



Decoherence of interacting electrons in disordered conductors: on the relation between influence functional and diagrammatic approaches

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We establish a connection between the influence functional approach of Golubev and Zaikin (GZ) and Keldysh diagrammatic perturbation theory for calculating the decoherence time τ_φ of interacting electrons in disordered metals; we show how the standard diagrams for the Cooperon self energy can be recovered from GZ's influence functional $e^{-(i\bar{S}_R+\bar{S}_I)}$. This allows us to shed light on GZ's claim that \bar{S}_R is irrelevant for decoherence: \bar{S}_R generates as many important self energy diagrams as \bar{S}_I ; GZ's neglect of \bar{S}_R is permissible only at high temperatures ($T \gtrsim \hbar/\tau_{el}$).

KEYWORDS: Decoherence, influence functional, diagrammatic perturbation theory, interactions, disorder

1. Introduction

A few years ago, Golubev and Zaikin (GZ) developed an influence functional approach for describing interacting fermions in a disordered conductor. Their key idea was as follows: to understand how the diffusive behavior of a given electron is affected by its interactions with other electrons in the system, which constitute its effective environment, the latter should be integrated out, leading to an influence functional (denoted by $e^{-(i\bar{S}_R+\bar{S}_I)}$ below) in the path integral $\int \mathcal{D}(\mathbf{R}\mathbf{P})$ describing its dynamics. To derive the effective action $i\bar{S}_R + \bar{S}_I$, GZ devised a strategy which, when implemented with sufficient care, *properly incorporates the Pauli principle* – this is essential, since both the particle and its environment originate from the same system of indistinguishable fermions, a feature which makes the present problem interesting and sets it apart from all other applications of the influence functional strategy that we are aware of.

GZ used their new approach to calculate the electron decoherence time $\tau_\varphi(T)$, as extracted from the magnetoconductance in the weak localization regime, and found it to be finite at zero temperature:^{1–4)} $\tau_\varphi(T \rightarrow 0) = \tau_\varphi^0$, in apparent agreement with some experiments.⁵⁾ However, this result contradicts the standard view, based on the work of Altshuler, Aronov and Khmel'nitskii⁶⁾ (AAK), that $\tau_\varphi(T \rightarrow 0) = \infty$, and hence elicited a considerable and ongoing controversy,⁷⁾ with pertinent critique coming, in particular, from Ref.^{8,9)}

The fact that GZ's final results for $\tau_\varphi(T)$ are controversial, however, does not imply that their influence functional approach, as such, is fundamentally flawed. To the contrary, having repeated their calculations in detail, we have come to the conclusion that their strategy is sound in principle and that an influence functional of the form $e^{-(i\bar{S}_R+\bar{S}_I)}$ which they found can indeed be derived without making non-standard approximations. In fact, it can be shown, and this is our main result, that *the standard Keldysh diagrammatic expressions for the self energy of the Cooperon can be obtained from $i\bar{S}_R + \bar{S}_I$* . However, when *applying* this influence functional to the problem of decoherence, GZ make a semiclassically-motivated ap-

proximation according to which the effects of \bar{S}_R for decoherence can be neglected. We recount a simple back-of-the-envelope argument,¹⁰⁾ due to F. Marquardt, to suggest that neglecting \bar{S}_R is permissible only for $T \gtrsim \hbar/\tau_{el}$, where τ_{el} is the elastic mean free time. We shall also show that in diagrammatic language, neglecting \bar{S}_R corresponds to neglecting several important diagrams contributing to the Cooperon self energy, as first pointed out in Ref.⁹⁾

Equation numbers from GZ's papers,^{1–4)} will be prefaced, when cited below, by I, II, III or IV, respectively.

2. The model

We consider a disordered system of interacting fermions, with Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_i$, where

$$\hat{H}_0 = \int dx \hat{\psi}^\dagger(x) h_0(x) \hat{\psi}(x), \quad (1)$$

$$\hat{H}_i = \frac{e^2}{2} \int dx_1 dx_2 : \hat{\psi}^\dagger(x_1) \hat{\psi}(x_1) : \tilde{V}_{12}^{\text{int}} : \hat{\psi}^\dagger(x_2) \hat{\psi}(x_2) :$$

Here $\int dx = \sum_\sigma \int d\mathbf{r}$, and $\hat{\psi}(x) \equiv \hat{\psi}(\mathbf{r}, \sigma)$ is the electron field operator for creating a spin- σ electron at position \mathbf{r} , with the following expansion in terms of the exact eigenfunctions $\psi_\lambda(x)$ of $h_0(x) = \frac{\hbar^2}{2m} \nabla_{\mathbf{r}}^2 + V_{\text{imp}}(\mathbf{r}) - \mu$:

$$\hat{\psi}(x) = \sum_\lambda \psi_\lambda(x) \hat{c}_\lambda, \quad [h_0(x) - \xi_\lambda] \psi_\lambda(x) = 0. \quad (2)$$

The interaction potential $\tilde{V}_{12}^{\text{int}} = \tilde{V}^{\text{int}}(|\mathbf{r}_1 - \mathbf{r}_2|)$ acts between the normal-ordered densities at \mathbf{r}_1 and \mathbf{r}_2 .

3. Influence functional for interacting electrons

GZ proposed a strategy (whose steps and approximations are recapitulated in the Appendix), that allows the DC conductivity, σ_{DC} , to be expressed [see Eq. (A.3)] in terms of path integrals of the following general form [cf. (II.53), (IV.31)]:

$$\tilde{C}_{12', \bar{2}1'} = \int_{2'_{F, \bar{2}B}}^{1_{F, 1'_B}} \mathcal{D}(\mathbf{R}\mathbf{P}) e^{-[i\bar{S}_R+\bar{S}_I](t_1, t_2)/\hbar}. \quad (3)$$

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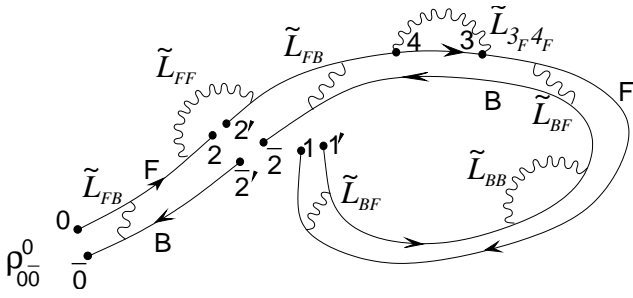


Fig. 1. A pair of paths contributing to the weak localization correction, illustrating Eq. (A.3). The interaction lines, generated by expanding the influence functional $e^{-(i\bar{S}_R + \bar{S}_I)}$ as in Eq. (A.14), are labelled according to Eq. (A.13).

