Relational Observables in Asymptotically Safe Gravity

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Quantum gravity, hydrodynamics and emergent cosmology

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“There are no local observables in Quantum Gravity”

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"It is impossible to compare results stemming from different theories of Quantum Gravity, since there are no observables.”

“There are no local observables in Quantum Gravity”

"It is impossible to compare results stemming from different theories of Quantum Gravity, since there are no observables.”

“Asymptotic Safety is a powerful conjecture, but it cannot be tested, since it does not furnish a device to make predictions for physical observables.”

Relational Observables

Ingredients

Asymptotically Safe Gravity

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Relational Observables

Asymptotically Safe Gravity

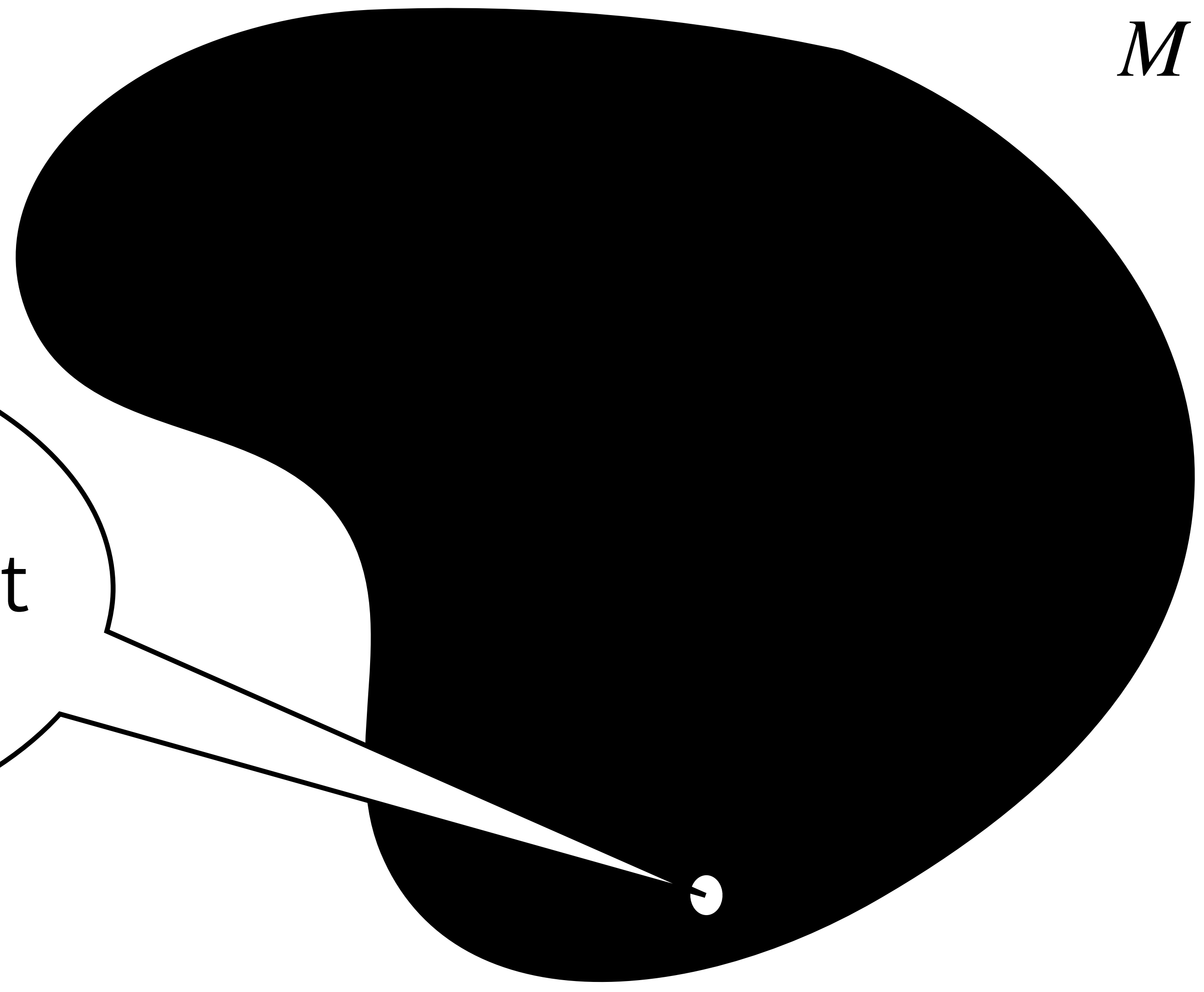
Tool

Composite Operator Flow

Relational Observables

Example: The Ricci scalar

$$R(x)?$$



Can I measure the curvature at that point?

