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Protected helical transport in magnetically doped quantum wires: Beyond the one-dimensional paradigm

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One-dimensional (1D) quantum wires, which are functionalized by magnetic ad-atoms, can host ballistic helical transport. Helicity protects transport from an undesirable influence of material imperfections, and it makes the magnetically doped wire a very promising element for nanoelectronics and spintronics. However, fabricating purely 1D conductors is experimentally very challenging and not always feasible. In this paper, we show that the protected helical transport can exist even in quasi-1D wires. We model the quasi-1D magnetically doped wire as two coupled dense 1D Kondo chains. Each chain consists of itinerant electrons interacting with localized quantum magnetic moments—Kondo impurities. We have analyzed the regimes of weak, intermediate, and strong interchain coupling, and we found conditions necessary for the origin of the aforementioned protected transport. Our results may pave the way for experimental realizations of helical states in magnetically doped wires.

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One major stepping stone in the progress of nanoelectronics and spintronics is the reduction of destructive effects caused by material imperfections, e.g., backscattering and localization. One-dimensional (1D) conductors are especially sensitive to such undesirable effects that suppress ballistic transport [1]. One possibility for protected transport is provided by the helicity of conduction electrons. Helicity, $h = \mathrm{sgn}(p)\,\mathrm{sgn}(\sigma)$, reflects the lock-in relation between the electron's momentum, p, and spin, σ. Transport in a quantum wire is helical and, hence, protected when all conduction electrons have the same helicity.

Physical mechanisms, which yield helical states, generally fall into two categories. The first category includes topological insulators (TI) [2–4]. 1D conducting helical modes can exist on edges of 2D TI [5–7]. Modern experiments show helicity of hinge states in high-order TI [8–10]. Protection of the helical edge transport is expected to be ideal, though it is not robust in reality [11–19].

The second category includes systems in which the helical states are governed by interactions, e.g., the hyperfine interaction between nuclear magnetic moments and itinerant electrons [20–24], and the spin-orbit interaction (SOI) in a combination with either magnetic fields [25,26] or Coulomb interactions [27]. Several experiments confirmed the existence of helical states in interacting systems [25,26,28,29].

Another promising platform for protected helical transport is provided by magnetically doped 1D quantum wires [30–33]. It is somewhat similar to the successful realization of topological superconductivity [34–36]. Despite the solid theoretical background, experiments demonstrating helical transport in magnetically doped 1D wires are still missing. The main obstacle hampering these experiments in traditional materials (GaAs or SiGe) is the nontriviality of methods used to produce 1D conductors [37,38].

The goal of this paper is to show that the strict one-dimensionality is not necessary, and the protected helical states can emerge also in quasi-1D samples.

Model. Magnetically doped quantum wires can be described by the well-known theoretical model of a Kondo chain (KC)—a 1D array of localized quantum magnetic impurities interacting with 1D itinerant electrons [39–47]. The physics of the KC is governed by two competing, mutually exclusive effects: the Kondo effect and the indirect, Ruderman-Kittel-Kasuya-Yosida (RKKY), exchange interaction between the impurities. The dominant effect can be found from a comparison of relevant energy scales: the Kondo temperature, T_K, and the RKKY energy, E_{RKKY} [48]. If $T_K > E_{\text{RKKY}}$, the Kondo screening dominates; magnetic impurities are screened individually. This leads to a Kondo insulator at half-filling and a heavy fermion phase away from half-filling [39,49]. In the opposite case $T_K < E_{\text{RKKY}}$, the RKKY interaction dominates and governs interimpurity correlations. One can translate the above inequality to distances and show that RKKY overwhelms the Kondo effect in dense KCs, where the (mean) interimpurity distance ξ_s is smaller than a crossover scale ξ_c: $\xi_s \leqslant \xi_c \propto \xi_0(\rho_0 J^2/T_K)^{1/2}$ [30–33]. Here J, ρ_0, and ξ_0 are the Kondo coupling, the density of states, and the lattice spacing, respectively. The RKKY-dominated regime is typical in 1D systems [32,50]. We have recently shown that helical spin ordering and protected transport can exist in the dense and incommensurate KC with a small Kondo coupling, which can be anisotropic (easy-plane anisotropy) [30,31] or isotropic [32,33].

