

QUANTUM REFERENCE FRAMES: A RELATIONAL PERSPECTIVE ON NONCLASSICAL SPACETIME

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F. Giacomini, E. Castro Ruiz, Ā. Brukner, *Nat. Commun.* **10**(494), 2019

A. Vanrietvelde, P. A. Höhn, F. Giacomini, E. Castro Ruiz, *Quantum* **4**(225), 2020

F. Giacomini, E. Castro Ruiz, Ā. Brukner, *Phys. Rev. Lett.* **123**(9), 2019

E. Castro Ruiz, F. Giacomini, A. Belenchia, Ā. Brukner, *Nat. Commun.* **11**(1), 2020

L. F. Streiter, F. Giacomini, Ā. Brukner, *Phys. Rev. Lett.* **126**(23), 2021

F. Giacomini, Ā. Brukner, arXiv:2012.13754 (2020)

F. Giacomini, *Quantum* **5**, 508 (2021)

F. Giacomini, Ā. Brukner, *AVS Quantum Sci.* **4**, 015601 (2022)

C. Cephollaro, F. Giacomini, arXiv:2112.03303 (2021)

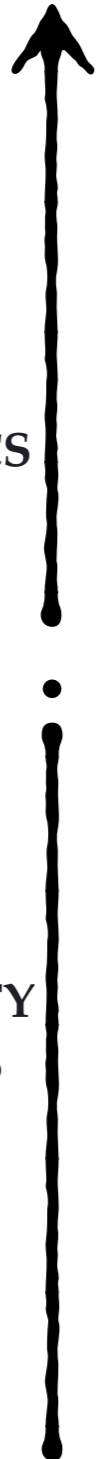
Overstreet, Kim, Curti, Asenbaum, Kasevich, Giacomini, arXiv:2209.02214 (2022)

Quantum Gravity, Hydrodynamics and Emergent Cosmology

University of Munich, 9 December 2022

Quantum aspects of spacetime

What replaces the classical notion of spacetime when gravity acquires quantum properties?



**HIGH ENERGIES:
PLANCK-SCALE PHYSICS**

•

**LOW ENERGIES:
PERTURBATIVE GRAVITY
QUANTUM PARTICLES**

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Image credits: Perimeter Institute

**HIGH ENERGIES:
PLANCK-SCALE PHYSICS**

QUANTUM SPACETIME "FUZZINESS"

- Spin foams
- Quantum Cosmology
- Modified dispersion relations
- (...)

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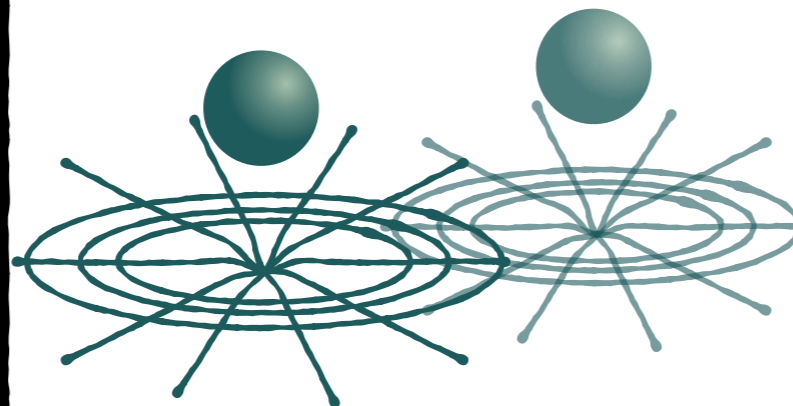
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**HIGH ENERGIES:
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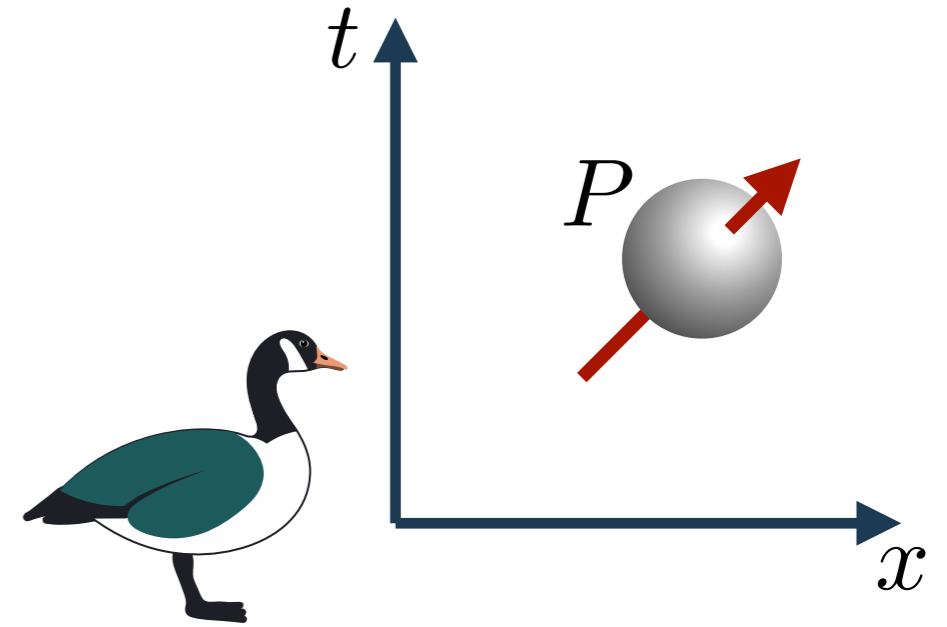
NONCLASSICAL SPACETIME

- Quantum Time and quantum clocks
- Indefinite causal structures
- Lack of classical reference frames
- (...)



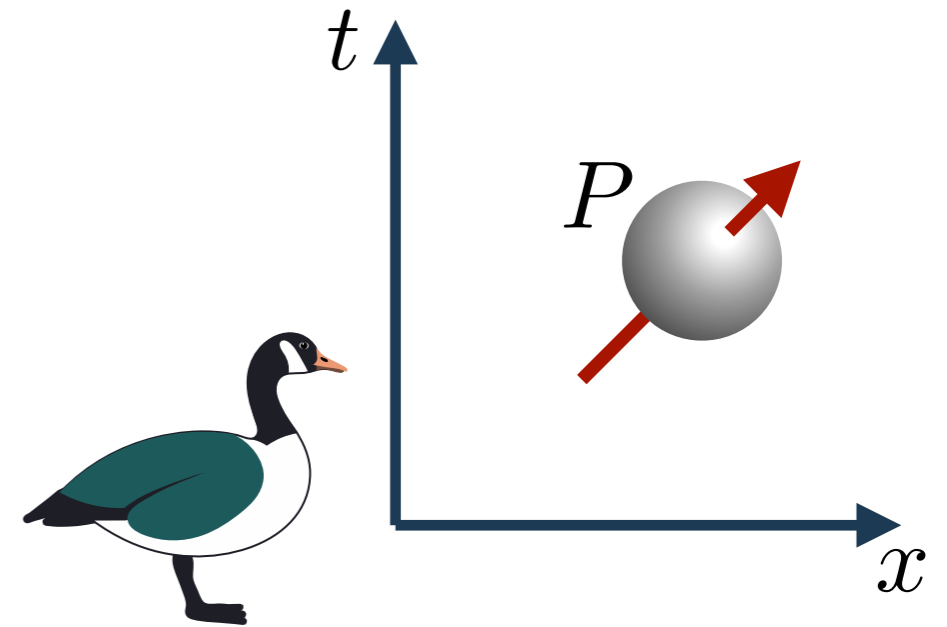
Concrete scenarios with
immediate physical meaning

What are reference frames?



What are reference frames?

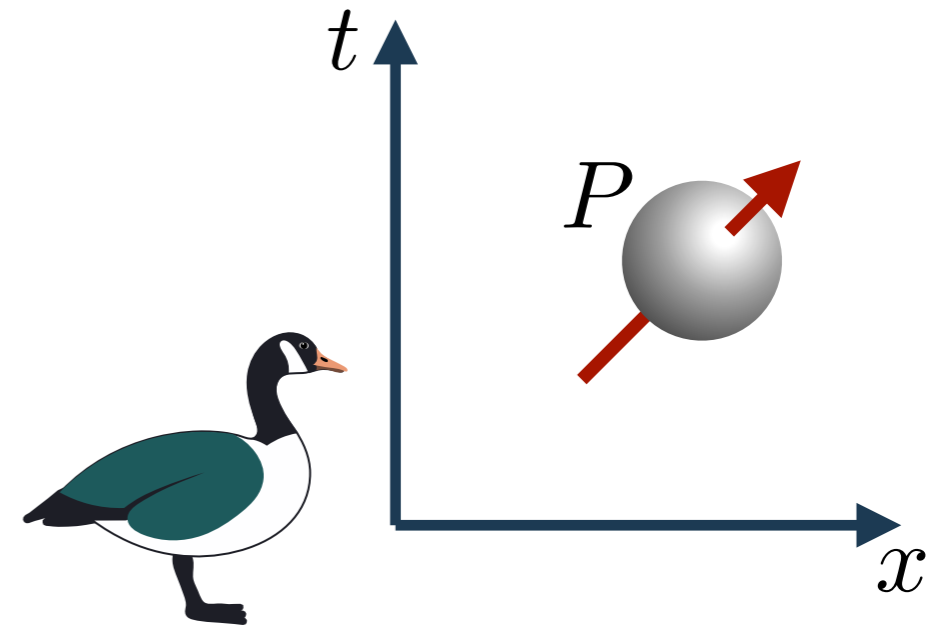
Reference frames are abstract entities, used to fix the point of view from which observations are carried out.



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The laws of physics are the same regardless of the choice of the reference frame.
(Principle of covariance)



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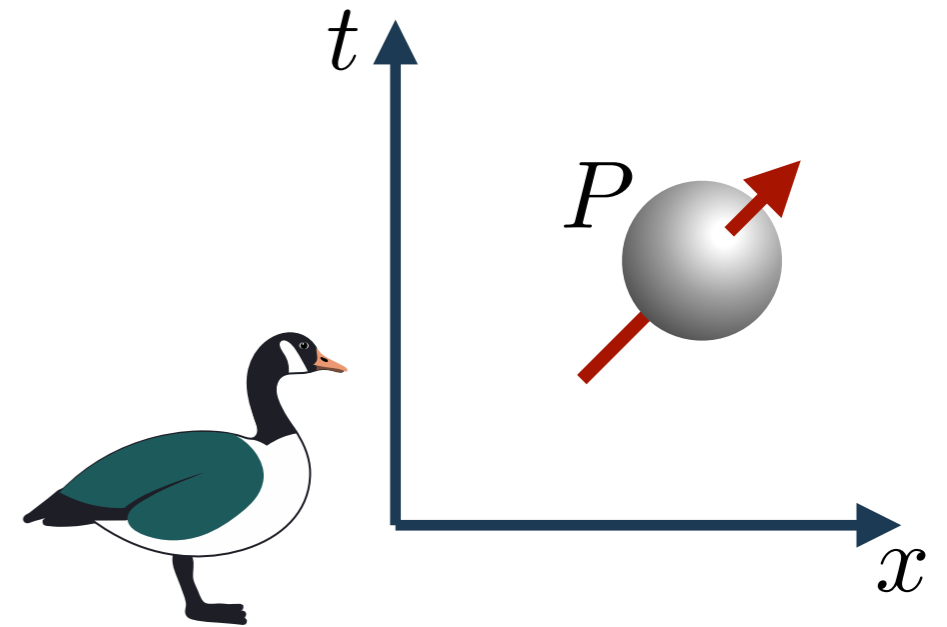
The laws of physics are the same regardless of the choice of the reference frame.

