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Crossover from weak- to strong-coupling regime in dispersive circuit QED

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Abstract – We study the decoherence of a superconducting qubit due to the dispersive coupling to a damped harmonic oscillator. We go beyond the weak qubit-oscillator coupling, which we associate with a *phase Purcell effect*, and enter into a strong coupling regime, with qualitatively different behavior of the dephasing rate. We identify and give a physicaly intuitive discussion of both decoherence mechanisms. Our results can be applied, with small adaptations, to a large variety of other physical systems, *e.g.* trapped ions and cavity QED, boosting theoretical and experimental decoherence studies.

Introduction. – With a thrust from applications in quantum computing, the manipulation of quantum states in superconducting nanocircuits has made tremendous progress over the last decade [1–9]. A crucial step for these successes is the understanding of decoherence and the design of good measurement schemes. The latter is a particular challenge as the detector is made using the same technology as the system being detected, *i.e.* the qubit. Also, the measurement timescale cannot be considered to be infinitesimally short as compared to the intrinsic scales of the qubit evolution. Thus, understanding the measurement process is crucial both fundamentally and for improving experiments.

A specifically attractive development is the emergence of circuit quantum electrodynamics (cQED) [10–17], where effective Hamiltonians, similar to those of the coherent light-matter interaction of quantum optics and in particular of cavity QED, can be realized in the microwave frequency domain. There are many approaches to realize the qubit, including flux and charge, and the cavity, including a superconducting quantum interference device (SQUID) or a coplanar waveguide.

In this context, measurement protocols making use of dispersive qubit-oscillator interactions [1,2] are useful for reducing the backaction on the qubit [18]. For example, in the flux qubit-SQUID combination, as in the Delft setup of refs. [1,19], the SQUID behaves like a harmonic oscillator. Its inductive coupling to the flux qubit leads to a frequency shift depending on the qubit state $\Omega_{\uparrow,\downarrow} = \sqrt{\Omega^2 \pm \Delta^2}$. Here, Ω is the bare oscillator frequency and Δ is the quadratic frequency shift. A measurement of the SQUID resonance frequency provides information of the qubit state. While the manipulation of the qubit is usually performed at the optimum working point [3], the readout can and should be performed in quantum nondemolition measurement, *i.e.* in the pure dephasing limit.

In this letter we study the decoherence of a qubit due to the dispersive coupling to a damped harmonic oscillator, taking the Delft setup as an example though our results may be adapted to several physical systems. In the Purcell effect a narrow oscillator linewidth enhances the absorption of the resonant photon emitted by the two-level atom and thus the energy relaxation of the latter. In the weak qubit-oscillator coupling regime (WQOC), we explain the behavior of dephasing in terms of a similar process, the phase Purcell effect. This regime is characterized, as we will be show later, by $\Delta/\Omega < \sqrt{\kappa/\Omega}/(1+n(\Omega))^{1/4}$, where $n(\Omega)$ is the Bose function at the frequency Ω and environment temperature T. The main result of this work lays beyond the WQOC, in a regime where fast qubit-oscillator entanglement plays the dominant role. We find a qualitatively different behavior of the dephasing rate. The divergence of the qubit dephasing rate $1/\tau_\phi \propto 1/\kappa$ when the oscillator decay rate $\kappa \to 0$ is lifted by the onset of the strong-coupling regime.

The Hamiltonian describing the Delft setup [19] can be written as

$$\hat{H} = \underbrace{\frac{E}{2}\hat{\sigma}_z + \hbar\Omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \frac{\hbar\Delta^2}{4\Omega}(\hat{a}+\hat{a}^\dagger)^2\hat{\sigma}_z}_{\hat{H}_S} + \hat{H}_D. \quad (1)$$

Here, $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators of the harmonic oscillator, $\hat{\sigma}_z$ acts in the Hilbert space of the qubit and $\hat{H}_D$ describes the damping of the oscillator. A full-length derivation of Hamiltonian (1) and discussion of the approximations used is given in ref. [20]. It basically derives the Hamiltonian from the equations of motion of the Josephson phases across the junctions and truncates the SQUID potential to the second order.