To demonstrate that helical transport can exist in quasi-1D wires, we consider two tunneling-coupled dense and incommensurate KCs (Fig. 1). This simplest (minimal) quasi-1D model can provide a proof of principle since it possesses a nontrivial degree of freedom: The magnetic impurities in

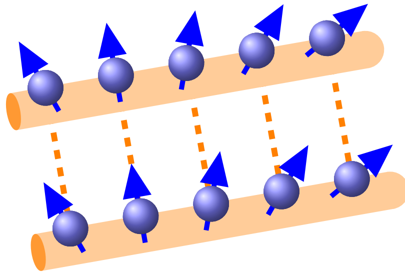

FIG. 1. The quasi-1D doped quantum wire is modeled by two coupled Kondo chains, each consisting of itinerant electrons (orange tubes) and an array of localized quantum magnetic impurities (blue spins). Electrons can tunnel at every site of the electronic lattice into the neighboring wire; tunneling is indicated by dashed lines.

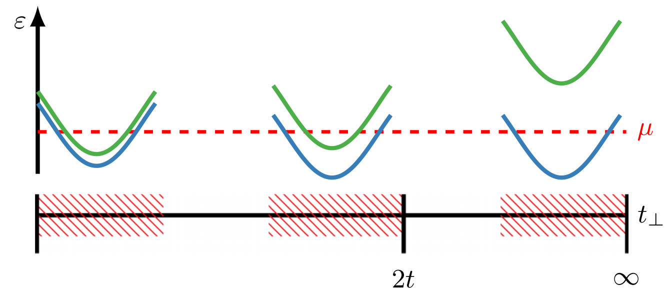

FIG. 2. Three regimes of interchain tunneling (marked by shaded areas). Left panel: tunneling is weak, $t_\perp \ll 2t$, and one comes across four Fermi points that almost coincide in pairs. Central panel: tunneling is larger, $t_\perp \lesssim 2t$, and all four Fermi points are well separated. Right panel: tunneling is strong, $2t \ll t_\perp$, and energy bands are separated by the gap and there are at most two Fermi points.

different KCs are correlated only via tunneling, and it is *a priori* not clear whether they form a global helical ordering, which has the same handedness in each KC [51]. Such correlations can protect ballistic transport in the quasi-1D system. The Hamiltonian of our quasi-1D model reads

$$\hat{H} = \hat{H}^{(1)}_{\rm KC} + \hat{H}^{(2)}_{\rm KC} + \hat{H}_{\rm tun},\ \hat{H}_{\rm tun} = -t_\perp c^{\dagger(1)}_j c^{(2)}_j + \text{H.c.},$$
$$\hat{H}^{(n)}_{\rm KC} = -t(c^\dagger_j c_{j+1})^{(n)} + J_a (c^\dagger_j S^a_j \sigma_a c_j)^{(n)} + \text{H.c.}, \qquad (1)$$

where $\hat{H}^{(1,2)}_{\rm KC}$ are the Hamiltonians of the uncoupled KCs; $\hat{H}_{\rm tun}$ describes the electron tunneling; $c_j = (c_{j\uparrow}, c_{j\downarrow})^T$ is a spinor; $c_{j\sigma}$ $(c^\dagger_{j\sigma})$ annihilates (creates) an electron with spin $\sigma =\uparrow,\downarrow$ at a lattice site j of a given chain marked by $n = 1, 2$; t $(t_\perp)$ is the intra (inter) chain hopping strength; J_a is the strength of the Kondo interaction in the $a = x, y, z$ direction; $\mathbf{S}$ is an impurity spin s operator; and σ_a are the Pauli matrices [52]. For simplicity, we do not distinguish lattice constants ξ_0 and ξ_s, we assume that the individual KCs have the same parameters, and we focus on zero temperature, $T \to 0$. We explore the case of the easy-plane magnetic anisotropy $J_x = J_y = J \gg J_z \to 0$ with a small coupling constant, $sJ \ll t$, and incommensurate band fillings. This setup is relevant for the search of protected transport [30,31] and much simpler for the theoretical study than the isotropic case [32,33]. Note that Kondo-like renormalizations are suppressed and can be neglected in the dense KCs whose physics is dominated by the RKKY interaction [31].

Three regimes of the tunneling-coupled KC. The noninteracting part of the Hamiltonian (1), $\hat{H}_0 \equiv \hat{H}|_{J=0}$, has the spectrum $\varepsilon_\pm(k) = -2t\cos(k\xi_0) \mp t_\perp - \mu$ [53]. The value of $t_\perp$ determines three different regimes: the strong-, intermediate-, and weak-interchain tunneling; see Fig. 2.

If tunneling is strong, $2t \ll t_\perp$, there are two bands separated by a large gap of order $t_\perp$. Without loss of generality, we can place the chemical potential, μ, in the lower band and take into account the electron-spin interaction perturbatively by using the smallness $sJ/t_\perp \ll t/t_\perp \ll 1$. We will show that such a perturbation yields only small and inessential corrections to the physics of the helical 1D wire described in Refs. [30,31].