(Principle of covariance)

Translation $\hat{U}_T = e^{\frac{i}{\hbar} X_0 \hat{p}}$

Galilean boost $\hat{U}_B = e^{\frac{i}{\hbar} v \hat{G}} \quad \hat{G} = \hat{p}t - m\hat{x}$

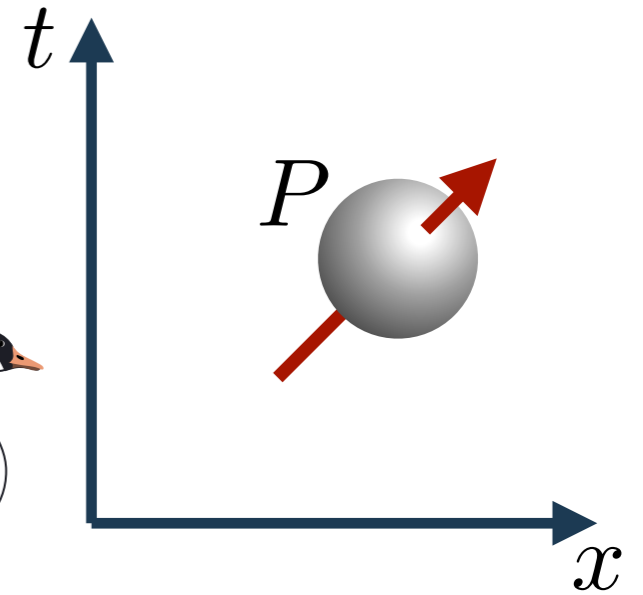
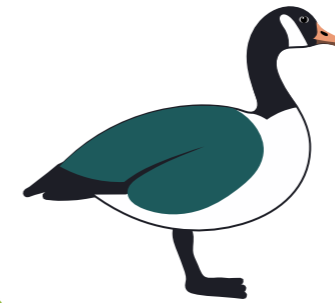
⋮



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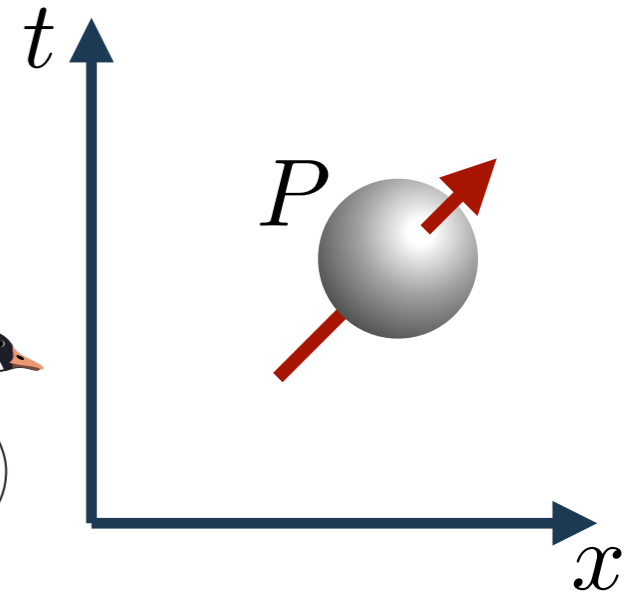
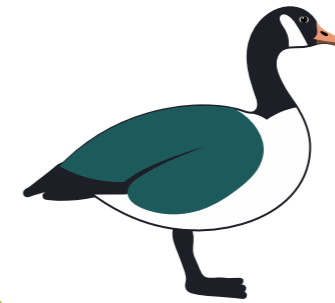
The reference frame enters the transformation as a **parameter**.

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Covariance of physical laws

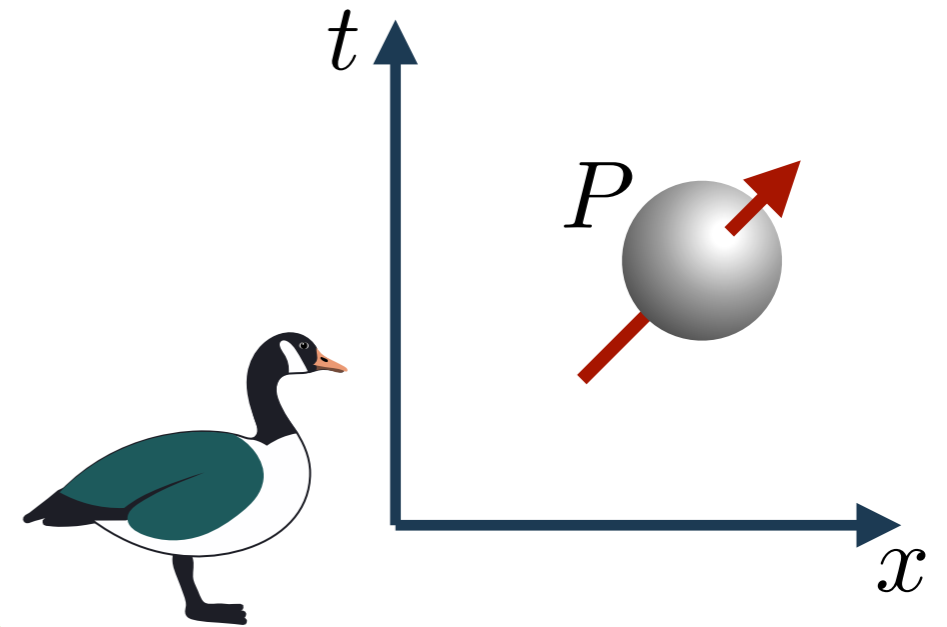
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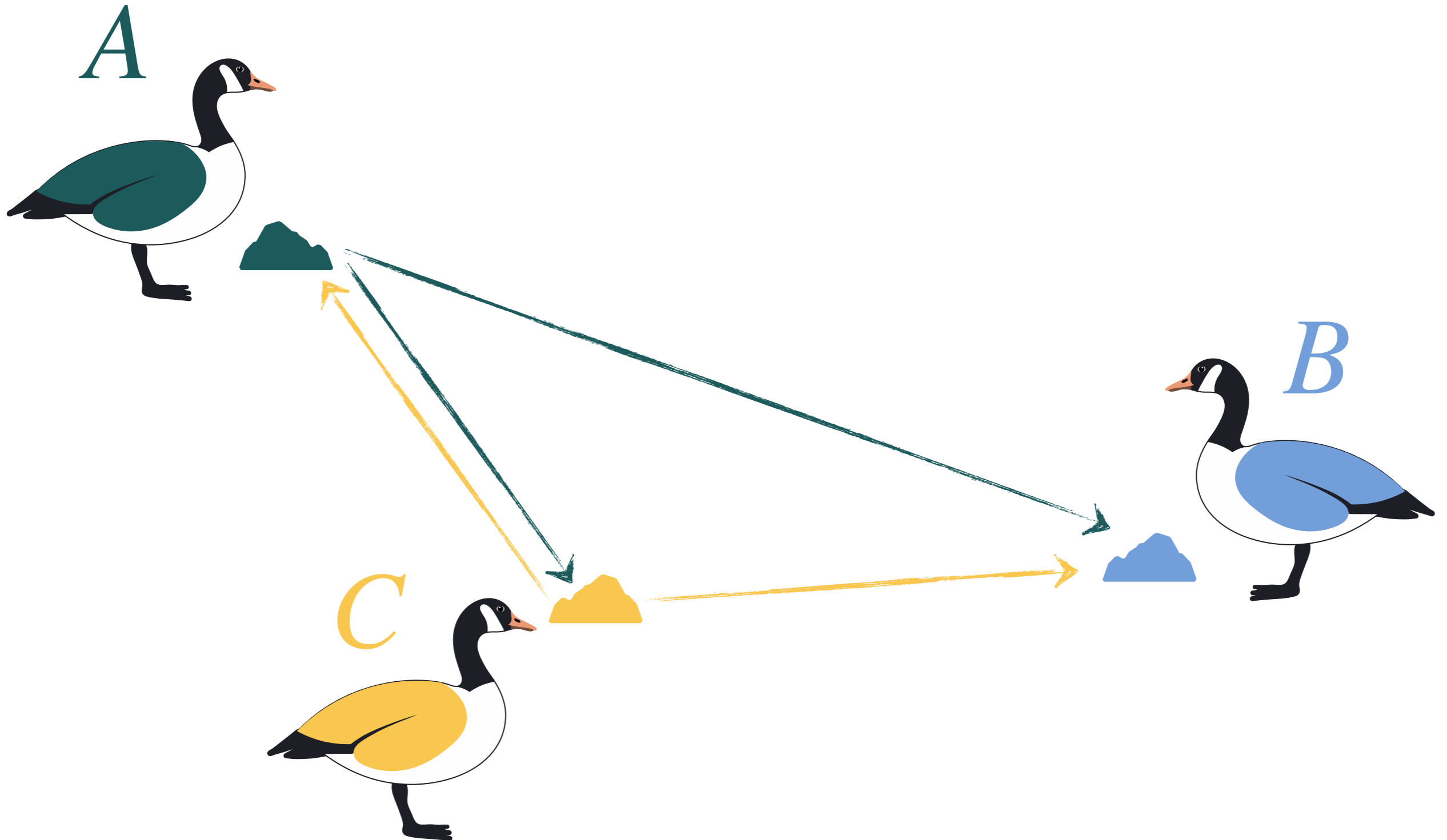
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Symmetry

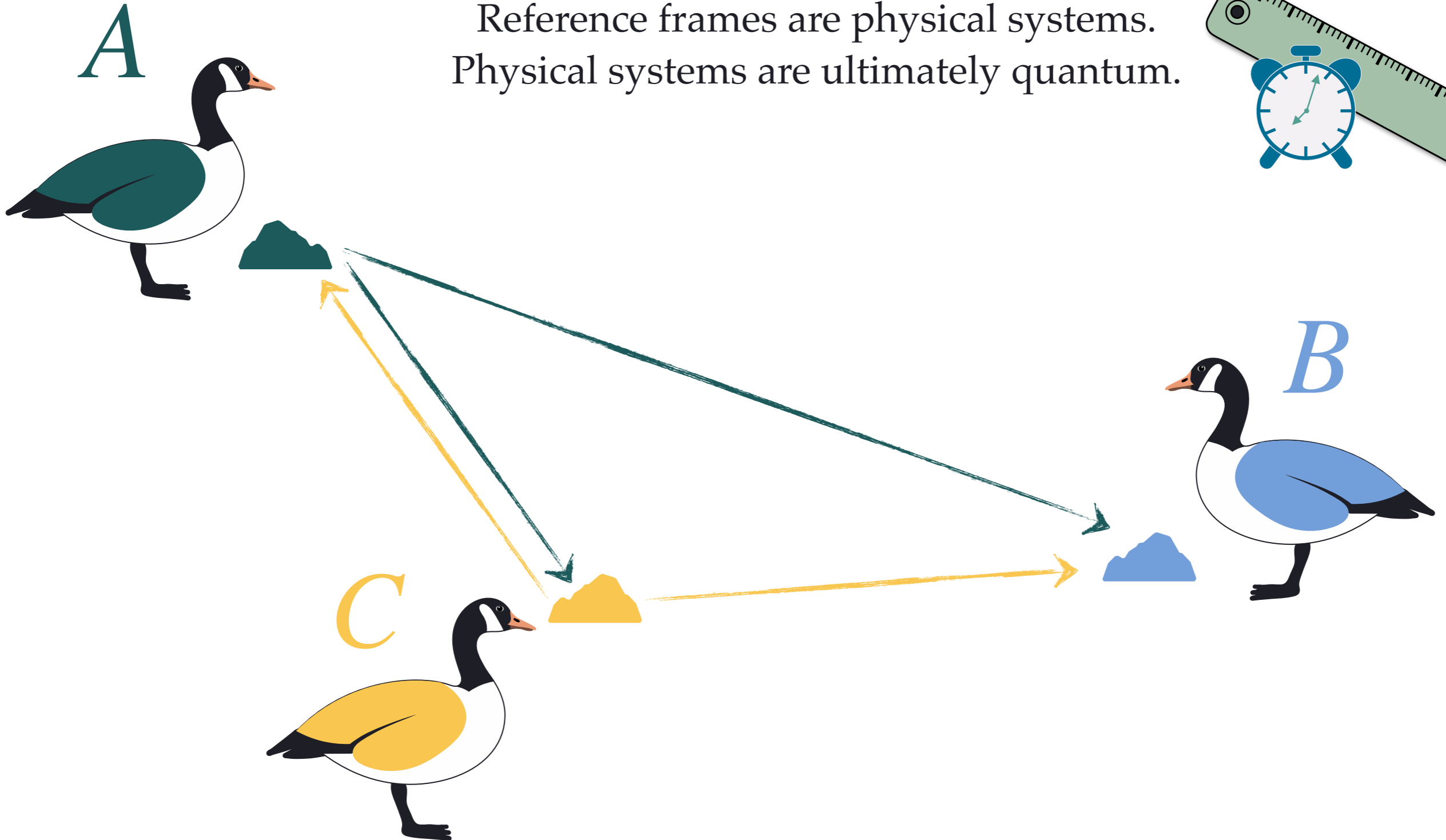
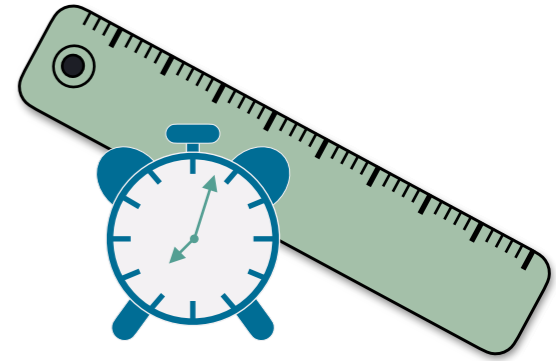
$$\hat{H}' = \hat{H}$$

What are reference frames?