We will show that key experiments [1,2] are performed outside the WQOC. Moreover, a very recent experiment [21] explicitly relies on the use of a strong dispersive coupling regime. We demonstrate that the dephasing rate $1/\tau_\phi \propto 1/\kappa$ for WQOC, and $1/\tau_\phi \propto \kappa$ at strong coupling. We discuss the crossover between these regimes and its dependence on κ and temperature T. We provide physical interpretations of both regimes, the former as a phase Purcell effect and the latter as the onset of qubit-oscillator entanglement. The results of the present study may be extended straightforwardly to any system with similar dispersive qubit-oscillator coupling: the charge-qubit–coplanar wave guide system (see Yale setup [2]), trapped ions [22] and 3D microwave cavity QED [23], quantum dots [24], among others.

Method. – In studying the qubit dephasing we are facing the challenge of a complex non-Markovian environment consisting in the main oscillator (*i.e.* SQUID) and the ohmic bath. Moreover, the qubit couples to a non-Gaussian variable of its environment. Therefore the tools developed for Gaussian baths [25] cannot be applied in this system for arbitrary strong coupling between the qubit and the oscillator.

We study the qubit dynamics under the Hamiltonian (1) for arbitrary Δ/Ω, assuming essentially the dimensionless oscillator decay rate κ/Ω as the *only* small parameter. In this regime we avoid over-damping of the oscillator and the strong backaction on the system which this would cause. We give in the following a brief description of the crucial steps and approximations of the calculation. We model the damping, associated with the oscillator decay rate κ, in the Caldeira-Leggett way by a bath of harmonic oscillators

$$\hat{H}_D = \underbrace{\sum_j \hbar\omega_j\left(\hat{b}_j^\dagger\hat{b}_j + \frac{1}{2}\right)}_{\hat{H}_B} + \underbrace{\sum_j \frac{\hbar(\hat{a}+\hat{a}^\dagger)}{2\sqrt{m\,\Omega}}\frac{\lambda_j(\hat{b}_j^\dagger + \hat{b}_j)}{\sqrt{m_j\,\omega_j}}}_{\hat{H}_I} + \hat{H}_c, \quad (2)$$

with $J(\omega) = \sum_j \lambda_j^2\hbar/(2m_j\omega_j)\delta(\omega-\omega_j) = m\hbar\kappa\omega\Theta(\omega-\omega_c)/\pi$ and $\hat{H}_c$ the counter term [26–28], where Θ is the Heaviside step function and ω_c an intrinsic high-frequency cut-off. Our starting point is the Born-Markov master equation in the weak coupling to the bath limit for the reduced density matrix $\hat{\rho}_S$ in the qubit-oscillator Hilbert space

$$\begin{aligned}\dot{\hat{\rho}}_S(t) &= \frac{1}{i\hbar}\left[\hat{H}_S, \hat{\rho}_S(t)\right] \\ &\quad + \int_0^t \frac{\mathrm{d}t'}{(i\hbar)^2}\,\mathrm{Tr}_B\Big[\hat{H}_I, [\hat{H}_I(t,t'), \hat{\rho}_S(t)\otimes\hat{\rho}_{B0}]\Big].\end{aligned}$$

This approach is valid at finite temperatures $k_BT \gg \hbar\kappa$, for times $t \gg 1/\omega_c$ [28,29], which is the limit we will discuss henceforth. We start from a standard factorized initial state for all subsystems. We express $\hat{\rho}_S(t)$ in the qubit basis and represent its elements, which are still oscillator operators, in phase space as

$$\hat{\rho}_S = \begin{pmatrix}\hat{\rho}_{\uparrow\uparrow} & \hat{\rho}_{\uparrow\downarrow} \\ \hat{\rho}_{\downarrow\uparrow} & \hat{\rho}_{\downarrow\downarrow}\end{pmatrix}, \qquad \hat{\rho}_{\sigma\sigma'} = \int \frac{\mathrm{d}^2\alpha}{\pi}\,\chi_{\sigma\sigma'}(\alpha,\alpha^*,t)\hat{D}(-\alpha),$$

where $\chi_{\sigma\sigma'}$ is the characteristic Wigner function and $\hat{D} = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ the displacement operator [30]. Independent of our work, ref. [31] has used a different phase-space representation to calculate the qubit dephasing rate. We characterize the qubit coherence by $C(t) = \langle\hat{\sigma}_x \otimes \hat{\mathbb{1}}\rangle = 2\mathrm{Re}\,\mathrm{Tr}\hat{\rho}_{\uparrow\downarrow}(t)$ which can be easily shown to be $C(t) = 8\pi\mathrm{Re}\chi_{\uparrow\downarrow}(0,0,t)$. After a rather long but essentially straightforward calculation, one obtains for $\chi_{\uparrow\downarrow}$ a *generalized* Fokker-Planck equation