Try to specify the point $\mathscr{P}$ where a particular event happens.

The scalar curvature at $\mathscr{P}$ is then an observable!

Diffeomorphism invariant theory

Local observables

Rovelli (1991)
Dittrich (2006): arXiv:gr-qc/0507106
Tambornino (2012): arXiv:1109.0740
RF, Percacci (2020): arXiv:2012:04507
Goeller, Höhn, Kirklin (2022), arXiv: 2206.01193

Diffeomorphism invariant theory

Local observables

add matter fields

construct a physical coordinate system

such that

perform a transformation

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Diffeomorphism invariant theory

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Measurements of local fields are possible!

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RF, Percacci (2020): arXiv:2012:04507

Goeller, Höhn, Kirklin (2022), arXiv: 2206.01193

Physical coordinate frame

$\phi^a(x)$ dynamical fields

Metric $g_{\mu\nu}$

Matter fields

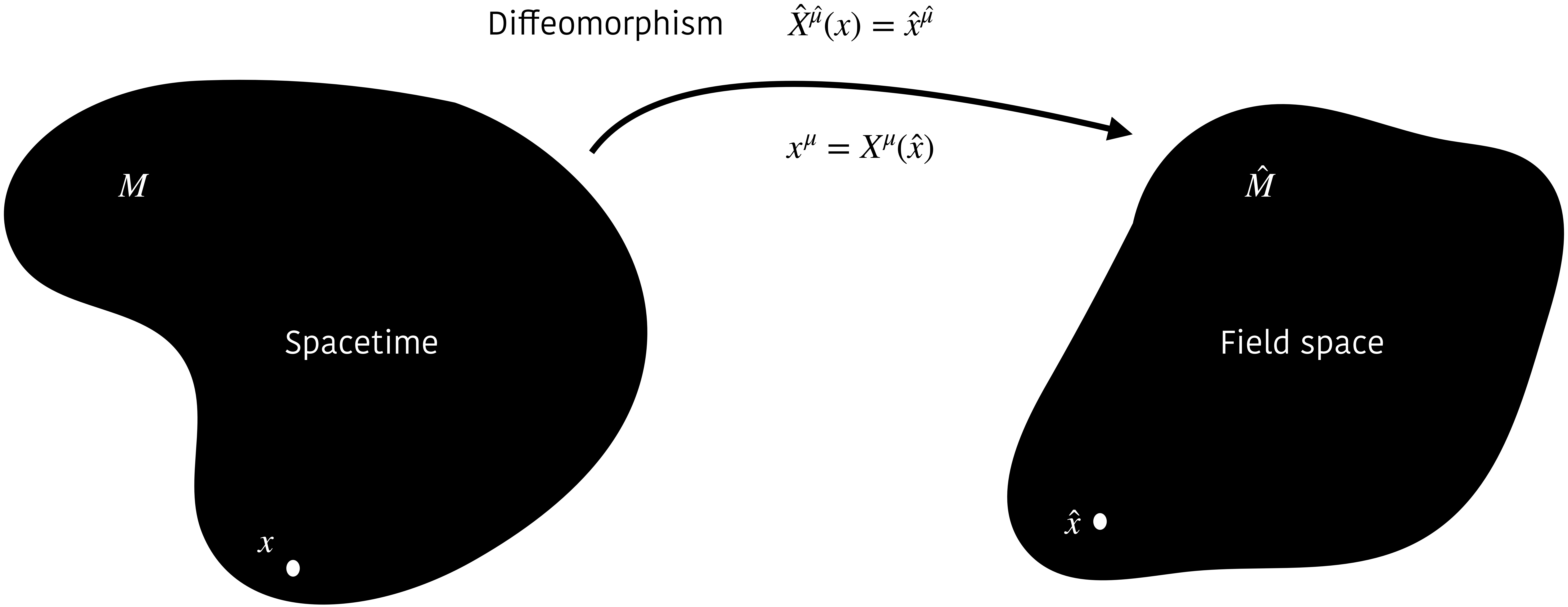
Physical coordinate frame: $\hat{X}^{\hat{\mu}}(x) = \hat{X}^{\hat{\mu}}(\phi(x), \partial\phi(x), \cdots), \quad \hat{\mu} = 0,1,2,3$