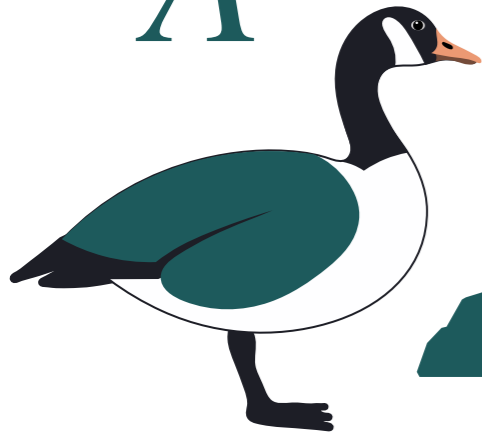
What are reference frames?

Reference frames are physical systems.
Physical systems are ultimately quantum.

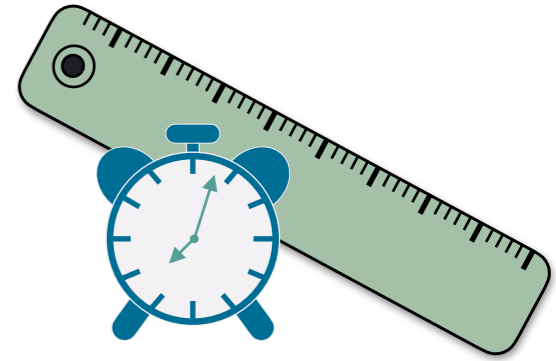


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A



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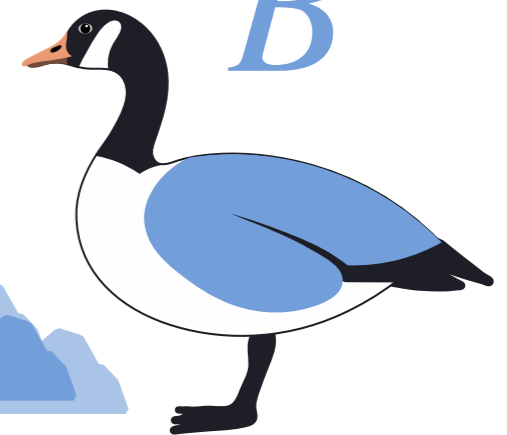


Can we “attach” a reference frame to an object whose state is in a superposition of classical states (in some basis)?

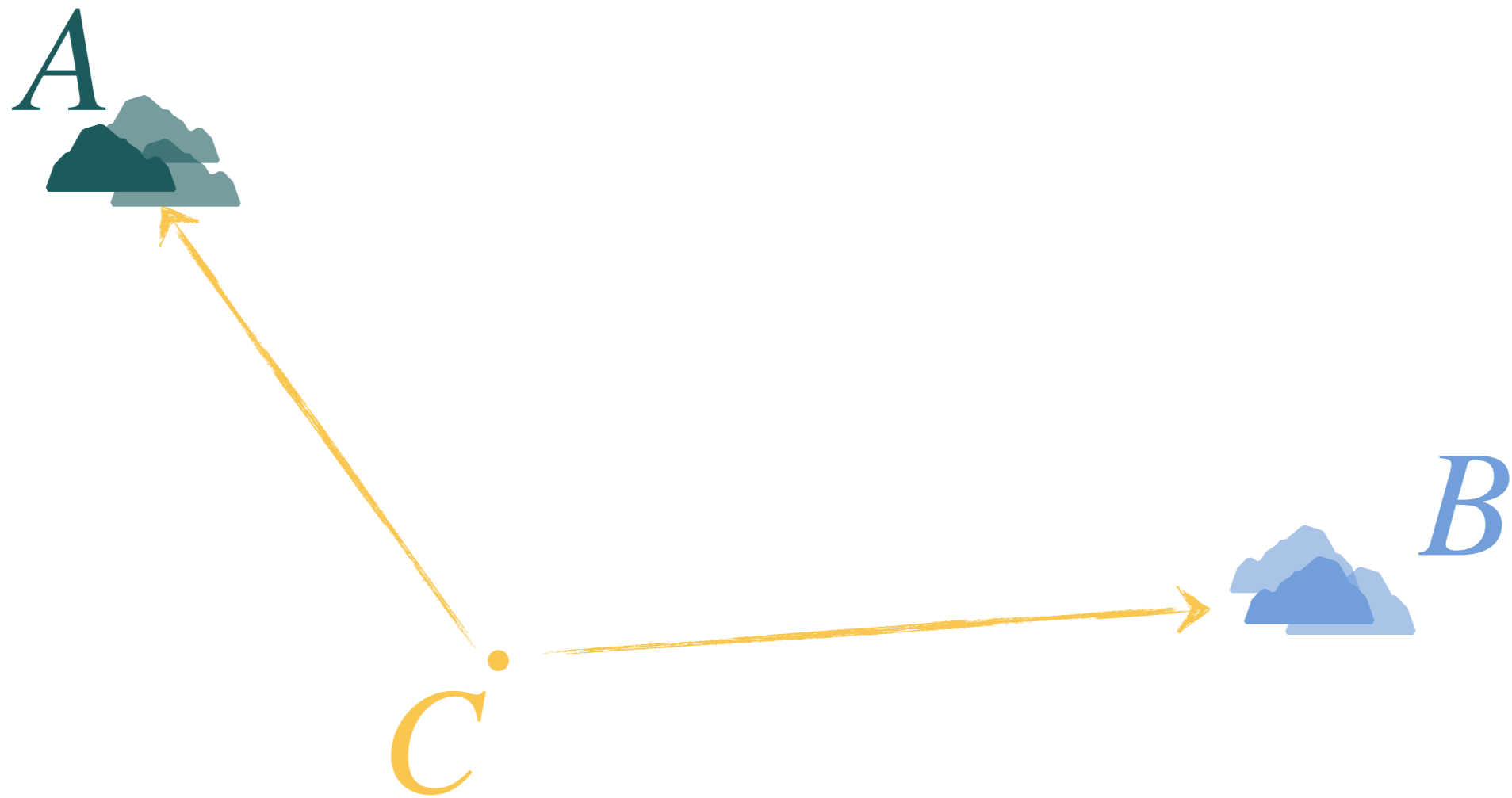
C



B



What are reference frames?



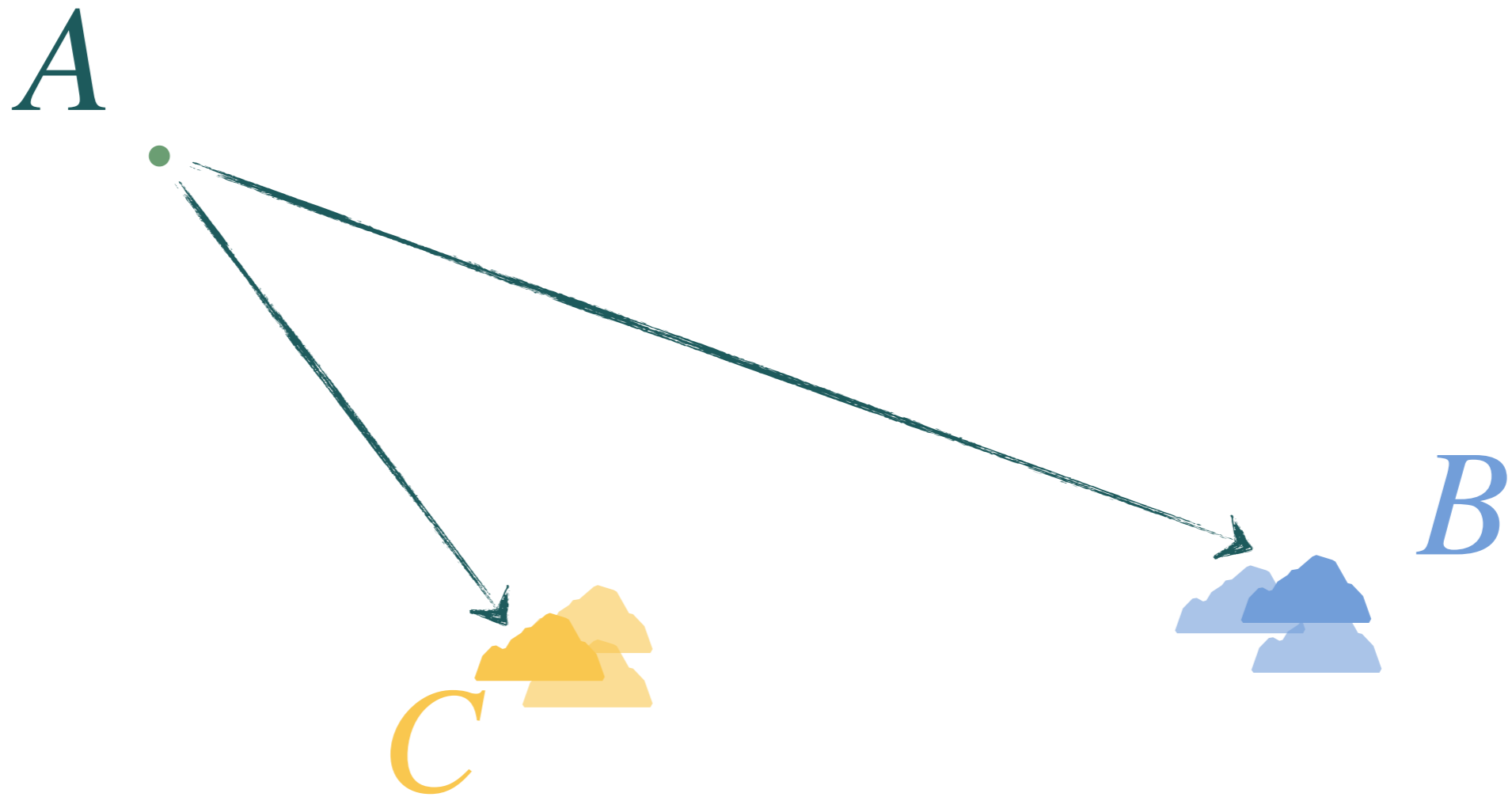
What are reference frames?

A

B

C

What are reference frames?



