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Information return in Hawking radiation, a Gaussian quantum information theory perspective

Sommerfeld Theory Colloquium @ LMU

Erik Aurell

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Motivation and outline

Entropy is a concept of statistical physics. It is not simple. Perhaps a non-black-hole expert can contribute to black hole physics from this perspective. At least, a renewed sense of mystery.

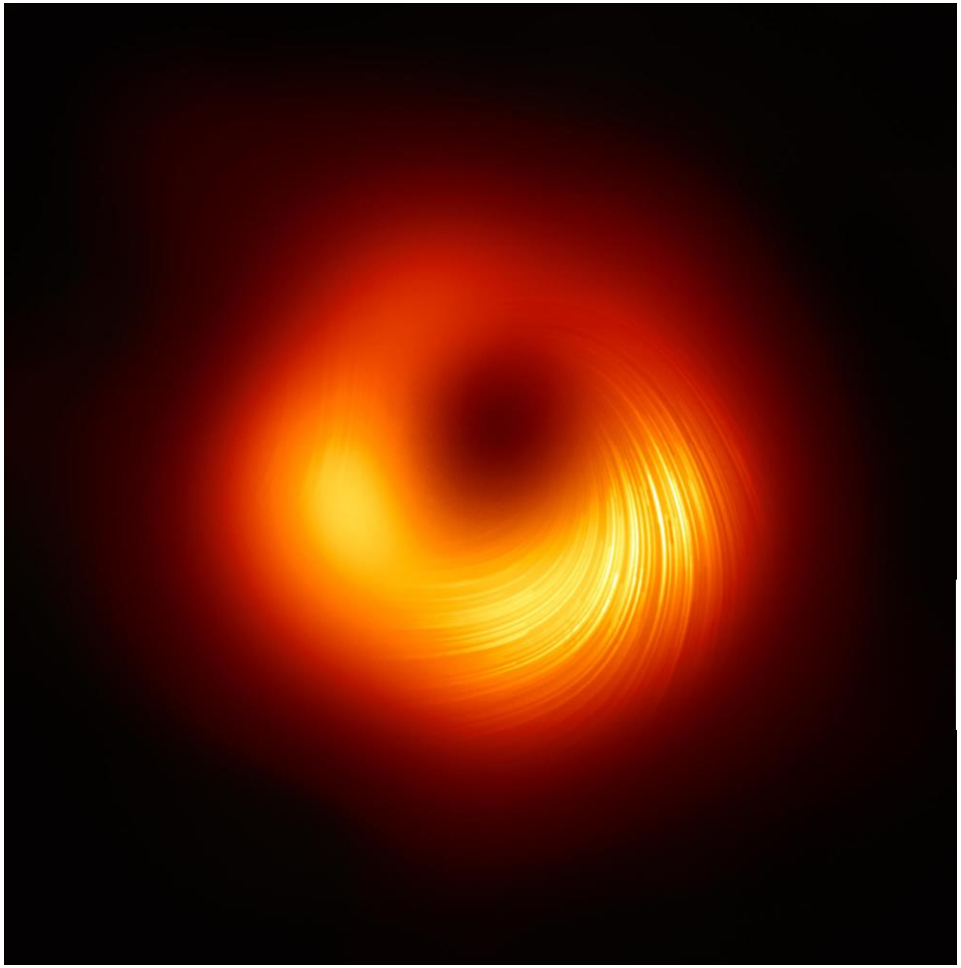
Thermal marginals vs global pure states.

EA, Michał Eckstein & Paweł Horodecki, *JCAP* 2022 (01), 014

Loose ends, and a conjecture.

EA, Michał Eckstein & Paweł Horodecki, *Found. Phys*. **51**:54 (2021)
EA [arXiv:2206.11870]

Black holes are real objects in our universe



Stars like the sun end as white dwarfs. Bigger stars as neutron stars or black holes.

At the center of (almost) every galaxy (including ours) there is very big black hole.

Rendering of the black hole M87*
Event Horizon Telescope (EHT)

Collisions of black holes have been observed in gravitational waves by LIGO-VIRGO.