$$\begin{aligned}\dot{\chi}_{\uparrow\downarrow}(\alpha,\alpha^*,t) = \Big(&(\alpha(k_1+i\Omega) + \alpha^*k_1)\,\partial_\alpha \\ &+ (\alpha^*(k_2-i\Omega) + \alpha k_2)\,\partial_{\alpha^*} - \frac{i\Delta^2}{2\Omega}(\partial_\alpha - \partial_{\alpha^*})^2 \\ &+ p(\alpha+\alpha^*)^2\Big)\chi_{\uparrow\downarrow}(\alpha,\alpha^*,t), \end{aligned} \quad (3)$$

where

$$k_{1,2} = -\frac{\kappa}{4}\left(2 \mp \frac{\Omega_\uparrow}{\Omega}(1+2n_\uparrow) \pm \frac{\Omega_\downarrow}{\Omega}(1+2n_\downarrow)\right), \quad (4)$$

$$p = -\frac{\kappa}{8\Omega}\left(\Omega_\uparrow(1+2n_\uparrow) + \Omega_\downarrow(1+2n_\downarrow)\right) - \frac{i\Delta^2}{8\Omega} \quad (5)$$

and $n_\sigma = n(\Omega_\sigma)$ is the Bose function. To solve eq. (3) we make a Gaussian ansatz for $\chi_{\uparrow\downarrow}$:

$$\chi_{\uparrow\downarrow} = A(t)\exp(-M(t)\alpha^2 - N(t)\alpha^{*2} - Q(t)\alpha\alpha^*). \quad (6)$$

This ansatz includes coherent and thermal states. In the following we assume the oscillator to be initially in a thermal state, in equilibrium with its environment. This implies $Q(0) = 1/2 + n(\Omega)$ and $M(0) = N(0) = 0$. Due to the quadratic (pure dephasing) form of the Hamiltonian (1), obtain a closed system of ordinary differential equations for the parameters of the Gaussian ansatz, see also ref. [20]. This system can be easily solved perturbatively in Δ in the weak-coupling regime, or numerically (for arbitrarily strong coupling) and we can extract the dephasing time τ_ϕ from the strictly exponential long-time tail of $C(t)=8\pi\mathrm{Re}A(t)$.

Weak qubit-oscillator coupling. – Before solving eq. (3) in a general manner, we revisit the case of small Δ. Up to the lowest non-vanishing order Δ^4, the analytically calculated WQOC dephasing rate is

$$\frac{1}{\tau_\phi}=\Delta^4\frac{n(\Omega)\,(n(\Omega)+1)}{\Omega^2}\left(\frac{\kappa}{\kappa_m^2}+\frac{1}{\kappa}\right), \qquad (7)$$

where $\kappa_m=\sqrt{2k_BT\Omega/(\hbar(1+2\,n(\Omega)))}$. The term $1/\kappa$ exactly reproduces the Golden Rule dephasing rate of ref. [19], and is similar to the result of ref. [32]. These previous results have been obtained considering only the two-point correlator of the fluctuating observable $(a+a^\dagger)^2$, *i.e.* assuming an Gaussian environment. The crossover point κ_m from $1/\kappa$ to κ in eq. (7) is, at the Delft parameters [1], comparable to Ω, *i.e.*, κ would dominate over $1/\kappa$ only in a regime where the Born approximation fails. Nevertheless, since the golden rule limit $\lim_{\kappa\to\infty}1/\tau_\phi=0$ is unphysical, such a term was to be expected.

In the WQOC regime, the enhancement of dephasing by weak coupling to the environment is analogous to the enhancement of spontaneous emission by the narrow cavity lines in the *resonant* Purcell effect, see refs. [33,34]. In the pure dephasing case we have no energy exchange between the qubit and the oscillator. Qubit decoherence is caused by fluctuations of $(\hat{a}+\hat{a}^\dagger)^2$. Since we are in the WQOC regime, the stronger coupling between the oscillator and the environment causes equilibrium between the oscillator and the bath on a shorter time scale than the qubit dephasing. In equilibrium, the main contribution to the fluctuations of $(\hat{a}+\hat{a}^\dagger)^2$ is the exchange of photons between oscillator and bath. The process is analogous to equilibrium fluctuations in canonical thermodynamics. A virtual photon returning from the environment is at resonance with the oscillator. The absorbtion of this photon, like in the resonant Purcell effect will be enhanced by narrow oscillator lines. Therefore, the entire dephasing process will be enhanced when the coupling to the environment is weak and this mechanism can be viewed as a phase Purcell effect. We give a more detailed discussion of this effect in the appendix.