Superpositions of spacetimes

LIGHTEST GRAVITY SOURCE: 90 mg

Article | [Published: 10 March 2021](#)

Measurement of gravitational coupling between millimetre-sized masses

[Tobias Westphal](#) , [Hans Hepach](#), [Jeremias Pfaff](#) & [Markus Aspelmeyer](#) 

[Nature](#) 591, 225–228 (2021) | [Cite this article](#)

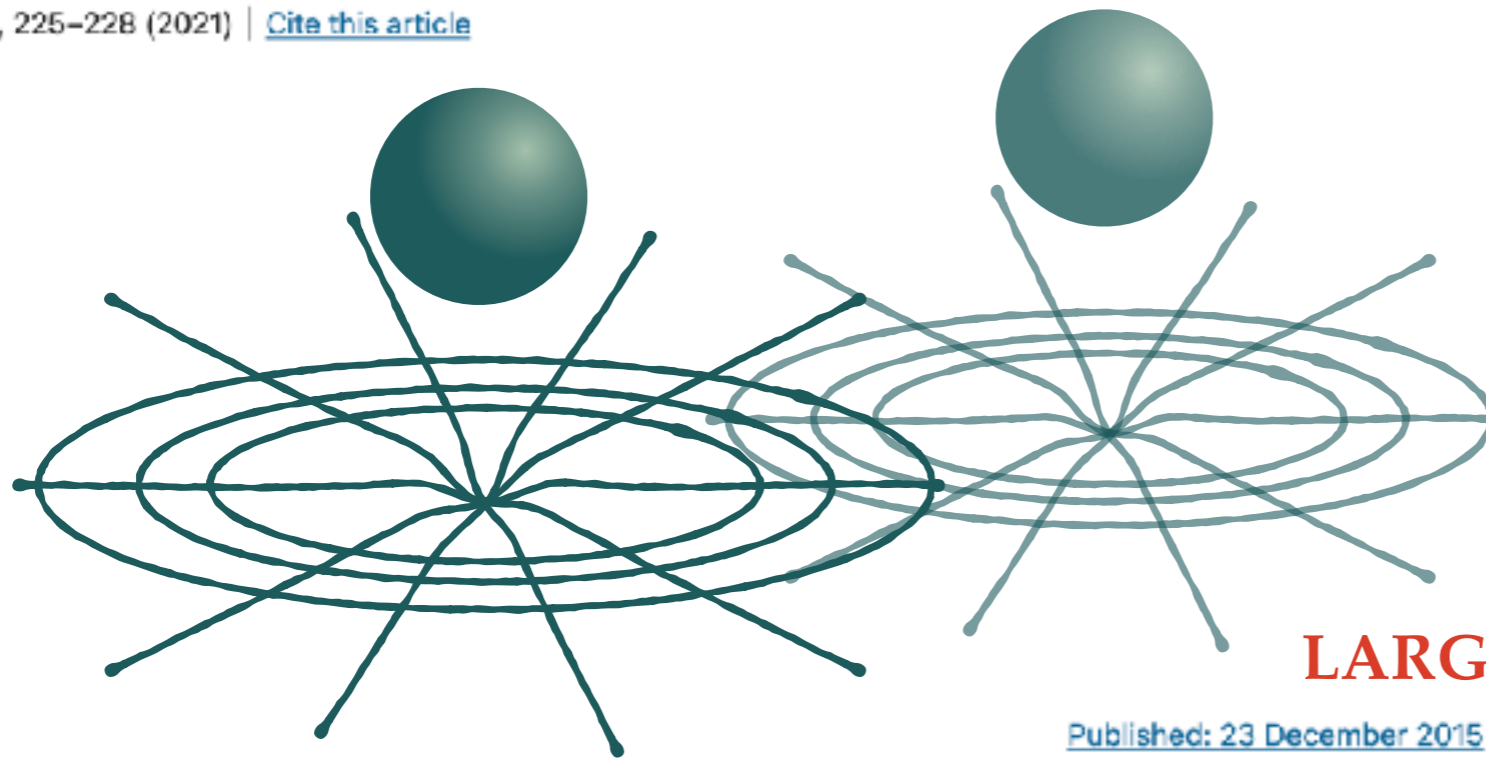
HEAVIEST SUPERPOSED MASS: $10^{-20}g$

Letter | [Published: 23 September 2019](#)

Quantum superposition of molecules beyond 25 kDa

[Yaakov Y. Fein](#), [Philipp Geyer](#), [Patrick Zwick](#), [Filip Kialka](#), [Sebastian Pedalino](#), [Marcel Mayer](#), [Stefan Gerlich](#) & [Markus Arndt](#) 

[Nature Physics](#) 15, 1242–1245 (2019) | [Cite this article](#)



LARGEST SUPERPOSITION: 0.5 m

[Published: 23 December 2015](#)

Quantum superposition at the half-metre scale

[T. Kovachy](#), [P. Asenbaum](#), [C. Overstreet](#), [C. A. Donnelly](#), [S. M. Dickerson](#), [A. Sugarbaker](#), [J. M. Hogan](#) & [M. A. Kasevich](#) 

[Nature](#) 528, 530–533 (2015) | [Cite this article](#)

$$m \cdot \Delta x \approx 10^{-25} g \cdot m \quad (\text{Aspelmeyer, 2203.05587})$$