The other two cases of the intermediate, $t_\perp \lesssim 2t$, or small, $t_\perp \ll 2t$, tunneling, can possess four Fermi points. In the former case, the Fermi points are well separated and one has to take into account all electron-spin interactions nonperturbatively. If tunneling is weak, the four Fermi points almost coincide in pairs, and small $t_\perp$ can be treated as a perturbation for two decoupled KCs.

We rewrite the Kondo interaction in the eigenbasis of $\hat{H}_0$:

$$\hat{H}_{\rm int} = (J/2)[c^\dagger_\nu S^b_+ \sigma_b c_\nu + c^\dagger_\nu S^b_- \sigma_b c_{-\nu} + \text{H.c.}], \qquad (2)$$

where $b = x, y$, $\mathbf{S}_\pm = \mathbf{S}^{(1)} \pm \mathbf{S}^{(2)}$, and $\nu = +(-)$ labels the lower (upper) band. The Kondo interaction enables intra- and interband scatterings. We will use the functional integral formulation of the theory on the imaginary-time contour and analyze the three cases shown in Fig. 2. The localized spins in this approach are conveniently parametrized by a normalized vector field [54].

Strong tunneling, $J \ll 2t \ll t_\perp$. If μ belongs to the lower band and $T = 0$, transitions between the bands are virtual and result only in a small renormalization of parameters of the conduction band [55]. To show this, we integrate out the fermions from the upper band perturbatively. This yields a mass term for the propagator of the conduction electrons from the lower band: $\Sigma_- = J^2(S^b_-)^2\langle\psi_-\bar{\psi}_-\rangle \simeq -(J^2/2t_\perp)(S^b_-)^2 + O(J^2/t^2_\perp)$ [56]; $\psi_\pm$ are fermionic fields. Σ_- governs a shift of μ and enables a weak spin conserving backscattering. Both effects are parametrically small compared to those governed by the intraband Kondo interaction. Therefore, the interband transitions can be neglected, and the Lagrangian density of the electrons in the lower band reduces to

$$\mathcal{L}^{(\rm ST)}_+ \simeq \bar{\psi}_+[-i\omega + \epsilon_+(k) - \mu + (J\rho_s/2)S^b_+\sigma_b]\psi_+, \qquad (3)$$

where ω is the fermionic Matsubara frequency and ρ_s is the spin density. Below, we will change to the continuous limit with $\rho_s = $ const and absorb ρ_s in the coupling constant: $J' \equiv J\rho_s/2$. Equation (3) describes a single KC where the itinerant electrons interact with the composite spins $\mathbf{S}_+$. This theory can be studied by using the approach developed in Refs. [30,31] for 1D KC. It can be straightforwardly proven that model (3) supports protected helical transport.

Intermediate tunneling, $t_\perp \lesssim 2t$. Let us analyze the case in which four Fermi points (two in the lower band and two in the upper band with Fermi momenta $\pm k^{(\pm)}_F$, respectively) coexist and are well separated, $\delta k_F \equiv k^{(+)}_F - k^{(-)}_F \sim \tilde{k}_F \equiv (k^{(+)}_F + k^{(-)}_F)/2$. We have to single out slow modes. We linearize the dispersion relation of the noninteracting system around the Fermi points and introduce smooth left (L) and right (R) moving modes in a standard way. These fermionic modes are described by the Lagrangian $\mathcal{L}_0 = \bar{R}_\nu \partial_+^{(\nu)} R_\nu + \bar{L}_\nu \partial_-^{(\nu)} L_\nu$, with $\partial_\pm^{(\nu)} = \partial_\tau \mp i v_F^{(\nu)} \partial_x$ being the chiral derivative. The Fermi velocity depends on the band index: $v_F^{(\nu)} = 2t\xi_0 \sin(k_F^{(\nu)}\xi_0)$.

The physics of the dense KCs is governed by backscattering of the fermions [30–33] described by

$$\mathcal{L}_{\rm bs}^{\nu\nu'} = J' R_\nu^\dagger S_\pm^b \sigma_b L_{\nu'} e^{2ik_F^{\nu\nu'}x} + \text{H.c.} \qquad (4)$$

$k_F^{\nu\nu'} = \tilde{k}_F$ for the interband backscattering (S_-), $\nu = -\nu'$, and $k_F^{\nu\nu'} = \tilde{k}_F + \nu\delta k_F/2$ for the intraband one (S_+), $\nu = \nu'$. Backscattering opens a gap in the spectrum of fermions and thus reduces the ground-state (GS) energy of the entire system [30–33].

