the Weinberg angle being $\sin^2\theta_w(M_s) = 3/8$.

Assuming $g_{st} = g_X$, one obtains for the overall radius R and the internal radii R_s, R_w

$$M_sR = 5.32, \quad M_sR_s = 2.6, \quad M_sR_w = 3.3.$$

III) Gauge coupling unification



III) Gauge coupling unification

In general besides the chiral matter string theory contains also additional vector-like matter.

This is also localized on the intersection loci of the $D6$ branes and also comes with multiplicity n_{ij} with $i,j \in \{a,b,c,d\}$.

One finds the following contribution to B

$$
\begin{aligned}
B &= 12 - 2\,n_{aa} - 4\,n_{ab} + 2\,n_{a'c} + 2\,n_{a'd} - 2\,n_{bb} + 2\,n_{c'c} \\
&\quad +2\,n_{c'd} + 2\,n_{d'd}.
\end{aligned}
$$

B does not depend on the number of weak Higgs multiplets n_{bc}.

<u>Example A:</u>

If we have a model with a second weak Higgs field, i.e. $n_{bc} = 1$, we still get $B = 12$ but with

$$(b_3, b_2, b_1) = (3, -2, -12).$$

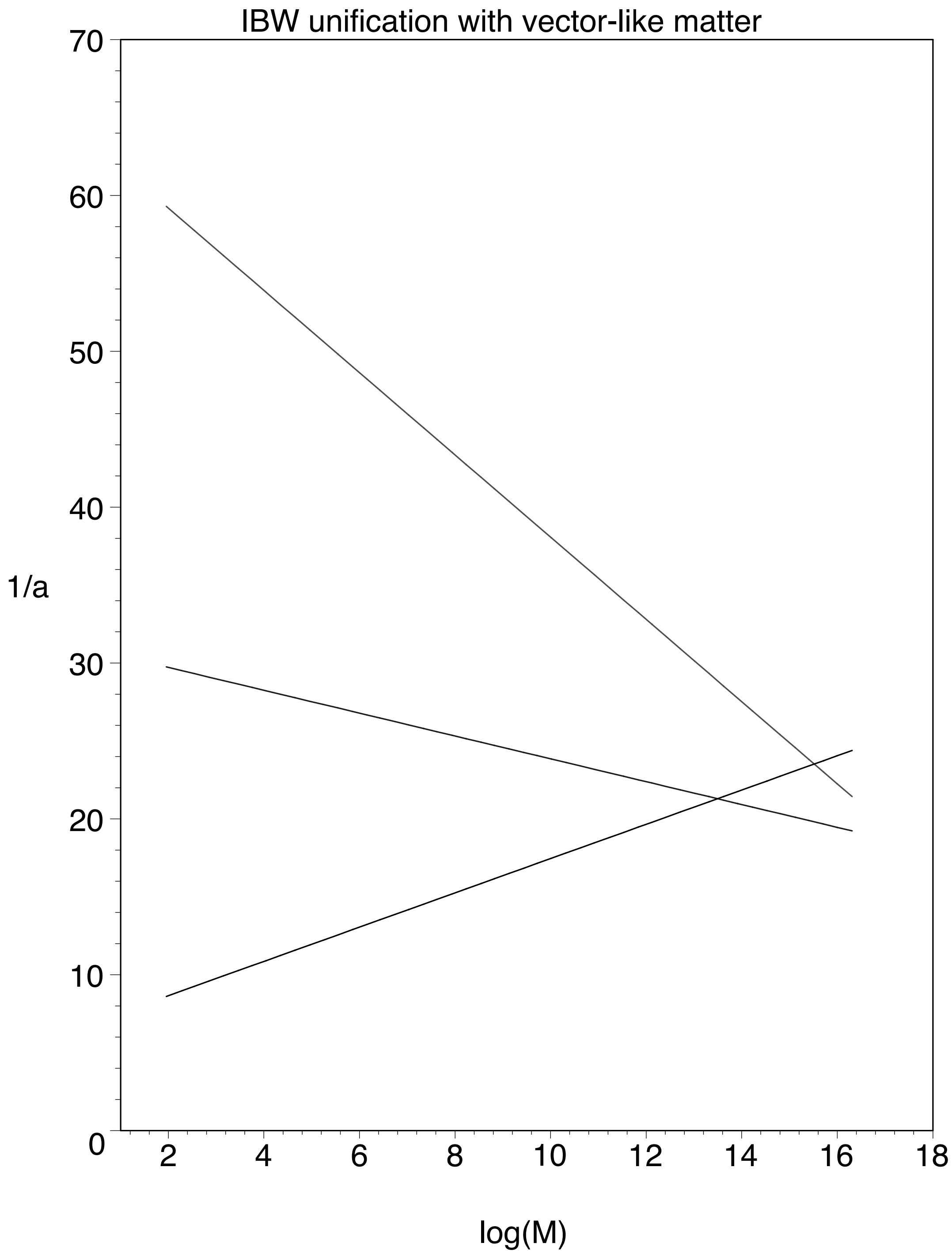
The gauge couplings "unify" at the scale