1

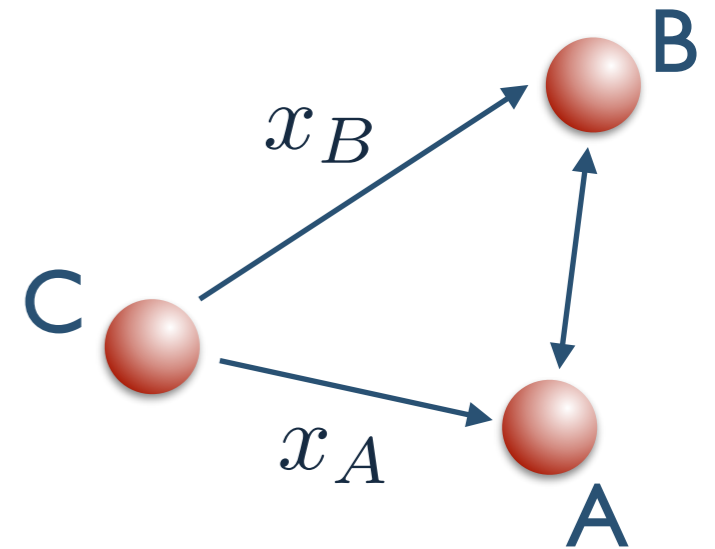
**QUANTUM REFERENCE FRAMES IN
QUANTUM MECHANICS**

Quantum reference frames

Transformation to relative coordinates

$$x_A \mapsto -q_C$$

$$x_B \mapsto q_B - q_C$$



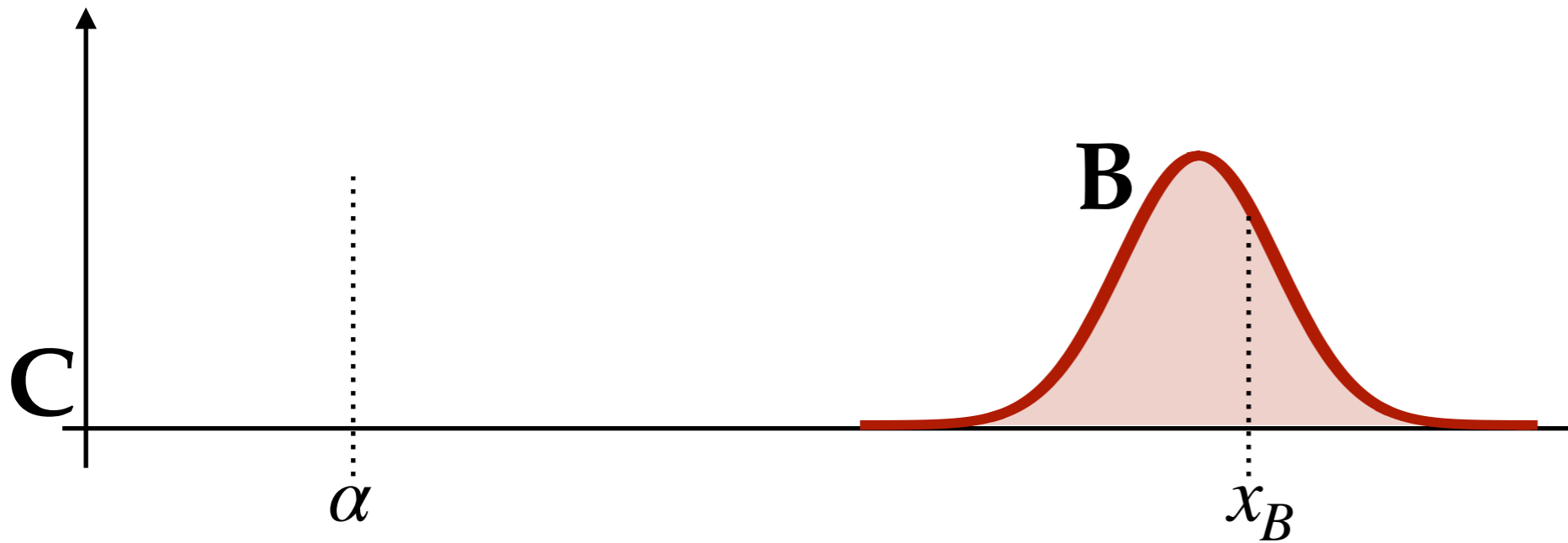
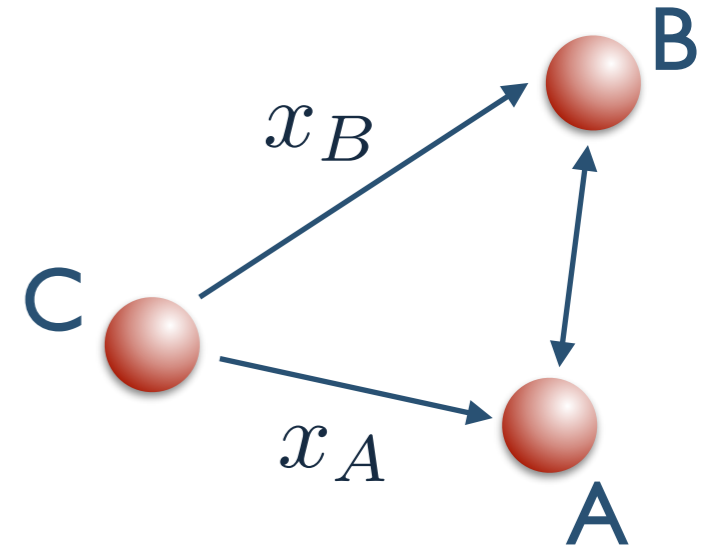
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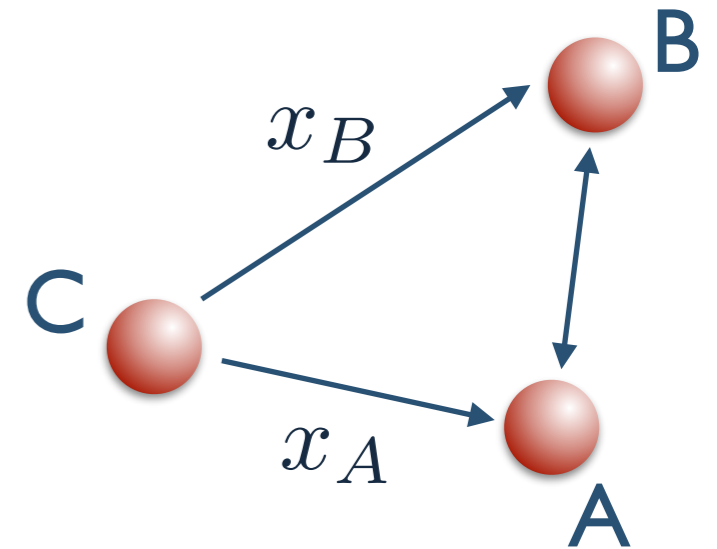
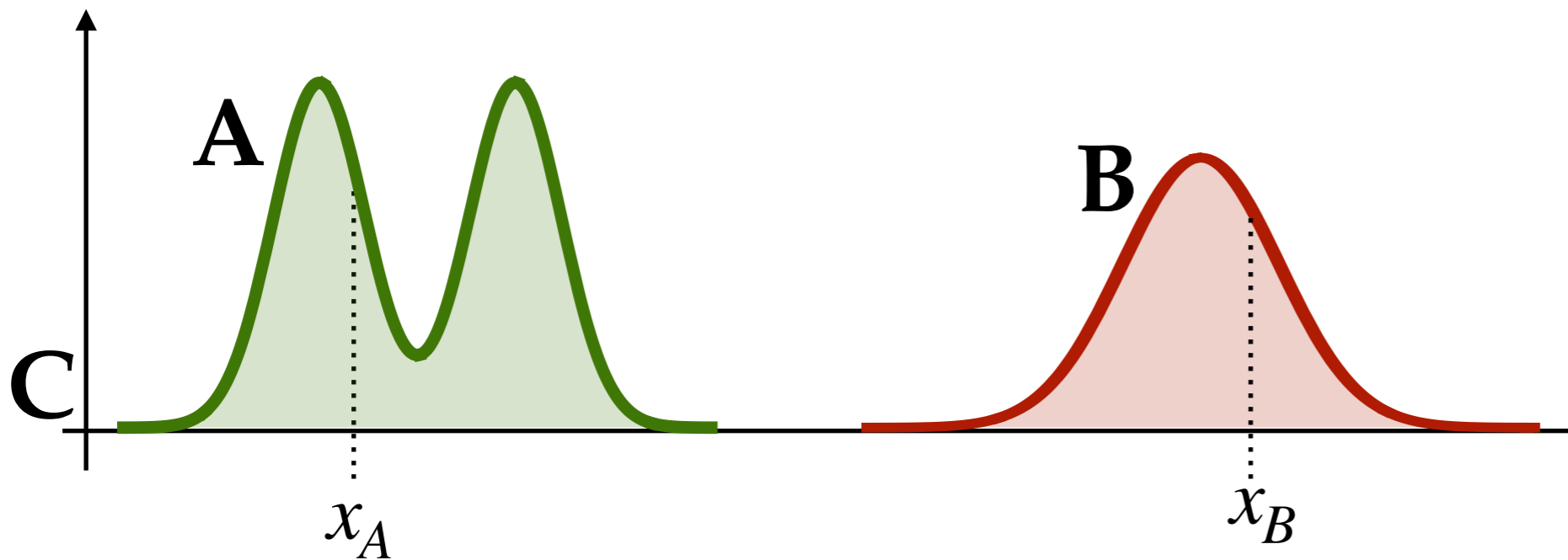
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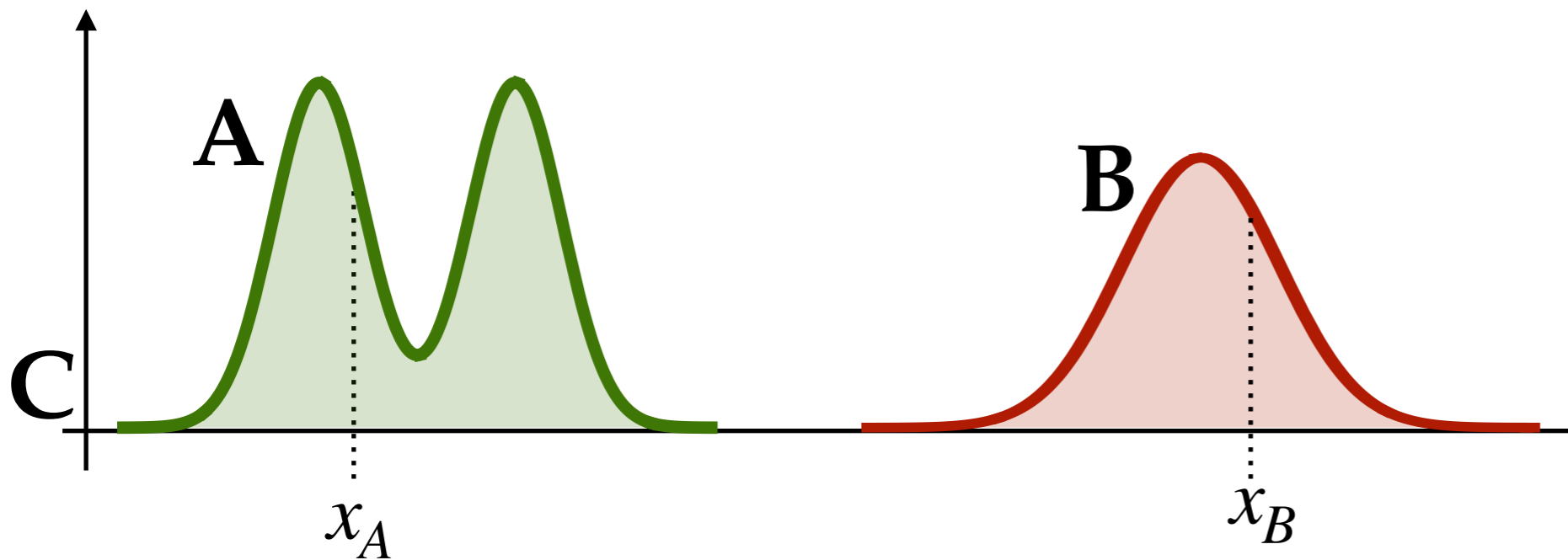
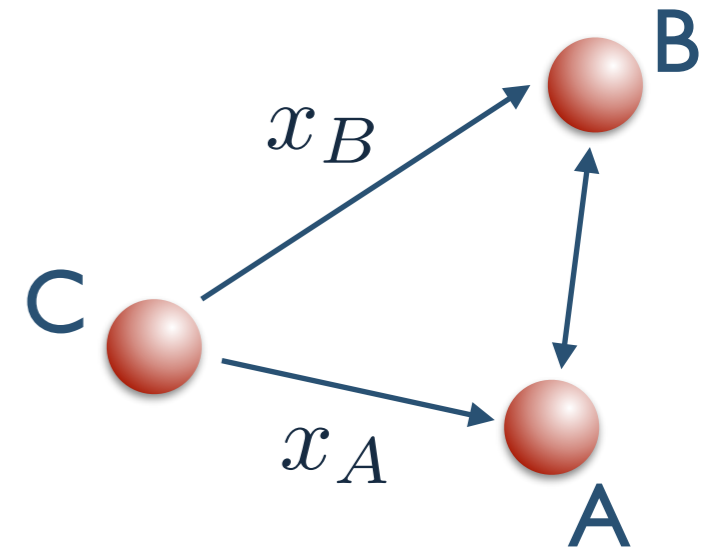
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$$e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B} |\phi\rangle_A |\psi\rangle_B$$

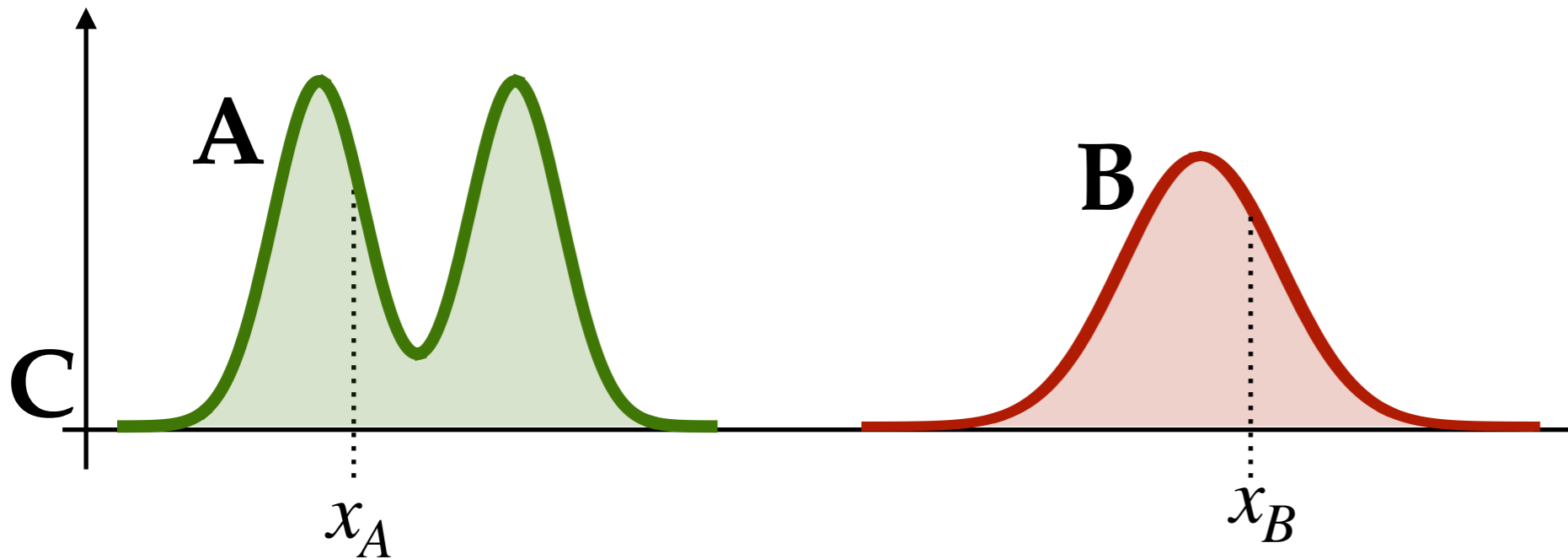
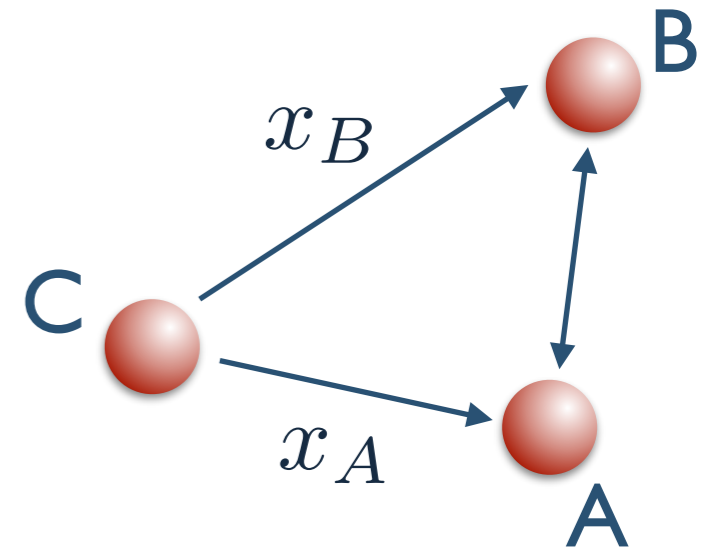
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$$\hat{S}_x = \mathcal{P}_{AC} e^{\frac{i}{\hbar}\hat{x}_A\hat{p}_B}$$

$$\mathcal{P}_{AC}\hat{x}_A\mathcal{P}_{AC}^\dagger = -\hat{q}_C$$

parity-swap operator

$$\rho_{BC}^{(A)} = \hat{S}_x \rho_{AB}^{(C)} \hat{S}_x^\dagger$$

Quantum reference frames

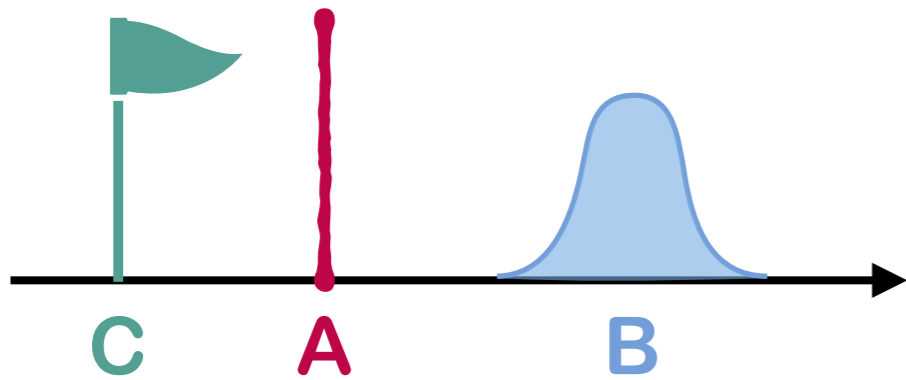
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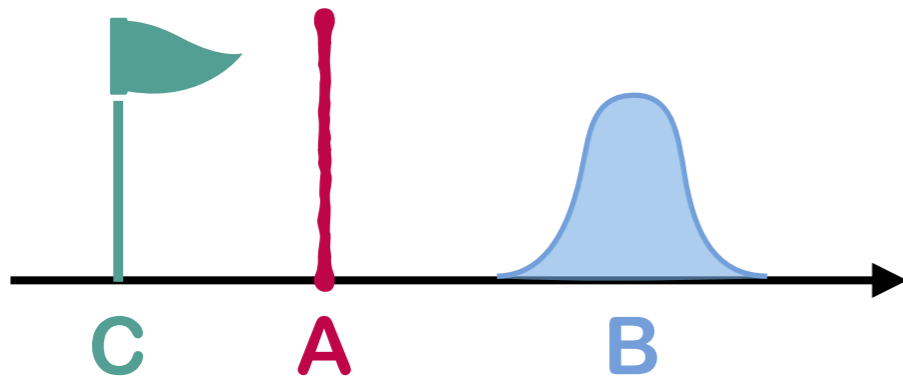


