T-duality I: Constructions

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Abstract

This first part of two talks aims to introduce T-Duality and to highlight some problems.

Contents

1 Introduction

Dualities are among the most interesting features of string theory as they reflect a rich symmetry structure non-existent in point-particle theories. T-Duality or Target-Space Duality can be considered a geometric duality identifying different string backgrounds as providing identical theories. They have led to the discovery of D-branes, of mirror symmetry and can be used as solution generator. The latter application has also revealed the existence of so-called non-geometric backgrounds which exceed the established notions of geometry.

The plan of the two talks is to provide an introduction to T-duality. But instead of discussing it as a symmetry of the mass spectrum of the string – which is restricted to cases where we can actually solve the string EOM's explicitly – we will provide. The first talk will cover methods for determining T-duals and the second talk will cover some applications.

A very good general (but a little out-dated) reference is the review by [1].

2 Target-space dualities

The most conventional method for obtaining duality goes back to [2, 3] and has been clarified and extended by [4]. It relies on *gauging isometries* of the two-dimensional sigma model.

To this end, let Σ be the two-dimensional worldsheet and M the d-dimensional target-space with an embedding $X : \Sigma \hookrightarrow M$. Having local coordinates $\{x^a\}$ on M, the pulled-back coordinates are denoted $X^*x^a \equiv X^a$. Moreover, Σ is assumed to be equipped with coordinates $\{\tau, \sigma\}$, the metric $h = \mathrm{diag}(-1, 1)$ (conformal gauge) and the volume element $d\tau\wedge d\sigma$, such that the Hodge-star is given by $\alpha\wedge\star\beta = h(\alpha, \beta)d\tau\wedge d\sigma$. The target-space is equipped with the background (G, B) consisting of a metric and a two-form field.[1] Then the string sigma-model reads

$$S(X; G, B) = \frac{1}{4\pi\alpha'}\int_\Sigma \Big[G(X)_{ab}\, dX^a \wedge \star dX^b + \underbrace{B(X)_{ab}\, dX^a \wedge dX^b}_{=X^*B}\Big]\,. \tag{2.1}$$

Being a quantum correction and breaking classical Weyl invariance, the dilaton will be discussed separately later. It is worth mentioning that

- the equations of motion for S can be considered as two-dimensional geodesic equation on a Riemann-cartan space:

$$d \star dX^a + \Gamma^a{}_{bc}\, dX^b \wedge \star dX^c = \tfrac{1}{2}\, G^{am}\, H_{mbc}\, dX^b \wedge dX^c \tag{2.2}$$

  with $\Gamma^a{}_{bc}$ the coefficients of the Levi-Civita connection on TM and $H = dB$,

- and that S is subject to the two constraints

$$\begin{aligned} G_{ab}(\partial_\tau X^a \partial_\tau X^b + \partial_\sigma X^a \partial_\sigma X^b) &= 0\,, \\ G_{ab}\, \partial_\tau X^a \partial_\sigma X^b &= 0\,. \end{aligned} \tag{2.3}$$

As to spacetime symmetries, S is invariant under B-field gauge transformations only if $\partial\Sigma = \emptyset$. Otherwise, appropriately transforming $U(1)$-gauge fields have to be included at the endpoints of the string in order to restore gauge invariance, i.e. $S_A = \frac{1}{4\pi\alpha'}\int_{\partial\Sigma} X^*A$ with $\delta_{\text{gauge}}A = -\omega$ for $\delta_{\text{gauge}}B = d\omega$. For spacetime diffeomorphisms we will be more explicit: Infinitesimally, they are generated by vector fields k_i (with $i \in \{1, \ldots, n\}$ a label) via

$$X^a \to X^a + \epsilon^i\, k_i^a\,. \tag{2.4}$$

Varying the action with respect to these diffeomorphisms $S \to S + \delta S$ gives

$$\delta S(X; G, B) = \frac{\epsilon^i}{4\pi\alpha'}\int_\Sigma \big[(L_{k_i}G)_{ab}\, dX^a \wedge \star dX^b + (L_{k_i}B)_{ab}\, dX^a \wedge dX^b\big] \tag{2.5}$$

with L_{k_i} the Lie-derivative with respect to k_i. Thus spacetime diffeomorphism invariance involves