The symbol $\int \mathcal{D}(\mathbf{R}\mathbf{P})$ is a shorthand for the following coordinate-momenta double path integral,

$$\int_{\mathbf{r}'_2, \bar{\mathbf{2}}_B}^{\mathbf{r}'_1, \bar{\mathbf{1}}_B} \mathcal{D}(\mathbf{R}\mathbf{P}) = \int_{\mathbf{R}^F(t_2)=\mathbf{r}'_2}^{\mathbf{R}^F(t_1)=\mathbf{r}'_1} \mathcal{D}\mathbf{R}^F(t_3) \int \mathcal{D}\mathbf{P}^F(t_3) \times \int_{\mathbf{R}^B(t_2)=\mathbf{r}_2}^{\mathbf{R}^B(t_1)=\mathbf{r}'_1} \mathcal{D}\mathbf{R}^B(t_3) \int \mathcal{D}\mathbf{P}^B(t_3) e^{i[\bar{S}_0^F - \bar{S}_0^B](t_1, t_2)/\hbar}, \quad (4)$$

which, when taken by itself, gives the amplitude for a free electron to propagate from \mathbf{r}'_2 at time t_2 to \mathbf{r}'_1 at t_1 , times the amplitude for a free electron to propagate from \mathbf{r}'_1 at time t_1 to \mathbf{r}_2 at t_2 [corresponding to the loop parts of the paths in Fig. 1], in the absence of interactions with other electrons. We shall call these the forward and backward paths, respectively, and label them by an index $a = F, B$. The corresponding free actions $\bar{S}_0^a = \bar{S}_0^{F/B}$ are given in Eq. (A.4). The weak localization correction to the conductivity, $\sigma_{\text{DC}}^{\text{WL}}$, arises from contributions to σ_{DC} for which the coordinates $\mathbf{r}_1, \mathbf{r}'_1, \mathbf{r}_2$ and \mathbf{r}'_2 all lie close together. We henceforth consider only this case. Then $\tilde{C}_{12', \bar{2}1'}$ is just the Cooperon propagator, dominated by contributions from those classical paths for which path B is the time-reversed version of path F . The effect of the other electrons on this propagation is encoded in the influence functional $e^{-(i\bar{S}_R + \bar{S}_I)}$ occurring in Eq. (3). The effective action $i\bar{S}_R + \bar{S}_I$ turns out to have the form

$$\bar{S}_{R/I}(t_1, t_2) = \sum_{a, a'=F, B} \int_{t_2}^{t_1} dt_3 \int_{t_2}^{t_3} dt_4 \bar{L}_{3_a 4_{a'}}^{R/I}, \quad (5)$$

where the $\bar{L}_{3_a 4_{a'}}^R$ are functions of the coordinates and momenta $\mathbf{R}^a(t_3), \mathbf{P}^a(t_3)$ and $\mathbf{R}^{a'}(t_4), \mathbf{P}^{a'}(t_4)$ that occur in the path integral [cf. (II.54,55)]:

$$\bar{L}_{3_a 4_{a'}}^R = s_{a'} \tilde{R}(t_3 - t_4, \mathbf{R}^a(t_3) - \mathbf{R}^{a'}(t_4)) \times \frac{1}{2} [1 - 2\bar{\rho}_0^a(\mathbf{R}^{a'}(t_4), \mathbf{P}^{a'}(t_4))], \quad (6a)$$

$$\bar{L}_{3_a 4_{a'}}^I = s_a s_{a'} \tilde{I}(t_3 - t_4, \mathbf{R}^a(t_3) - \mathbf{R}^{a'}(t_4)), \quad (6b)$$

Here s_a stands for $s_{F/B} = \pm 1$, $\bar{\rho}_0^a(\mathbf{R}, \mathbf{P})$ is the single-particle density matrix in a mixed position-momentum representation [cf. Eq. (A.6)], while $\tilde{R}(t, \mathbf{R})$ and $\tilde{I}(t, \mathbf{R})$ are real functions [given by Eq. (A.7)] that are, respectively, proportional to retarded and Keldysh parts of the

interaction propagators ($\tilde{R} = \mathcal{L}^R, \tilde{I} = i\frac{1}{2}\mathcal{L}^K$).

Via the influence functional, Eqs. (3) to (6) concisely incorporate the effects of interactions into the path integral approach. \bar{S}_I describes the *classical* part of the effective environment, and corresponds to the contribution calculated by AAK.⁶⁾ With \bar{S}_R , GZ succeeded additionally also include the quantum part of the environment, and in particular, via the occurrence of the density matrix $\bar{\rho}_0^a(\mathbf{R}, \mathbf{P})$ in Eq. (6a), to properly account for the Pauli principle. Note, though, that Eqs. (3) to (6) all refer to a given impurity configuration; impurity averaging still has to be performed, and in the path integral formalism it is by no means easy to do this properly.

4. GZ's strategy for determining τ_φ

To calculate the decoherence time τ_φ , GZ argue as follows: the effective action in Eq. (3) in general causes the Cooperon to decay with increasing time, say as $\tilde{C}_{12', \bar{2}1'} \sim \tilde{C}_{12', \bar{2}1'}^0 e^{-f_d(t_1 - t_2)}$ [cf. (II.66), (IV.10)], where $f_d(t)$ is an increasing function of time; τ_φ is the time scale characterizing this decay, set by $f_d(\tau_\varphi) \simeq 1$. To obtain the function f_d , GZ evoke a standard semiclassical argument: since the path integral is dominated by the saddle point paths of the free action \bar{S}_0^a , i.e. by the set of *classical*, time-reversed diffusive paths, they take f_d to be (i) the disorder average¹¹⁾ $\langle \rangle_{\text{dis}}$ of the sum over all classical paths $\langle \rangle_{\text{cl}}$ of the effective action evaluated along such a path ($i\bar{S}_R^{\text{cl}} + \bar{S}_I^{\text{cl}}$), but (ii) without including any non-classical paths:

$$f_d(t_1 - t_2) \equiv \left\langle \left\langle (i\bar{S}_R^{\text{cl}} + \bar{S}_I^{\text{cl}})(t_1, t_2) \right\rangle_{\text{cl}} \right\rangle_{\text{dis}} \quad (7)$$

[cf. (III.22), (IV.11)]. Moreover, in the spirit of semiclassical approximations, they (iii) take the limit¹²⁾ $\hbar \rightarrow 0$ in the single-particle density matrix occurring in Eq. (6a) for \bar{S}_R , i.e., they replace $\bar{\rho}_0^a(\mathbf{R}, \mathbf{P})$ of Eq. (A.6) by the ‘‘occupation number’’ $n_0(h_0(\mathbf{R}, \mathbf{P}))$, where $n_0(\xi) = 1/[e^{\xi/T} + 1]$ is the Fermi function [cf. (II.43), (II.68)].