From C: $|x\rangle_A |\phi\rangle_B$

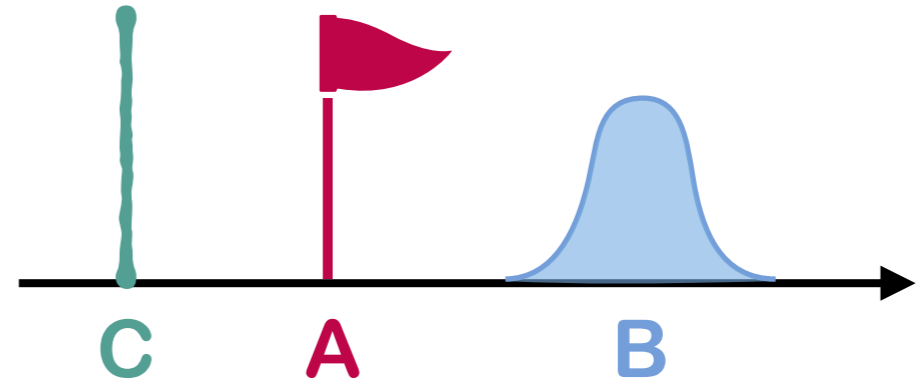
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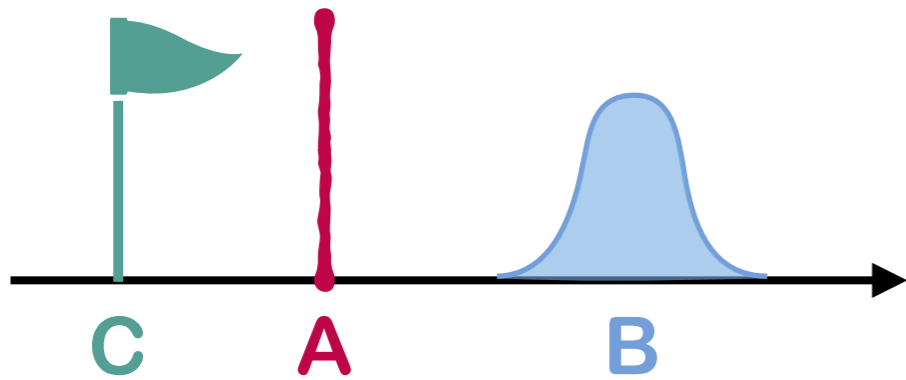


From A: $e^{-\frac{i}{\hbar} x \hat{p}_B} |\phi\rangle_B | -x\rangle_C$

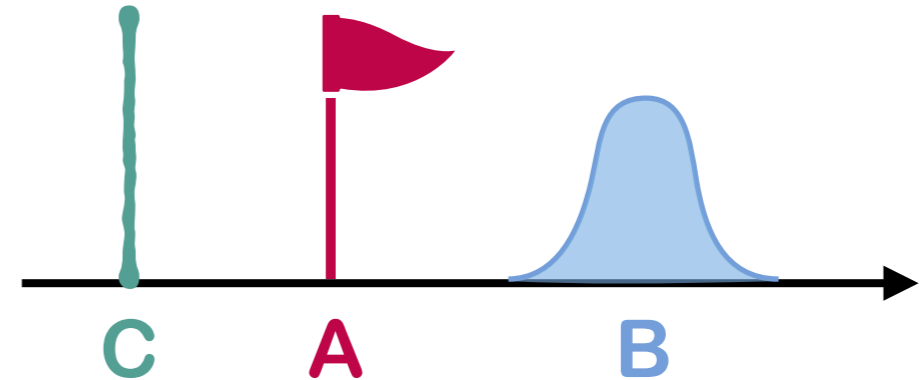
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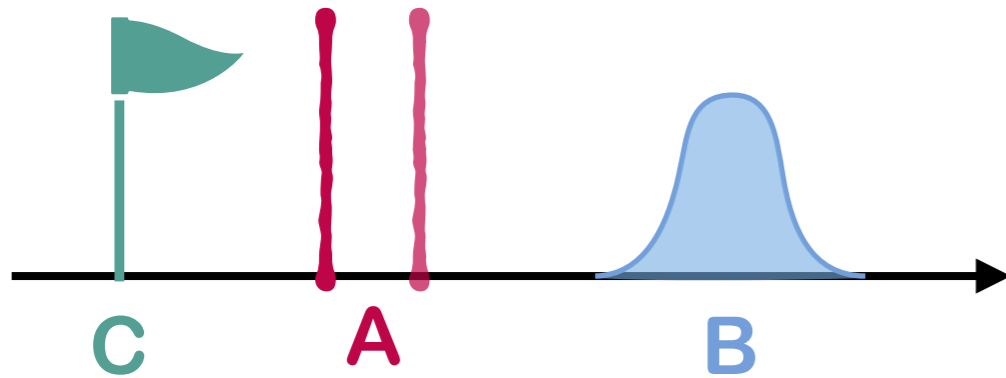
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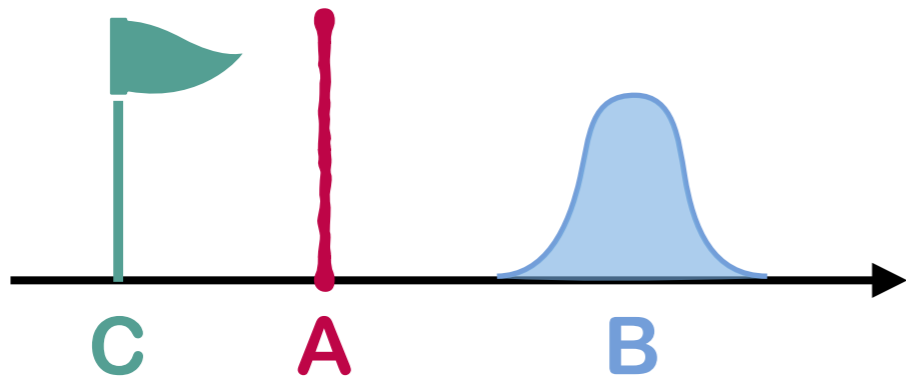


From C: $\frac{1}{\sqrt{2}} (|x_1\rangle_A + |x_2\rangle_A) |\phi\rangle_B$

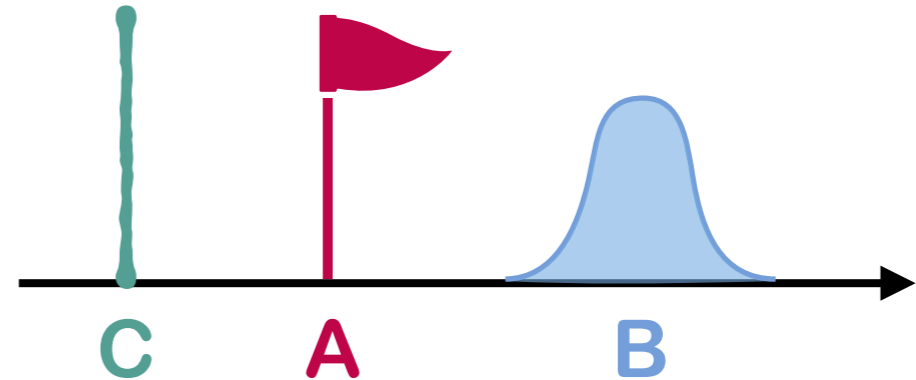
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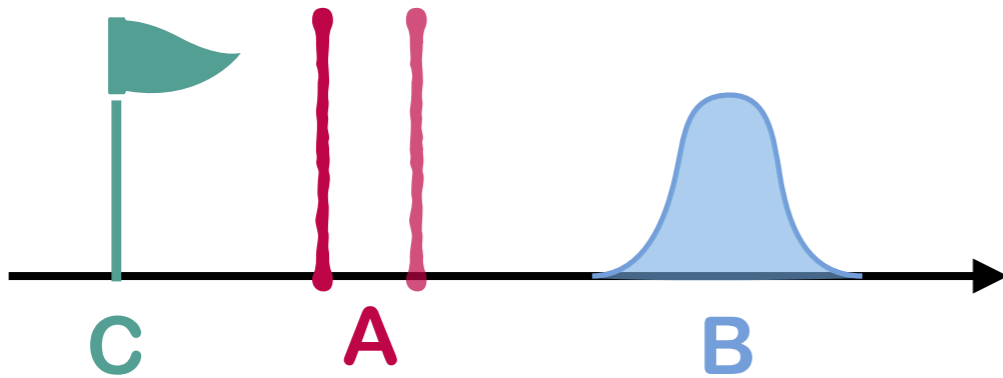
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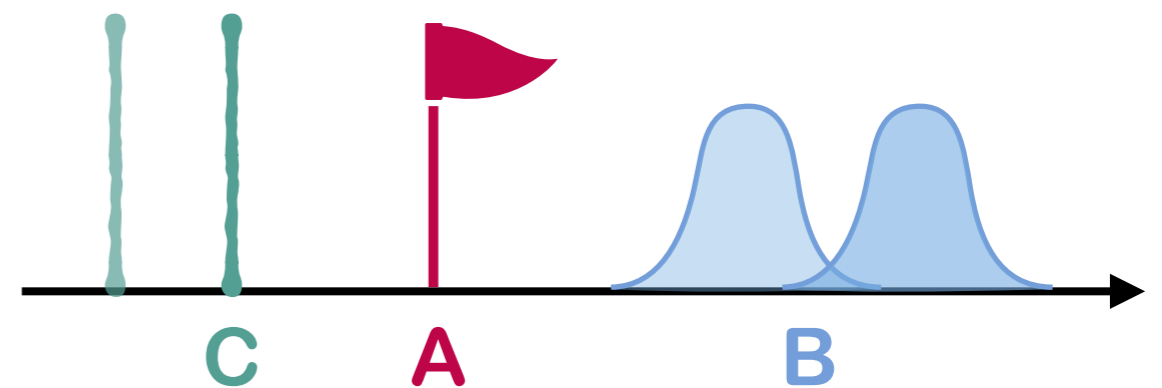
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From A: $\frac{1}{\sqrt{2}} \left(e^{-\frac{i}{\hbar} x_1 \hat{p}_B} |\phi\rangle_B | -x_1\rangle_C + e^{-\frac{i}{\hbar} x_2 \hat{p}_B} |\phi\rangle_B | -x_2\rangle_C \right)$

Extended Galilean transformations...

$$x' = x - X(t)$$

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Spatial translations

$$X(t) = X_0$$

$$\hat{U}_x$$

Galilean boosts

$$X(t) = vt$$

$$\hat{U}_v$$

Accelerated reference frame

$$X(t) = \frac{1}{2}at^2$$

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Hamiltonian of
the system

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In QRFs:
Hamiltonian of the system **AND** of the QRF!

$$\hat{H} = \frac{\hat{p}^2}{2m} \quad \rightarrow \quad \hat{H}' = \frac{\hat{p}^2}{2m} + m\ddot{X}\hat{x}$$

...and their superpositions!