$\hat{X}^{\hat{\mu}}(x)$ are 4 scalars

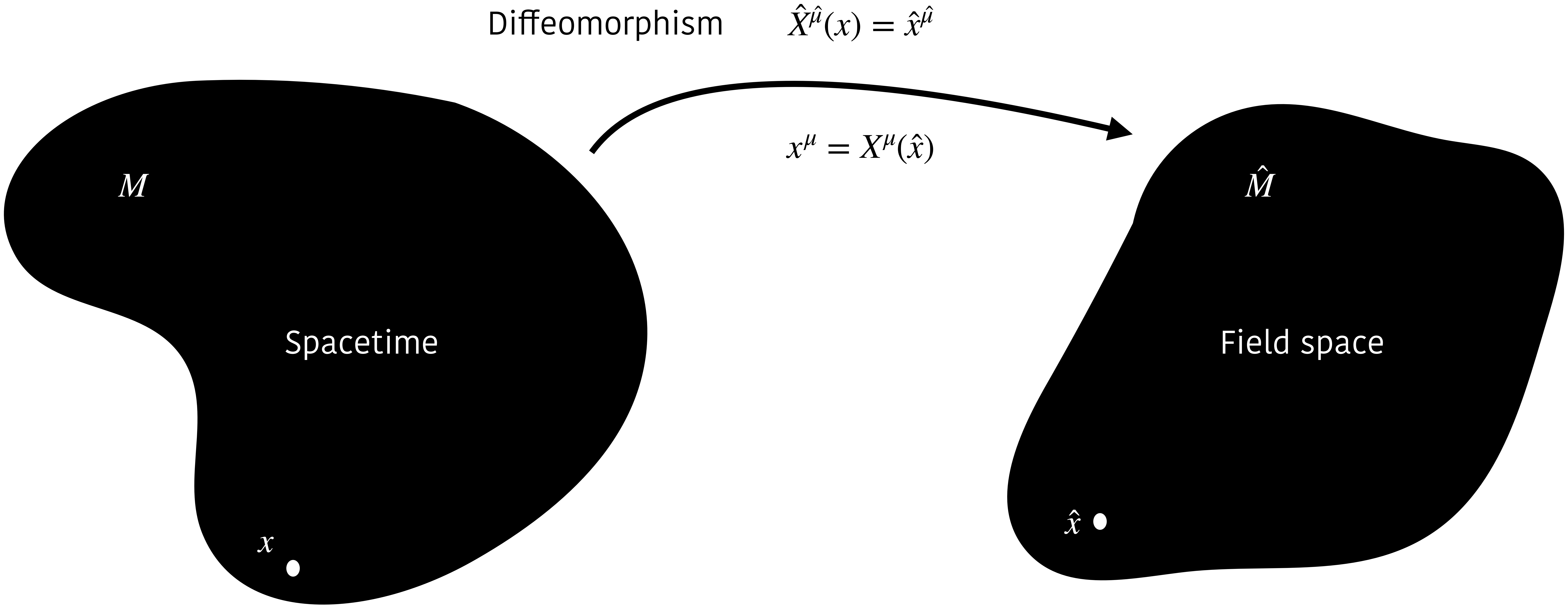
The point $\mathscr{P}$ is labelled by the values that $\hat{X}^{\hat{\mu}}(x)$'s take at $\mathscr{P}$.

ASSUME Invertible map

Physical coordinate frame



Physical coordinate frame



A good choice of scalars $\hat{X}^{\hat{\mu}}$ may define a good coordinate system for all field configurations ϕ.

Diffeomorphism transformation

Transformation under diffeomorphism $\xi^\mu(x)$

$$\phi^a_\xi = T^a[\phi, \xi]$$

Scalar field

$$\psi_\xi(x) = \psi(\xi(x))$$

Metric

$$g_{(\xi)\mu\nu}(x) = \partial_\mu \xi^\lambda(x) \partial_\nu \xi^\rho(x) g_{\lambda\rho}(\xi(x))$$

Composition identity: $T^a[\phi_\xi, \xi'] = T^a[\phi, \xi \circ \xi']$

Relational Observables

- Transformation under diffeomorphisms → transform the dynamical fields into the physical frame:

$$\hat{\phi}^{\hat{a}} = \phi_X^{\hat{a}}(\hat{x}) = T^{\hat{a}}[\phi, X]$$

Set of local observables at each point $\hat{x}$

- Check that they are diff-invariant

$$\hat{\phi}_\xi^{\hat{a}} = T^{\hat{a}}[\phi_\xi, X_\xi] = T^{\hat{a}}[\phi_\xi, \xi^{-1} \circ X] = T^{\hat{a}}[\phi, X] = \hat{\phi}^{\hat{a}}$$

$$x^\mu = X^\mu(\hat{x})$$

$$X_\xi(\hat{x}) = (\xi^{-1} \circ X)(\hat{x})$$

$X(\hat{x})$ are not scalars

By composition!