Strong qubit-oscillator coupling. – The dephasing rate (7) obtained in the small κ and WQOC limit diverges for $\kappa\to 0$, *i.e.*, in the absence of an environment. The solution to this apparent contradiction lies beyond the WQOC, therefore we solve eq. (3) numerically using again the Gaussian ansatz for $\chi_{\uparrow\downarrow}$.

Figure 1 shows the dependence of the dephasing rate on, Δ for various values of κ. The dimensionless parameter $\hbar\Omega/k_BT$ is 2, similar to the Delft and Yale setups. As predicted by eq. (7) for $\kappa\ll\kappa_m$ and small Δ, the dephasing rate is proportional to Δ^4/κ. Further increasing Δ, we observe a saturation of the dephasing rate which marks

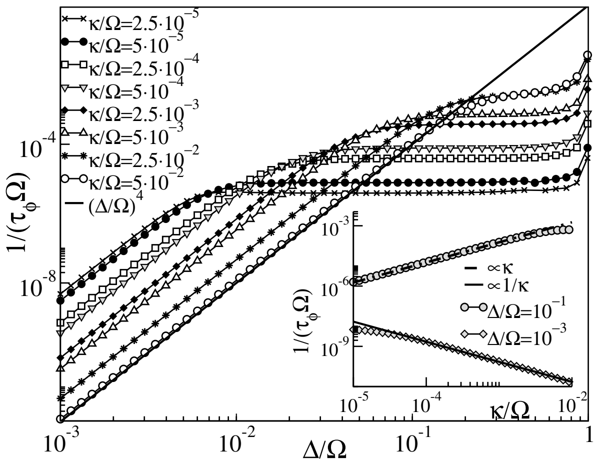

Fig. 1: Dephasing rate $1/\tau_\phi$ as a function of Δ for different values of κ. Power law Δ^4 growth at low Δ crosses over to Δ-independence at strong coupling. Inset: Dephasing rate as a function of κ in the weak-coupling regime ($\Delta/\Omega=10^{-3}$) showing $1/\tau_\phi\propto\Delta^4/\kappa$ and the strong-coupling regime ($\Delta/\Omega=10^{-1}$) with $1/\tau_\phi\propto\kappa$ dependence. Here $\hbar\Omega/k_BT=2$ similar to experiments.

the onset of the strong coupling regime. This regime is analogous to the strong coupling in linear cavity QED. Here $1/\tau_\phi$ is proportional to κ.

At strong qubit-oscillator coupling the oscillator couples to the qubit stronger than it couples to the heat bath, such that one cannot use the effective bath concept of WQOC. As the qubit-oscillator system becomes entangled, a fundamentally different dephasing mechanism emerges. The eigenstates of the Hamiltonian (1) at $\kappa=0$ are the dressed states $\{|\sigma,m_\sigma\rangle\}$, where $|m_\sigma\rangle$ are the number states of the oscillator with frequency Ω_σ. Opposed to WQOC, these dressed states are built in the strong coupling regime on a shorter time scale than the re-thermalization of the oscillator. In the evolution from thermal state of oscillator with frequency Ω to an equilibrium between the new oscillator with Ω_σ and the bath, the state in the narrower potential tends to absorb and the one in the wider potential to emit photons to the bath in an incoherent manner, causing fluctuations of $(\hat{a}+\hat{a}^\dagger)^2$ and thus qubit decoherence. Thus we expect $1/\tau_\phi\propto\kappa n(\Omega)$. This simple picture is confirmed by numerical results in fig. 2, for a wide range of values of κ. The inset of fig. 2 shows the crossover from strong coupling rate κ to WQOC rate $1/\kappa$. This indicates that, for fixed Δ, as κ decreases, Δ stops being “small” and the WQOC limit breaks down. Thus, approaching $\kappa=0$ for any given Δ we eventually leave the domain of validity for eq. (7) avoiding the divergence at $\kappa\to 0$. As expected, dephasing will vanish as we go to a finite quantum system (qubit $\otimes$ single oscillator) at $\kappa=0$. We observe that the criterion of “small” Δ in WQOC is valid only relative to κ. Using $1/\tau_\phi=\kappa n(\Omega)$ in the strong-coupling regime and the $1/\kappa$ term of $1/\tau_\phi$ in eq. (7) in the weak-coupling regime, we determine the position of the crossover Δ_c between the

