Hybrid representation of spin foams

Sebastian Steinhaus

sebastian.steinhaus@uni-jena.de

Theoretisch-Physikalisches Institut
Friedrich-Schiller-Universität Jena

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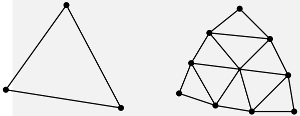
Two challenges in spin foam quantum gravity

Challenge 1: Defining the continuum limit

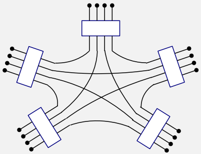


- Spin foams: **path integral approach**
  - **Regularized** by triangulation (2-complex)
- How does the theory **depend** on the choice of regulator?

Formulate **renormalization group flow** by relating **boundary states**.

Challenge 2: Performing calculations

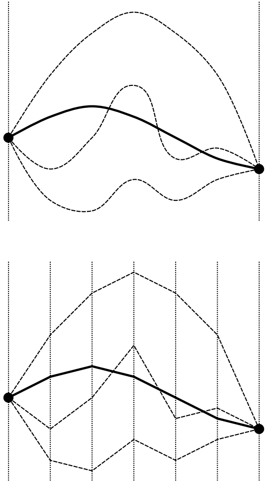
- Spin foam amplitudes: **difficult to compute** beyond **quantum regime**
  - Numerical costs for **grow exponentially**
- Can we find **efficient approximations**?



Hybrid method: Utilize **semi-classical** amplitude to **accelerate** calculations

Outline

Spin foam gravity [Rovelli, Reisenberger, Barrett, Crane, Freidel, Livine, Krasnov, Perez, Speziale, Engle, Pereira, Kaminski...]



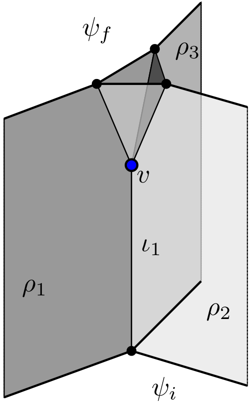
- **Path integral of geometries**
- Regulator: **Discretization** / 2-complex
- **Quantum geometric** building blocks
  - (Constrained) **topological quantum field theory**
  - **Discrete** area spectrum
- Physical content: **Transition amplitudes**
  - Assign an amplitude $\mathcal{A} \sim e^{iS_{\text{EH}}} + e^{-iS_{\text{EH}}}$ to each geometry
  - **Single building block** $\sim$ **discrete gravity** [Conrady, Freidel '08, Barrett, Dowdall, Fairbairn, Gomes, Hellmann '09, Kaminski, Kisielowski, Sahlmann '17, Liu, Han '18, Simão, S.St. '21]
  - Quantum amplitudes (not Wick-rotated)

Derived from **general relativity**

No reference to background geometry

Aim to implement **diffeomorphism symmetry**

Spin foam gravity - Basics

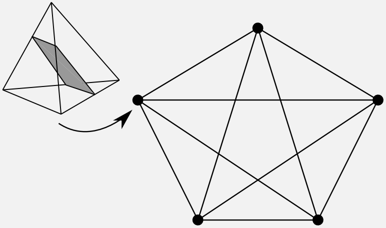


- Regulator: (dual) **2-complex** Δ^*
  - Vertices v, edges e, faces f
- Coloured with group theoretic data $\{\rho_f, \iota_e\}$
- Boundary graph $\sim$ **3D geometry**
  - **Polyhedra** $\sim$ intertwiner ι_e
  - **Area** of face $\sim$ representation ρ_f
- **Evolution**: bulk geometry
  - History interpolating between boundaries
- **Sum over all histories**
  - Sum over all ι and ρ
  - Assign amplitude to each history
- **Amplitude functionals**: $\mathcal{A}_b : \mathcal{H}_b \to \mathbb{C}$
  - From initial to final state: $\mathcal{H}_i \otimes \mathcal{H}_f^*$: $\langle \psi_f, \psi_i \rangle_{\mathcal{A}}$

Partition function and geometric interpretation

- **Amplitudes locally** assigned to building blocks

$$Z(\Delta^*) = \sum_{\rho_f, \iota_e} \prod_{f \in \Delta^*} \mathcal{A}_f(\rho_f) \prod_{e \in \Delta^*} \mathcal{A}_e(\iota_e) \prod_{v \in \Delta^*}$$

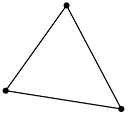


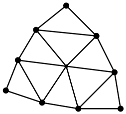
Quantum space-time as a **superposition of quantum geometric building blocks**

Outline

Challenge 1: Defining continuum limit

- **Discretisation dependence**
  - **Diffeomorphism symmetry?** [Dittrich, Bahr '09, Bahr, S.St. '15]
- **Ambiguities** in definition of 4D models
  - Implementation of **simplicity constraints**
- **Background independence** – no scale!
  - Physical content are **transition amplitudes**
  - **Boundary states** / geometry encode **scale**
  - Refinement of discretisation implies refinement of boundary
- For **results to agree**, we must assign different amplitudes.