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$$\hat{U}_T = \sum_i \hat{U}_{x_i} \otimes |x_i\rangle\langle x_i|$$

$$\hat{U}_G = \sum_i \hat{U}_{v_i} \otimes |v_i\rangle\langle v_i|$$

$$\hat{U}_A = \sum_i \hat{U}_{a_i} \otimes |a_i\rangle\langle a_i|$$

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Controlled superposition of the transformation on some additional Hilbert space

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Controlled superposition of the transformation on some additional Hilbert space

Operational meaning

Quantum Reference Frames

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Controlled superposition of the transformation on some additional Hilbert space

Operational meaning

Quantum Reference Frames

How do the laws of motion change before and after the transformation?

Extension of the covariance of physical laws

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Controlled superposition of the transformation on some additional Hilbert space

Operational meaning

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How do the laws of motion change before and after the transformation?

Extension of the covariance of physical laws

Superposition of translations
Superposition of Galilean boosts

...and their superpositions!

$$\hat{U}_T = \sum_i \hat{U}_{x_i} \otimes |x_i\rangle\langle x_i|$$
$$\hat{U}_G = \sum_i \hat{U}_{v_i} \otimes |v_i\rangle\langle v_i|$$
$$\hat{U}_A = \sum_i \hat{U}_{a_i} \otimes |a_i\rangle\langle a_i|$$

Controlled superposition of the transformation on some additional Hilbert space

Operational meaning

Quantum Reference Frames

How do the laws of motion change before and after the transformation?

Extension of the covariance of physical laws

Superposition of translations
Superposition of Galilean boosts

Extended symmetries of the dynamics

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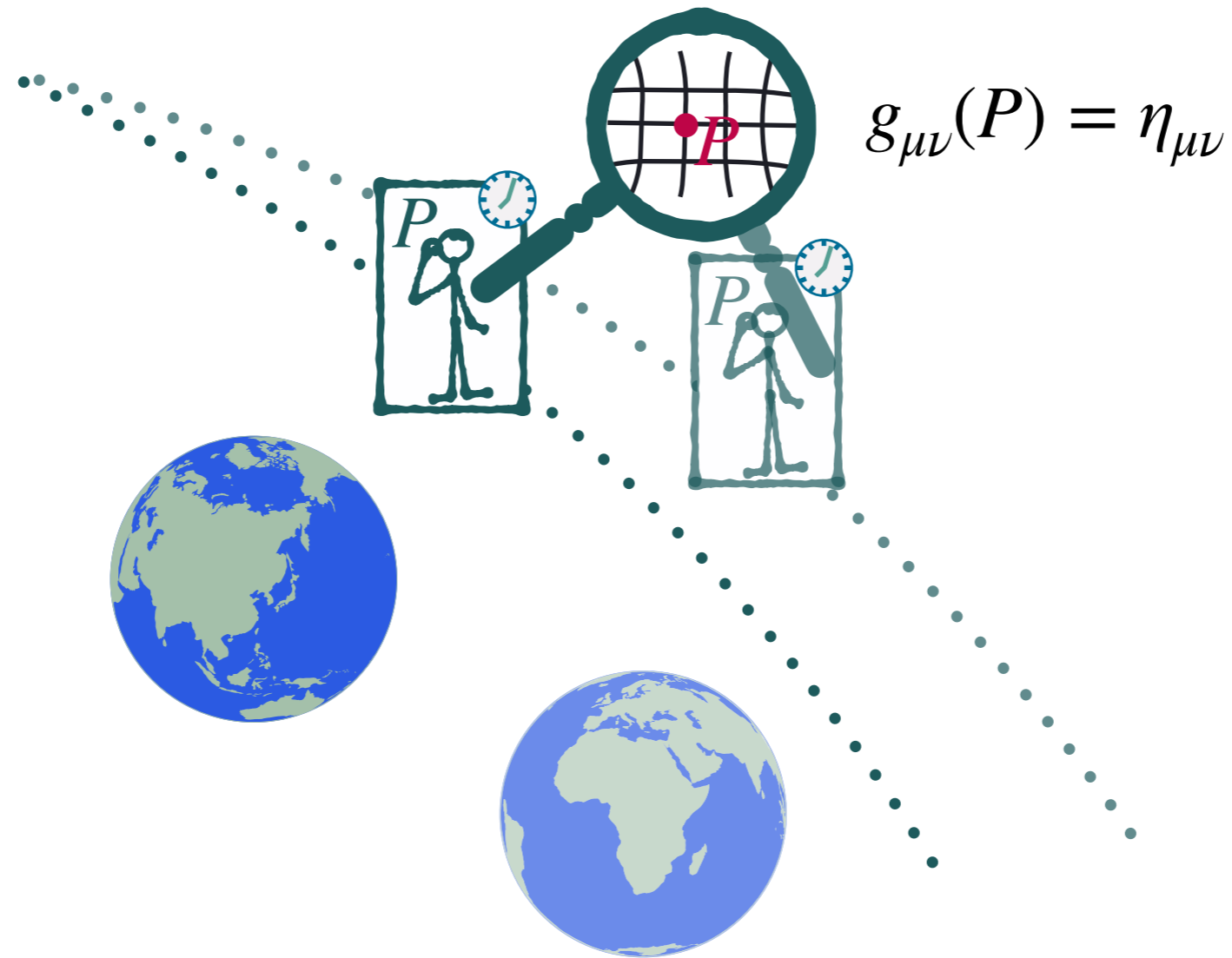
Extended symmetries of the dynamics

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B} \quad \hat{H}_{BC}^{(A)} = \frac{\hat{p}_B^2}{2m_B} + \frac{\hat{p}_C^2}{2m_C}$$

2

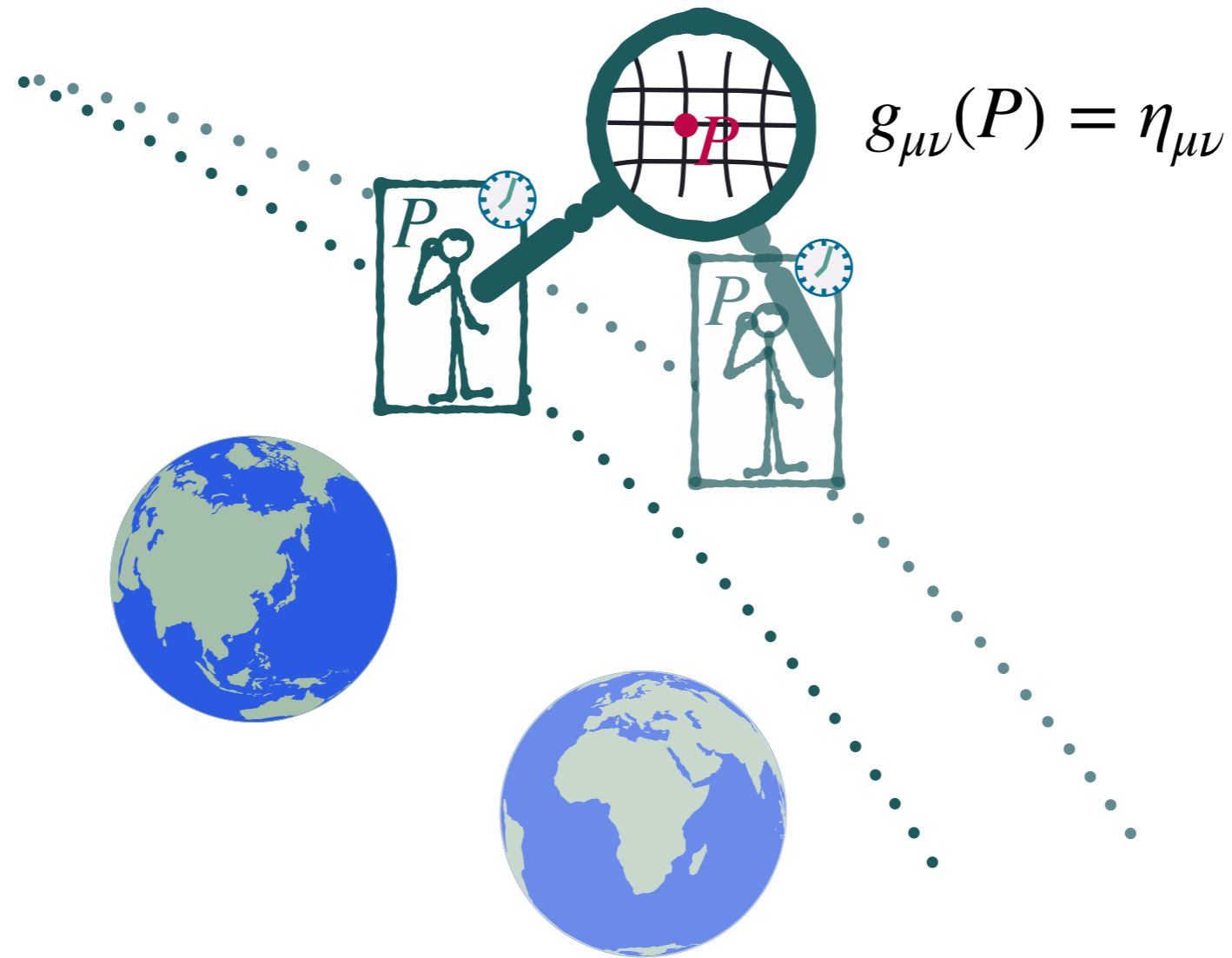
QUANTUM REFERENCE FRAMES AND GRAVITY?