Relational Observables

- Transformation under diffeomorphisms → transform the dynamical fields into the physical frame:

$$\hat{\phi}^{\hat{a}} = \phi^{\hat{a}}_X(\hat{x}) = T^{\hat{a}}[\phi, X]$$

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$$\hat{\phi}^{\hat{a}}_\xi = T^{\hat{a}}[\phi_\xi, X_\xi] = T^{\hat{a}}[\phi_\xi, \xi^{-1} \circ X] = T^{\hat{a}}[\phi, X] = \hat{\phi}^{\hat{a}}$$

By composition!

$$x^\mu = X^\mu(\hat{x})$$

$$X_\xi(\hat{x}) = (\xi^{-1} \circ X)(\hat{x})$$

$X(\hat{x})$ are not scalars

Relational observables

Now we can take any composite operator $\hat{A}[\phi]$

$$\hat{A}[\phi] = A_X[\phi] = A[\hat{\phi}]$$

Frame field: $e_\mu^{\hat{\mu}}(x) = \partial_\mu \hat{X}^{\hat{\mu}}(x)$

Invariant volume element: $\tilde{e} = \det e_\mu^{\hat{\mu}}$

$$\delta(X(\hat{x}), x) = \tilde{e}(x)\, \delta(\hat{x}, \hat{X}(x))$$

Product of $e_\mu^{\hat{\mu}}(x)$ and $e_{\hat{\mu}}^{\mu}(x)$ depending on the Relational Observable.
The indices I run over the rank of the tensor structure.

Relational Observable

$$\hat{A}^{\hat{I}}(\hat{x}) = \int \mathrm{d}^4x\, \tilde{e}(x)\delta(\hat{x}, \hat{X}(x)) E_I^{\hat{I}}(x) A^I(x)$$

Relational Observables - Examples

$$\delta(X(\hat{x}), x) = \tilde{e}(x)\, \delta(\hat{x}, \hat{X}(x))$$

Relational Ricci scalar

$$\hat{R}(\hat{x}) = R(X(\hat{x})) = \int d^d x\, \delta(X(\hat{x}), x)\, R(x) = \int d^d x\, \tilde{e}(x)\, \delta(\hat{x}, \hat{X}(x))\, R(x)$$

Relational inverse metric

$$\hat{g}^{\hat{\mu}\hat{\nu}}(\hat{x}) = e^{\hat{\mu}}_{\mu}(X(\hat{x}))e^{\hat{\nu}}_{\nu}(X(\hat{x}))g^{\mu\nu}(X(\hat{x})) = \int d^d x\, \delta(X(\hat{x}), x)\, g^{\mu\nu}(x)$$

$$= \int d^d x\, \tilde{e}(x)\, e^{\hat{\mu}}_{\mu}(X(\hat{x}))\, e^{\hat{\nu}}_{\nu}(X(\hat{x}))\, \delta(\hat{x}, \hat{X}(x))\, g^{\mu\nu}(x)$$

Relational Observables

Is there too much freedom?

Which physical coordinate system?

Which composite operator?

Relational Observables

Is there too much freedom?

Which physical coordinate system?

Which composite operator?

No observables!

Relational Observables

Is there too much freedom?

Which physical coordinate system?

Which composite operator?

~~No observables!~~

To many observables!