We are interested in the low-energy physics whose Lagrangian does not contain $2k_F$ oscillations. Our strategy is to absorb them into spin configurations and find the configuration that minimizes the GS energy by maximizing backscattering. We decompose the spin variables into slow and fast components [56]:

$$\mathbf{S}^{(n)}/s = \mathbf{m}_n + \left[\mathbf{e}_1^{(n)} \cos(Qx) + \mathbf{e}_2^{(n)} \sin(Qx)\right]\sqrt{1-\mathbf{m}_n^2}.$$

Here $\mathbf{m}_n = \sin(\alpha^{(n)})[\mathbf{e}_1^{(n)} \times \mathbf{e}_2^{(n)}]$, $2k_F^{(-)} \leqslant Q \leqslant 2k_F^{(+)}$, and $\mathbf{e}_{1,2}^{(n)}$ are two orthonormal vectors that lie almost in the plane defined by the magnetic anisotropy ("easy plane"). These two vectors are parametrized by the in-plane polar angle, $\psi^{(n)}$, and by another angle describing small out-of-plane fluctuations, $\theta^{(n)}$. Oscillating terms allow one to absorb $2k_F$ oscillations from the backscattering and thus are needed to minimize the GS energy. The angle $\alpha^{(n)}$ weighs the zero mode $\mathbf{m}_n$ and has the semiclassical value $\alpha_{\rm cl}^{(n)} \to 0$. Deviations of $\alpha^{(n)}$ from this value are small. $\theta^{(n)}$ and $\alpha^{(n)}$ are massive variables and they can be integrated out in the Gaussian approximation [30,31].

Equation (4) contains oscillations with three different wave vectors, $2k_F^{(+)} = 2k_F^{(++)}$, $2k_F^{(-)} = 2k_F^{(--)}$, and $2\tilde{k}_F = 2|k_F^{(+-)}|$, which are of the same order in the intermediate tunneling regime and correspond to various intra- and interband scatterings; see Fig. 3. By tuning Q, one can absorb into the spin configuration only one of these vectors; the other two result in fast oscillations that do not contribute to the low-energy theory. The remaining smooth part of the backscattering opens the helical gap [see Eq. (6) below] in the fermionic spectrum. If $Q = 2k_F^{(\pm)}$, the gap is opened only in one (either lower or upper) band. The choice $Q = 2\tilde{k}_F$ results in doubling the number of gapped fermionic modes. Moreover, it provides the maximal value of all gaps [56]. We thus conclude that the GS energy reaches its minimum at $Q = 2\tilde{k}_F$. After inserting this choice into Eq. (4) and neglecting oscillating terms, we arrive at

$$\mathcal{L}_{bs}^{\nu\nu'} \simeq R_\nu^\dagger[\hat{\Delta}^{(1)} - \hat{\Delta}^{(2)}]L_{\nu'} + \text{H.c.}, \quad \nu \neq \nu', \qquad (5)$$

where $\hat{\Delta}^{(n)}$ are scattering amplitudes of the respective 1D KCs [30–33]: $\hat{\Delta}^{(n)}/\tilde{J} = e^{i\psi^{(n)}} \sin^2(\theta^{(n)}/2)\sigma_- - e^{-i\psi^{(n)}}\cos^2(\theta^{(n)}/2)\sigma_+$; $\tilde{J} = s\cos(\alpha^{(n)})J'$. Next, we use the classical value $\alpha_{\rm cl}^{(n)} = 0$ and look for the classical value of $\theta^{(n)}$. We anticipate that $\theta_{\rm cl}^{(n)} = 0$ or π [30,31].

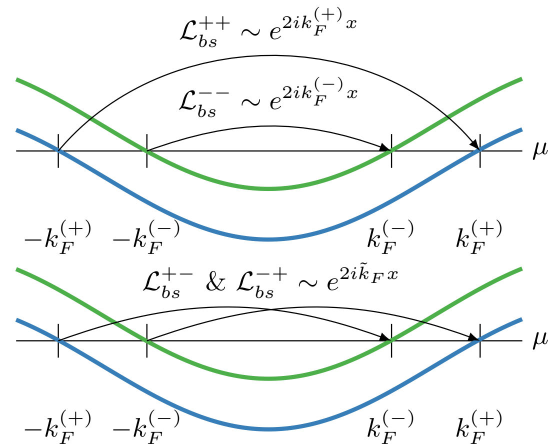

FIG. 3. Upper/lower panels: intraband/interband scattering processes and corresponding oscillating factors in Eq. (4).