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Black hole information paradox

Ways out?

Take a large piece of matter in a pure quantum state.

Gravity acts. The matter collapses in a black hole.

The black hole evaporates through Hawking radiation.

At the end the black hole is gone, and all that remains is thermal radiation.

A pure state has developed into a mixed state. This breaks unitarity.

Fundamental information loss: actually the dynamics of a quantum black hole is not unitary.

Physics at horizon: firewalls or other physics stops the collapse.

Information return in Hawking radiation: entanglement between modes of the Hawking radiation.

Remnants: evaporation is not complete, something remains that keeps the information.

from S.B. Giddings "Comments on information loss and remnants" Phys Rev D 49:4078 (1988)

Planck units

When discussing black holes, Planck units are particularly useful. The Schwarzschild black hole radius is $2l_p \frac{M}{m_p}$, its Hawking temperature is $\frac{m_p c^2}{8\pi k_B}\frac{m_p}{M}$, etc.

$$[c] = \frac{L}{T} \qquad \text{Planck mass}: \quad m_p = \sqrt{\frac{\hbar c}{G}} \approx 2.2 \times 10^{-8}\ \text{kg},$$

$$[\hbar] = \frac{ML^2}{T} \qquad \text{Planck length}: \quad l_p = \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \times 10^{-35}\ \text{m},$$

$$[G] = \frac{L^3}{MT^2} \qquad \text{Planck time}: \quad t_p = \sqrt{\frac{\hbar G}{c^5}} \approx 5.4 \times 10^{-44}\ \text{s}.$$

m_P is about the mass of a grain of sand, the scale of objects of everyday life. It is however very large for an elementary particle mass.

The other two are very small, and "It is natural to suppose also that $[l_p]$ determines the limit of applicability of present-day notions of space and causality."

—A.D. Sakharov, *Dokl. Akad. Nauk* **177**:70-71 (1967)

Temperature to entropy

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Hawking radiation has never been observed. Still, if it would not exist physics would need to be changed drastically. Either quantum fields in not very strong gravitational fields would behave differently than one would think, or black holes would be different objects than one reads in textbooks.

If we do not consider the drastic possibilities, in thermodynamics temperature is the partial derivative of entropy with respect to energy. The energy of a black hole as seen from far away is its mass $\cdot\, c^2$. Hence

$$\frac{1}{k_B T_H} = \frac{\partial S_{BH}}{c^2 \partial M} \quad \Rightarrow \quad S_{BH} = 4\pi k_B \cdot \left(\frac{M}{m_P}\right)^2$$

Such an entropy, with the same dependence on mass but with an undetermined pre-factor, was introduced by Bekenstein in 1972.

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A basic mystery

(you can get all numbers from Google)