- the vanishing of δS. For $\partial\Sigma = \emptyset$ this means

$$\begin{gathered} L_{k_i}G = 0 \\ L_{k_i}B = d\nu_i \Longleftrightarrow \iota_{k_i}H = -d(\iota_{k_i}B - \nu_i) \equiv -d\xi_i \end{gathered} \tag{2.6}$$

[1] From the point of view of Σ, G and B are the couplings of the bosonic fields X^a.

with ν_i a one-form. For open strings this has to be supplemented by $L_{k_i}A = df_i - \nu_i$ for $f_i \in C^\infty(M)$; since it can be implemented straight-forwardly, it will be neglected in the following.

- The associated conserved current is given by

$$J_i = (\iota_{k_i}G)_a \star dX^a + (\iota_{k_I}B - \nu_i)_a \, dX^a = \star\left[X^*(\iota_{k_i}G)\right] + X^*\xi_i \,. \tag{2.7}$$

  The EOM can be used to show conservation, i.e. $dJ_i = 0$.

- We assume multiple non-abelian isometries $\mathcal{I}$ with Lie algebra

$$\mathfrak{i}: \quad \langle \{k_i\}_{i=1}^n \rangle \quad \text{with} \quad [k_i, k_j] = f^m{}_{ij}\, k_m \,. \tag{2.8}$$

- Consistency with the Lie algebra $\mathfrak{i}$ yields further conditions; we can evaluate

$$-f^m{}_{ij}\, d\xi_m = f^m{}_{ij}\, \iota_{k_m} H = \iota_{[k_i,k_j]}H = [L_{k_i}, \iota_{k_j}]H = -L_{k_i} d\xi_j \,. \tag{2.9}$$

  Since Lie and exterior derivative commute, the left-hand side of this equation must be closed, i.e.

$$df^m{}_{ij} \wedge d\xi_m = 0 \qquad \Longrightarrow \qquad \boxed{L_{k_i}\xi_j = f^m{}_{ij}\,\xi_m} \quad \text{(up to exact terms)}\,; \tag{2.10}$$

  For simplicity we assume constant structure coefficients in the following.

These isometries are global symmetries from the worldsheet point of view, provided that the conditions above hold. For obtaining a dual sigma model, the idea is to start with making this global symmetries local – they will be gauged (see [5, 6]).

2.1 The gauged sigma model

We now consider the local transformation $X^a \to X^a + \epsilon^i(X) k_i^a(X)$. The aim is to find an action stemming from S which is invariant under it. Let us see what happens to the action under this transformation. The basic ingredients are:

$$\begin{aligned} \delta dX^a &= \epsilon^i\, dk_i^a + k_i^a\, d\epsilon^i \\ \delta f(X) &= \epsilon^i\, k_i^a \partial_a f \\ \delta k_i &= \epsilon^j L_{k_j} k_i = k_m\, f^m{}_{ni}\, \epsilon^n \,. \end{aligned} \tag{2.11}$$

The last variation is a consequence of k_i being a target-space vector field. Thus, the difference to the considerations above is the appearance of $d\epsilon^i$-terms, which spoils the invariance. For example, the kinetic term transforms as

$$\delta\big(G_{ab} dX^a \wedge \star dX^b\big) = \underbrace{\epsilon^i (L_{k_i}G)_{ab} dX^a \wedge \star dX^b}_{=0 \text{ by (2.6)}} + 2k_i^a\, G_{ab}\, d\epsilon^i \wedge \star dX^b \,. \tag{2.12}$$

As is known from basic QFT, the unwanted term can be cancelled by introducing an appropriate gauge field

$$A = A^i\, k_i \;\in \Gamma(T^*M \otimes \mathfrak{i}) \quad \text{with} \quad \delta A = [A, \epsilon] - d\epsilon\,; \tag{2.13}$$

in components this means $\delta A^i = f^i{}_{mn}\, A^m \epsilon^n - d\epsilon^i$. Then the partial derivative is extended the covariant derivative

$$DX^a = dX^a + k_i^a\, A^i \quad \text{with} \quad \delta DX^a = \epsilon^i\, dk_i^a \tag{2.14}$$

as desired. The appropriate inclusion of the gauge field follows the basic rule of thumb "for every $d\epsilon^i$ in the variation, add this terms with an A^i to the action" and is as follows

