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# Exact mapping of the 2+1 Dirac oscillator onto the Jaynes-Cummings model: Ion-trap experimental proposal

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We study the dynamics of the 2+1 Dirac oscillator exactly and find spin oscillations due to a *Zitterbewegung* of purely relativistic origin. We find an exact mapping of this quantum-relativistic system onto a Jaynes-Cummings model, describing the interaction of a two-level atom with a quantized single-mode field. This equivalence allows us to map a series of quantum optical phenomena onto the relativistic oscillator and vice versa. We make a realistic experimental proposal, in reach with current technology, for studying the equivalence of both models using a single trapped ion.

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Current technology has allowed the implementation of the paradigmatic nonrelativistic quantum harmonic oscillator in a single trapped ion [1], one of the most fundamental toy models in any quantum mechanical textbook. However, its relativistic version, the so-called Dirac oscillator [2,3], remains still far from any possible experimental consideration for different fundamental and technical reasons. We will show here that available experimental tools may allow the implementation of the relativistic Dirac oscillator in a single nonrelativistic trapped ion.

The Dirac oscillator was introduced as an instance of a relativistic wave equation such that its nonrelativistic limit leads to the well-known Schrödinger equation for the harmonic oscillator. This is achieved by introducing the following coupling in the Dirac equation:

$$i\hbar\frac{\partial|\Psi\rangle}{\partial t}=[c\boldsymbol{\alpha}(\mathbf{p}-\mathrm{i}m\beta\omega\mathbf{r})+\beta mc^2]|\Psi\rangle, \qquad (1)$$

where $|\Psi\rangle$ is the Dirac four-component bispinor corresponding to a quantum relativistic spin-$\frac{1}{2}$ particle, like the electron, $c$ is the speed of light, $m$ is the particle rest mass, and $\alpha_j$ and $\beta$ are Dirac matrices in the standard representation. The interacting Hamiltonian is linear in both momentum $p^j$ and position $r^j$, $j=x,y,z$, and $\omega$ turns out to be the harmonic oscillator frequency. We remark that when $\omega=0$ we recover the standard Dirac equation [4]. The Dirac oscillator looks like a particular gauge transformation $\mathbf{p}\rightarrow\mathbf{p}-\frac{e}{c}\mathbf{A}$ that is linear in position, but the presence of the i and the $\beta$ matrix makes a crucial difference. Demanding the correct energy-momentum relation for a relativistic free particle $E=\sqrt{p^2c^2+m^2c^4}$, these matrices are $4\times4$ dimensional and must obey a Clifford algebra given by the anticommutation relations

$$\alpha_j\alpha_k+\alpha_k\alpha_j=2\delta_{jk},$$

$$\alpha_j\beta+\beta\alpha_j=0. \qquad (2)$$

There has been a growing interest in simulating quantum relativistic effects in other physical systems, such as black hole evaporation in Bose-Einstein condensates [5] and the Unruh effect in an ion chain [6]. Another astonishing relativistic prediction is the *Zitterbewegung* [4], a helicoidal motion realized by the average position of a relativistic fermion, which has been discussed in the context of condensed matter systems [7] and the free-particle Dirac equation in a single ion [8].

Here, we shall be concerned with the Dirac oscillator in 2+1 dimensions, since it is in this setting where we can establish a precise equivalence with the Jaynes-Cummings (JC) model [9]. In two spatial dimensions, the solution to the Clifford algebra (2) is given by the $2\times2$ Pauli matrices $\alpha_x=\sigma_x$, $\alpha_y=\sigma_y$, and $\beta=\sigma_z$. In this case, $|\Psi\rangle$ can be described by a two-component spinor which mixes spin-up and -down components with positive and negative energies, and the Dirac oscillator equation is

$$i\hbar\frac{\partial|\Psi\rangle}{\partial t}=\left[\sum_{j=1}^{2}c\sigma_j(p^j-\mathrm{i}m\sigma_z\omega r^j)+\sigma_z mc^2\right]|\Psi\rangle. \qquad (3)$$

In this paper, we shall provide the complete (eigenstates and energies) and exact solution of the two-dimensional (2D) Dirac oscillator in order to study its relativistic dynamics, where certain collapses and revivals in the spin degree of freedom appear as a consequence of *Zitterbewegung*. In addition, we derive an exact mapping of the 2+1 Dirac oscillator onto the JC model, an archetypical quantum optical system. Furthermore, we propose the simulation of this relativistic dynamics in a single trapped ion, a physical setup possessing outstanding coherence features.