Asymptotic Safety

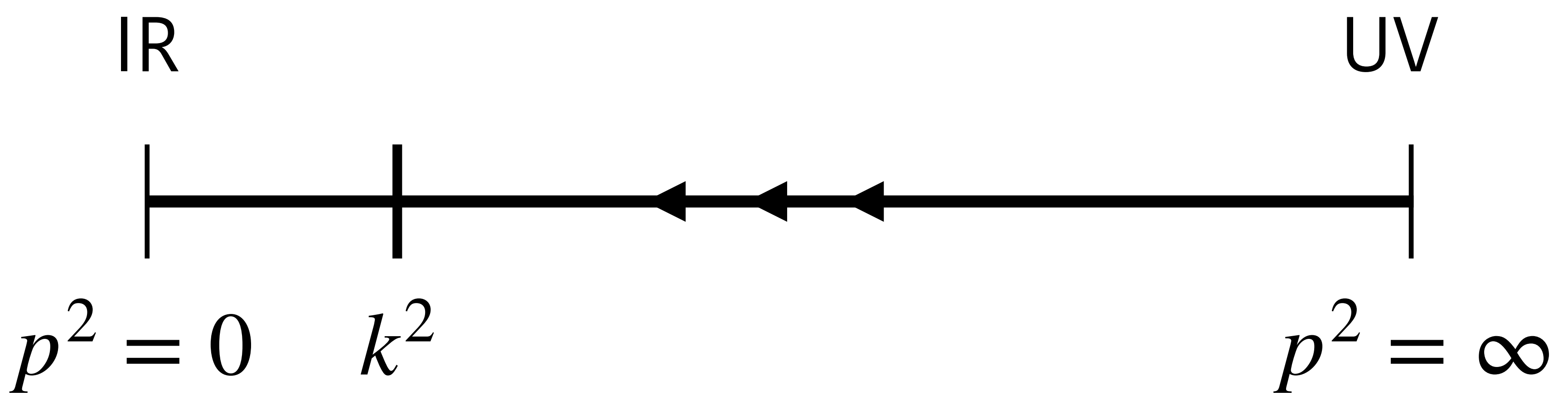
FRGE and Asymptotic Safety

Generating functional for connected Green's functions

$$e^{W[J]} = \int \mathscr{D}\varphi e^{-S[\varphi] + \int d^dxJ\varphi} \longrightarrow e^{W_k[J]} = \int \mathscr{D}\varphi e^{-S[\varphi] - \Delta S_k[\varphi] + \int d^dxJ\varphi}$$

Smooth cutoff functional

$$\Delta S_k[\phi] = \frac{1}{2}\int_x \phi(-x)\mathscr{R}_k(-\nabla^2)\phi(x)$$



$$\Gamma[\phi] = \int d^dxJ\phi - W[J] \longrightarrow \Gamma_k[\phi] = \int d^dxJ\phi - W_k[J] - \Delta S_k[\phi]$$