Within the approximations (i), (ii) and (iii), GZ find that $\bar{S}_R^{\text{cl}} \simeq 0$ for any given pair of classical, time-reversed paths, and hence conclude that \bar{S}_R is ‘‘irrelevant’’ for decoherence, which is thus determined by \bar{S}_I alone [cf. discussion before (III.22), or after (IV.31).] They thus calculate f_d purely from \bar{S}_I^{cl} [see, e.g., (IV.12)], and from $f_d(\tau_\varphi) \simeq 1$ find, in 1 dimension for example, [cf. (II.76,77)]

$$\frac{1}{\tau_\varphi} = \frac{e^2 \sqrt{2D}}{\sigma_{\text{DC}}} \int_{1/\tau_\varphi}^{\omega_{\text{max}}} \frac{d\omega}{2\pi} \frac{\coth(\omega/2T)}{\sqrt{\omega}} \quad (8)$$

$$= \frac{e^2 \sqrt{2D}}{\pi \sigma_{\text{DC}}} (2T \sqrt{\tau_\varphi} + \sqrt{\omega_{\text{max}}}). \quad (9)$$

Note that the frequency integral has an ultraviolet divergence at large ω , and hence has to be cut off by hand.¹³⁾ GZ cut it off at $\omega_{\text{max}}^{\text{GZ}} = 1/\tau_{\text{el}}$, the inverse elastic mean free scattering time, arguing¹³⁾ that at higher ω (smaller times) the ‘‘approximation of electron diffusion becomes incorrect’’ [cf. paragraph before (II.76)]. This leads to a finite decoherence time at zero temperature in Eq. (9), $1/\tau_\varphi(T \rightarrow 0) \neq 0$. In contrast, according to the phi-

losophy of AAK,⁶⁾ one should take $\omega_{\max}^{\text{AAK}} = T/\hbar$, since frequencies larger than T/\hbar would correspond to *virtual* excitations of the environment, which are believed not to contribute to decoherence. This would yield the standard result $1/\tau_\varphi(T \rightarrow 0) = 0$. Thus, *the controversy centers on the question whether the higher frequency modes do contribute to low temperature decoherence or not.*

5. On the importance of \bar{S}_R for decoherence

Although we agree with GZ's influence functional (3) to (6), *we disagree with GZ's central conclusion that \bar{S}_R is irrelevant for decoherence.* Firstly, influence functionals have the general feature that deducing decoherence properties from \bar{S}_I alone is reliable only for highly excited states: as emphasized in a very insightful recent paper by F. Marquardt,¹⁴⁾ neglecting \bar{S}_R would amount to neglecting dissipative effects by which the quantum system can give back to the environment some of the energy which it picks up from the classical part of the environment; this neglect of dissipation would thus cause the system to heat up beyond what is allowed by detailed balance (an effect which can be ignored for decoherence only if the system is already highly excited).

Secondly, and more specifically: even if one puts aside all reservations^{11,12)} about approximations (i) and (iii) [although these reservations *are* serious], and accepts GZ's result that $\bar{S}_R^{\text{cl}} \simeq 0$ for classical paths, a question remains about approximation (ii): what about fluctuations, i.e. contributions from paths in the path integral that deviate slightly from the classical paths"? If the "leading" contribution vanishes, the "subleading" one becomes of interest! Indeed, it is natural to expect that such close-to-classical paths can produce a significant contribution missed by GZ's semiclassics, say $\delta\bar{S}_R \neq 0$, due to the following intrinsic problem (IP): *\bar{S}_R depends on the density matrix $\bar{\rho}_0(\mathbf{R}, \mathbf{P})$, which at low temperatures is a very sharp function of \mathbf{R} and \mathbf{P} , and hence very sensitive to fluctuations of (or approximations inside¹²⁾) its arguments.* This intrinsic problem sets the present problem apart from other, exactly solvable models such as the Caldeira-Leggett model, and renders meaningless any attempts³⁾ to justify neglecting \bar{S}_R here by comparisons to such other models.

Let us estimate¹⁰⁾ the importance of these fluctuations, by estimating $\delta\bar{S}_R$ for a segment of a diffusive path in which an electron travels ballistically with velocity v_F (and energy $\simeq \mu$) for a time τ_{el} between two scattering events at two impurities. Now, the close-to-classical paths that still contribute significantly to the path integral are those for which $(\bar{S}_0 - \bar{S}_0^{\text{cl}}) \lesssim \hbar$. Considering only the first term in Eq. (A.4) for \bar{S}_0 , we thus use

$$\hbar \simeq (\bar{S}_0 - \bar{S}_0^{\text{cl}}) \simeq \int_0^{\tau_{el}} dt \delta\mathbf{P} \cdot \dot{\mathbf{R}} \simeq \tau_{el} \delta P v_F, \quad (10)$$

or $\delta P \simeq \hbar/\tau_{el}v_F$ (which is just the uncertainty principle), to estimate the typical magnitude of momentum fluctuations around P_F . It follows that

$$\delta n_0(h_0(\mathbf{R}, \mathbf{P})) \simeq \frac{\delta h_0(\mathbf{R}, \mathbf{P})}{T} = \frac{P_F \delta P}{mT} = \frac{\hbar}{\tau_{el}T}. \quad (11)$$

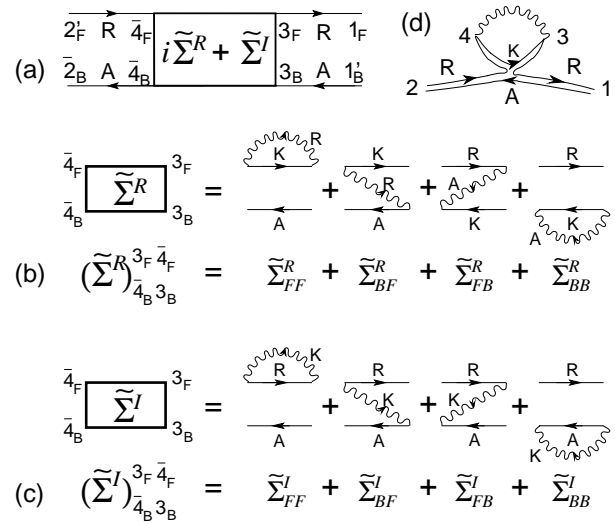


Fig. 2. First order contributions to the irreducible self energy of the Cooperon, illustrating (a) Eq. (A.15), and (b,c) Eqs. (A.16). (d): typical path contributing to $\tilde{\Sigma}_{FF}^R$; GZ neglect $\tilde{\Sigma}^R$ and hence such paths, as first pointed out in Ref.⁸⁾