The gap values (at fixed angles $\psi^{(n)}$) are different in the cases $\theta^{(1)} = \theta^{(2)}$ and $\theta^{(1)} \neq \theta^{(2)}$. For example, if $\theta^{(1)} = 0$, then

$$\theta^{(2)} = 0 : \hat{m}_- = 2i\tilde{J}e^{-i\tilde{\psi}}\sin(\delta\psi)\sigma_+, \qquad (6)$$

$$\theta^{(2)} = \pi : \hat{m}_- = -\tilde{J}(e^{i\psi^{(2)}}\sigma_- + e^{-i\psi^{(1)}}\sigma_+). \qquad (7)$$

Here $\hat{m}_- \equiv \hat{\Delta}^{(1)} - \hat{\Delta}^{(2)}$, $\tilde{\psi} \equiv (\psi^{(1)} + \psi^{(2)})/2$, and $\delta\psi \equiv (\psi^{(1)} - \psi^{(2)})/2$. The modulus of the eigenvalue of $\hat{m}$ reaches maximum in Eq. (6) at $\delta\psi = \pm\pi/2$ and becomes twice as large as that in Eq. (7). Therefore, we come across a mode locking of the in-plane spin polar angles that makes the spin configuration $\theta^{(1)} = \theta^{(2)}$ energetically favorable [56]. The phase factor $\tilde{\psi}$ in Eq. (6) can be gauged out. This leads to the expression for the gain (with respect to the noninteracting case, $J = 0$) of the GS energy [56]:

$$\delta E^{(\rm IT)} = -[4\xi_0\tilde{J}^2/\pi(v_{F_+} + v_{F_-})]\ln(2t/|\tilde{J}|). \qquad (8)$$

The analysis of the GS shows that the helical symmetry is spontaneously broken and a gap opens for fermions with a given helicity in both bands. As a result, we find gapless helical fermions with $h = -1$ for $\theta^{(1,2)} = 0$ (or $h = +1$ for $\theta^{(1,2)} = \pi$).

To finalize the derivation of the effective low-energy theory, we reinstate the Wess-Zumino term for the spin variables [54] and integrate out all massive fields approximately [56]. This yields the Lagrangian $\mathcal{L}^{(\rm IT)} = \mathcal{L}_{\rm LL}[\tilde{\psi}] + \sum_{\nu=\pm}\mathcal{L}_0[R_{\nu\downarrow}, L_{\nu\uparrow}]$.

Here $\mathcal{L}_{\rm LL} = [(\partial_\tau\tilde{\psi})^2 + (v_\psi\partial_x\tilde{\psi})^2]/2\pi K_\psi$ is the Luttinger liquid Lagrangian, which describes the slow, $v_\psi \ll v_F$, collective bosonic helical mode with the effective strong interactions, $K_\psi \ll 1$. Gapless fermionic modes have the same helicity in each band, $\nu = \pm$. This parametrically suppresses Anderson localization, which can be induced by an additional spinless disorder, with the disorder strength being $<J$ [56]. Thus, transport in these systems is protected by the helicity and remains ballistic in parametrically long samples.

Weak tunneling, $t_\perp \leqslant J \ll 2t$. If $t_\perp$ is small, the separation between the Fermi points shrinks and they almost coincide in pairs when $\delta k_F \approx 2t_\perp/\tilde{v}_F \ll \tilde{k}_F$. We start again from Eq. (4);

however, unlike the intermediate tunneling, δk_F oscillations are slow and cannot be neglected in the low-energy sector. This makes the number of gapped fermionic modes independent of the choice of Q. We retain $Q = 2\tilde{k}_F$ for convenience and repeat the steps resulting in Eq. (5). Slow δk_F oscillations yield now additional intraband scattering terms:

$$\mathcal{L}_{\rm bs}^{\nu\nu} \simeq R_\nu^\dagger[\hat{\Delta}^{(1)} + \hat{\Delta}^{(2)}]L_\nu e^{i\nu\delta k_F x} + \text{H.c.} \tag{9}$$

The slowly oscillating backscattering opens a gap at the energy that is shifted by $\delta k_F v_F$ from μ, leading to a small number of occupied (or empty) states above (or below) the gap [32,33]. These states are energetically split off by the gap and thus have no noticeable influence on the dc transport.