Effective Action

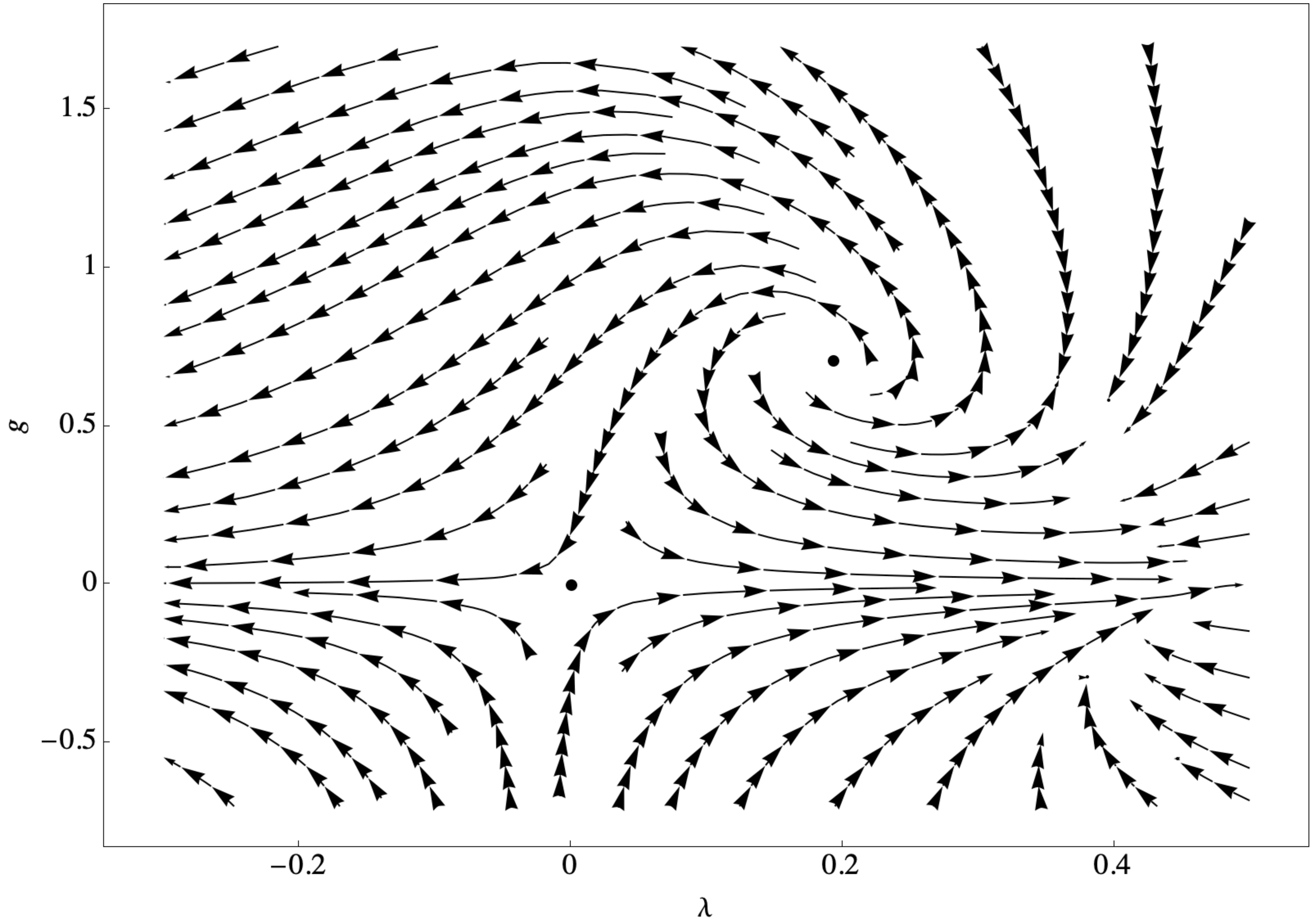
Effective Average Action

Functional Renormalization Group Equation

$$k\partial_k\Gamma_k[\varphi] = \frac{1}{2}\mathrm{Tr}\frac{k\partial_k\mathscr{R}_k}{\Gamma_k^{(2)} + \mathscr{R}_k}$$

FRGE and Asymptotic Safety

$$\Gamma_k = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N(k)}(R - 2\Lambda(k)) + \frac{1}{2}\delta_{AB}g^{\mu\nu}\partial_\mu\varphi^A\partial_\nu\varphi^B\right) + \text{Additional matter}$$



$$\begin{cases} k\partial_k g_k = \beta_g(g,\lambda) \\ k\partial_k \lambda_k = \beta_\lambda(g,\lambda) \end{cases}$$

$$\begin{cases} \beta_g(g_*,\lambda_*) = 0 \\ \beta_\lambda(g_*,\lambda_*) = 0 \end{cases}$$

$g_* = \lambda_* = 0$

Gaussian fixed point

$g_* > 0, \quad \lambda_* > 0$

Non-Gaussian fixed point

ASYMPTOTIC SAFETY

Composite Operator

Composite operator flow equation

Composite operator $\hat{\mathcal{O}}(x)$: function of the field and its derivatives

Expectation value of the observable

Source

$$\langle \hat{\mathcal{O}}(x) \rangle = -\frac{\delta}{\delta\varepsilon(x)} \int (d\hat{\chi})\ \mathrm{e}^{-S[\hat{\chi}] - \varepsilon \cdot \hat{\mathcal{O}}[\hat{\chi}]} \Bigg|_{\varepsilon=0}$$

Legendre transform + add regulator

FRG equation

$$\partial_t \Gamma_k[\phi, \varepsilon] = \frac{1}{2} \mathrm{Tr} \left[\left(\Gamma_{\mathrm{k}}^{(2,0)}[\phi, \varepsilon] + \mathscr{R}_{\mathrm{k}} \right)^{-1} \partial_{\mathrm{t}} \mathscr{R}_{\mathrm{k}} \right]$$

Regulator

Pawlowski (2007): arXiv:hep-th/0512261
Pagani, Reuter (2017): arXiv:1611.06522

Composite operator flow equation

Composite operator flow equation

Pagani equation

Regulator

$$\int d^d x\, \varepsilon\, \partial_t \mathcal{O}_k = -\frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}\left(\int \mathrm{d^d x}\, \varepsilon \mathcal{O}_k^{(2)}\right)\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}\partial_t \mathcal{R}_k\right]$$

Hessian of the composite operator

$$\lim_{k\to\infty} \mathcal{O}_k = \hat{\mathcal{O}}\,|_{\hat{\chi}\to\phi} \qquad \mathcal{O}_{k=0} = \langle \hat{\mathcal{O}} \rangle$$

Pagani, Reuter (2017): arXiv:1611.06522

Composite operator flow equation

Composite operator flow equation

Pagani equation

Regulator

$$\int d^d x \, \varepsilon \, \partial_t \mathcal{O}_k = -\frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}\left(\int d^d x \, \varepsilon \mathcal{O}_k^{(2)}\right)\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}\partial_t \mathcal{R}_k\right]$$

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$$\lim_{k\to\infty} \mathcal{O}_k = \hat{\mathcal{O}}\,|_{\hat{\chi}\to\phi}$$

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Solving the flow equation for the composite operator gives us a concrete method to compute expectation values of observables.