- **kinetic term:** It can be treated via *minimal coupling*. By the last equation the variation of the minimally coupled kinetic term becomes

$$\delta\big(G_{ab}\, DX^a \wedge \star DX^b\big) = \epsilon^i (L_{k_i} G)_{ab} DX^a \wedge \star DX^b = 0\,. \tag{2.15}$$

- **B-field term:** Minimal coupling does in general not work since

$$\begin{aligned} \delta\big(B_{ab} DX^a \wedge DX^b\big) &= \epsilon^i (L_{k_i} B)_{ab} DX^a \wedge DX^b \\ &= \epsilon^i(X)\, \partial_{[a}\nu_{b]i}\, DX^a \wedge DX^b \end{aligned} \tag{2.16}$$

  is not a total derivative anymore. To gauge this term an annoying procedure of adding terms which again produce new terms leading to more unwanted terms has to be followed. It can be found in the papers of [5, 6]. In particular, the consistency condition $L_{k_i}\xi_j = f^m{}_{ij}\xi_m$ has to be true outside of cohomology (i.e. no exact terms are allowed). Moreover, we encounter an *anomaly* $c_{(ij)}$ with $c_{ij} = \iota_{k_i}\xi_j$.

After the lengthy procedure which also involves the inclusion of a new scalar field λ_i we end up with

$$\begin{aligned} S_{\mathrm{g}}(G, B, A; \xi, \lambda) = \frac{1}{4\pi\alpha'} \int_\Sigma \Big[& G_{ab}\, DX^a \wedge \star DX^b + B_{ab}\, dX^a \wedge dX^b \\ & - 2\big(\xi_i + d\lambda_i\big) \wedge A^i - \big(c_{[ij]} + \lambda_m f^m{}_{ij}\big) A^i \wedge A^j \Big] \end{aligned} \tag{2.17}$$

with

- the transformation rules

$$\delta_\epsilon X^a = \epsilon^i\, k_i^a \quad , \quad \delta_\epsilon \lambda_i = c_{(ij)}\, \epsilon^j - f^m{}_{ni}\, \lambda_m\, \epsilon^n \quad , \quad \delta_\epsilon A^i = f^i{}_{mn}\, A^m\, \epsilon^n - d\epsilon^i\,,$$

- the total variation

$$\delta S_{\rm g} = -\frac{1}{4\pi\alpha'}\int_\Sigma \epsilon^j c_{(ij)}\, f^i{}_{mn}\, A^m \wedge A^n, \tag{2.18}$$

  which can be derived using the Jacobi identity for $\mathfrak{i}$, i.e. $f^l{}_{[i|m} f^m{}_{|jk]} = 0$; thus, at least for the abelian case further conditions are avoided,

- the new terms are of particularly nice form: we added $-d\lambda_i \wedge A^i - \frac{1}{2}\lambda_m f^m{}_{ij} A^i \wedge A^j$; upon an integration by parts this gives $\lambda_i F^i$ with

$$F = dA + [A \wedge A] \quad \in \Gamma(\Lambda^2 T^* M \times \mathfrak{i})\,, \tag{2.19}$$

  i.e. the field strengths associated to A with λ_i Lagrange multipliers. However, as we will see below, it is important for the cancellation of possible holonomies to keep the $A^i \wedge d\lambda_i$-term.