Next, we use the value $\alpha_{\rm cl}^{(n)} = 0$ and look for the optimal spin configuration with $\theta_{\rm cl}^{(n)} = 0$ or π. The intraband scattering introduces a new gap structure. In addition to Eqs. (6) and (7), we find for $\theta^{(1)} = 0$

$$\theta^{(2)} = 0 : \hat{m}_+ = -2\tilde{J}e^{-i\tilde{\psi}}\cos(\delta\psi)\sigma_+, \tag{10}$$

$$\theta^{(2)} = \pi : \hat{m}_+ = \tilde{J}(e^{i\psi^{(2)}}\sigma_- - e^{-i\psi^{(1)}}\sigma_+), \tag{11}$$

where $\hat{m}_+ = \hat{\Delta}^{(1)} + \hat{\Delta}^{(2)}$. We integrate out the gapped fermions, expand the result perturbatively in $t_\perp/t \ll 1$, and find the expression for the (relative) GS energy of the weakly coupled KCs [56]:

$$\begin{aligned}\delta E^{(\rm WT)} \approx & -(2\xi_0\tilde{J}^2/\pi\tilde{v}_F)\ln(2t/|\tilde{J}|) \\ & \times \{1 + [t_\perp/t\ \sin(\tilde{k}_F\xi_0)]^2 \\ & \times [1 + 2\cos^2(\delta\psi)\cot^2(\tilde{k}_F\xi_0)]\},\end{aligned} \tag{12}$$

with $\tilde{v}_F = v_F(\tilde{k}_F)$ and $\xi_0 t_\perp/\tilde{v}_F \ll 1$. The energy gain due to the mode locking, $\delta\psi \simeq 0$ or π, manifests itself in Eq. (12) starting from the term $O(t_\perp/t)^2$ and guarantees that the helical phase provides the minimum of the GS energy. The low-energy theory is described by $\mathcal{L}^{(\rm IT)}$ with $v_{F_+} = v_{F_-} = \tilde{v}_F$. We conclude that transport is helical and protected in weakly coupled KCs.

Vanishing tunneling. If $t_\perp \ll J \ll 2t$, the perturbative corrections to the GS energy in Eq. (12) become beyond the accuracy of calculations. If one naively neglects them, our model is reduced to two uncoupled 1D KCs whose GS is degenerate, either $\theta^{(1)} = \theta^{(2)} = 0$ or $\theta^{(1)} = 0,\ \theta^{(2)} = \pi$. The latter configuration corresponds to the phase where gapless fermions have opposite helicity in different wires. Clearly, two channels with opposite helicity form a usual (nonhelical) spinful conducting channel where transport is not protected. However, this artificial degeneracy does not mean there is a violation of the helical protection, which can be reinstated via cumbersome analysis with higher accuracy. We prefer to avoid unnecessary technical complications. To this end, we note that even a weak intrinsic Dresselhaus SOI [57], which typically exists in GaAs quantum wires, removes this ambiguity and generates corrections to $\delta E^{(\rm WT)}$, which again drive the system to the helical phase with protected transport [56].

Conclusions. We have shown that strict one-dimensionality is not a necessary prerequisite for the formation of a helical phase with protected transport in nanowires functionalized by magnetic adatoms. To demonstrate this statement, we have studied the simplest theoretical model of two dense magnetically anisotropic 1D Kondo chains coupled by the interchain tunneling of itinerant electrons. The anisotropy simplifies calculations; however, preliminary analysis shows that our conclusions remain valid also in the isotropic case. The ground state of our model is manifestly helical when the interchain tunneling is larger or of the order of the exchange coupling between the itinerant electrons and localized spins. The latter, in turn, must be much smaller than the width of the conduction band, but much larger than the temperature, $T \ll J \ll t$. These conditions are natural for experimental setups in which J can be tuned by using various magnetic adatoms and changing their density and proximity to the quantum wire. Small fluctuations of the Kondo couplings $J_{1,2} = \tilde{J} \pm \delta J,\ \delta J/\tilde{J} \ll 1$ cannot change our conclusions [56].

The global helicity is provided by the indirect (intra- and interwire) interaction between the localized spins. Adding more chains to the model can make the spin interaction weaker when the system approaches the 2D limit, but it cannot violate the helical protection in the quasi-1D samples. Our predictions are also stable with respect to a weak or moderate Coulomb interaction of the electrons; cf. Ref. [31]: the electrostatic repulsion enhances the RKKY interaction and thus can only make interspin correlations and the helical protection of transport stronger. Thus, our results substantially expand predictions made for purely 1D wires [30,31], and they could facilitate experimental studies of protected transport in various magnetically doped nanostructures.

