Comment on "Theoretical Analysis of the Transmission Phase Shift of a Quantum Dot in the Presence of Kondo Correlations"

In a recent Letter [1], Jerez, Vitushinsky, and Lavagna (JVL) propose an interpretation of the measurements [2] of the transmission phase shift, $\delta_{\rm ABI}$, through a quantum dot (QD) in the Kondo regime, as deduced from placing the QD in a double-slit Aharonov-Bohm interferometer (ABI). Describing the QD (coupled to two reservoirs via one-dimensional leads) by the single level Anderson model (SLAM), JVL argue that the zero magnetic field ($H = 0$) conductance through the QD is given by $G \propto \sin^2(\delta_G)$, with $\delta_G = \delta_{\rm ABI}/2$. This Comment questions the validity of this result for the SLAM, since it fails in several exactly known limits. Whether the SLAM describes the experiment [2] is irrelevant to the theoretical problem posed here [3].

Without interactions ($U = 0$), JVL's SLAM has the exact scattering solution $A_\ell e^{ikx} + B_\ell e^{-ikx}$ to the left of the QD, and $A_r e^{-ikx} + B_r e^{ikx}$ to its right, with

$$\begin{bmatrix} B_\ell \\ B_r \end{bmatrix} = S_k \begin{bmatrix} A_\ell \\ A_r \end{bmatrix}, \tag{1}$$

where $E_k = -2t\cos k$ [we use the notations of [1]] and

$$S_k = \begin{bmatrix} -1 + 2i\sin k\mathcal{G}_k V_L^2/t & 2i\sin k\mathcal{G}_k V_L V_R/t \\ 2i\sin k\mathcal{G}_k V_L V_R/t & -1 + 2i\sin k\mathcal{G}_k V_R^2/t \end{bmatrix} \equiv -e^{i\delta_k}\begin{bmatrix} \cos\delta_k - iv_-\sin\delta_k & iv_+\sin\delta_k \\ iv_+\sin\delta_k & \cos\delta_k + iv_-\sin\delta_k \end{bmatrix}, \tag{2}$$

where $\mathcal{G}_k = [E_k - \epsilon_0 + e^{ik}\Gamma/2]^{-1}$, with $\Gamma = 2(V_L^2 + V_R^2)/t$, $\cot\delta_k = -[E_k - \epsilon_0 + (\Gamma/2)\cos k]/[(\Gamma/2)\sin k]$, while $v_+ = \sin 2\theta$ and $v_- = \cos 2\theta$, with $\tan\theta = V_L/V_R$. When $H = 0$, S_k does not depend on the spin index σ.

Although JVL start with our Eq. (2) (with $V_L = V_R$, i.e., $v_+ = 1$, $v_- = 0$), they replace this equation (at the Fermi level, $k = k_F$) by their Eq. (4),

$$S_{k_F\sigma}^{\rm JVL} = e^{i\delta}\begin{bmatrix} \cos\delta_\sigma & i\sin\delta_\sigma \\ i\sin\delta_\sigma & \cos\delta_\sigma \end{bmatrix}, \tag{3}$$

with the modified overall phase $\delta = \delta_\uparrow + \delta_\downarrow$ (and a different sign). In fact, they multiply Eq. (2) by an additional factor, $-C_\sigma = -e^{i\delta_{-\sigma}}$, which they attribute to generalizations of Levinson's theorem. Although the conductance is still given by $G \propto \sum_\sigma \sin^2\delta_\sigma$, $\delta_{\rm ABI}$ is then claimed to be equal to δ. For $H = 0$, one has $\delta_\uparrow = \delta_\downarrow = \delta/2$, and therefore JVL conclude that $\delta_G = \delta/2 = \delta_{\rm ABI}/2$.

However, for $U = 0$ Eq. (3) contradicts the exact solution (2), which does *not* contain the factor $-C_\sigma$. More generally, at zero-temperature but $U \neq 0$, one has $G \propto \sum_\sigma {\rm Im}\,\mathcal{G}_{d\sigma}(0) \propto \sum_\sigma \sin^2\delta_\sigma$, where $\mathcal{G}_{d\sigma}(0) \equiv e^{i\delta_\sigma}|\mathcal{G}_{d\sigma}(0)|$ is the exact local Green's function of the SLAM at $k = k_F$ [4,5]. Moreover, Eq. (2) of [5] shows generally that the complex transmission amplitude $T_{d\sigma}$ through a SLAM QD is proportional to $\mathcal{G}_{d\sigma}(0)$, implying that $\delta_{\rm ABI} = \arg T_{d\sigma} \equiv \delta_\sigma$ is *the same* as δ_G, again contradicting JVL's $\delta_G = \delta_{\rm ABI}/2$ [6].

We conclude that JVL's Eq. (4) does *not* follow from the SLAM. Either the SLAM is not compatible with the Levinson theorem, or the application of this theorem to the SLAM was done incorrectly. In either case, if JVL believe that their Eq. (4) is correct then they should supply its explicit derivation from a well-defined model.

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