$$M_s = 2.02 \cdot 10^{16}\,\text{GeV}.$$

However they are not all equal at that scale

$\alpha_s(M_s) = 0.041, \quad \alpha_w(M_s) = 0.052, \quad \alpha_Y(M_s) = 0.028.$

III) Gauge coupling unification



Example B: intermediate scale model

For models with gravity mediated supersymmetry breaking (hidden anti-branes) the string scale is naturally in the intermediate regime $M_s \simeq 10^{11}$GeV.

Choosing vector-like matter

$$n_{a'a} = n_{a'd} = n_{d'd} = 2, \quad n_{bb} = 1$$

leads to $B = 18$.

The string scale turns out to be

$$M_s = 3.36 \cdot 10^{11} \text{GeV}.$$

The running of the couplings with

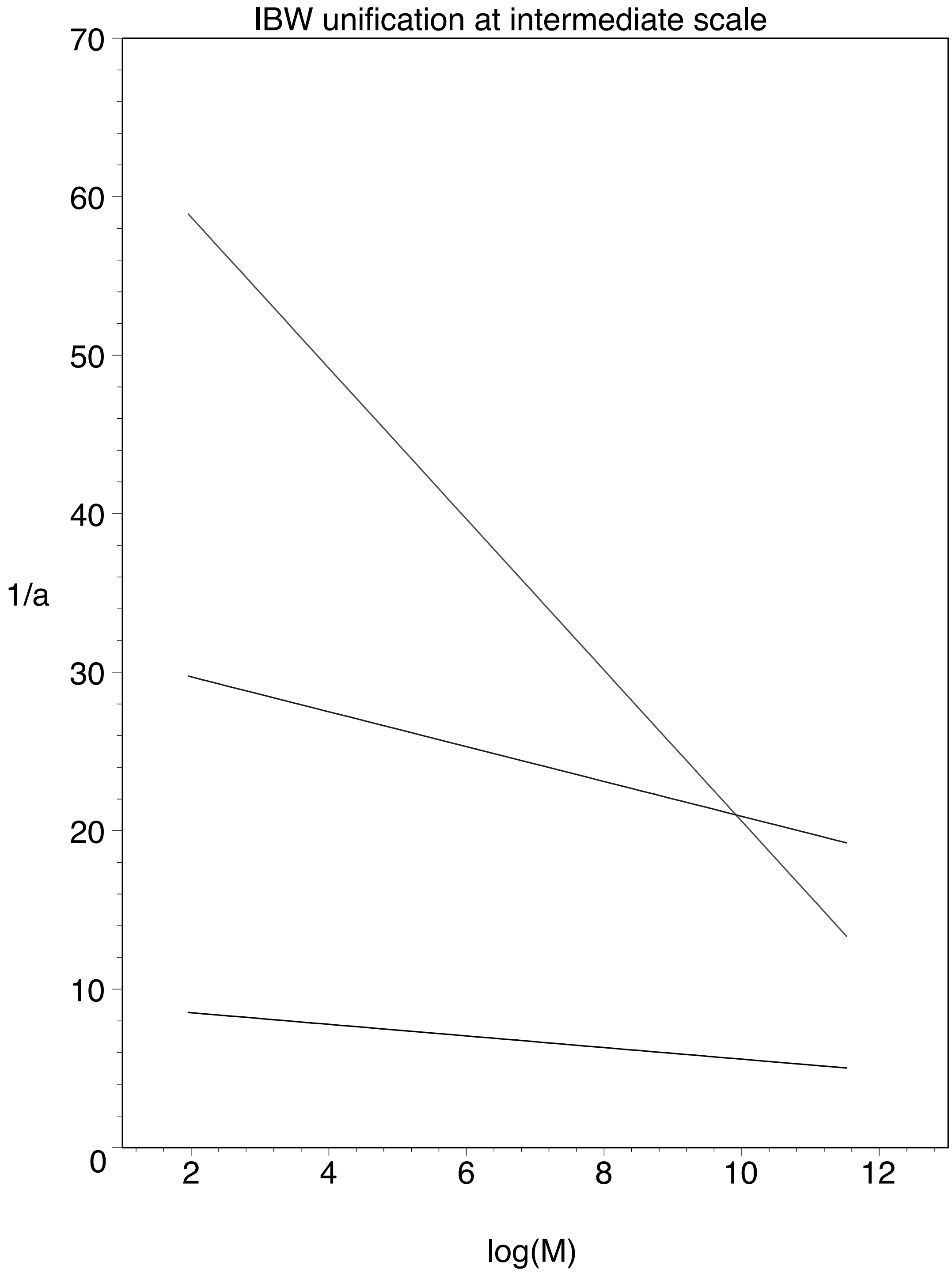
$$(b_3, b_2, b_1) = (-1, -3, -65/3)$$