Entropy of a star of the size of the sun: $10^{35}\frac{\mathrm{J}}{\mathrm{K}}$

Entropy of a black hole of solar mass: $10^{54}\frac{\mathrm{J}}{\mathrm{K}}$

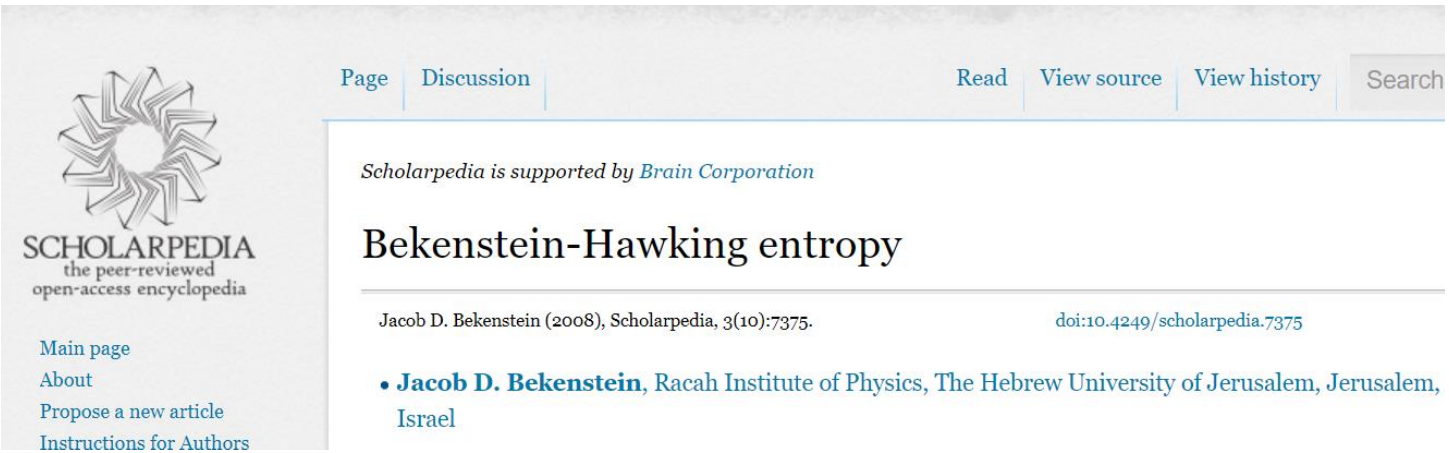
$$S = \log \mathcal{N}?$$

Which phase space volume $\mathcal{N}$ can increase by a factor 10^{19} in a collapse to a black hole?

BH entropy grows quadratically with mass. More than 99.99999% of the entropy in the universe today has been estimated to be S_{BH} of black holes in the center of galaxies.

Egan & Lineweaver, "A larger estimate of the entropy of the universe", Astrophys. J. 710:1825 (2010)

Bekenstein reviewed this question in 2008



One should start by going through the explanations Bekenstein surveyed. I will enumerate them as Bekenstein did, but group them in a different order.

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(1/3) explanations that depend on new physics

4. Black hole entropy is a conserved quantity connected with coordinate invariance of the gravitational action.

6. Black hole entropy counts the number of states or excitations of a fundamental string.

7. Black hole entropy is equivalent to the thermal entropy of the radiation residing on the boundary of the spacetime containing the black hole.

The first of these (no. 4) is by Wald, something in the same direction was proposed by Penrose. No. 6 is a theory of quantum gravity as a limit of another theory. All three suppose physics beyond quantum field theory in curved space-time.

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(2/3) explanations which focus on the horizon

2. Black hole entropy is the entropy of entanglement between degrees of freedom inside and outside the horizon.

3. Black hole entropy counts the number of horizon gravitational states.

5. Black hole entropy is thermal entropy of the gas of quanta constituting the thermal atmosphere of the black hole.

No. 2 implies the question why there is no such large entropy for an arbitrary closed surface. Explanations of this type are possibly also verifiable/falsifiable in future observations on astrophysical black holes and their surroundings.

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(3/3) an explanation which faces the question

1. Black hole entropy counts the number of internal states of matter and gravity.

Bekenstein's extended explanation however doesn't mention $\mathcal{N}$:

> As mentioned earlier, the perception that a particular black hole (specific M, J and Q) can be formed in many ways originally suggested the notion of black hole entropy; in this approach the internal states of matter and/or gravity are the sought for microstates. Examples of this viewpoint are provided by Frolov and Novikov (1993) and by Mukhanov (2003).

The arguments of Frolov and Novikov (1993) and Mukhanov (2003) count the possible initial states of the matter that went down the black hole. There are many for the same black hole. First such estimate: Bekenstein in *Phys. Rev. D* **7**:2333 (1973).