Now, *GZ neglect $\delta\bar{S}_R$ relative to \bar{S}_I^{cl} ; since this requires $\delta n_0 \lesssim 1$, it can be reliable only for $T \gtrsim \hbar/\tau_{el}$.* For smaller T , the contributions of $\delta\bar{S}_R$ should become important.¹⁵⁾

Having realized that $\delta\bar{S}_R$ is important at low temperatures, the natural next question is: can $\delta\bar{S}_R$ cancel the ultraviolet divergence arising from \bar{S}_I^{cl} ? We believe it does, but showing this will require a more accurate calculation than GZ's, that does not resort to semiclassical arguments and avoids approximations (i) to (iii).

6. Obtaining diagrams from influence functional

As a first step in that direction, we have explored the connection between GZ's influence functional and standard diagrammatic perturbation theory. The connection turns out to be remarkably simple: upon performing the momentum integrals in $\int \mathcal{D}(\mathbf{R}\mathbf{P})$ and expanding the resulting influence functional (some details are given in App. A.2), one generates a Dyson-like equation for the Cooperon [Eq. (A.14)], with a self energy whose lowest order¹⁶⁾ irreducible diagrams [given by Eq. (A.16)] are depicted diagrammatically in Fig. 2. Remarkably, *the resulting diagrams coincide precisely with those obtained by standard Keldysh diagrammatic perturbation theory*, as depicted, e.g., in Fig. 2 of Ref.⁹⁾ (There, impurity lines needed for impurity averaging are also depicted; in our Fig. 2, they are suppressed). This fact, which is our main new result, is a strong indication that the expressions of Eqs. (3) to (6) for the influence functional are sound, and the approximations made during its derivation reasonable [steps (5) and (6) in App. A.1, and Ref.¹⁷⁾].

Moreover, this fact also allows us to shed new light on the roles of \bar{S}_R and \bar{S}_I in the effective action. \bar{S}_I gives rise to the terms $\tilde{\Sigma}_I$, which contain a factor $\coth(\hbar\omega/2T)$ [arising from the factor $\bar{I}(\mathbf{k}, \omega)$, cf. Eqs. (A.7b) and (A.13)], and \bar{S}_R gives rise to the terms $\tilde{\Sigma}_R$, which contain a factor $\tanh[(\epsilon_\lambda - \omega)/2T]$ [arising from the factor $(\tilde{\delta} - 2\tilde{\rho}^0)$ in Eq. (A.13)]. Now, the sum of all the self-energy diagrams for $i\tilde{\Sigma}_R + \tilde{\Sigma}_R$ in Fig. 2 has been eval-

uated in Ref.,⁹⁾ in the energy-momentum representation and for those choices of energy and momentum arguments that are relevant for determining the Cooperon lifetime; the result [Eqs. (3,4) of Ref.⁹⁾] was found to be *not* ultraviolet divergent, since the coth and tanh functions always occur in the combination

$$\int \frac{d\omega}{2\pi} \{ \tanh[(\varepsilon_\lambda - \omega)/2T] + \coth(\omega/2T) \} \dots, \quad (12)$$

so that the frequency integral is cut off at $\omega_{\max} \simeq T/\hbar$, as anticipated by AAK.⁶⁾ We expect this cancellation of UV divergencies from $i\bar{S}_R$ and \bar{S}_I to occur not only in first order perturbation theory, but in every order,²²⁾ since the structure, $e^{-(i\bar{S}_R + \bar{S}_I)}$, of the influence functional is such that the self energy contribution $i\tilde{\Sigma}_R$ and $\tilde{\Sigma}_I$ *always* occur in the combination $i\tilde{\Sigma}_R + \tilde{\Sigma}_I$.¹⁶⁾

Conversely, we conclude that GZ's neglect of $\delta\bar{S}_R$ corresponds to neglecting the contribution of \bar{S}_R to the Cooperon self energy, i.e., *GZ neglect all the diagrams of Fig 2(b)*, as first pointed out in Ref.⁹⁾ As argued above, we believe that this is allowed only if $T \gtrsim \hbar/\tau_{el}$.

7. Conclusion

Our analysis can be summarized as follows: GZ's influence functional strategy is sound in principle; when implemented with sufficient care, it properly incorporates, via \bar{S}_R , the Pauli principle. However, GZ neglect the latter by neglecting $\delta\bar{S}_R$ during the semiclassical calculation of τ_φ . This can work only for large temperatures.

A complete, first-principle evaluation of τ_φ would be obtained if one sums up the Dyson-like equation (A.14) for the Cooperon in the presence of disorder, either diagrammatically or by using path integral techniques, but *without* neglecting $\delta\bar{S}_R$. To the best of our knowledge, this program has not yet been fully carried out to the end. Thus, at least in the eyes of the present author, the fate of τ_φ as $T \rightarrow 0$ for the present model has not yet been worked out in complete, conclusive detail. At present we see no reason to believe, though, that the result will disagree with the conclusions of AAK.

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Note added in proof:— Very recently, GZ defended their theory yet again²³⁾ in cond-mat/020814.

Appendix: Appendix

This appendix summarizes, without derivations, useful technical results that are alluded to in the main text.