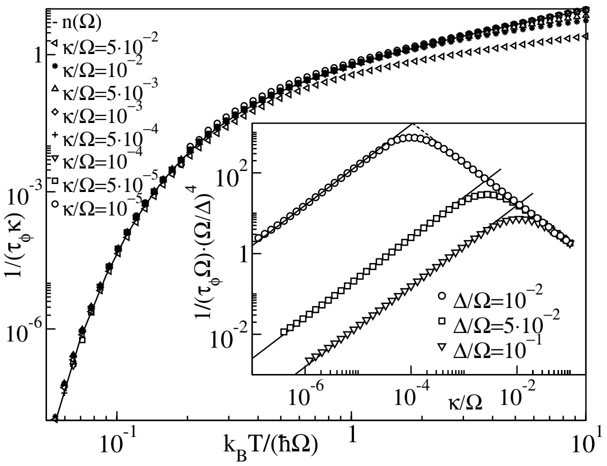

Fig. 2: Scaling plot of the dephasing rate $1/\tau_\phi$ as a function of temperature. For $\Delta/\Omega = 0.3$, *i.e.* in the strong-coupling regime (see fig. 1), for a wide range of κ's we show that $1/(\tau_\phi\kappa)$ is proportional to the Bose function $n(\Omega)$. Inset: dephasing rate $1/\tau_\phi$ as a function of κ for different values of Δ and $\hbar\Omega/k_BT = 2$. Continuous lines correspond to $\kappa n(\Omega)$, dashed lines correspond to $n(\Omega)(n(\Omega)+1)\Delta^4/(\Omega^2\kappa)$.

two regimes

$$\frac{\Delta_c}{\Omega} = \sqrt{\frac{\kappa}{\Omega}}\frac{1}{(1+n(\Omega))^{1/4}}. \tag{8}$$

The position of the cross-over is controlled by the ratio of the coupling strengths between the three subsystems *i.e.* Δ^2/Ω and κ. Note that, with the *in situ* tuning of the qubit-SQUID coupling, available in the Delft experiment, the position of the cross-over could be tested experimentally. Using the parameters from ref. [1,2] one finds $(\Delta/\Delta_c)_{\mathrm{Yale}} \approx 1.4$ and $(\Delta/\Delta_c)_{\mathrm{Delft}} \approx 1.3$ *i.e.*, the strong-coupling regime finds application in both setups.

If the oscillator is weakly driven off-resonance, as is the case in the dispersive measurement, the qualitative behavior remains the same as in fig. 1, as shown in ref. [20]. In general a tunneling $\hat{\sigma}_x$ term may occur in eq. (1) and lead to energy relaxation as well as further reducing the matrix elements containing the dephasing rate. We expect that, as long as the energy splitting E of the qubit is off-resonance with the oscillator, which in our case means $|E-2\Omega| \gg \kappa$, the effect of the relaxation is rather weak and dephasing still dominates. On resonance, we expect a similar Purcell to strong coupling crossover as for the dephasing channel.

Our results have applications in other systems with similar dispersive qubit-oscillator coupling, *e.g.*, the Yale setup [2], in the off-resonant dispersive regime. There, the system is described by a similar (eq. (12) in ref. [32]) quadratic coupling $\hat{a}^\dagger\hat{a}$ between qubit and cavity and a pure dephasing Hamiltonian. In particular, a strong dispersive regime of this system has been utilized to resolve number states of the electromagnetic field in ref. [21]. The terms $\hat{a}^2$ and $\hat{a}^{\dagger 2}$ in eq. (1) do not play a central role for our physical predictions, as confirmed by the numerical simulations. We expect our results, with minor adaptations, to be applicable to various cavity systems, *e.g.* quantum dot or atom-based quantum optical schemes [23,24]. The dispersive coupling of Hamiltonian (1) could have implications for the generation of squeezed states, quantum memory in the frame of quantum information processing, measurement and post-selection of the number states of the cavity.