Considering the spinor $|\Psi\rangle:=[|\psi_1\rangle,|\psi_2\rangle]^t$, Eq. (3) becomes a set of coupled equations

$$(E-mc^2)|\psi_1\rangle=c[(p_x+\mathrm{i}m\omega x)-\mathrm{i}(p_y+\mathrm{i}m\omega y)]|\psi_2\rangle,$$

$$(E+mc^2)|\psi_2\rangle=c[(p_x-\mathrm{i}m\omega x)+\mathrm{i}(p_y-\mathrm{i}m\omega y)]|\psi_1\rangle. \qquad (4)$$

In order to find the solutions, it is convenient to introduce the following chiral creation and annihilation operators:

$$a_r:=\frac{1}{\sqrt{2}}(a_x-\mathrm{i}a_y),\quad a_r^\dagger:=\frac{1}{\sqrt{2}}(a_x^\dagger+\mathrm{i}a_y^\dagger),$$

 

$$a_l := \frac{1}{\sqrt{2}}(a_x + \mathrm{i}a_y), \quad a_l^\dagger := \frac{1}{\sqrt{2}}(a_x^\dagger - \mathrm{i}a_y^\dagger), \tag{5}$$

where $a_x$, $a_x^\dagger$, $a_y$, and $a_y^\dagger$, are the usual annihilation and creation operators of the harmonic oscillator $a_i^\dagger = \frac{1}{\sqrt{2}}\left(\frac{1}{\Delta}r^i - \mathrm{i}\Delta/\hbar p^i\right)$ and $\Delta = \sqrt{\hbar/m\omega}$ represents the ground-state oscillator width. The orbital angular momentum may also be expressed as

$$L_z = \hbar(a_r^\dagger a_r - a_l^\dagger a_l), \tag{6}$$

which leads to a physical interpretation of $a_r^\dagger$ and $a_l^\dagger$. These operators create a right or left quantum of angular momentum, respectively, and are known hence as circular creation-annihilation operators. Equations (4) can be rewritten in the language of these circular operators

$$|\psi_1\rangle = \mathrm{i}\frac{2mc^2\sqrt{\xi}}{E - mc^2}a_l^\dagger|\psi_2\rangle,$$

$$|\psi_2\rangle = -\mathrm{i}\frac{2mc^2\sqrt{\xi}}{E + mc^2}a_l|\psi_1\rangle, \tag{7}$$

where $\xi := \hbar\omega/mc^2$ controls the nonrelativistic limit. In order to find the energy spectrum we obtain the associated Klein-Gordon equation from Eqs. (7) as follows:

$$(E^2 - m^2c^4)|\psi_1\rangle = 4m^2c^4\xi a_l^\dagger a_l|\psi_1\rangle,$$

$$(E^2 - m^2c^4)|\psi_2\rangle = 4m^2c^4\xi(1 + a_l^\dagger a_l)|\psi_2\rangle. \tag{8}$$

These equations can be simultaneously diagonalized writing the spinor in terms of the left chiral quanta basis $|n_l\rangle = \frac{1}{\sqrt{n_l!}}(a_l^\dagger)^{n_l}|\mathrm{vac}\rangle$, where $n_l = 0, 1, \ldots$,

$$(E_{n_l}^2 - m^2c^4)|n_l\rangle = 4m^2c^4\xi n_l|n_l\rangle,$$

$$(E_{n_l'}^2 - m^2c^4)|n_l'\rangle = 4m^2c^4\xi(1 + n_l')|n_l'\rangle. \tag{9}$$

Since both components $|\psi_1\rangle$ and $|\psi_2\rangle$ belong to the same solution, the energies must be the same $E_{n_l'} = E_{n_l}$. This physical requirement sets up a constraint on the quantum numbers $n_l =: n_l' + 1$. Note that, following Eq. (6), the state $|n_l\rangle$ corresponds to a negative angular momentum. The energy spectrum can be described as follows:

$$E = \pm E_{n_l} = \pm mc^2\sqrt{1 + 4\xi n_l}. \tag{10}$$

To find the corresponding eigenstates, we go back to Eq. (7), and after normalization we arrive at the expression for the positive and negative energy eigenstates:

$$|\pm E_{n_l}\rangle = \begin{bmatrix} \sqrt{\dfrac{E_{n_l} \pm mc^2}{2E_{n_l}}}|n_l\rangle \\ \mp\mathrm{i}\sqrt{\dfrac{E_{n_l} \mp mc^2}{2E_{n_l}}}|n_l - 1\rangle \end{bmatrix}, \tag{11}$$

where the quantum number is now restricted to $n_l = 1, 2, \ldots$. In this way, we have solved the two-dimensional Dirac oscillator describing the energy spectrum and the eigenstates in terms of circular quanta. The distinction between Dirac and Klein-Gordon eigenstates is an important point in order to understand the dynamics of the $2+1$ Dirac oscillator and its realization in an ion trap.

These eigenstates (11) can be expressed transparently in terms of two-component Pauli spinors $|\chi_\uparrow\rangle$ and $|\chi_\downarrow\rangle$:

$$|+E_{n_l}\rangle = \alpha_{n_l}|n_l\rangle|\chi_\uparrow\rangle - \mathrm{i}\beta_{n_l}|n_l - 1\rangle|\chi_\downarrow\rangle,$$

$$|-E_{n_l}\rangle = \beta_{n_l}|n_l\rangle|\chi_\uparrow\rangle + \mathrm{i}\alpha_{n_l}|n_l - 1\rangle|\chi_\downarrow\rangle, \tag{12}$$

where $\alpha_{n_l} := \sqrt{\frac{E_{n_l}+mc^2}{2E_{n_l}}}$ and $\beta_{n_l} := \sqrt{\frac{E_{n_l}-mc^2}{2E_{n_l}}}$ are real. From this expression we observe that the energy eigenstates present entanglement between the orbital and spin degrees of freedom. This property is extremely important since the initial state $|\Psi(0)\rangle := |n_l - 1\rangle|\chi_\downarrow\rangle = \mathrm{i}\beta_{n_l}|+E_{n_l}\rangle - \mathrm{i}\alpha_{n_l}|-E_{n_l}\rangle$ superposes positive- and negative-energy components, and this is the fundamental ingredient that leads to *Zitterbewegung* in relativistic quantum dynamics. This phenomenon, due to the interference of positive and negative energies, has never been observed experimentally. The reason is that the amplitude of these rapid oscillations lies below the Compton wavelength, where pair creation is allowed, and the one-particle interpretation falls down.

Now, the evolution of this initial state can be expressed as $|\Psi(t)\rangle = \mathrm{i}\beta_{n_l}|+E_{n_l}\rangle e^{-\mathrm{i}\omega_{n_l}t} - \mathrm{i}\alpha_{n_l}|-E_{n_l}\rangle e^{\mathrm{i}\omega_{n_l}t}$, where

$$\omega_{n_l} := \frac{E_{n_l}}{\hbar} = \frac{mc^2}{\hbar}\sqrt{1 + 4\xi n_l} \tag{13}$$

describes the frequency of oscillations. Writing this evolved state in the language of Pauli spinors,

$$|\Psi(t)\rangle = \left(\cos\omega_{n_l}t + \frac{\mathrm{i}}{\sqrt{1 + 4\xi n_l}}\sin\omega_{n_l}t\right)|n_l - 1\rangle|\chi_\downarrow\rangle + \left(\sqrt{\frac{4\xi n_l}{1 + 4\xi n_l}}\sin\omega_{n_l}t\right)|n_l\rangle|\chi_\uparrow\rangle, \tag{14}$$

we observe an oscillatory dynamics between $|n_l\rangle|\chi_\uparrow\rangle$ and $|n_l - 1\rangle|\chi_\downarrow\rangle$. The initial state, $|n_l - 1\rangle|\chi_\downarrow\rangle$, which has spin down and $n_l - 1$ quanta of left orbital angular momentum, evolves exchanging a quantum of angular momentum from the spin to the orbital motion.

The dynamics described in Eq. (14) is completely similar to the atomic Rabi oscillations occurring in the Jaynes-Cummings model, though arising from a completely different reason. Whereas the Rabi oscillations in the Jaynes-Cummings model are caused by the interaction of a quantized electromagnetic field with a two-level atom, the relativistic oscillations are caused by the interference of positive- and negative-energy states and therefore constitute a clear signature of *Zitterbewegung* [4].