2.2 Going back to the original action and global issues

The original action S is returned from $S_{\rm g}$ if we set $A = 0$; this, however, is not well-defined since A is a gauge field and the right-hand side is gauge invariant. Thus we need a procedure consistent with gauge transformations. The newly introduced scalar field λ can be considered a Lagrange multiplyer and its EOM from $S_{\rm g}$ is

$$F = 0 \qquad \Longleftrightarrow \qquad dA = -[A \wedge A]\,. \tag{2.20}$$

Locally, this can be solved by a pure gauge field

$$A = d \ln g = g^{-1} dg \qquad \text{with } g \in C^\infty(M) \otimes \mathcal{I}; \tag{2.21}$$

this makes sense as the logarithm – being the inverse of the exponential map – is a map $\ln : \mathcal{I} \to \mathfrak{i}$, i.e. it maps the group to its Lie algebra. This now allows for choosing a gauge in which $A = 0$: a general gauge transformation acts as

$$A \to A' = h^{-1} A h + h^{-1} dh = h^{-1}(g^{-1} dg) h + h^{-1} dh; \tag{2.22}$$

thus $A' = 0$ for $h = g^{-1}$. Therefore, in this gauge we **locally** obtain the original action.

The subtleties of the procedure lie in the global properties.[2] For Σ simply-connected (a two-sphere, i.e. string tree-level) there are no problems. However, if Σ has genus g (i.e. string g-loop), there might be non-trivial monodromies or, in other words, non-trivial holonomies of the gauge connection. We want to find a mechanism to rid the gauge field of its holonomies. For simplicity of the formulas, we assume Σ to be a

[2] I find it hard do find a good reference. I think the canonical one is [7], but different aspects can also be found in [4, 1, 8, 9, 10, 11] and for sure many more.

torus, i.e. $g = 1$, and choose the cycles a, b to be the generators of its first homology. Then A_i might have a holonomy

$$h^i_{a,b} = \frac{1}{\sqrt{\alpha'}} \oint_{a,b} A\,. \tag{2.23}$$

This can be cared of by adding appropriate δ-functions

$$\delta(h^i_a)\delta(h^i_b) \sim \sum_{n_a,n_b\in\mathbb{Z}} e^{\frac{i}{\sqrt{\alpha'}}(n_a \oint_a A + n_b \oint_b A)} \tag{2.24}$$

to the path integral. The addition of the delta functions is a task which we assign to the newly introduced field λ_i and is the reason why we write the Lagrange-multiplier term in this strange manner. Indeed, using *Riemann's bilinear identity* we find

$$\frac{1}{2\pi\alpha'} \int_\Sigma d\lambda_i \wedge A^i = \frac{1}{2\pi\alpha'} \left(\oint_a d\lambda_i \oint_b A^i - \oint_b d\lambda_i \oint_a A^i \right); \tag{2.25}$$

thus, if the Lagrange multipliers are multi-valued with

$$\oint_a d\lambda_i = \ell_s\, n_b \qquad \text{and} \qquad \oint_b d\lambda_i = -\ell_s\, n_a \tag{2.26}$$

and $\ell_s = 2\pi\sqrt{\alpha'}$ the string length, we automatically get the desired terms in the path integral; the summation is contained in the measure. However, recall that the Lagrange multipliers transform under gauge transformations $\delta\lambda_i = c_{(ij)}\epsilon^j - f^m{}_{in}\lambda_m\epsilon^n$. In the absence of the anomaly $c_{(ij)}$ this means that they transform in the adjoint representation $\lambda \to g^{-1}\lambda g$. Therefore, λ is gauge-invariant for abelian isometries and can without problems be equipped with the suitable periodicities. However, for non-abelian isometries the gauge-dependence is in general incompatible with a particular choice of periodicities.

The upshot of this short discussion is that non-abelian isometries on non-simply connected worldsheets spoil the way back to the original action. However, the "way back" is essential for obtaining dual theories as it is necessary for their equivalence. Therefore we assume the isometries to be abelian in the following.

2.3 The abelian-dual theory: the Buscher rules

Since we have already restricted ourselves to abelian isometries, we can very well restrict further to a single isometry. We also choose *adapted coordinates* in which the single generator is $k = \partial/\partial X^0$ and fix two gauges:

- Since we have now a single abelian gauge field A, it can be gauge-fixed to $A - dX^0$. In particular, this gives $DX^0 = A$ and $DX^a = dX^a$ for $a \neq 0$.