A.1 Kubo formula in terms of path integrals

The Kubo formula for the DC conductivity of a d -dimensional conductor can be expressed as

$$\sigma_{DC} = - \lim_{\omega \rightarrow 0} \frac{e}{d} \sum_{\sigma_1} \int dx_2 \mathbf{j}_{11'} \cdot \mathbf{r}_2 \tilde{G}_{11',22}^R(\omega_0) \Big|_{x_1=x_{1'}},$$

$$\tilde{G}_{11',22'}^R(\omega_0) = \int_{-\infty}^{\infty} dt_{12} e^{i\omega_0 t_{12}} \theta(t_{12}) \tilde{C}_{[11',22']}, \quad (A.1)$$

$$\tilde{C}_{[11',22']} \equiv \frac{1}{\hbar} \langle [\hat{\psi}^\dagger(t_1, x_{1'}) \hat{\psi}(t_1, x_1), \hat{\psi}^\dagger(t_2, x_{2'}) \hat{\psi}(t_2, x_2)] \rangle_H,$$

where $\mathbf{j}_{11'} \equiv \frac{-ie\hbar}{2m} (\nabla_1 - \nabla_{1'})$ and a uniform applied electric field $\mathbf{E}(\omega_0)$ was represented using a scalar potential, $V_{\text{ext}}(\omega_0, \mathbf{r}_2) = -\mathbf{r}_2 \cdot \mathbf{E}(\omega_0)$. A path integral representation for $\tilde{C}_{[11',22']}$ can be derived using the following strategy, adapted¹⁸⁾ from GZ's Ref.:²⁾ (1) introduce a source term, in which an artificial source field $\tilde{v}_{2'2}$ couples to $\hat{\psi}^\dagger(t_2, x_{2'}) \hat{\psi}(t_2, x_2)$, and write $\tilde{C}_{[11',22]}$ as the linear response of the single-particle density matrix $\tilde{\rho}_{11'} = \langle \hat{\psi}^\dagger(t_1, x_{1'}) \hat{\psi}(t_1, x_1) \rangle_H$ to the source field $\tilde{v}_{2'2}$. (2) Decouple the interaction using a Hubbard-Stratonovitch transformation, thereby introducing a functional integral over real scalar fields $V_{F/B}$, the so-called “interaction fields”; these then constitute a dynamic, dissipative environment with which the electrons interact. (3) Derive an equation of motion for $\tilde{\rho}_{11'}^V$, the single-particle density matrix for a given, fixed configuration of the fields $V_{F/B}$, and linearize it in $\tilde{v}_{2'2}$, to obtain an equation of motion for the linear response $\delta\tilde{\rho}_{11'}^V(t)$ to the source field. (4) Formally integrate this equation of motion by introducing a path integral $\int \mathcal{D}(\mathbf{R}\mathbf{P})$ over the coordinates and momenta of the single degree of freedom associated with the single-particle density matrix $\delta\tilde{\rho}_{11'}^V$. (5) Use the RPA-approximation to bring the effective action S_V that governs the dynamics of the fields $V_{F/B}$ into a quadratic form. (6) Neglect the effect of the interaction on the single-particle density matrix wherever it still occurs, i.e. replace $\tilde{\rho}_{ij}^V$ by the free single-particle density matrix

$$\tilde{\rho}_{ij}^0 = \langle \hat{\psi}^\dagger(x_j) \hat{\psi}(x_i) \rangle_0 = \sum_\lambda \psi_\lambda^*(x_j) \psi_\lambda(x_i) n_0(\xi_\lambda), \quad (A.2)$$

where $\langle \hat{O} \rangle_0 = \text{Tr}[e^{-\beta\hat{H}_0} \hat{O}] / \text{Tr}[e^{-\beta\hat{H}_0}]$. (7) Perform the functional integral (which steps (5) and (6) have rendered Gaussian) over the fields $V_{F/B}$; the environment is thereby integrated out, and its effects on the dynamics of the single particle are encoded in an influence functional of the form $e^{-(i\bar{S}_R + \bar{S}_I)}$. The final result of this strategy is that $\tilde{C}_{[11',22]}$ can be written as¹⁷⁾ [cf. (II.49)]

$$\lim_{t_0 \rightarrow -\infty} \int dx_{\bar{2}} \{ \tilde{J}_{12',2\bar{2},\bar{2}1'}(t_1, t_2; t_0) - \tilde{J}_{1\bar{2},2\bar{2}',\bar{2}1'}(t_1, t_2; t_0) \},$$

$$\tilde{J}_{12',2\bar{2}',\bar{2}1'}(t_1, t_2; t_0)_V = \int_{2'_F, \bar{2}_B}^{1'_F, 1'_B} \mathcal{D}(\mathbf{R}\mathbf{P}) \quad (A.3)$$

$$\times \int_{0_F, \bar{0}_B}^{2_F, \bar{2}_B} \mathcal{D}(\mathbf{R}\mathbf{P}) \frac{1}{\hbar} \int dx_0 dx_{\bar{0}} \tilde{\rho}_{0\bar{0}}^0 e^{-[i\bar{S}_R + \bar{S}_I](t_1, t_0)/\hbar}.$$

where $t_{\bar{2}} = t_2$, i.e. the indices 2 and $\bar{2}$ refer to the same time t_2 , but the integration variable $x_{\bar{2}}$ is independent of x_2 . The symbol $\int \mathcal{D}(\mathbf{R}\mathbf{P})$ stands for the double path integral defined in Eq. (4). The complex weighting functional $e^{i(\bar{S}_0^F - \bar{S}_0^B)}$ occurring therein involves the action for a single, free electron, in the mixed coordinate-momentum representation,

$$\bar{S}_0^a(t_i, t_j)[\mathbf{R}^a(t_3), \mathbf{P}^a(t_3)] \quad (\text{A.4})$$

$$= \int_{t_j}^{t_i} dt_3 [\mathbf{P}^a(t_3) \cdot \partial_{t_3} \mathbf{R}^a(t_3) - \bar{h}_0(\mathbf{R}^a(t_3), \mathbf{P}^a(t_3))],$$

$$\bar{h}_0(\mathbf{R}^a, \mathbf{P}^a) = \frac{P^{a2}}{2m} + V_{\text{imp}}(\mathbf{R}^a) - \mu. \quad (\text{A.5})$$

The first line of Eq. (A.3) corresponds to Eq. (3), and its significance as describing Cooperon propagation along two paths from $\mathbf{r}_{2'}$ at time t_2 to \mathbf{r}_1 at t_1 , and $\mathbf{r}_{1'}$ at time t_1 to \mathbf{r}_2 at t_2 , is explained after Eq. (4). The integrals in the second line of Eq. (A.3) “initialize” this Cooperon¹⁹ at an initial time $t_0 \rightarrow -\infty$, by including forward and backward propagation [see the left (non-loop) part of Fig. 1] between the times t_0 and t_2 , from and to some initial positions \mathbf{r}_0 and $\bar{\mathbf{r}}_0$, which are weighted by the initial, free single-particle density matrix $\tilde{\rho}_{00}^0$ of Eq. (A.2).