To clarify this issue further, we calculate the time evolution of the following physical observables, which catch the full essence of the system dynamics,

$$\langle L_z\rangle_t = -(n_l-1)\hbar - \frac{4\xi n_l}{1+4\xi n_l}\hbar\sin^2\omega_{n_l}t,$$

$$\langle S_z\rangle_t = -\frac{\hbar}{2} + \frac{4\xi n_l}{1+4\xi n_l}\hbar\sin^2\omega_{n_l}t,$$

$$\langle J_z\rangle_t = \hbar\left(\frac{1}{2}-n_l\right), \tag{15}$$

where $J_z=L_z+S_z$ stands for the $z$ component of the total angular momentum. The latter relations describe a certain oscillation in the spin and orbital angular momentum, while the total angular momentum is conserved due to the existent invariance under rotations around the $z$ axis. It is important to highlight that these oscillations have a pure relativistic nature. In the nonrelativistic limit $\xi \ll 1$, these oscillations become vanishingly small:

$$\langle L_z\rangle_t = -(n_l-1)\hbar - 4\xi n_l\hbar\sin^2\Omega_{n_l}t + O(\xi^2),$$

$$\langle S_z\rangle_t = -\frac{\hbar}{2} + 4\xi n_l\hbar\sin^2\Omega_{n_l}t + O(\xi^2), \tag{16}$$

where $\Omega_{n_l} := mc^2(1+2\xi n_l)/\hbar$ is the nonrelativistic oscillation frequency. In this limit, negative-energy components are negligible and the *Zitterbewung* disappears.

The results discussed so far allow a precise mapping between two seemingly unrelated models: the Jaynes-Cummings model of quantum optics and the 2D Dirac oscillator. Starting from Eq. (7), we may write the Dirac oscillator Hamiltonian as

$$\begin{aligned} H &= 2imc^2\sqrt{\xi}(a_l^\dagger|\psi_2\rangle\langle\psi_1| - a_l|\psi_1\rangle\langle\psi_2|) + mc^2\sigma_z \\ &= \hbar(g\sigma^- a_l^\dagger + g^*\sigma^+ a_l) + mc^2\sigma_z, \end{aligned} \tag{17}$$

where $\sigma^+$ and $\sigma^-$ are the spin raising and lowering operators and $g := 2imc^2\sqrt{\xi}/\hbar$ is the coupling strength between orbital and spin degrees of freedom. In quantum optics, this Hamiltonian describes a Jaynes-Cummings interaction, which has been studied in cavity QED and trapped ions [1,10], among others. Within this novel perspective, the electron spin can be associated with a two-level atom and the orbital circular quanta with the ion quanta of vibration—i.e., phonons. As we will see below, the central result of Eq. (17) allows both physical systems, the JC model and the 2D Dirac oscillator, to exchange a wide range of important applications.

We will show now how to implement the dynamics of Eq. (3) in a single ion inside a Paul trap, which was shown to follow the dynamics of Eq. (17). The Dirac spinor will be described by two metastable internal states $|g\rangle$ and $|e\rangle$ as follows $|\Psi\rangle := |\psi_1\rangle|e\rangle + |\psi_2\rangle|g\rangle$, while the circular angular momentum modes will be represented by two ionic vibrational modes $a_x$ and $a_y$. Current technology allows an overwhelming coherent control of ionic internal and external degrees of freedom [1]. There, three paradigmatic interactions, the carrier, red-sideband, and blue-sideband excitations, can be implemented at will, independently or simultaneously [11]. For example, using appropriately tuned lasers, it is possible to produce the interactions

$$H_i^{\mathrm{JC}} = \hbar\eta_i\tilde{\Omega}_i[\sigma^+ a_i e^{i\phi} + \sigma^- a_i^\dagger e^{-i\phi}] + \hbar\delta_i\sigma_z,$$

$$H_i^{\mathrm{AJC}} = \hbar\eta_i\tilde{\Omega}_i[\sigma^+ a_i^\dagger e^{i\varphi} + \sigma^- a_i e^{-i\varphi}], \tag{18}$$

where $\{a_i, a_i^\dagger\}$, with $i=x,y$, are the phonon annihilation and creation operators in directions $x$ and $y$, $\nu_i$ are the natural trap frequencies, $\eta_i := k_i\sqrt{\hbar/2M\nu_i}$ are the associated Lamb-Dicke parameters depending on the ion mass $M$ and the wave vector $\mathbf{k}$, $\delta_i$ and $\tilde{\Omega}_i$ are the excitation coupling strengths, and $\phi$ and $\varphi$ are the red- and blue-sideband phases. We remark that the term $\hbar\delta_i\sigma_z$, in $H_i^{\mathrm{JC}}$ of Eq. (18), stems from a detuned JC excitation.

