PHYSICAL REVIEW B, VOLUME 65, 241315(R)

Momentum-resolved tunneling into fractional quantum Hall edges

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(Received 8 March 2002; published 24 June 2002)

Tunneling from a two-dimensional contact into quantum-Hall edges is considered theoretically for a case where the barrier is extended, uniform, and parallel to the edge. In contrast to previously realized tunneling geometries, details of the microscopic edge structure are exhibited directly in the voltage and magnetic-field dependence of the differential tunneling conductance. In particular, it is possible to measure the dispersion of the edge-magnetoplasmon mode, and the existence of additional, sometimes counterpropagating, edge-excitation branches could be detected.

DOI: 10.1103/PhysRevB.65.241315 PACS number(s): 73.43.Jn, 71.10.Pm, 73.43.Cd

The quantum-Hall (QH) effect[1] arises due to incompressibilities developing in two-dimensional electron systems (2DES) at special values of the electronic sheet density n_0 and perpendicular magnetic field B for which the *filling factor* $\nu = 2\pi\hbar c n_0/|eB|$ is equal to an integer or certain fractions. The microscopic origin of incompressibilities at fractional ν is electron-electron interaction. Laughlin's trial-wave-function approach[2] successfully explains the QH effect at $\nu = \nu_{1,p} \equiv 1/(p+1)$ where p is a positive even integer. Our current microscopic understanding of why incompressibilities develop at many other fractional values of the filling factor, e.g., $\nu_{m,p} \equiv m/(mp+1)$ with nonzero integer $m \neq \pm 1$, is based on hierarchical theories.[3–5]

The underlying microscopic mechanism responsible for creating charge gaps at fractional ν implies peculiar properties of low-energy excitation in a finite quantum-Hall sample which are localized at the boundary.[6] For $\nu = \nu_{m,p}$, m branches of such edge excitations[7–10] are predicted to exist which are realizations of strongly correlated chiral one-dimensional electron systems called *chiral Luttinger liquids* (χLL). Extensive experimental efforts were undertaken recently to observe χLL behavior because this would yield an independent confirmation of our basic understanding of the fractional QH effect. In all of these studies,[11–16] current-voltage characteristics yielded a direct measure of the energy dependence of the *tunneling density of states* for the QH edge. This quantity generally contains information on global dynamic properties as, e.g., excitation gaps and the orthogonality catastrophe, but lacks any momentum resolution. Power-law behavior consistent with predictions from χLL theory was found[11,12,15] for the edge of QH systems at the Laughlin series of filling factors, i.e., for $\nu = \nu_{1,p}$. However, at hierarchical filling factors, i.e., when $\nu = \nu_{m,p}$ with $|m| > 1$, predictions of χLL theory are, at present, not supported by experiment.[13,14] This discrepancy inspired theoretical works, too numerous to cite here, from which, however, no generally accepted resolution emerged. Current experiments[16] suggest that details of the edge potential may play a crucial rôle. New experiments are needed to test the present microscopic picture of fractional-QH edge excitations.

Here we consider a tunneling geometry that is particularly well suited for that purpose, see Fig. 1, and which has been realized recently for studying the integer QH effect in cleaved-edge overgrown semiconductor heterostructures.[17] In contrast to previous experiments, it provides a *momentum resolved* spectral probe of QH edge excitations.[33] With both the components of canonical momentum parallel to the barrier and energy being conserved in a single tunneling event, strong resonances appear in the differential tunneling conductance dI/dV as a function of the transport voltage and applied magnetic field. Similar resonant behavior for tunneling via extended uniform barriers has been used recently[18–21] to study the electronic properties of low-dimensional electron systems. It has also been suggested as a tool to observe spin-charge separation in Luttinger liquids[22] and the interaction-induced broadening of electronic spectral functions at single-branch QH edges.[23] Here we find that the number of resonant features in dI/dV corresponds directly to the number of chiral edge excitations present. Edge-magnetoplasmon dispersion curves can be measured and power laws related to χLL behavior be observed. Momentum-resolved tunneling spectroscopy in the presently considered geometry thus constitutes a powerful probe to