Two kinds of entropy

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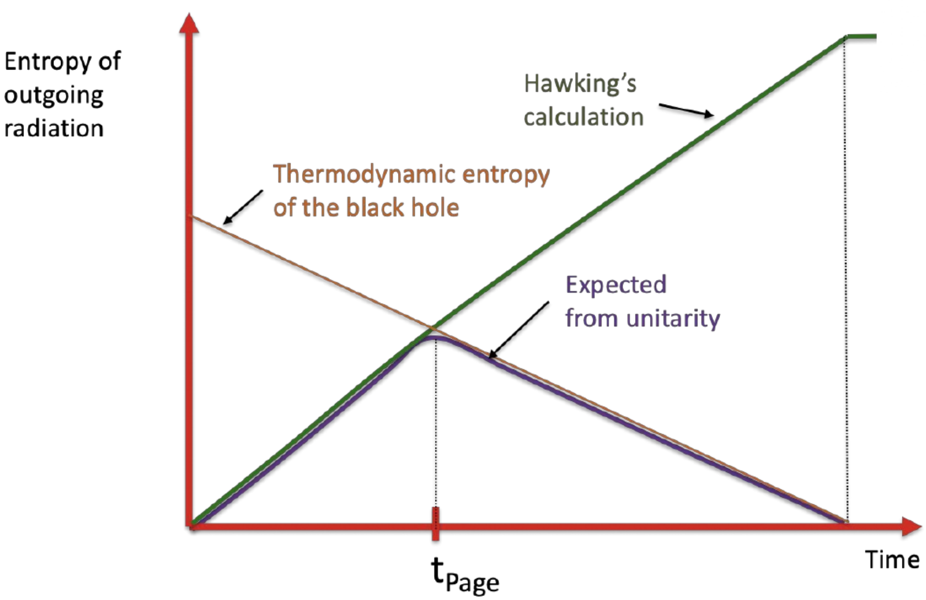
1. *Information-theoretic entropy (“fine-grained entropy”)* does not change under unitary evolution.

2. *Thermodynamic entropy (“coarse-grained entropy”).* tends to a maximum given external constraints.

Suppose initially a pure state black hole. It has zero entropy. The final Hawking radiation after evaporation must then also have zero information-theoretic entropy. This leads to constraints usually called the “Page curve”.

Don Page. Phys Rev Lett 71:3743 (1993); figure from A. Almheiri et al., arXiv:2006.06872



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Is BH entropy then a thermodynamic entropy?

Argument 1: The entropy of Hawking radiation was computed by Zurek in 1982 from classical thermodynamics. The answer is $\frac{4}{3}S_{BH}$; a more detailed calculation was later done by Page. The factor $\frac{4}{3}$ can be attributed to evaporation being an irreversible process. Going backwards, S_{BH} should also be a thermodynamic entropy.

Argument 2: The (ordinary) entropy of a star is so much less than black hole entropy. The von Neumann entropy (Shannon entropy) of all the matter and fields in the star region cannot reasonably increase by a factor about 10^{20} in a gravitational collapse.

Argument 3: A late paper by Hawking can be interpreted this way.

S. Hawking "Information Preservation and Weather Forecasting for Black Holes", [arXiv:1401.5761]

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Two views of thermodynamic entropy

Entropy as ignorance (Jaynes, Mandelbrot,…). "*The subjective interpretation of thermodynamics*". The black hole is classically characterized by mass, electric charge and angular momentum. All other information was lost in the collapse. Such a black hole entropy as ignorance was estimated by Bekenstein and others *op cit*.

Entropy of ensemble (Boltzmann, Khinchin, Lebowitz, Vulpiani,…). "*The objective interpretation of thermodynamics*". The ensembles of equilibrium statistical mechanics describe the distribution of outcomes of reasonably simple experiments that can in principle be done on a high-dimensional system. This second view is the dominant one in modern statistical mechanics.

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Thermal marginals vs global pure states.

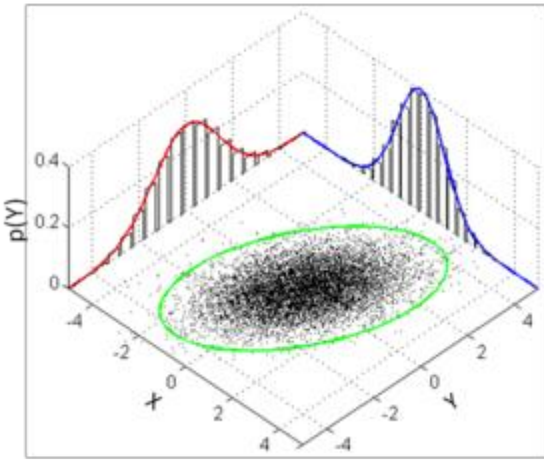
EA, Michał Eckstein & Paweł Horodecki, *JCAP* 2022 (01), 014