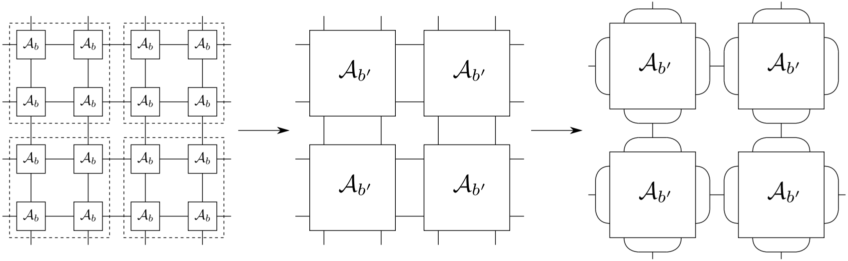
Consistent boundary formulation [Dittrich, S.St. '13, Dittrich '14, S.St. 2020, Asante, Dittrich, S.St. 2022]

Improve spin foam amplitudes by **coarse graining**

Renormalization group flow of amplitudes across discretisations

Consistent boundary formulation [Dittrich, S.St. '13, Dittrich '14]

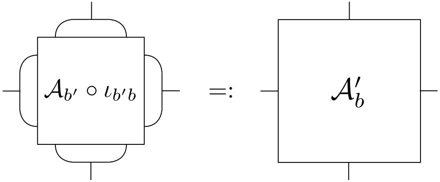
- **Amplitude functional** $\mathcal{A}_b : \mathcal{H}_b \to \mathbb{C}$
- Coarse graining involves **two main steps**:
  - **Summing** over **fine** degrees of freedom $\to \mathcal{A}_{b'}$ with finer boundary data
  - Define **embedding maps** $\iota_{bb'} : \mathcal{H}_b \to \mathcal{H}_{b'}$



- Embedding maps **prescribe** how to **add degrees of freedom**
  - **Kinematical maps**: Ashtekar-Lewandowski vacuum [Ashtekar, Isham '92, Ashtekar, Lewandowski '95] and BF vacuum [Dittrich, Geiller '15, Bahr, Dittrich, Geiller '15]

Spin foam RG equations

- Compute **effective amplitude** $\mathcal{A}'_b$ with embedding maps:



Spin Foam RG equations

$$\mathcal{A}'_b := \mathcal{A}_{b'} \circ \iota_{b'b}$$

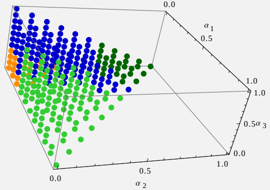
- **Renormalization group flow** of **amplitudes** $\mathcal{A} \to \mathcal{A}' \to \mathcal{A}'' \to \ldots$ across different complexes Δ^*
  - Uncover **phase diagram** / universality classes of dynamics
  - Must hold for **all boundary states** $\to$ **all scales**!

Search for fixed point and **2nd order phase transition**.

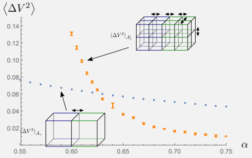
Realizations of boundary formulation [S.St '20, Asante, Dittrich, S.St. '22]

Tensor Network Renormalization [Levin, Nave '07, Gu, Wen '09]

- **Phase diagram** from RG flow of tensors
  - **Embedding maps** (and truncations) constructed from **dynamics**
- Applications to spin foams:
  - 2D Analogue spin foams [Dittrich, Eckert, Martin-Benito '11, Dittrich, Martin-Benito, Schnetter '13, Dittrich, Martin-Benito, S.St. '13, Dittrich, Schnetter, Seth, S.St. '16]
  - **3D Lattice gauge theory** [Dittrich, Mizera, S.St. '14, Delcamp, Dittrich, '16, Cunningham, Dittrich, S.St. '20]



Restricted spin foam models [Bahr, S.St. '15, Bahr, S.St. '16, Bahr, Klöser, Rabuffo '17, Bahr, Rabuffo, S.St. '18]