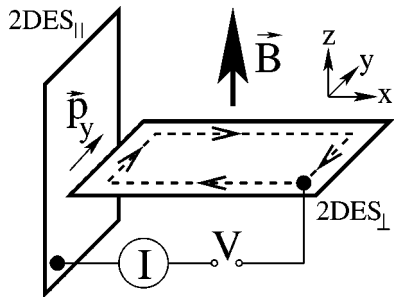

FIG. 1. Schematic picture of tunneling geometry. Two mutually perpendicular two-dimensional electron systems are realized, e.g., in a semiconductor heterostructure. An external magnetic field is applied such that it is perpendicular to one of them ($2\mathrm{DES}_\perp$) but in plane for the other one ($2\mathrm{DES}_\parallel$). When $2\mathrm{DES}_\perp$ is in the quantum-Hall regime, chiral edge channels form along its boundary (indicated by broken lines with arrows). Where they run parallel to $2\mathrm{DES}_\parallel$, electrons tunnel between edge states in $2\mathrm{DES}_\perp$ and plane-wave states in $2\mathrm{DES}_\parallel$ with the *same* quantum number p_y of momentum component parallel to the barrier. Experimentally, the differential tunneling conductance dI/dV is measured.

characterize the QH edge microscopically.

To compute the tunneling conductances, we apply the general expression for the current obtained to lowest order in a perturbative treatment of tunneling,[27]

$$I(V)=\frac{e}{\hbar^2}\sum_{\vec{k}_\parallel,n,X}|t_{\vec{k}_\parallel,n,X}|^2\int\frac{d\varepsilon}{2\pi}\{n_{\rm F}(\varepsilon)-n_{\rm F}(\varepsilon+eV)\}$$
$$\times A_\parallel(\vec{k}_\parallel,\varepsilon)A_\perp(n,X,\varepsilon+eV). \quad (1)$$

Here $A_\parallel$ and $A_\perp$ denote single-electron spectral functions for $2{\rm DES}_\parallel$ and $2{\rm DES}_\perp$, respectively. (See Fig. 1). We use a representation where electron states in the first are labeled by a two-dimensional wave vector[34] $k_\parallel=(k_y,k_z)$, while the quantum numbers of electrons in $2{\rm DES}_\perp$ are the Landau-level index n and guiding-center coordinate X in x direction. We assume that $2{\rm DES}_\parallel$ is located at $x=0$. The simplest form of the tunneling matrix element $t_{\vec{k}_\parallel,n,X}$ reflecting translational invariance in y direction is

$$t_{\vec{k}_\parallel,n,X}=t_n(X)\,\delta(k_y-k), \quad (2)$$

where $k\equiv X/l^2$ with the magnetic length $l=\sqrt{\hbar c/|eB|}$. The dependence of $t_n(X)$ on X results from the fact that an electron from $2{\rm DES}_\perp$ occupying the state with quantum number X is spatially localized on the scale of l around $x=X$. The overlap of its tail in the barrier with that of states from $2{\rm DES}_\parallel$ will drop precipitously as X/l gets large. Finally, $n_{\rm F}(\varepsilon)=[\exp(\varepsilon/k_{\rm B}T)+1]^{-1}$ is the Fermi function. In the following, we use the expression $A_\parallel(\vec{k}_\parallel,\varepsilon)=2\pi\delta(\varepsilon-E_{\vec{k}_\parallel})$ which is valid for a clean system of noninteracting electrons.[35] Here $E_{\vec{k}_\parallel}$ denotes the electron dispersion in $2{\rm DES}_\parallel$.