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[1] T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, 2003).
[2] S.-Q. Shen, *Topological Insulators: Dirac Equation in Condensed Matter* (Springer, Singapore, 2017).
[3] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. **82**, 3045 (2010).
[4] X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. **83**, 1057 (2011).
[5] B. A. Bernevig and S.-C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006).
[6] B. A. Bernevig, T. L. Hughes, and S.-C. Zhang, Science **314**, 1757 (2006).
[7] C. Liu, T. L. Hughes, X.-L. Qi, K. Wang, and S.-C. Zhang, Phys. Rev. Lett. **100**, 236601 (2008).
[8] I. K. Drozdov, A. Alexandradinata, S. Jeon, S. Nadj-Perge, H. Ji, R. J. Cava, B. A. Bernevig, and A. Yazdani, Nat. Phys. **10**, 664 (2014).
[9] A. Murani, A. Kasumov, S. Sengupta, Y. A. Kasumov, V. T. Volkov, I. I. Khodos, F. Brisset, R. Delagrange, A.

Chepelianskii, R. Deblock, H. Bouchiat, and S. Guéron, Nat. Commun. **8**, 15941 (2017).
[10] F. Schindler, Z. Wang, M. G. Vergniory, A. M. Cook, A. Murani, S. Sengupta, A. Y. Kasumov, R. Deblock, S. Jeon, I. Drozdov, H. Bouchiat, S. Guéron, A. Yazdani, B. A. Bernevig, and T. Neupert, Nat. Phys. **14**, 918 (2018).
[11] I. Knez, R.-R. Du, and G. Sullivan, Phys. Rev. Lett. **107**, 136603 (2011).
[12] I. Knez, C. T. Rettner, S.-H. Yang, S. S. P. Parkin, L. J. Du, R. R. Du, and G. Sullivan, Phys. Rev. Lett. **112**, 026602 (2014).
[13] E. M. Spanton, K. C. Nowack, L. J. Du, G. Sullivan, R. R. Du, and K. A. Moler, Phys. Rev. Lett. **113**, 026804 (2014).
[14] M. König, S. Wiedmann, C. Brüne, A. Roth, H. Buhmann, L. W. Molenkamp, X.-L. Qi, and S.-C. Zhang, Science **318**, 766 (2007).
[15] M. König, H. Buhmann, L. W. Molenkamp, T. Hughes, C.-X. Liu, X.-L. Qi, and S.-C. Zhang, J. Phys. Soc. Jpn. **77**, 031007 (2008).
[16] J. I. Väyrynen, F. Geissler, and L. I. Glazman, Phys. Rev. B **93**, 241301(R) (2016).
[17] B. L. Altshuler, I. L. Aleiner, and V. I. Yudson, Phys. Rev. Lett. **111**, 086401(R) (2013).
[18] O. M. Yevtushenko, A. Wugalter, V. I. Yudson, and B. L. Altshuler, Europhys. Lett. **112**, 57003 (2015).
[19] O. M. Yevtushenko and V. I. Yudson, arXiv:1909.08460.
[20] B. Braunecker, P. Simon, and D. Loss, Phys. Rev. Lett. **102**, 116403 (2009).
[21] J. Klinovaja, P. Stano, A. Yazdani, and D. Loss, Phys. Rev. Lett. **111**, 186805 (2013).
[22] B. Braunecker, P. Simon, and D. Loss, Phys. Rev. B **80**, 165119 (2009).
[23] J. Klinovaja, G. J. Ferreira, and D. Loss, Phys. Rev. B **86**, 235416 (2012).
[24] J. Klinovaja, M. J. Schmidt, B. Braunecker, and D. Loss, Phys. Rev. Lett. **106**, 156809 (2011).
[25] C. H. L. Quay, T. L. Hughes, J. A. Sulpizio, L. N. Pfeiffer, K. W. Baldwin, K. W. West, D. Goldhaber-Gordon, and R. de Picciotto, Nat. Phys. **6**, 336 (2010).
[26] S. Heedt, N. Traverso Ziani, F. Crépin, W. Prost, S. Trellenkamp, J. Schubert, D. Grützmacher, B. Trauzettel, and T. Schäpers, Nat. Phys. **13**, 563 (2017).
[27] N. Kainaris and S. T. Carr, Phys. Rev. B **92**, 035139 (2015).
[28] C. P. Scheller, T.-M. Liu, G. Barak, A. Yacoby, L. N. Pfeiffer, K. W. West, and D. M. Zumbühl, Phys. Rev. Lett. **112**, 066801 (2014).
[29] J. Kammhuber, M. C. Cassidy, F. Pei, M. P. Nowak, A. Vuik, Ö. Gül, D. Car, S. R. Plissard, E. P. A. M. Bakkers, M. Wimmer, and L. P. Kouwenhoven, Nat. Commun. **8**, 478 (2017).
[30] A. M. Tsvelik and O. M. Yevtushenko, Phys. Rev. Lett. **115**, 216402 (2015).