Pagani, Reuter (2017): arXiv:1611.06522

Composite operator flow equation

Expansion needed

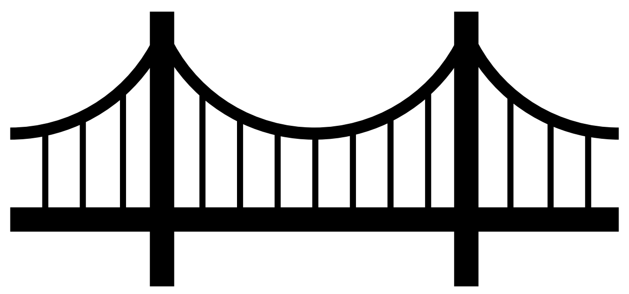
$$\mathcal{O}_k(x) = \sum_i a_i(k)\mathcal{O}_i(x)$$

Stability matrix

$$S_{ij} = \frac{\partial \beta_i}{\partial a_j} = d_i\delta_{ij} + \gamma_{ij}\,, \quad \partial_t a_j = \sum_i a_i \gamma_{ij}$$

$$-\frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}\left(d^dx\, \varepsilon(x)\mathcal{O}_i^{(2)}(x)\right)\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1}\partial_t\mathcal{R}_k\right] = \sum_j \gamma_{ij}\int d^dx\, \varepsilon(x)\mathcal{O}_j(x)$$

Matrix of scaling dimensions



Composite operator

$\hat{\mathcal{O}}(x)$

$\hat{=}$

Relational observable

$\hat{A}[\phi]$

Relational Effective Average Action

$$\mathcal{O}_k(\hat{x}) = \sum_i a_{\hat{I}}(k)\hat{A}_i^{\hat{I}}(\hat{x})$$

function of $\hat{x}$

$$\int \mathrm{d}^4\hat{x}\,\varepsilon(\hat{x})\mathcal{O}_k(\hat{x}) = \sum_i a_{\hat{I}_i}(k)\int \mathrm{d}^4\hat{x}\,\varepsilon(\hat{x})\hat{A}_i^{\hat{I}_i}(\hat{x}) = \int \mathrm{d}^4 x\,\tilde{e}(x)\,\varepsilon(\hat{X}(x))\underbrace{\sum_i a_{\hat{I}_i}(k)E_{iI_i}^{\hat{I}_i}(x)A_i^{I_i}(x)}_{\mathcal{L}_k^{\text{rel.}}(x)}$$

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integral over x with the volume element given by $\tilde{e}$ rather than the usual density $\sqrt{-g}$.

Relational Effective Average Action

$$\Gamma_k^{\text{rel.}} \equiv \int \mathrm{d}^4x\,\tilde{e}(x)\mathscr{L}_k^{\text{rel.}}(x) := \int \mathrm{d}^4x\,\tilde{e}(x)\sum_i a_{\hat{I}_i}(k)E_{iI_i}^{\hat{I}_i}(x)A_i^{I_i}(x)$$

Relational Effective Average Action

$$\mathcal{O}_k(\hat{x}) = \sum_i a_{\hat{I}}(k)\hat{A}_i^{\hat{I}}(\hat{x})$$

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Relational Effective Average Action	$\Gamma_k^{\text{rel.}} \equiv \int d^4x\,\tilde{e}(x)\mathcal{L}_k^{\text{rel.}}(x) := \int d^4x\,\tilde{e}(x)\sum_i a_{\hat{I}_i}(k)E_{iI_i}^{\hat{I}_i}(x)A_i^{I_i}(x)$
Flow of the Relational Effective Average Action	$\partial_t\Gamma_k^{\text{rel.}} = -\frac{1}{2}\text{Tr}\left[\left(\Gamma_{\text{k}}^{(2)}+\mathcal{R}_{\text{k}}\right)^{-1}\left(\Gamma_{\text{k}}^{\text{rel.(2)}}\right)\left(\Gamma_{\text{k}}^{(2)}+\mathcal{R}_{\text{k}}\right)^{-1}\partial_{\text{t}}\mathcal{R}_{\text{k}}\right]$

Hessian of the relational EEA

Flow of the Relational Effective Average Action

Distinguish between Γ_k and $\Gamma_k^{\text{rel.}}$

$$\Gamma_k[g,\dots;\varepsilon] = \Gamma_k[g,\dots] + \varepsilon\,\Gamma_k^{\text{rel.}}[g,\dots] + O(\varepsilon^2)$$

$$\boxed{\Gamma_k \equiv \int \mathrm{d}^4x \sqrt{-g(x)}\,\mathscr{L}_k(x)}$$

$$\partial_t \Gamma_k[\phi,\varepsilon] = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_{\mathrm{k}}^{(2,0)}[\phi,\varepsilon] + \mathscr{R}_{\mathrm{k}}\right)^{-1} \partial_{\mathrm{t}}\mathscr{R}_{\mathrm{k}}\right]$$

$$\boxed{\Gamma_k^{\text{rel.}} \equiv \int \mathrm{d}^4x\, \tilde{e}(x)\mathscr{L}_k^{\text{rel.}}(x)}$$

$$\partial_t \Gamma_k^{\text{rel.}} = -\frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_{\mathrm{k}}^{(2)} + \mathscr{R}_{\mathrm{k}}\right)^{-1}\left(\Gamma_{\mathrm{k}}^{\text{rel.}(2)}\right)\left(\Gamma_{\mathrm{k}}^{(2)} + \mathscr{R}_{\mathrm{k}}\right)^{-1} \partial_{\mathrm{t}}\mathscr{R}_{\mathrm{k}}\right]$$