- We have the condition $L_{\partial/\partial X^0} B = d\nu$ for the Killing vector $\partial/\partial X^0$ to generate an isometry. Gauge transformations $B \to B + d\omega$ therefore imply transformations $\nu \to \nu + L_{\partial/\partial X^0}\omega$. Thus we can find a gauge with $\nu = 0$ (the equation $\nu + L_{\partial/\partial X^0}\omega = 0$ for ω is an ODE). This implies $\xi = B_{0a} dX^a$.

Taking these gauges into account, the gauged sigma model simplifies to

$$\begin{aligned} S_{\mathrm{g}} = \frac{1}{4\pi\alpha'} \int_\Sigma \Big[& G_{ab}\, dX^a \wedge \star dX^b + B_{ab}\, dX^a \wedge dX^b + 2\, d\lambda \wedge dX^0 \\ & G_{00}\, A \wedge \star A + 2\, G_{0a}\, A \wedge \star dX^a + 2\, B_{0a}\, A \wedge dX^a + -2\, d\lambda \wedge A \Big] \end{aligned} \tag{2.27}$$

with $a, b \neq 0$ and G as well as B independent of X^0 by (2.6). Apart from integrating out the periodic Lagrange multiplier λ which gives back the initial theory, we can also integrate-out the gauge field. Its EOM is

$$\star A = -\frac{1}{G_{00}} \left(G_{0a} \star dX^a + B_{0a} dX^a + d\lambda \right). \tag{2.28}$$

Plugging this back into the action above gives – upon interpreting $d\lambda = -d\widetilde{X}^0$ as new coordinate – the sigma model action $S(\widetilde{X}^0, X^a; g, b)$ with the new background given by

$$\begin{aligned} g_{00} &= \frac{1}{G_{00}}\,, \qquad g_{0a} = -\frac{B_{0a}}{G_{00}}\,, \qquad g_{ab} = G_{ab} - \frac{G_{a0}G_{0b} + B_{a0}B_{0b}}{G_{00}} \\ & \qquad\qquad\qquad b_{0a} = -\frac{G_{0a}}{G_{00}}\,, \qquad b_{ab} = b_{ab} - \frac{G_{a0}B_{0b} + B_{a0}G_{0a}}{G_{00}} \end{aligned} \tag{2.29}$$

for $a, b \neq 0$. These are the *Buscher rules*. Some comments:

- The action above contains the term $d\lambda \wedge dX^0$. However, in order to exchange X^0 with $\widetilde{X}^0$ properly, this term has to vanish in the path integral. Keep in mind that due to the multi-valuedness of λ this is not a total derivative. Using Riemann's bilinear identity and (2.26) we find the contribution

  $$\exp\left(\frac{i}{2\pi\alpha'} \int_\Sigma d\lambda \wedge dX^0 \right) = \exp\left(\frac{i n_b}{\sqrt{\alpha'}} \oint_b dX^0 - \frac{i n_a}{\sqrt{\alpha'}} \oint_b dX^0 \right) \tag{2.30}$$

  to the path integral. It vanishes, if we obtain multiples of 2π; thus we have to assure that

  $$\oint_\gamma dX^0 \in \ell_s \mathbb{Z} \tag{2.31}$$

  for γ any non-trivial cycle.

- The latter consistency condition shows that the initial coordinate X^0 as well as the new coordinate $\widetilde{X}^0 = \lambda$ have to contain winding as multiple of the string length. Thus both X^0 and $\widetilde{X}^0$ can be interpreted as coordinates on a circle-direction of the target-space: we have exchanged a circle of unit radius with another circle of unit radius. If we would have started with a circle of radius R, the winding of X^0 would be integer multiples of $R\ell_s$. Then, in order for the cancellation of the holonomies to still work out, $\lambda = \widetilde{X}^0$ would require winding as multiples of $\frac{1}{R}\ell_s$: duality inverts the radius of the circle!

- More generally, the last argument indicates that abelian duality is restricted to toroidally-compactified target spaces. In the approach followed here, this conclusion is a consequence of a careful treatment of global obstructions.

- The new background is independent of $\widetilde{X}^0$ because the old one was independent of X^0. In general, one can show that abelian duality preserves the isometries – we can go back the same way we arrived at the new action. If we would have ignored the global issues discussed above and would have derived the non-abelian "dual", we would have observed the loss of some of the initial isometries and therefore the possibility to go back to the initial model. This is another incarnation of the global problems.