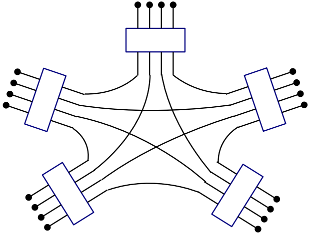


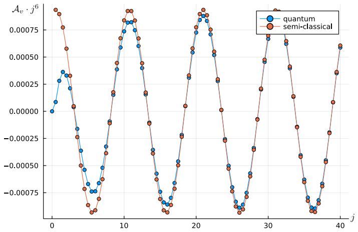
- **Define a subset** of full 4D spin foam path integral
  - Coherent intertwiners restricted to **cuboids / frusta**
- Indications for **UV attractive fixed point**
- **Spectral dimension** [S.St., Thürigen '18, Jercher, S.St., Thürigen w.i.p]
- **Scalar matter** coupled to spin foams [Ali, S.St. '22]

Outline

Challenge 2: Performing calculations

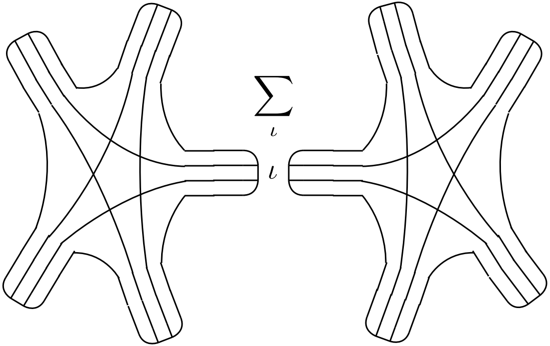
- **Recent progress** in spin foam calculations:
  - **Vertex amplitude:** numerical algorithm for EPRL/FK model [Donà, Fanizza, Sarno, Speziale '19, Gozzini '21]
  - **Sum over configurations:** effective spin foam algorithm [Asanta, Dittrich, Haggard PRL '20, Asante, Dittrich, Padua-Argüelles '21]
  - **Observables**: MCMC on Lefshetz thimbles [Han, Huang, Liu, Qu, Wan '20]
- **Analytical expression** for vertex amplitude [Conrady, Freidel '08, Barrett, Dowdall, Fairbarn, Gomes, Hellmann '09, '10, Kaminski, Kisielowski, Sahlmann '17, Han, Liu '18, Simão, S.St. '21]
  - **Coherent** boundary data [Livine, Speziale '07]
- **Critical points** dominate for large spins ($j \gg 1$)
  - $\mathrm{SU}(2)$ representations j
  - **Closure condition**: geometric 3D tetrahedra
  - **Bivector constraint**: gluing of tetrahedra
- **Exponential suppression** away from critical points





Gluing constraints [Asante, Simão, S.St. '22]

- Consider **two vertices** glued together



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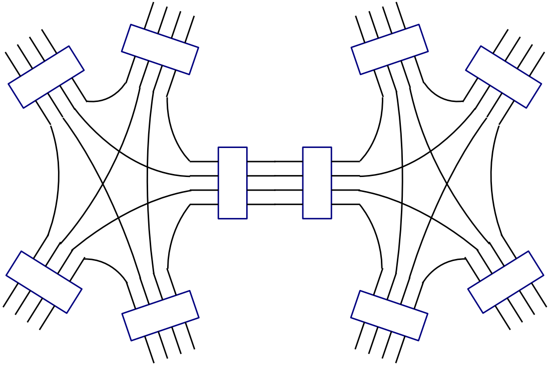

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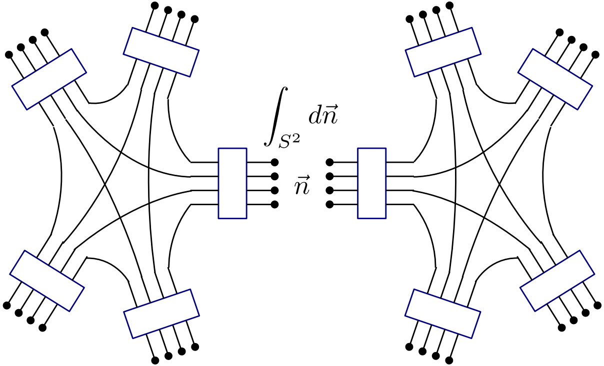

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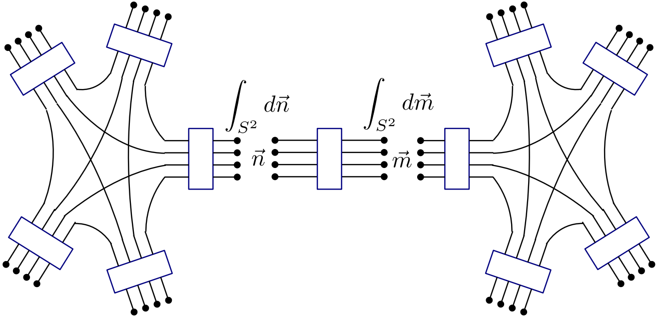


Equip each vertex with **independent set of coherent data**.
Interpolate between them by **gluing constraints**.