Conclusion. – We have presented a concise theory of the dephasing of a qubit coupled to a dispersive detector spanning both strong and weak coupling. The phase-space method applied is based on treating the oscillator on the same level of accuracy as the qubit. We have discussed the dominating decoherence mechanism at weak qubit-oscillator coupling, where the linewidth of the damped oscillator plays the main role, analogous to the Purcell effect. At strong qubit-oscillator coupling we have identified a qualitatively different behavior of the qubit dephasing and discussed it in terms of the onset of the qubit-oscillator entanglement. We have provided a criterion delimitating the parameter range at which these processes dominate the qubit dephasing.

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Appendix. – Assuming the WQOC limit we use Fermi's golden rule in an otherwise exact manner to prove the analogy between the weak qubit-oscillartor coupling regime and the Purcell effect. One can map the damped oscillator by an exact normal mode transformation [35] onto an *effective* heat bath of *decoupled* oscillators denoted by $\hat{c}_j, \hat{c}_j^\dagger$ and with a spectral density

$$J_{\mathrm{eff}}(\omega) = \frac{2\kappa\omega}{(\omega^2-\Omega^2)^2+\kappa^2\omega^2}. \tag{A.1}$$

J_{eff} corresponds to the effective density of electromagnetic modes in the cavity introduced in regular linear cavity QED for describing the Purcell effect. The WQOC decoherence rate is proportional to the two-point correlation function of the environmental operator coupling to the qubit [28,36], in our case

$$S_2(\omega) = \left\langle \hat{X}^2(t)\hat{X}^2(0)\right\rangle_\omega - \left(\langle \hat{X}^2\rangle\right)^2, \tag{A.2}$$

where $\hat{X}$ is the sum of the *effective* bath coordinates $\hat{X} = \sum_j \sqrt{\hbar/(2m_j\omega_j)}(\hat{c}_j + \hat{c}_j^\dagger)$. For the pure dephasing situation described by the Hamiltonian (1) we only need to study $1/\tau_\phi \propto S_2(\omega \to 0^+)$ because the qubit energy conservation implies energy conservation within its effective environment. The last term of eq. (A.2) removes the noise bias. This is important since dephasing is caused only by processes that leave a trace in the bath [37], *i.e.* the exchanged boson spends a finite time in the environment. Terms of the structure $\left\langle \hat{c}_i^\dagger(t)\hat{c}_j^\dagger(t)\hat{c}_k\hat{c}_l\right\rangle$, $\left\langle \hat{c}_i(t)\hat{c}_j(t)\hat{c}_k^\dagger\hat{c}_l^\dagger\right\rangle$ contribute to $S_2(0)$ only when $\omega_i=\omega_j=0$, which are modes with density $J_{\rm eff}\simeq 2\kappa\omega/\Omega^4$ each, leading to terms are of order κ^2. Up to linear order in κ, the only terms in $S_2(\omega\to 0^+)$ that fulfill the energy conservation and leave a trace in the bath are of the structure $\left\langle \hat{c}_l^\dagger(t)\hat{c}_j(t)\hat{c}_j^\dagger\hat{c}_l\right\rangle$, including the permutations among the operators taken at time t and those taken at time 0. The terms contributing to $S_2(\omega\to 0^+)$ satisfy the condition $|\omega_l-\omega_j|\to 0^+$. Physically this corresponds to infinitesimal energy fluctuations which leave a trace in the bath. Or, in other words, the photon absorbed at $t=0$, $\hat{c}_l$, should spend finite time in the bath and be emitted back only at the later time t, but at the same time the energy change in the environment *e.g.* caused by $\hat{c}_j^\dagger\hat{c}_l$ should remain undetectable within the energy-time uncertainty at every time, therefore in the Golden Rule (long time) limit $\omega_l\approx\omega_j$. Taking the continuum limit we thus have

$$1/\tau_\phi \propto \int_0^\infty \mathrm{d}\omega\, J_{\rm eff}(\omega)(1+n(\omega))J_{\rm eff}(\omega)n(\omega). \qquad \text{(A.3)}$$

The integral in eq. (A.3) can be rewritten as the convolution $K(\omega')=\int \mathrm{d}\omega J_{\rm eff}(\omega)n(\omega)J_{\rm eff}(\omega'-\omega)n(\omega'-\omega)$ for $\omega'\to 0$. Using eq. (A.1), $K(\omega')$ becomes a function with resonances at $\omega'=0$ and $\omega'=2\Omega$. The the height of these resonances and consequently $1/\tau_\phi\propto S_2(0)$ increases with decreasing κ, thus matching the behavior of the dephasing rate (7). At the same time, the tail of the peak at 2Ω enhances $S_2(0)$ when κ increases. This corresponds to the κ term in eq. (7). Analogous to $1/\tau_\phi$ in eq. (7), $S_2(\omega\to 0^+)$ vanishes for $T\to 0$.