The spectral function of electrons in $2{\rm DES}_\perp$ depends crucially on the type of QH state in this layer. At integer ν, when single-particle properties dominate and disorder broadening is neglected, it has the form

$$A_\perp(n,X,\varepsilon)\equiv A_n(k,\varepsilon)=2\pi\delta(\varepsilon-E_{nk}), \quad (3)$$

where E_{nk} is the Landau-level dispersion. Strong correlations present at fractional ν alter the spectral properties of edge excitations. In the low-energy limit, it is possible to linearize the lowest-Landau-level dispersion around the Fermi point $k_{\rm F}$. At the Laughlin series $\nu=1/(p+1)$ and for short-range interactions present at the edge, the spectral function was found[9,28] to be

$$A_{\frac{1}{p+1}}(q,\varepsilon)=\frac{z}{p!}\left(\frac{q}{2\pi/L_y}\right)^p\delta(\varepsilon-r\hbar v_{\rm e}q). \quad (4)$$

Here $q\equiv k-k_{\rm F}$, $r=\pm$ distinguishes the two chiralities of edge excitations, L_y is the edge perimeter, $v_{\rm e}$ the edge-magnetoplasmon velocity, and z an unknown normalization constant. The power-law prefactor of the δ function in Eq. (4) is a manifestation of χLL behavior.

The main focus of our work is the sharp QH edge at hierarchical filling factors. Here we provide explicitly the momentum-resolved spectral functions for $\nu=\nu_{\pm2,p}$. Microscopic theories[7,9] predict the existence of two Fermi points $k_{\rm Fo}$ and $k_{\rm Fi}$ which correspond to outer and inner single-branch chiral edges of QH fluids at Laughlin-series filling factors $\nu_{\rm o}^{\pm}=1/(p\pm1)$ and $\nu_{\rm i}^{\pm}=\pm1/[(2p\pm1)(p\pm1)]$, respectively. The negative sign of $\nu_{\rm i^-}$ indicates that the inner edge mode is counterpropagating. We have used the χLL bosonization ansatz[9] and standard methods[29] routinely applied in the study of one-dimensional electron systems to compute the spectral functions. As these have not been obtained before, we briefly discuss their main features here.

According to χLL theory, the existence of two Fermi points gives rise to a discrete infinite set of possible electron tunneling operators at the edge. This is because an arbitrary number N of fractional-QH quasiparticles with charge equal to $e\nu_{\rm o}^{\pm}$ can be transferred to the inner edge after an electron has tunneled into the outer one.[9] Each of these processes gives rise to a separate contribution to the electronic spectral function at the edge which is of the general form

$$A^{(N)}_{\nu_{\pm2,p}}(q,\varepsilon)=\frac{2\pi z}{\Gamma(\eta_1^{(N)})\Gamma(\eta_2^{(N)})}\left(\frac{L_y/2\pi\hbar}{|v_1\mp v_2|}\right)^{\eta_1^{(N)}+\eta_2^{(N)}-1}$$
$$\times|\varepsilon-r\hbar v_1q|^{\eta_2^{(N)}-1}|\varepsilon\mp r\hbar v_2q|^{\eta_1^{(N)}-1}$$
$$\times\{\Theta(r\hbar v_1q-\varepsilon)\Theta(\pm\varepsilon-r\hbar v_2q)$$
$$+\Theta(\varepsilon-r\hbar v_1q)\Theta(r\hbar v_2q\mp\varepsilon)\}. \quad (5)$$

Here $q\equiv k-k_{\rm F}^{(N)}$, where $k_{\rm F}^{(N)}=k_{\rm Fo}-N\nu_{\rm o}^{\pm}(k_{\rm Fo}-k_{\rm Fi})$. The velocities $v_1>v_2>0$ of normal-mode edge-density fluctuations and the exponents $\eta_{1,2}^{(N)}$ depend strongly on microscopic details of the edge, e.g., the self-consistent edge potential and interedge interactions. We focus here on the experimentally realistic case when inner and outer edges are strongly coupled and the normal modes correspond to the familiar[30] charged and neutral edge-density waves. In this limit, we have[30,31] $v_1=v_{\rm c}\sim\mathcal{O}(\ln[L_y/l])$, $v_2=v_{\rm n}\sim\mathcal{O}(1)$ (where c and n denote charged and neutral, respectively), and the exponents assume universal values: $\eta_1^{(N)}=\eta_{\rm c}\equiv p\pm1/2$, $\eta_2^{(N)}=\eta_{\rm n}^{(N)}\equiv(2N\pm1)^2/2$. Note that exponents are generally larger than unity except for $N=0,\mp1$ where $\eta_2^{(N)}=1/2$. In the latter case, an algebraic singularity appears in the spectral function. This is illustrated in Fig. 2. Such divergences will be visible as strong features in the differential tunneling conductance; see below. Contributions to the spectral function for all other values of N do not show such divergences and will give rise only to a featureless background in the conductance.