Hybrid algorithm idea [Asante, Simão, S.St. '22]

- **Approximate** each vertex by **asymptotic** formula
  - Only (regions around) **critical points** contribute

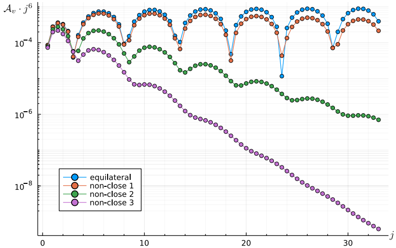
Critical points in Lorentzian EPRL [Barrett, Dowdall, Fairbarn, Gomes, Hellmann '09, Donà, Fanizzo, Sarno, Speziale '19]

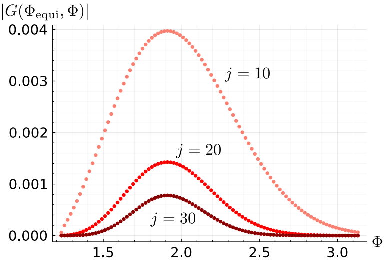
- **Lorentzian 4-simplices**
- Vector geometries (degenerate)

- **Gluing constraints** interpolate between vertices
  - Constraints peaked on **closing and matching tetrahedra**
  - Analytical formula away from critical points!

Non-matching, semi-classical vertices

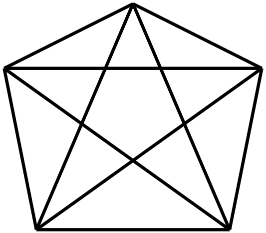
Non-metricity: **Torsion** degrees of freedom [Asante, Dittrich, Haggard '20]





Non-metricity: area vs. length variables

- Spin foams use **area**, not **length variables**.
  - Asymptotic formula: **area Regge calculus**
  - Discrete area spectrum
- Example: **single 4-simplex** [Asante, Dittrich, Haggard '18]
  - **Uniquely** determined by its **10 edge lengths**
  - 10 triangle areas: potentially multiple **length configurations**
- **Simplicity constraints** $\sim$ metricity
  - Partially **weakly imposed**: related to Immirzi parameter γ
- **Effective spin foams** [Asanta, Dittrich, Haggard PRL '20, Asante, Dittrich, Padua-Argüelles '21]
  - Rapidly **oscillating** amplitude can **"wash out"** constraint



Flatness problem [Bonzom '09, Han '13, Hellmann, Kaminksi '13, Oliveira '18, Donà, Gozzini, Sarno '20, Engle, Kaminski, Oliveira '20, Engle, Rovelli '21]

Small γ **lessens flatness problem** [Gozzini '21]

Hybrid representation might give new insights into γ **dependence**

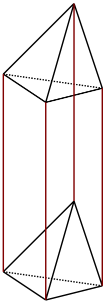
Summary

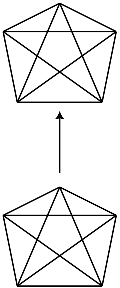
- Brief introduction to **spin foam models**
  - Defined on cellular complex $\Delta^* \sim$ regulator
  - **Quantum amplitudes** for **quantum geometric building blocks**
- Challenge 1: **consistent boundary formulation**
  - **Relate transitions** on different boundaries via embedding maps
  - **Renormalization group flow of amplitudes**
- Challenge 2: **hybrid algorithm**
  - **New representation** of spin foam partition function
  - Use asymptotic formula to accelerate calculations
  - **Gluing constraints**
  - New regime: **semi-classical, non-matching vertices**

Better understanding of **quantum space-time à la spin foams**

Outlook

- **Implement** and test **hybrid algorithm** [Asante, Simão, S.St. w.i.p.]
- **Hybrid representation**: justify / develop **simplified models**
  - Effective spin foam models [Asante, Dittrich, Haggard '20]
- **Tackle larger triangulations**
  - Spectral dimension of triangulation
  - **Cosmology** (for $\Lambda > 0$) [Liu, Williams '15]
  - Spin foam amplitudes as embedding maps [Dittrich, Hoehn '09, Dittrich, Hoehn '11, Dittrich, Hoehn '13, Hoehn '14]
- Define a **theory space** of spin foam models
  - **Flow in Immirzi parameter** γ?
- **Effective continuum theories**
  - Area metric theory [Dittrich '21, Dittrich, Kogios '22, Borissova, Dittrich '22]





Thank you for your attention!