[31] D. H. Schimmel, A. M. Tsvelik, and O. M. Yevtushenko, New J. Phys. **18**, 053004 (2016).
[32] A. M. Tsvelik and O. M. Yevtushenko, Phys. Rev. B **100**, 165110 (2019).
[33] A. M. Tsvelik and O. M. Yevtushenko, New J. Phys. **22**, 053013 (2020).
[34] B. E. Feldman, M. T. Randeria, J. Li, S. Jeon, Y. Xie, Z. Wang, I. K. Drozdov, B. Andrei Bernevig, and A. Yazdani, Nat. Phys. **13**, 286 (2017).
[35] M. M. Desjardins, L. C. Contamin, M. R. Delbecq, M. C. Dartiailh, L. E. Bruhat, T. Cubaynes, J. J. Viennot, F. Mallet, S. Rohart, A. Thiaville, A. Cottet, and T. Kontos, Nat. Mater. **18**, 1060 (2019).
[36] B. Jäck, Y. Xie, J. Li, S. Jeon, B. A. Bernevig, and A. Yazdani, Science **364**, 1255 (2019).
[37] L. Pfeiffer, H. L. Störmer, K. W. Baldwin, K. W. West, A. R. Goñi, A. Pinczuk, R. C. Ashoori, M. M. Dignam, and W. Wegscheider, J. Cryst. Growth **127**, 849 (1993).
[38] R. Mizokuchi, R. Maurand, F. Vigneau, M. Myronov, and S. De Franceschi, Nano Lett. **18**, 4861 (2018).
[39] H. Tsunetsugu, M. Sigrist, and K. Ueda, Rev. Mod. Phys. **69**, 809 (1997).
[40] G. Honner and M. Gulácsi, Phys. Rev. Lett. **78**, 2180 (1997).
[41] E. Novais, E. Miranda, A. H. Castro Neto, and G. G. Cabrera, Phys. Rev. B **66**, 174409 (2002).
[42] N. Shibata and K. Ueda, J. Phys.: Condens. Matter **11**, R1 (1999).
[43] J. C. Xavier and E. Miranda, Phys. Rev. B **70**, 075110 (2004).
[44] J. Maciejko, Phys. Rev. B **85**, 245108 (2012).
[45] E. Novais, E. Miranda, A. H. Castro Neto, and G. G. Cabrera, Phys. Rev. Lett. **88**, 217201 (2002).
[46] M. Troyer and D. Würtz, Phys. Rev. B **47**, 2886 (1993).
[47] O. Zachar, S. A. Kivelson, and V. J. Emery, Phys. Rev. Lett. **77**, 1342 (1996).
[48] S. Doniach, Physica B+C **91**, 231 (1977).
[49] P. Coleman, Heavy fermions: Electrons at the edge of magnetism, in *Handbook of Magnetism and Advanced Magnetic Materials*, edited by H. Kronmüller, S. Parkin, M. Fähnle, S. Maekawa, and I. Zutic (American Cancer Society, 2007).
[50] O. M. Yevtushenko and V. I. Yudson, Phys. Rev. Lett. **120**, 147201 (2018).
[51] We remind readers that, in the 1D case, spins form a helix in a transverse plane. This is reflected by the spin susceptibility $\langle S_l^+ S_m^- \rangle \sim e^{\pm 2ik_F \xi_s |l-m|}$, $S^\pm = S^x \pm iS^y$, which has either a $+2k_F$ or $-2k_F$ component depending on the helix handedness [31] .
[52] We implicitly assume summation over all repeated indices.
[53] The new band operators $c_\pm = \frac{1}{\sqrt{2}}(c_1 \pm c_2)$ are the (anti)symmetric linear combinations of the old operators. The lower + band is thus accompanied by a downward shift in energy $\varepsilon_+ = \varepsilon_0 - t_\perp$ and vice versa.
[54] A. Tsvelik, *Quantum Field Theory in Condensed Matter Physics* (Cambridge University Press, Cambridge, 2003).
[55] The case in which the chemical potential belongs to the upper band can be treated analogously.
[56] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.102.161102 for a derivation of the effective Lagrangian in the strong tunneling limit; the separation of fast and slow spin variables; a derivation of the ground-state energy equation in the intermediate and weak tunneling regime; the computation of the ground-state energy of gapped 1D Dirac fermions; the renormalization of the Luttinger parameter K_ψ; spinless disorder; the effect of Dresselhaus spin-orbit interaction; and asymmetrically doped Kondo chains.
[57] G. Dresselhaus, Phys. Rev. **100**, 580 (1955).