Building on advances in continuous-variable quantum information, in particular

J. Eisert et al "Gaussian Quantum Marginal Problem"
Commun Math Phys **280**:263 (2008)

The marginals problem

classical



$$P_X(x) = \sum_y P_{X,Y}(x,y)$$

The marginals of a pure classical state are pure.

quantum

$$|\psi\rangle_{AB} = \sum_i \alpha_i |i\rangle_A |i\rangle_B$$

$$\rho_A = \mathrm{Tr}_B[|\psi\rangle\langle\psi|] \quad \rho_B = \mathrm{Tr}_A[|\psi\rangle\langle\psi|]$$

Quantum marginals are typically not pure. There exists a general theory on the existence (or not) of a total pure quantum state with given (mixed) marginals:

A. Klyachko [arXiv:quant-ph/0409113]

It is quite complicated.

Gaussian Quantum Marginal Problem (1/2)

("*when physics gets difficult, try harmonic oscillators*" [anonymous])

Gaussian oscillator states are completely specified by a correlation matrix C. For quantum states C satisfies Robertson-Schrödinger uncertainty relation.

$$\mathbf{z} = (q_1, p_1, q_2, p_2, \cdots, q_N, p_N)$$

$$C_{ij} = \mathrm{Tr}\left[\left(\hat{z}_i \hat{z}_j + \hat{z}_j \hat{z}_i\right)\hat{\rho}\right]$$

$$C + i\Omega \geq 0 \quad \Omega = \mathbf{1}_{ij} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

A real symmetric C can be put in diagonal form by a symplectic transformation S. Gaussian pure states have $d_1 = d_2 = \cdots = d_N = 1$.

$$C = S\,\mathrm{diag}(d_1, d_1, d_2, d_2, \cdots, d_N, d_N)\, S^T$$

The density matrix marginalized over one mode is also Gaussian, and given by $\begin{pmatrix} C_{2k-1,2k-1} & C_{2k-1,2k} \\ C_{2k,2k-1} & C_{2k,2k} \end{pmatrix} = S_k^{(1)}\,\mathrm{diag}(c_k, c_k)\left(S_k^{(1)}\right)^T$

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Gaussian Quantum Marginal Problem (2/2)

Given *symplectic spectra* of C and its 2-block diagonal $\mathrm{diag}_2(C)$:

$$\mathrm{sspec}(C) = (d_1, d_2, \cdots, d_N) \quad , 1 \leq d_1 \leq d_2 \leq \cdots \leq d_N$$

$$\mathrm{sspec}\big(\mathrm{diag}_2(C)\big) = (c_1, c_2, \cdots, c_N) \quad , 1 \leq c_1 \leq c_2 \leq \cdots \leq c_N$$

J. Eisert et al "Gaussian Quantum Marginal Problem" *Commun Math Phys* **280**:263 (2008):

Necessary conditions, and a construction using 2-mode operations

$$\sum_{j=1}^{k} c_j \geq \sum_{j=1}^{k} d_j \text{ for } k = 1, .., \mathrm{N}, \qquad \text{and} \qquad c_N - d_N \leq \sum_{m<N} c_m - \sum_{m<N} d_m$$

$$S = \left(\mathbf{1}_{2(N-1)} \oplus S^{(2)}_{N-1,N}\right) \circ \cdots \circ \left(\mathbf{1}_2 \oplus S^{(2)}_{2,k_2} \oplus \mathbf{1}_{2(N-2)}\right) \circ \left(S^{(2)}_{1,k_1} \oplus \mathbf{1}_{2(N-1)}\right)$$

Global purity?