Equivalence Principle in QRFs



**Reconciliation of EEP and Principle of
Linear Superposition**

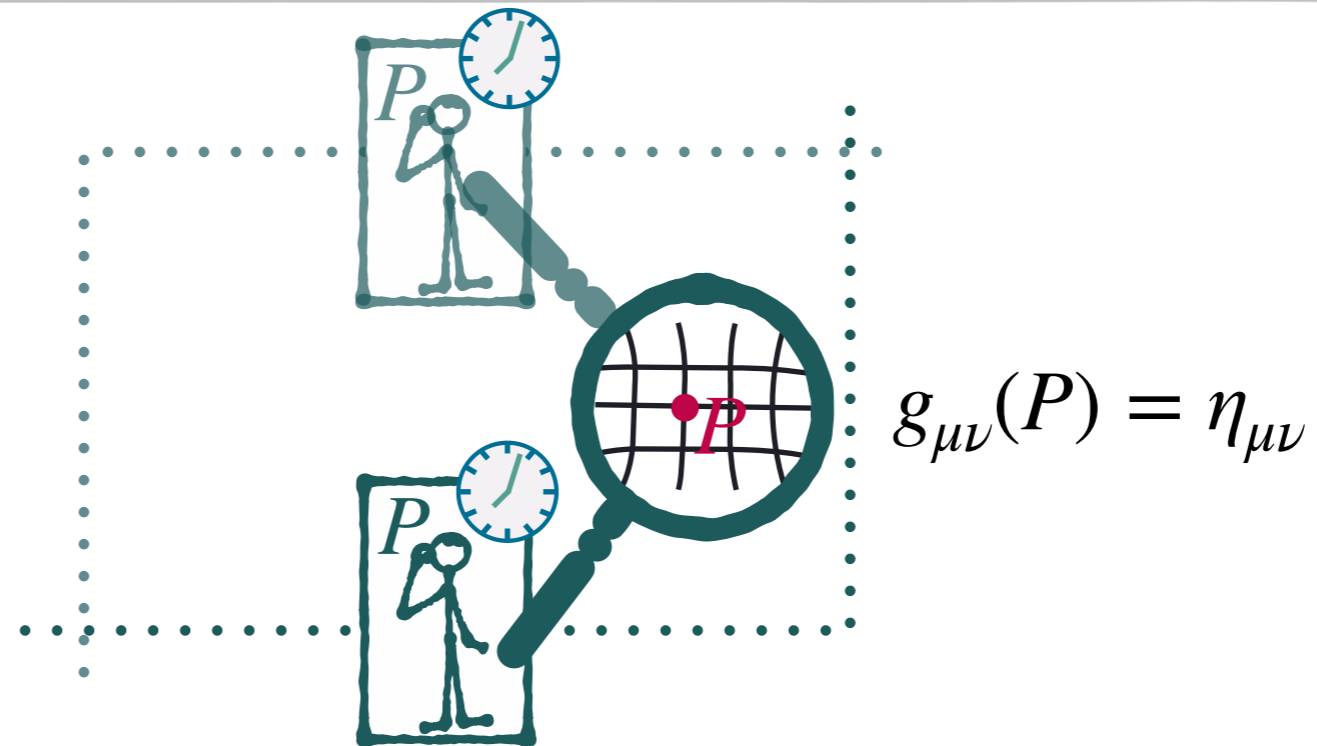
Equivalence Principle in QRFs



Reconciliation of EEP and Principle of
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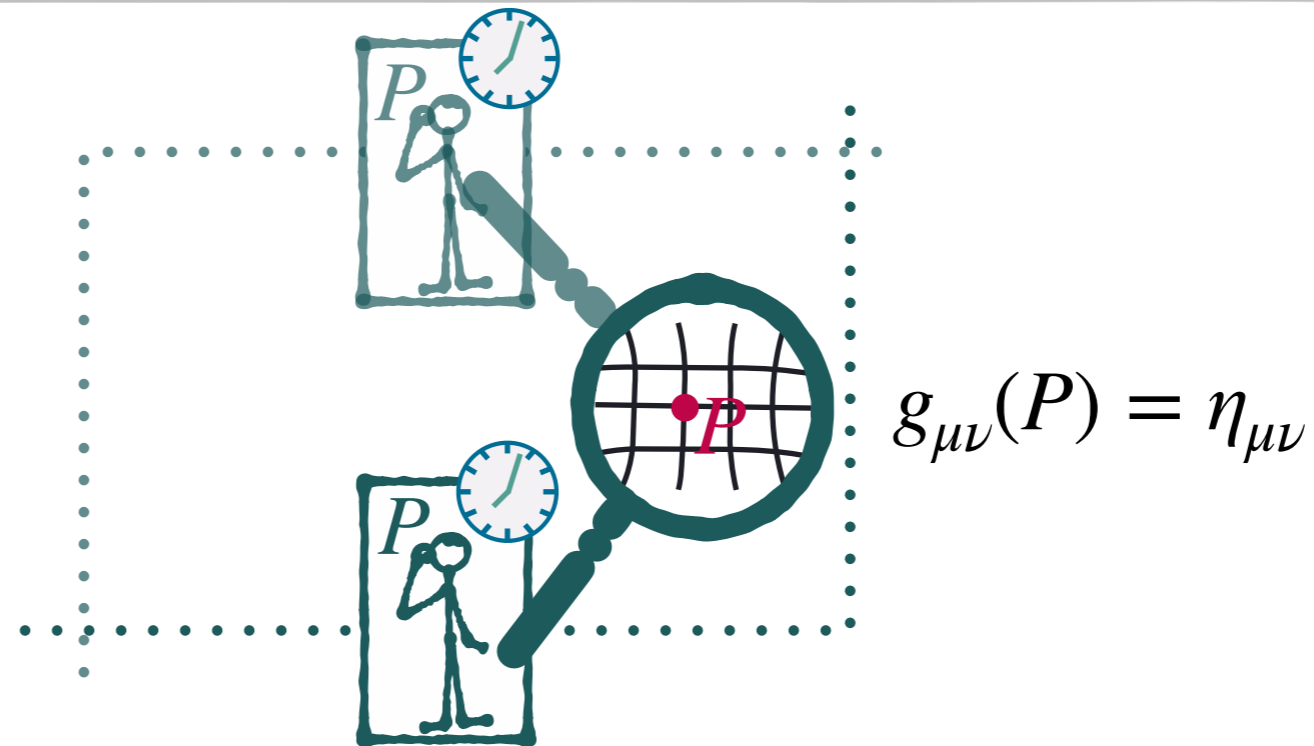
**Overcomes Penrose's spontaneous
state reduction**

Test of the generalised EEP



Test of EEP for QRFs in atom interferometer
with quantum clocks

Test of the generalised EEP



Test of EEP for QRFs in atom interferometer
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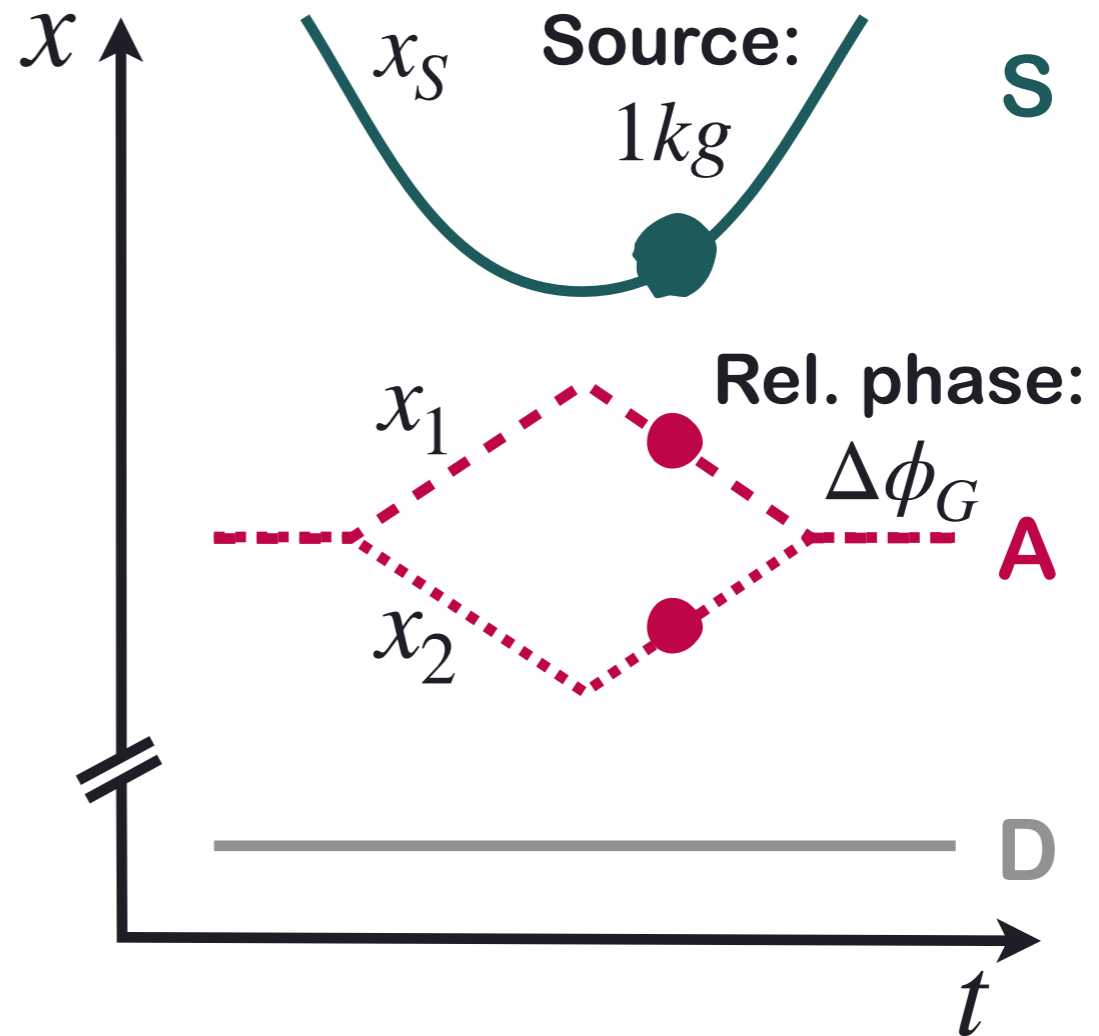
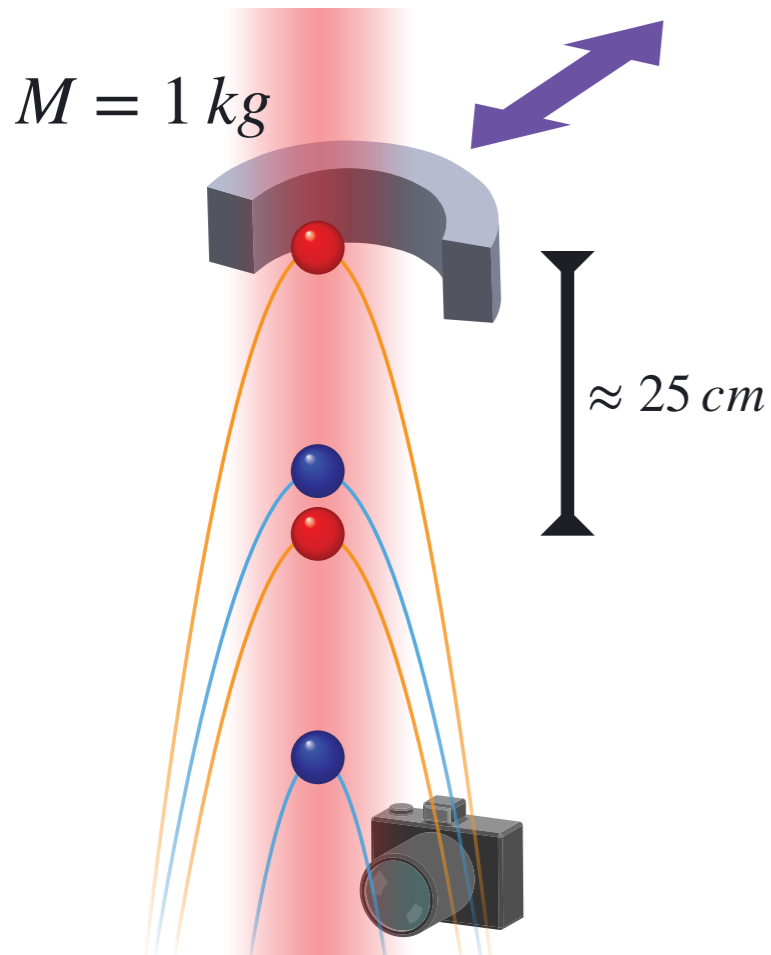
If EEP for QRF not valid, it is not possible to
define time evolution in a QRF

Nonclassical spacetime



Is the superposition of gravitational fields a relative concept?

Gravitational Aharonov-Bohm experiment



Gravitational action difference:

$$\Delta\phi_G = \frac{1}{\hbar} \int_0^T dt [V(x_2 - x_S) - V(x_1 - x_S)]$$

Phase shift beyond linear regime

What we know from experiments

- Gravity is tested for masses as light as 90 mg

Westphal, Hepach, Pfaff, Aspelmeyer, Nature (2021)

- Equivalence Principle is valid up to experimental resolution (10^{-15})

MICROSCOPE Mission, September 2022

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B. Abbott et al., PRL (2016)

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- Classical gravity (tungsten) and Quantum Theory (atom) are compatible beyond the linear regime

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Which assumptions should we add to have a superposition of gravitational fields?

Three fundamental principles

1. EXISTENCE OF GRAVITATIONAL FIELDS:

Any massive particle that is well-localized at a position x_0 sources a gravitational field \mathbf{g} with functional form $\mathbf{g}(\mathbf{x} - \mathbf{x}_0)$

2. FIELD ENERGY PRINCIPLE:

The phase of an interferometer is a function of the energies of the fields that interact with the interfering particle

3. QUANTUM RELATIVITY PRINCIPLE:

*The laws of physics take the same form in every reference frame, including the reference frames associated with quantum particles
(quantum reference frames)*

Summary

Operational and relational formalism for **quantum reference frames**:
associate a reference frame to a quantum system.

In quantum mechanics:

Frame-dependence of entanglement and superposition

Generalisation of covariance

Generalisation of the weak equivalence principle

Operational definition of the rest frame of a quantum system (relativistic spin)

In gravity:

Generalisation of the Einstein Equivalence Principle

Penrose decoherence

Thank you for your attention!