With spectral functions for $2{\rm DES}_\perp$ at hand, we are now able to calculate tunneling transport. We focus first on the case when $2{\rm DES}_\perp$ is in the QH state at $\nu=1$. For realistic situations, the differential tunneling conductance dI/dV as a function of voltage V and magnetic field B will exhibit two lines of strong maxima whose positions in V-B space are given by the equations

$$E_{0k_V}=\varepsilon_{\rm F\perp}, \quad (6a)$$

$$E_{0k_{\rm F\parallel}}=\varepsilon_{\rm F\perp}+eV. \quad (6b)$$

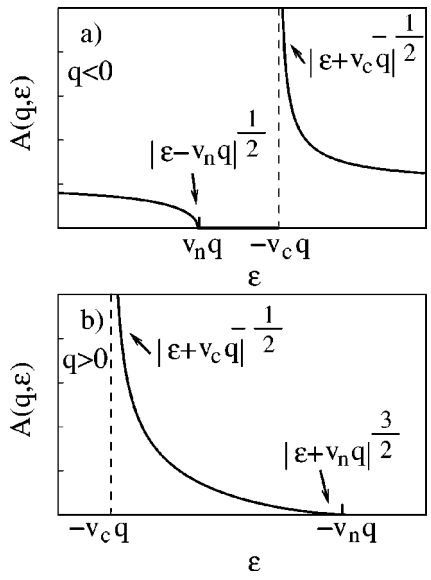

FIG. 2. Spectral functions for two-branch hierarchical fractional-QH edges at bulk filling factor 2/3 [a)] and 2/5 [b)], where the charged (edge-magnetoplasmon) mode is assumed to be left-moving. (a) We show $A^{(0)}_{2/3}(q,\varepsilon)\equiv A^{(1)}_{2/3}(q,\varepsilon)$ for a fixed value of q. Note the similarity with the spectral function of a spinless Luttinger liquid.[24,25] The only difference is that, in our case, velocities of right-moving and left-moving plasmon modes are not equal. (b) $A^{(0)}_{2/5}(q,\varepsilon)\equiv A^{(-1)}_{2/5}(q,\varepsilon)$ at fixed q. It is reminiscent of the spectral function for a spinful χLL exhibiting spin-charge separation[25,26] but differs due to the absence of any algebraic divergence at $-v_n q$.

Here $k_V=\sqrt{2m(\varepsilon_{F\|}-eV)/\hbar^2}$ and $k_{F\|}$, the Fermi wave vector in $2\mathrm{DES}_\|$, are the extremal wave vectors for which momentum-resolved tunneling occurs. Fermi energies in $2\mathrm{DES}_{\perp,\|}$ are denoted by $\varepsilon_{F\perp,\|}$. Equations (6) can be used to extract the lowest-Landau-level dispersion E_{0k} from maxima in the experimentally obtained dI/dV, thus enabling microscopic characterization of real QH edges.

When $2\mathrm{DES}_\perp$ is in a QH state at a Laughlin-series filling factor $\nu_{1,p}$, it supports a single branch of edge excitations just like at $\nu=1$, and the calculation of the differential tunneling conductance proceeds the same way. The major difference is, however, the vanishing of spectral weight at the Fermi point of the edge; compare Eqs. (3) and (4). This results in the suppression of maxima described by Eq. (6a), while those given by Eq. (6b) remain. The intensity of the latter rises along the curve as a power law with exponent p.