REFERENCES

[1] Bertet P. *et al.*, *Phys. Rev. Lett.*, **95** (2005) 257002.
[2] Wallraff A. *et al.*, *Nature*, **431** (2004) 162.
[3] Vion D. *et al.*, *Science*, **296** (2002) 286.
[4] Duty T. *et al.*, *Phys. Rev. B*, **69** (2004) 140503(R).
[5] Makhlin Y., Schön G. and Shnirman A., *Rev. Mod. Phys.*, **73** (2001) 357.
[6] Pashkin Y. *et al.*, *Nature*, **421** (2003) 823.
[7] McDermott R. *et al.*, *Science*, **307** (2005) 1299.
[8] Oliver W. *et al.*, *Science*, **310** (2005) 5754.
[9] Plourde B. *et al.*, *Phys. Rev. B*, **72** (2005) 060506(R).
[10] Buisson O. and Hekking F., *Macroscopic Quantum Coherence and Quantum Computing*, edited by Averin D., Ruggerio B. and Silvestrini P. (Kluwer, New York) 2001, p. 137.
[11] You J. and Nori F., *Phys. Rev. B*, **68** (2003) 064509.
[12] Goorden M., Thorwart M. and Grifoni M., *Phys. Rev. Lett.*, **93** (2004) 267005.
[13] Wilhelm F. K., *Phys. Rev. B*, **68** (2003) 060503.
[14] Kleff S., Kehrein S. and von Delft J., *Phys. Rev. B*, **70** (2004) 014516.
[15] Yang C., Chu S. and Han S., *Phys. Rev. Lett.*, **92** (2004) 117902.
[16] Sarovar M. *et al.*, *Phys. Rev. A*, **72** (2005) 062327.
[17] Mariantoni M. *et al.*, cond-mat/0509737 (2005).
[18] Lupascu A. *et al.*, *Phys. Rev. Lett.*, **93** (2004) 177006.
[19] Bertet P. *et al.*, cond-mat/0507290 (2005).
[20] Serban I., Solano E. and Wilhelm F. K., *Phys. Rev. B*, **76** (2006) 104510.
[21] Schuster D. I. *et al.*, *Nature*, **445** (2006) 515.
[22] Leibfried D. *et al.*, *Rev. Mod. Phys.*, **75** (2003) 281.
[23] Raimond J., Brune M. and Haroche S., *Rev. Mod. Phys.*, **73** (2001) 565.
[24] Balodato A. *et al.*, *Science*, **308** (2005) 1158.
[25] Weiss U., *Quantum Dissipative Systems* (World Scientific, Singapore) 1999.
[26] Ingold G., *Quantum Transport and Dissipation* (Wiley-VCH, Weinheim) 1998.
[27] Cohen-Tannoudji C., Dupont-Roc J. and Grynberg G., *Atom-Photon Interactions: Basic Processes and Applications* (Wiley-VCH, Weinheim) 1998.
[28] Wilhelm F. *et al.*, *Manipulating Quantum Coherence in Solid State Systems, NATO-ASI Ser.*, Vol. **244**, edited by Flatté M. and Ţifrea I. (Springer, Dordrecht) 2007.
[29] Alicki R., Lidar D. and Zanardi P., *Phys. Rev. A*, **73** (2006) 052311.
[30] Cahill K. and Glauber R., *Phys. Rev.*, **177** (1969) 1882.
[31] Gambetta J. *et al.*, *Phys. Rev. A*, **74** (2006) 042318.
[32] Blais A. *et al.*, *Phys. Rev. A*, **69** (2004) 062320.
[33] Purcell E. M., *Phys. Rev.*, **69** (1946) 681.
[34] Haroche S. and Kleppner D., *Phys. Today*, No. 42 (1989) 24.
[35] Ambegaokar V., *J. Stat. Phys.*, **125** (2006) 1187.
[36] Slichter C., *Principles of Magnetic Resonance, Springer Ser. Solid State Sci.*, Vol. **1** (Springer, Berlin) 1996.
[37] Leggett A., *J. Phys. C*, **14** (2002) 415.