- In [4] it was shown that the abelian dual of a conformal (quantum) theory is also conformal, i.e. abelian duality holds to all orders in α'. If also possible holonomies are treated correctly, duality also holds to all orders in the string coupling

2.4 The Dilaton

So far we have neglected the contribution of the Dilaton. The contribution to the string sigma model is of higher order in α' and therefore a quantum correction

$$S_{\text{dil}} = \frac{1}{4\pi} \int_\Sigma \phi(X)\, R^{(2)} \star 1\,, \tag{2.32}$$

where $R^{(2)}$ is the Ricci scalar on the worldsheet. This term breaks the classical Weyl invariance of the theorie, but proves to be a valuable contribution in retaining Weyl invariance in the quantum theory: the classical lack of Weyl invariance is compensated by a one-loop contribution. This gives rise to the lowest-order string EOMs

$$\begin{aligned}
0 &= R_{ab} + 2\,\nabla_a\nabla_b\phi - \tfrac{1}{4}\,H_{amn}\,H_b{}^{mn} + \mathcal{O}(\alpha')\,,\\
0 &= G^{ab}\,\nabla_a\phi\,\nabla_b\phi - \tfrac{1}{2}G^{ab}\,\nabla_a\nabla_b\phi - \tfrac{1}{24}\,H_{abc}\,H^{abc} + \mathcal{O}(\alpha')\,,\\
0 &= \tfrac{1}{2}\,\nabla^m H_{mab} - \nabla^m\phi\,H_{mab} + \mathcal{O}(\alpha')\,.
\end{aligned} \tag{2.33}$$

Now suppose we have a Ricci-flat background without B-field and dilaton. This certainly satisfies the string EOMs. However, if the metric is chosen suitably, T-duality can produce a B-field via the Buscher rules but the dual metric is still Ricci flat. Thus the H-terms in the EOM have to be compensated by the introduction of a suitable dilaton field.

The change of the dilaton can be derived by carefully integrating-out the gauge field A in the path integral, which changes the measure [3]. However, a very simple way to obtain the transformation of the dilaton is to demand invariance of the measure factor:

$$\sqrt{|\det G|}\, e^{-2\phi} \quad \xrightarrow{\text{Buscher}} \quad \sqrt{|\det g|}\, e^{-2\tilde{\phi}} = \sqrt{|\det G|}|G_{00}|^{-1}\, e^{-2\tilde{\phi}}\,. \tag{2.34}$$

The new factor is compensated by a shift of the dilaton

$$\tilde{\phi} = \phi - \tfrac{1}{2}\ln G_{00}\,. \tag{2.35}$$

2.5 Topology changes by T-duality

The Busher rules show that if G_{00} is vanishing a some point, this point becomes a singularity in the dual theory. The general statement is that a fixed point of the killing vector are interchanged with singularities by duality.

A very simple example is given by spacetimes of the form $M = \mathbb{R}^2 \times \mathbb{R}^{1,d-3}$ with flat metric

$$ds^2 = dr^2 + r^2\, d\phi^2 + d\mathbf{x}^2\,. \tag{2.36}$$

Thus the plane is equipped with the euclidean metric in spherical coordinates $r \in [0, \infty)$, $\phi \in [0, 2\pi)$ and the $d\mathbf{x}^2$ is the flat Minkowski metric. The Ricci scalar of this metric is $R = 0$ and $\partial/\partial\phi$ is generates an isometry and has a fixed point at $r = 0$. Thus we can perform T-duality along the angular direction and get

$$d\tilde{s}^2 = dr^2 + r^{-2}\, d\tilde{\phi}^2 + d\mathbf{x}^2 \tag{2.37}$$

by the Buscher rules. Global considerations also restrict the dual coordinate $\tilde{\phi}$ to be periodic. This metric has Ricci scalar $\widetilde{R} = -2r^{-2}$ and therefore a singularity at $r = 0$. This is the most simple example of topology changes by T-duality.

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