The effective action $i\bar{S}_R + \bar{S}_I$ in the influence functional $e^{-(i\bar{S}_R + \bar{S}_I)}$ in Eq. (A.3) is given by Eqs. (5) and (6). \bar{S}_R depends via Eq. (6a) on the single-particle density matrix in a mixed coordinate-momentum representation,²⁰

$$\tilde{\rho}_0^F(\mathbf{R}^F, \mathbf{P}^F) = \int d\mathbf{r}^F e^{-i\mathbf{p}^F \cdot \delta\mathbf{r}^F} \tilde{\rho}^0(\mathbf{R}^F + \delta\mathbf{r}^F, \mathbf{R}^F),$$

$$\tilde{\rho}_0^B(\mathbf{R}^B, \mathbf{P}^B) = \int d\mathbf{r}^B e^{-i\mathbf{p}^B \cdot \delta\mathbf{r}^B} \tilde{\rho}^0(\mathbf{R}^B, \mathbf{R}^B - \delta\mathbf{r}^B) \quad (\text{A.6})$$

where $\tilde{\rho}^0(\mathbf{r}_i, \mathbf{r}_j) = \tilde{\rho}_{ij}^0$ is defined in Eq. (A.2).

Furthermore, $i\bar{S}_R + \bar{S}_I$ depends on two purely real functions, $\tilde{R}(t, \mathbf{r})$ and $\tilde{I}(t, \mathbf{r})$, that are defined as follows via their Fourier transforms [cf.(II.56,57)]:

$$(\tilde{R}/\tilde{I})(t, \mathbf{R}) = \int \frac{d\mathbf{k}d\omega}{(2\pi)^{d+1}} e^{-i\omega t + i\mathbf{k} \cdot \mathbf{R}} (\tilde{R}/\tilde{I})(\omega, \mathbf{k}),$$

$$\tilde{R}(\omega, \mathbf{k}) = \frac{\tilde{V}^{\text{int}}(\mathbf{k})}{1 - \tilde{V}^{\text{int}}(\mathbf{k})\tilde{\chi}(\omega, \mathbf{k})}, \quad (\text{A.7a})$$

$$\tilde{I}(\omega, \mathbf{k}) = -\coth(\hbar\omega/2T) \text{Im} \tilde{R}(\omega, \mathbf{k}). \quad (\text{A.7b})$$

Here $\tilde{V}^{\text{int}}(\mathbf{q})$ and $\tilde{\chi}(\omega, \mathbf{k})$ are the Fourier transforms of the interaction potential $\tilde{V}^{\text{int}}(|\mathbf{r}_1 - \mathbf{r}_2|)$ and the charge susceptibility $\tilde{\chi}_{12}$, which can be written as

$$\tilde{\chi}_{12} = -ie^2\hbar(\tilde{G}_{12}^R\tilde{G}_{21}^K + \tilde{G}_{12}^K\tilde{G}_{21}^A). \quad (\text{A.8})$$

$\tilde{G}_{ij}^{R/A/K}$ are the retarded, advanced and Keldysh Green's functions of the noninteracting, disordered system:

$$\tilde{G}_{ij}^{R/A} = \mp \frac{i}{\hbar} \theta(\pm t_{ij}) \sum_{\lambda} \psi_{\lambda}^*(x_j) \psi_{\lambda}(x_i) e^{-i\xi_{\lambda} t_{ij}/\hbar}, \quad (\text{A.9a})$$

$$\tilde{G}_{ij}^K = -\frac{i}{\hbar} \sum_{\lambda} \psi_{\lambda}^*(x_j) \psi_{\lambda}(x_i) e^{-i\xi_{\lambda} t_{ij}/\hbar} [1 - 2n_0(\xi_{\lambda})].$$

A.2 Derivation of Cooperon self energy from influence functional

The path integrals $\int \mathcal{D}(\mathbf{R}\mathbf{P})$ of Eq. (3) can be given a precise definition in terms of the standard time-slicing procedure for path integrals, with one coordinate and one momentum integral for every time slice.²⁰ For each time slice (labeled by n , say), the momentum integral $\int d\mathbf{P}_n$ can then easily be performed, since it simply has the effect of converting the expressions occurring in the action for that time slice from the mixed coordinate-momentum representation to the coordinate-only representation. Thus, the free action $\bar{S}_0^a[\mathbf{R}, \mathbf{P}]$ is mapped to

$$\tilde{S}_0^a(t, t')[\mathbf{R}^a, \dot{\mathbf{R}}^a] = \int_{t'}^t dt_3 \left[\frac{1}{2} m \dot{\mathbf{R}}^{a2}(t_3) - V_{\text{imp}}(\mathbf{R}^a(t_3)) \right],$$

the density matrix at time slice n is converted²⁰ from $\tilde{\rho}_n^a(\mathbf{R}_n^a, \mathbf{P}_n^a)$ to $\tilde{\rho}^0(\mathbf{r}_n^a, \mathbf{r}_{n-1}^a)$, and $\tilde{C}_{12', \bar{2}1'}$ of Eq. (3) can be rewritten as

$$\tilde{C}_{12', \bar{2}1'} = \int_{2'_F, \bar{2}_B}^{1'_F, 1'_B} \mathcal{D}'(\mathbf{R}) e^{-[i\bar{S}_R + \bar{S}_I](t_1, t_2)/\hbar}, \quad (\text{A.10})$$

where the integral $\int \mathcal{D}'(\mathbf{R})$ is used as a shorthand for

$$\int_{2'_F, \bar{2}_B}^{1'_F, 1'_B} \mathcal{D}'(\mathbf{R}) = \int_{\mathbf{R}^F(t_2)=\mathbf{r}_{2'}}^{\mathbf{R}^F(t_1)=\mathbf{r}_1} \mathcal{D}'\mathbf{R}^F(t_3) \quad (\text{A.11})$$

$$\times \int_{\mathbf{R}^B(t_2)=\mathbf{r}_2}^{\mathbf{R}^B(t_1)=\mathbf{r}_{1'}} \mathcal{D}'\mathbf{R}^B(t_3) e^{i[\bar{S}_0^F - \bar{S}_0^B]/\hbar}.$$