leads to the values of the gauge couplings at the string scale

$$\alpha_s(M_s) = 0.199, \quad \alpha_w(M_s) = 0.052, \quad \alpha_Y(M_s) = 0.045.$$

Assuming $g_{st} \simeq 1$, one obtains for the radii

$$M_s R = 230, \quad M_s R_s = 1.7, \quad M_s R_w = 3.3.$$


IBW unification at intermediate scale
70
60
50
40
1/a
30
20
10
0
2
4
6
8
10
12
log(M)

Example C: Planck scale model

Interestingly for $B = 10$ one gets

$$\frac{M_s}{M_{pl}} = 1.24 \sim \sqrt{\frac{\pi}{2}}.$$

Choosing vector-like matter

$$n_{aa} = 1,$$

the beta-function coefficients read

$$(b_3, b_2, b_1) = (0, -1, -11).$$

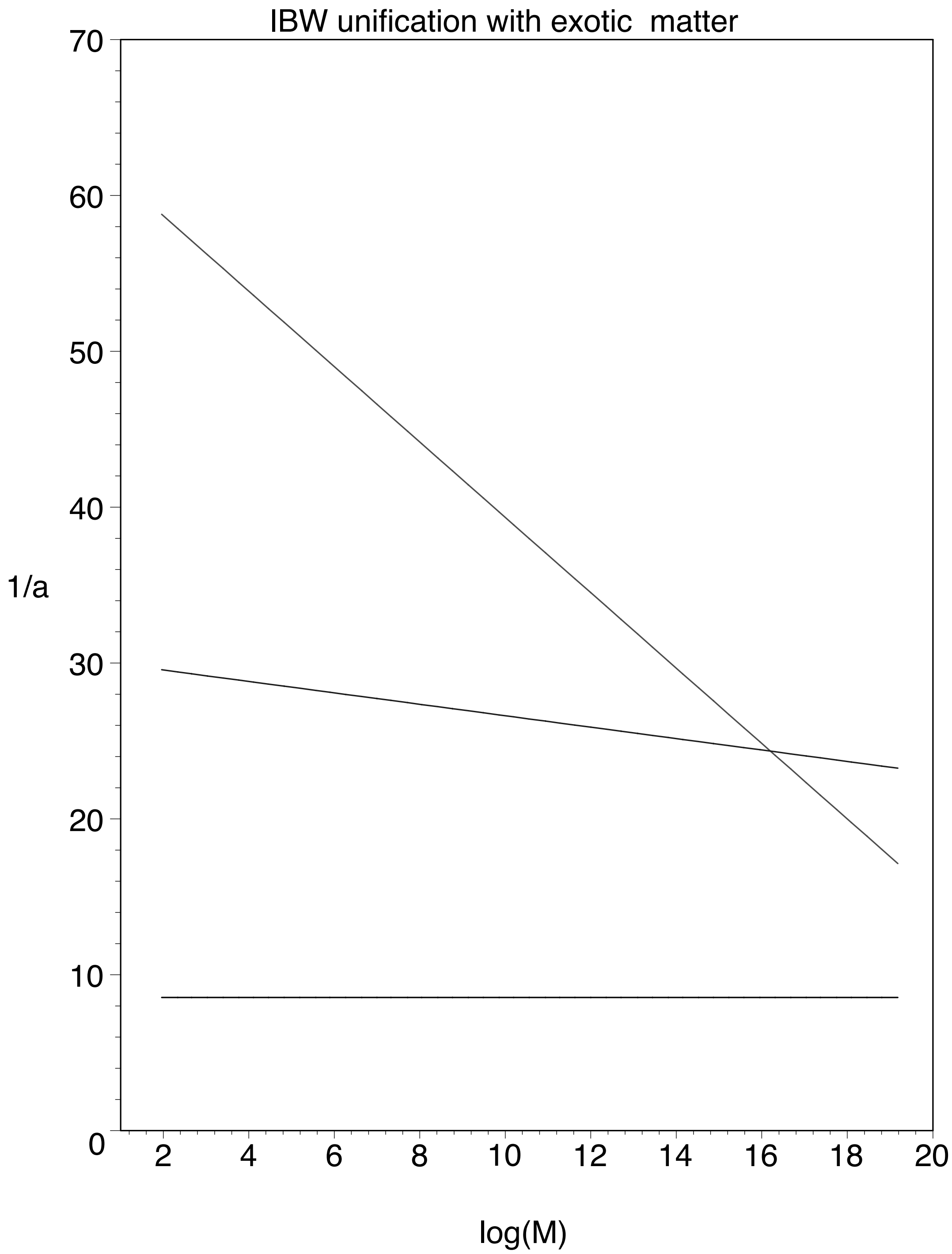
The couplings at the string scale turn out to be