Finally, we discuss the case of hierarchical filling factors $\nu_{\pm 2,p}$ which are expected to support two branches of edge excitations. To be specific, we consider filling factors 2/3 and 2/5. In both cases, there are many contributions to the spectral function and, hence, the differential tunneling conductance. However, only two of these exhibit algebraic singularities. It turns out that these singularities give rise to either a strong maximum or a finite step in the differential tunneling conductance, depending on the sign of voltage. (See Fig. 3.) The strong maximum results from a logarithmic divergence that occurs when $eV=\hbar v_c(k_F^{(N)}-k_{F\|})$. Both the maximum and the step edge follow the dispersion of the charged edge-magnetoplasmon mode and would therefore enable its

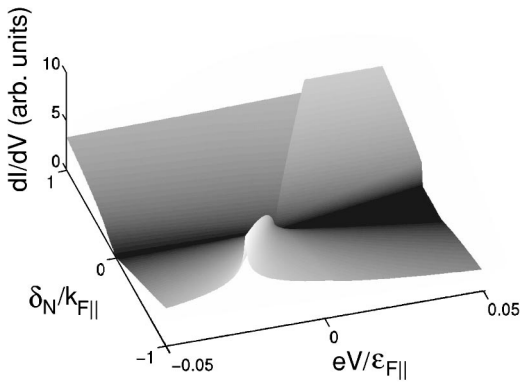

FIG. 3. Gray-scale plot of singular contributions to the differential conductance for tunneling into the two-branch QH edge at filling factor 2/3. A qualitatively similar plot is obtained for filling factor 2/5. Note the strong maximum rising as a power law for negative bias, which is continued as a step edge for positive bias. Its position in the eV-δ_N plane follows a line whose slope corresponds to the edge-magnetoplasmon velocity v_c. To obtain the plot, we have linearized the spectrum in $2\mathrm{DES}_\|$ and absorbed the magnetic-field dependence into the parameter $\delta_N=k_F^{(N)}-k_{F\|}$. As there are two such singular contributions to dI/dV with $N=0,1$ which have different δ_N, a doubling of resonant features shown in this plot would be observed experimentally.

experimental investigation. Most importantly, however, the two spectral functions with singularities exhibit them slightly shifted in guiding-center, i.e., k direction by an amount $\nu_o^{\pm}(k_{Fo}-k_{Fi})$. Hence, two maxima and a double-step feature should appear in the differential tunneling conductance whose distance in magnetic-field direction will be a measure of the separation of inner and outer edges. Observation of this doubling would yield an irrefutable confirmation of the expected multiplicity of excitation branches at hierarchical QH edges.

Experimental observation of our predictions requires sufficient momentum resolution for tunneling in real samples. Deviations from perfect momentum conservation can be quantified by a length scale $L_{rel}\leqslant L_y$ associated with the dominant source of momentum relaxation. To resolve structure in guiding-center space such as distinct integer QH edge branches with Fermi points having a distance $l^2\Delta k_F$ requires $L_{rel}>2\pi/\Delta k_F$. Edge-dispersion spectroscopy performed in the integer QH regime[17] indicates that this requirement can be fulfilled in real samples where $l^2\Delta k_F$ of the order of a few l is expected. Multiple edge-excitation branches due to reconstruction[32] at a smooth edge could be observed in a similar fashion. Verification of the two-branch structure of edges at $\nu=\nu_{\pm 2,p}$ requires $L_{rel}>2\pi/(\nu_o^{\pm}|k_{Fo}-k_{Fi}|)$ and realization of a sharp edge in the fractional QH regime.

In conclusion, we have calculated the differential conductance for momentum-resolved tunneling from a 2DES into a QH edge. Maxima exhibited at $\nu=1$ follow two curves in V-B parameter space whose expression we give in terms of the lowest-Landau-level dispersion. Their explicit form enables edge-dispersion spectroscopy. At Laughlin-series filling factors, χLL behavior results in the suppression of one of these maxima and characteristic power-law behavior exhib-

ited by the other one. The multiplicity of edge modes at hierarchical filling factors corresponds directly to the multiplicity of maxima in the differential tunneling conductance. Their observation would constitute an important test of χLL theory.

We thank M. Grayson and M. Huber for many useful discussions and comments on the manuscript. This work was supported by DFG Grant No. ZU 116 and the DIP project of BMBF. U.Z. enjoyed the hospitality of Sektion Physik at LMU München when finishing this work.

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