The effective action $i\tilde{S}_R + \tilde{S}_I$ in Eq. (A.10) is found to have the following form²¹ [with $\tilde{\delta}_{ii} = \delta_{\sigma_i \sigma_i} \delta(\mathbf{r}_i - \mathbf{r}_i)$ and $(\tilde{R}/\tilde{I})_{\bar{i}a\bar{j}a'} = (\tilde{R}/\tilde{I})(t_i - t_j, \mathbf{r}_i^a(t_i) - \mathbf{r}_j^a(t_j))$]:

$$\tilde{S}_{R/I}(t_1, t_2) = \sum_{aa'} \int_{t_2}^{t_1} dt_{3a} \int_{t_2}^{t_{3a}} dt_{4a'} \tilde{L}_{3a4a'}^{R/I}, \quad (\text{A.12})$$

$$\tilde{L}_{3_F4_F}^{R/I} = \begin{cases} \frac{1}{2} \tilde{\delta}_{3_F3_F} (\tilde{\delta} - 2\tilde{\rho}^0)_{4_F4_F} \tilde{R}_{3_F4_F}^{R/I} \\ \tilde{\delta}_{3_F3_F} \tilde{\delta}_{4_F4_F} \tilde{I}_{3_F4_F}^{R/I}, \end{cases} \quad (\text{A.13a})$$

$$\tilde{L}_{3_B4_F}^{R/I} = \begin{cases} -\frac{1}{2} \tilde{\delta}_{3_B3_B} (\tilde{\delta} - 2\tilde{\rho}^0)_{4_F4_F} \tilde{R}_{3_B4_F}^{R/I} \\ -\tilde{\delta}_{3_B3_B} \tilde{\delta}_{4_F4_F} \tilde{I}_{3_B4_F}^{R/I}, \end{cases} \quad (\text{A.13b})$$

$$\tilde{L}_{3_F4_B}^{R/I} = \begin{cases} \frac{1}{2} \tilde{\delta}_{3_F3_F} (\tilde{\delta} - 2\tilde{\rho}^0)_{4_B4_B} \tilde{R}_{3_F4_B}^{R/I} \\ -\tilde{\delta}_{3_F3_F} \tilde{\delta}_{4_B4_B} \tilde{I}_{3_F4_B}^{R/I}, \end{cases} \quad (\text{A.13c})$$

$$\tilde{L}_{3_B4_B}^{R/I} = \begin{cases} -\frac{1}{2} \tilde{\delta}_{3_B3_B} (\tilde{\delta} - 2\tilde{\rho}^0)_{4_B4_B} \tilde{R}_{3_B4_B}^{R/I} \\ \tilde{\delta}_{3_B3_B} \tilde{\delta}_{4_B4_B} \tilde{I}_{3_B4_B}^{R/I}. \end{cases} \quad (\text{A.13d})$$

Now expand the effective action in powers of $\tilde{S}_{R/I}$,

$$\tilde{C}_{12', \bar{2}1'} = \sum_{N=0}^{\infty} \frac{1}{N!} \int_{2'_F, \bar{2}_B}^{1'_F, 1'_B} \mathcal{D}'(\mathbf{R}) \quad (\text{A.14})$$

$$\times \left[\frac{-1}{\hbar} \sum_{aa'} \int_{t_0}^{t_1} dt_{3a} \int_{t_0}^{t_{3a}} dt_{4a'} [i\tilde{L}_{3a4a'}^R + \tilde{L}_{3a4a'}^I] \right]^N,$$

and analyze the structure of this expansion, which can be viewed as a type of Dyson equation for the Cooperon. The $N = 0$ term simply yields free propagation between the specified end points, $\tilde{C}_{12',21'}^0 = \hbar^2 \tilde{G}^{R,1F2F} \tilde{G}_{2B,1B}^A$. The $N = 1$ term of Eq. (A.14) turns out to have the form of a forward-backward propagator sandwiching self-energy insertions,²¹⁾ say $\tilde{\Sigma}_{aa'}^{R/I}$, that are depicted diagrammatically in Fig. 2 and are given by:

$$\int_{2'_F, 2_B}^{1_F, 1'_B} \mathcal{D}'(\mathbf{R}) \tilde{L}_{aa'}^{R/I}(t_3, t_4) = \int dx_{3_F} dx_{3_B} dx_{4_F} dx_{4_B} \hbar^4 \tilde{G}^{R,1F3F} \tilde{G}_{2B,4B}^A \left(\tilde{\Sigma}_{aa'}^{R/I} \right)_{4B,3B}^{3F,4F} \tilde{G}^{R,4F2F} \tilde{G}_{3B,1B}^A. \quad (\text{A.15})$$

$$\begin{aligned} \left(\tilde{\Sigma}_{FF}^{R/I} \right)_{4B,3B}^{3F,4F} &= \hbar^2 (\tilde{G}^{K/R})_{3F,4F} \tilde{G}_{4B,3B}^A (\mathcal{L}^R / i\mathcal{L}^K)_{3F,4F} \\ \left(\tilde{\Sigma}_{BF}^{R/I} \right)_{4B,3B}^{3F,4F} &= -\hbar^2 (\tilde{G}^{K/R})_{3F,4F} \tilde{G}_{4B,3B}^A (\mathcal{L}^R / i\mathcal{L}^K)_{3B,4F} \\ \left(\tilde{\Sigma}_{FB}^{R/I} \right)_{4B,3B}^{3F,4F} &= -\hbar^2 \tilde{G}^{R,3F4F} (\tilde{G}^{K/A})_{4B,3B} (\mathcal{L}^R / i\mathcal{L}^K)_{3F,4B} \\ \left(\tilde{\Sigma}_{BB}^{R/I} \right)_{4B,3B}^{3F,4F} &= \hbar^2 \tilde{G}^{R,3F4F} (\tilde{G}^{K/A})_{4B,3B} (\mathcal{L}^R / i\mathcal{L}^K)_{3B,4B}. \end{aligned} \quad (\text{A.16})$$

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- 7) Most relevant references can be found in Ref.,⁴⁾ which gives a useful overview of the controversy from GZ's point of view.
- 8) I. Aleiner, B. L. Altshuler, and M. E. Gershenson, Waves and Random Media **9**, 201 (1999) [cond-mat/9808053].
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- 10) This argument is due to F. Marquardt (priv. communication).
- 11) Point (i), namely performing the disorder average in the exponent, $\langle e^{-(i\tilde{S}_R + \tilde{S}_I)} \rangle_{\text{dis}} \rightarrow e^{\langle -(i\tilde{S}_R + \tilde{S}_I) \rangle_{\text{dis}}}$, involves an approximation which, unless careful justification is provided, is *a priori* dubious for disordered systems, for which in general $\langle AB \rangle_{\text{dis}} \neq \langle A \rangle_{\text{dis}} \langle B \rangle_{\text{dis}}$.
- 12) Approximation (iii) is [as] problematic [as (ii)], because it occurs *inside* the argument of the density matrix $\tilde{\rho}_0^a(\mathbf{R}, \mathbf{P})$, which has an infinitely sharp step when $T \rightarrow 0$. In the coordinate representation, the density matrix can be written as $\tilde{\rho}_{ij}^0 = \left\{ \sum_{n=0}^{\infty} \frac{\partial^n n_0(0)}{n!} \left[\frac{(-i\hbar \nabla_i)^2 / 2m + V_{\text{imp}}(\mathbf{r}_i) - \mu}{2T} \right]^n \right\} \delta_{ij}$. Now, GZ neglect the fact that $\hbar \nabla_{\mathbf{r}_i}$ does not commute with $V_{\text{imp}}(\mathbf{r}_i)$, arguing that this is justified here in the semiclassical limit $\hbar \rightarrow 0$. We disagree, since in the above expression for $\tilde{\rho}_{ij}^0$ every neglected power of \hbar is accompanied by at least one power of $1/T$, i.e. the neglected terms are of order $\hbar/(T\tau_{\text{char}})$, where τ_{char} is a time scale characteristic of the disorder potential (rough estimates suggest $\tau_{\text{char}} \simeq \tau_{\text{el}}$). Thus, they can