$$\alpha_s(M_s) = 0.117, \quad \alpha_w(M_s) = 0.043, \quad \alpha_Y(M_s) = 0.035$$

leading to $\sin^2 \theta_w(M_s) = 0.445$.

For the scales of the overall Calabi-Yau volume and the 3-cycles we obtain

$$M_s R = 0.6, \quad M_s R_s = 1.9, \quad M_s R_w = 3.3.$$


IBW unification with exotic matter
70
60
50
40
30
20
10
0
1/a
2
4
6
8
10
12
14
16
18
20
log(M)

IV) Conclusions

What did we learn: though very hard, it seems possible to derive the SM from brane constructions!

The challenge remains to construct realistic supersymmetric IBW models with the chiral spectrum of the MSSM and only a mild amount of vector-like matter. (For recent progress via Gepner models see: *(Dijkstra, Huiszoon, Schellekens, hep-th/0403196)*

Other interesting topics:

- Intersecting D6-branes can be lifted to M-theory on a G_2 manifold.
  (Behrndt, Dall'Agata, D.L., Mahapatra, hep-th/0307117; Cvetic, Shiu, Uranga; Atiyah, Witten)

- Combine D-branes and flux compactications.
  (Blumenhagen, D.L., Taylor, hep-th/0303116; Cascales, Uranga; Behrndt, Cvetic)

- Proton decay amplitudes
  (Klebanov, Witten)

- Flavor changing neutral currents
  (Abel, Masip, Santiago; Abel, Owen)

V) Conclusions

- Strong one-loop thresold corresctions

  (D.L., S. Stieberger, hep-th/0302221)

- Higher point string disk amplitudes

  (D.L., P. Mayr, S. Stieberger, work in progress)

V) Conclusions

Important question: Does it make at all sense to construct 4-dim. string vacua without knowing the dynamical selection process which determines the unique string ground state (if it exists)?

(Preliminary) answer: Probably Yes!

Statistics of string/M theory vacua:
(M. Douglas, hep-th/0303194)

Assume that we can construct the SM spectrum from strings in several ways, where the SM couplings for each model are statistically, i.e. uniformly distributed.

SM fills the following volume in the space of coupling constants (measured in natural units):

$$\delta V_{SM} \sim 10^{-238}$$

Therefore we need at least $\mathcal{O}(10^{238})$ brane/flux string vacua with SM spectrum in order to make the statistical statement that string theory contains the SM.