A suitable combination of the above-introduced excitations (18), with proper couplings and relative phases, can reproduce the Hamiltonian

$$H = c[\sigma_x^{ge}p_x + \sigma_y^{ge}p_y] + m\omega c[\sigma_x^{ge}y - \sigma_y^{ge}x] + mc^2\sigma_z^{ge}, \tag{19}$$

with $\sigma_x^{ge} := |g\rangle\langle e| + |g\rangle\langle e|$, $\sigma_y^{ge} := -i(|e\rangle\langle g| - |e\rangle\langle g|)$, and $\sigma_z^{ge} := |e\rangle\langle e| - |g\rangle\langle g|$, with the following correspondence:

$$c = \sqrt{2}\eta\tilde{\Omega}\tilde{\Delta}, \quad mc^2 = \hbar\delta, \quad m\omega c = \hbar\sqrt{2}\eta\tilde{\Omega}\tilde{\Delta}^{-1}, \tag{20}$$

where $\tilde{\Delta} := \tilde{\Delta}_i$ is the width of the motional ground state, $\tilde{\Omega} := \tilde{\Omega}_i$, $\eta := \eta_i$, $\forall i=x,y$. The remarkable equivalence of the Dirac oscillator Hamiltonian (3) and the interaction in Eq. (19) shows that it is possible to reproduce the 2D Dirac oscillator, with all its quantum relativistic effects, in a controllable system as a single trapped ion.

For the sake of illustration, note that the effective terms appearing in Eq. (19) can be achieved by suitable linear combinations of $H_i^{\mathrm{JC}}$ and $H_i^{\mathrm{AJC}}$ in Eqs. (18),

$$i=x, \quad \delta_x=\delta, \quad \phi=\frac{3\pi}{2},$$

$$\varphi = \frac{\pi}{2} \rightarrow \sqrt{2}\hbar\eta\tilde{\Omega}\tilde{\Delta}\sigma_x^{ge}p_x + \hbar\delta\sigma_z^{ge},$$

$$i=y, \quad \delta_y=0, \quad \phi=0, \quad \varphi=\pi \rightarrow \sqrt{2}\hbar\eta\tilde{\Omega}\tilde{\Delta}\sigma_y^{ge}p_y,$$

$$i=x, \quad \delta_x=0, \quad \phi=\frac{\pi}{2}, \quad \varphi=\frac{\pi}{2} \rightarrow \sqrt{2}\hbar\eta\tilde{\Omega}\tilde{\Delta}^{-1}\sigma_y^{ge}x,$$

$$i=y, \quad \delta_y=0, \quad \phi=0, \quad \varphi=0 \rightarrow \sqrt{2}\hbar\eta\tilde{\Omega}\tilde{\Delta}^{-1}\sigma_x^{ge}y. \tag{21}$$

Note that in the trapped-ion picture, the important parameter $\xi=2(\eta\tilde{\Omega}/\delta)^2$ can take on all positive values, assuming available experimental parameters: $\eta\sim 0.1$, $\tilde{\Omega}\sim 0-10^6$ Hz, and $\delta\sim 0-10^6$ Hz [1]. The ability to experimentally tune these parameters will allow the experimenter to study otherwise inaccessible physical regimes that entail relativistic and nonrelativistic phenomena. Moreover, using similar techniques, the experimentalist could also introduce certain modifications to the relativistic Hamiltonian (19) that would entail novel phenomena.

Remarkably enough, the *Zitterbewegung* is encoded in the spin degree of freedom, and we can associate Rabi oscillations to the interference of positive- and negative-energy solutions. Setting the initial state $|0\rangle|\chi_{\downarrow}\rangle \leftrightarrow |0\rangle|g\rangle$, the evolution described in Eq. (15) leads to

$$\langle S_z \rangle_t = -\frac{\hbar}{2} + \frac{4\xi}{1+4\xi}\hbar \sin^2 \omega_1 t, \tag{22}$$

where $\omega_1 = \delta\sqrt{1+4\xi}$ [see Eq. (13)] stands for the frequency of the *Zitterbewegung* oscillations and can take on a wide variety of measurable values.