- be neglected only for $T \gtrsim \hbar/\tau_{\text{el}}$.
- 13) A complete theory would not need such an ad hoc cut-off. For example, to properly treat frequencies above $1/\tau_{\text{el}}$ it would be sufficient to use, when evaluating \tilde{S}_I^{cl} , the corresponding generalized expressions for the Cooperon propagator that hold also in the ballistic regime, see, e.g., Eq. (3.9) of G. Zala, B. N. Naroshny and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001). However, in our opinion such generalizations, though possible, are in fact unnecessary for the present problem: GZ's theory *is* complete if \tilde{S}_R is treated properly, since the combination $i\tilde{S}_R + \tilde{S}_I$ will "automatically" introduce an effective cutoff $\omega_{\text{max}} \simeq T/\hbar$ (see Eq. (12) below).
- 14) F. Marquardt, cond-mat/0207692.
- 15) Strictly speaking, this argument is slightly premature: one next needs to establish whether or not such $\delta n_0 \neq 0$ contributions, when summed over an entire diffusive path, could average to zero. While estimating this would be difficult, the main point to be made here, though, is simple: the fluctuations $\delta \tilde{S}_R$ *have* to be analyzed carefully, and cannot simply be thrown away by hand. Note that the conclusion $T \gtrsim \hbar/\tau_{\text{el}}$ is consistent with the estimate in Ref.,¹²⁾ and with the observation that GZ's results for $\tau_{\varphi}(T)$ agree with those of AAK only if $\omega_{\text{max}}^{\text{AAK}} \gtrsim \omega_{\text{max}}^{\text{GZ}}$.
- 16) By taking the irreducible self energy to be given only by the diagrams of Fig. 2, one neglects diagrams with crossed or overlapping interaction lines. However, these are smaller than those considered by at least one power of $1/g$, where g is the dimensionless conductance.
- 17) Here we have dropped a Hartree-like contribution $\tilde{C}_{[11',22']}^{\text{Hartree}}$ to $\tilde{C}_{[11',22']}$ which corresponds to (II.47) and is dropped by GZ, too. It vanishes in absence of interactions and corresponds to a renormalization of the conductance which does not contribute to decoherence.
- 18) We follow GZ's strategy, as presented in Ref.,²⁾ but the details of our notations (and unpublished derivations) deviate from GZ's whenever we believe that greater compactness, clarity or generality can thereby be achieved.
- 19) In Ref.,²⁾ GZ effectively set $t_0 = t_2$ in Eq. (A.3) [cf. (II-49)], thus they do not initialize in the infinite past. However, their approach *can* (JvD, unpublished) be generalized to the case $t_0 \rightarrow -\infty$ of Eq. (A.3), which is needed to achieve proper equilibration, and consistency with the standard Keldysh approach (for which t_0 is the initial time of the Keldysh contour).
- 20) By evoking the standard time-slicing definition for path integrals, it is possible to properly define $\int \mathcal{D}(\mathbf{R}\mathbf{P})$ as $\prod_{a=F,B} \prod_{n=1}^{M-1} \left(\int d\mathbf{r}_n^a \right) \prod_{n=1}^M \left(\int \frac{d\mathbf{p}_n^a}{(2\pi)^d} \right) e^{i(\dots)}$, etc. In doing so, we choose to use asymmetric conventions for $a = F$ or B : If \mathbf{r}_{n-1}^a and \mathbf{r}_n^a are the end-of-interval coordinates for time slice n , and $\delta\mathbf{r}_n^a = \mathbf{r}_n^a - \mathbf{r}_{n-1}^a$ is the relative coordinate [which is integrated over in the Fourier transforms of Eq. (A-6)], then we choose the "center-of-mass" coordinate as $\mathbf{R}_n^F = \mathbf{r}_{n-1}^F$ and $\mathbf{R}_n^B = \mathbf{r}_n^B$, respectively.
- 21) For each occurrence in Eq. (A-13) for $\tilde{L}^{R/I}$ of a pair of indices, one without bar, one with, e.g. 4_a and $\bar{4}_a$, the corresponding coordinates \mathbf{r}_4^a and $\mathbf{r}_{\bar{4}}^a$ are both associated with the *same* time t_4 , and integrated over, $\int d\mathbf{r}_4^a d\mathbf{r}_{\bar{4}}^a$, in the path integral $\int \mathcal{D}'(\mathbf{R})$. When evaluating Eq. (A-15) for \tilde{L}^I or \tilde{L}^R , one of these integrals can be used, respectively, to collapse the $\delta(\mathbf{r}_4^a - \mathbf{r}_{\bar{4}}^a)$ function in \tilde{L}^I , or to construct a Keldysh function from $\tilde{G}_{34}^K = \int dx_4 (\tilde{G}^R - \tilde{G}^A)_{34} (\delta - 2\bar{\rho}^0)_{44}$.
- 22) GZ have argued that since \tilde{S}_R and \tilde{S}_I are both real functionals if their arguments are real, "*i* \tilde{S}_R can never cancel any contribution from \tilde{S}_I " [discussion before (III.22)]. However, this argument overlooks the fact that \tilde{S}_R and \tilde{S}_I are both functionals of the path integral variables $\mathbf{R}(t_3)$ and $\mathbf{P}(t_3)$, and that the integration measure $e^{i(\tilde{S}_0^F - \tilde{S}_0^B)}$ in $\int \mathcal{D}(\mathbf{R}\mathbf{P})$ is *complex* (GZ's argument would be correct if the integration measure were real).
- 23) However, the analysis there (critized in cond-mat/0208264) is yet again semiclassical, and hence cannot properly deal with the intrinsic problem (IP) pointed out in section 5, namely that the density matrix becomes a sharp function at low temperatures. Thus, we believe that the expansions in Eqs. (40) to (42) of cond-mat/020814 will become singular at low T .