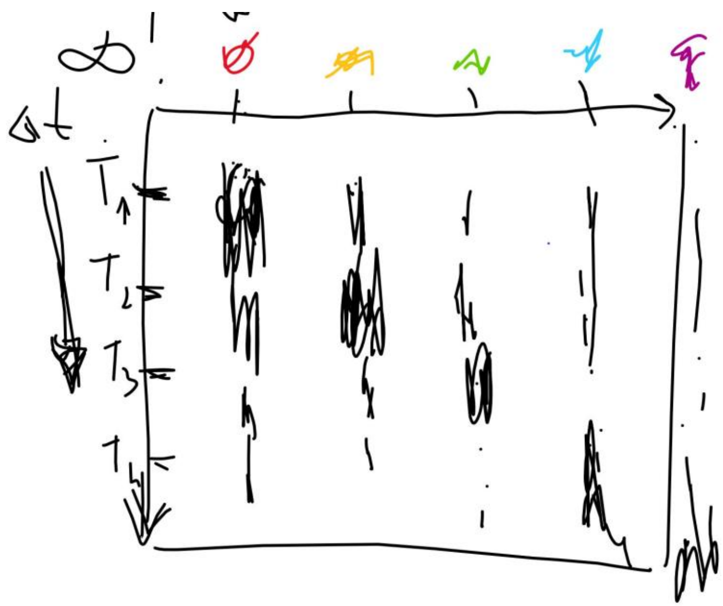
Translate the previous to marginal thermal states

$$\rho = \frac{1}{Z}\sum_n e^{-\beta\hbar\omega\left(n+\frac{1}{2}\right)}|n\rangle\langle n| \sim \exp\left(-\frac{1}{2(1+b)}(\hat{p}^2+\hat{q}^2)\right) \quad b = 2\left(e^{\beta\hbar\omega}-1\right)^{-1}$$

The symplectic spectrum of a globally pure Gaussian state is $d_1 = d_2 = \cdots = d_N = 1$. Hence Eisert et al conditions read

$$b_N \leq \sum_{m<N} b_m$$

These conditions are quite easy to satisfy for an initially large black hole, except perhaps for the very last emitted Hawking particles (photons).



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EA [arXiv:2206.11870]

What is the $\mathcal{N}$ opened by a gravitational collapse?

Open are the double doors of the horizon
Unlocked are its bolts

Akhnaten, Act I, Scene I


Pyramid of Unas (d. c. 2345 BC)
Wikipedia / Jon Bodsworth

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Gravitational chaos?

Homogeneous cosmology has an instability called Belinski–Khalatnikov–Lifshitz (BKL). The gravitational field then oscillates chaotically. The literature is not conclusive, but there are at least some who claim that a similar process happens in inhomogeneous collapse.

cf.
V. A. Belinskii, *Pisma v Zhurnal Eksperimentalnoi i Teoreticheskoi Fiziki* **56**:437 (1992)
L. Andersson et al *Phys. Rev. Lett*. **94:**051101 (2005)

There is hence a mechanism for generating information in a collapse to a black hole which does not exist in collapse to a neutron star. This produces extra entropy.

E.A. arXiv:2206.11870

However, unclear why that entropy should be S_{BM} and the same for every black hole with the same mass, charge and angular momentum.

Open problems black holes and marginal states

Mathematics: a total pure state which has Hawking one-mode marginals require multi-mode quantum correlations. How large? If everything is Gaussian perhaps one can estimate from random symplectic transformations giving C, constrained by $\mathrm{diag}_2(C)$. Work in progress (EA, Kieburg, Heckl, Horodecki & Jonsson).

Physics: when one Hawking particle is emitted the black hole recoils and moves in opposite direction. This entangles one Hawking particle with all later Hawking particles. Unfortunately it is a rather weak effect [Don Page, 1981]. Are there other such mechanisms. Which ones?

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What is a paradox?

(using philosophy as a device to finish this talk)

A tenet contrary to received opinion; an assertion contrary to appearance; a position in appearance absurd.

Samuel Johnson, *Dictionary*

Sometimes it happens, that as we are deceived in the position of terms, so also deception arises as to opinions…

Aristotle, *Prior Analytics* B:21

A slightly later passage adapted to the topic treated today

By universal knowledge [*quantum mechanics*] then we observe particulars [*quantum black holes*], but we do not know them by peculiar knowledge [*an accepted and tested theory of quantum gravity*], hence we may be deceived by them…

Aristotle, *idem*

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Vetenskapsrådet