In order to simulate this dynamics in an ion-trap tabletop experiment, the ion must be cooled down to its vibrational ground state $|0\rangle$, with a current efficiency above 99% [1]. To estimate the observable (22), one can make use of the powerful tool called electron shelving, where $\langle S_z \rangle_t = \frac{\hbar}{2}[2P_e(t)-1]$ can be obtained through a measurement of the probability of obtaining the ionic excited state, $P_e(t)$. This is usually performed with extraordinary precision using current technology. This measurement technique amounts to a great advance with respect to the work in Ref. [8], where it was proposed to simulate relativistic effects with measurements based on the position of an ion simulating a free Dirac particle. In our proposal, the measurement is different and more feasible since it is based on spin-Zitterbewegung, and not position-Zitterbewegung.

Another fundamental result of the JC model is the existence of collapses and revivals in the atomic population, which is direct evidence of the quantization of the electromagnetic field. This effect can be mapped to the Dirac oscillator if the initial state $|z\rangle|g\rangle$ is prepared, where $|z\rangle$ is an initial circular coherent state, $|\Psi(0)\rangle = e^{-|z|^2/2}\sum_{n_l=0}^{\infty}\frac{z^{n_l}}{\sqrt{n_l!}}|n_l\rangle|g\rangle$, with $z \in \mathbb{C}$. After an interaction time $t$,

$$\langle S_z \rangle_t = -\frac{\hbar}{2} + \hbar\sum_{n_l=0}^{\infty}\frac{4\xi(n_l+1)|z|^{2n_l}e^{-|z|^2}}{[1+4\xi(n_l+1)]n_l!}\sin^2(\omega_{n_l+1}t). \tag{23}$$

This expression can be understood as an interference effect of terms with different frequencies $\omega_{n_l+1}$ leading to collapses and revivals. A novel feature of the Dirac oscillator is the appearance of these collapses and revivals in the orbital circular motion of the particle, reflected in

$$\langle L_z \rangle_t = -\hbar|z|^2 - \hbar\sum_{n_l=0}^{\infty}\frac{4\xi(n_l+1)|z|^{2n_l}e^{-|z|^2}}{[1+4\xi(n_l+1)]n_l!}\sin^2(\omega_{n_l+1}t). \tag{24}$$

The generation of an initial circular coherent state requires two sequential applications of the technique described in Ref. [1] on an initial motional ground state. These two operations must have a relative phase such that $D_l(z) = D_x(z)D_y(-\mathrm{i}z)$, where $D_j(z) = \mathrm{e}^{za_j^{\dagger} - z^* a_j}$, $j = x, y$. The observable of Eq. (23) can be measured via a similar electron-shelving technique, while the observable of Eq. (24) needs a mapping of the collective motional state onto the internal degree of freedom [1].

It is worth mentioning that the chiral partner of the 2D Dirac oscillator Hamiltonian (3) can be obtained through the substitution $\omega \rightarrow -\omega$ and consists of right-handed quanta. This Hamiltonian presents similar features as those discussed above, and can be exactly mapped onto an anti-Jaynes-Cummings interaction $H = \hbar(ga_r\sigma^- + g^* a_r^{\dagger}\sigma^+) + mc^2\sigma_z$, with similar parameters. It is precisely this chirality which allows an exact mapping between the JC, AJC, and the left-handed and right-handed 2D Dirac oscillators. This essential property, missing in the 3D case, forbids an exact mapping of Eq. (1) onto a JC-like Hamiltonian. It is the lack of this mapping that makes a theoretical prediction of relativistic effects more difficult. Nevertheless, an experimental implementation, using similar ion trap techniques, would allow the measurement of novel effects in this 3D system.

In conclusion, we have demonstrated the exact mapping of the 2+1 Dirac oscillator onto a Jaynes-Cummings model, allowing an interplay between relativistic quantum mechanics and quantum optics. We gave two relevant examples: the *Zitterbewegung* and collapse-revival dynamics. In addition, we showed that the implementation of a 2D Dirac oscillator in a single trapped ion, with all analogies and measured observables, is in reach with current technology. This experimental implementation shall confirm the predicted relativistic phenomena and possibly measure nonpredicted ones.

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