Derivative expansion

Mixing and generation of new observables

$$\partial_t \Gamma_k^{\text{rel.}} = -\frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_{\mathrm{k}}^{(2)}+\mathscr{R}_{\mathrm{k}}\right)^{-1}\left(\Gamma_{\mathrm{k}}^{\text{rel.}(2)}\right)\left(\Gamma_{\mathrm{k}}^{(2)}+\mathscr{R}_{\mathrm{k}}\right)^{-1}\partial_{\mathrm{t}}\mathscr{R}_{\mathrm{k}}\right]$$

How to close the expansion?

Derivative expansion

Mixing and generation of new observables

$$\partial_t \Gamma_k^{\text{rel.}} = -\frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_{\mathrm{k}}^{(2)} + \mathscr{R}_{\mathrm{k}}\right)^{-1}\left(\Gamma_{\mathrm{k}}^{\text{rel.}(2)}\right)\left(\Gamma_{\mathrm{k}}^{(2)} + \mathscr{R}_{\mathrm{k}}\right)^{-1}\partial_{\mathrm{t}}\mathscr{R}_{\mathrm{k}}\right]$$

How to close the expansion?

Pick up terms with a finite number of derivatives acting on $\mathscr{L}_k^{\text{rel.}}(x)$

This means that the $\hat{X}^{\hat{\mu}}$'s only involve a finite number of derivatives.

$E_I^{\hat{I}}$ should be polynomial in derivatives

Derivative expansion

Mixing and generation of new observables

$$\partial_t \Gamma_k^{\text{rel.}} = -\frac{1}{2}\text{Tr}\left[\left(\Gamma_{\text{k}}^{(2)} + \mathscr{R}_{\text{k}}\right)^{-1}\left(\Gamma_{\text{k}}^{\text{rel.}(2)}\right)\left(\Gamma_{\text{k}}^{(2)} + \mathscr{R}_{\text{k}}\right)^{-1}\partial_{\text{t}}\mathscr{R}_{\text{k}}\right]$$

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$E_I^{\hat{I}}$ should be polynomial in derivatives

Natural basis $\hat{g}^{\mu\nu}, \quad \hat{R}^{\mu\nu}, \cdots$

only upper indices!

At each finite order in the derivative expansion, we do not find relational observables corresponding to tensors with lower indices.

First application

Inverse relational metric and relational curvature

$$\Gamma_k^{\text{rel.}} = \int \mathrm{d}^4\hat{x} \left(\alpha_0(k) + \alpha_R(k)\hat{R}(\hat{x}) + \alpha_1(k)\delta_{\hat{\mu}\hat{\nu}}\hat{g}^{\hat{\mu}\hat{\nu}}(\hat{x}) \right)$$

Topological term

Relational Ricci scalar

Relational inverse metric

$$\Gamma_k^{\text{rel.}} = \int \mathrm{d}^4x\, \tilde{e} \left(\alpha_0(k) + \alpha_R(k)R + \alpha_1(k)\delta_{\hat{\mu}\hat{\nu}} g^{\mu\nu}(\partial_\mu \hat{X}^{\hat{\mu}})(\partial_\nu \hat{X}^{\hat{\nu}}) \right)$$

Recipe

1 Find the Fixed Points

2 Identify the relational observables

Compute the flow of the relational observables

3 Evaluate scaling dimensions at Fixed Points

Recipe

1 Find the Fixed Points

$$\Gamma_k = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N(k)}(R - 2\Lambda(k)) + \frac{1}{2}\delta_{\mathrm{AB}}g^{\mu\nu}\partial_\mu\varphi^{\mathrm{A}}\partial_\nu\varphi^{\mathrm{B}}\right) + \dots$$

2 Identify the relational observables

$$\varphi^{\hat{\mu}} = \hat{X}^{\hat{\mu}}$$

Compute the flow of the relational observables

$$\Gamma_k^{\mathrm{rel.}} = \int \mathrm{d}^4x\,\tilde{e}\left(\alpha_0(k) + \alpha_R(k)R + \alpha_1(k)\delta_{\hat{\mu}\hat{\nu}}g^{\mu\nu}(\partial_\mu\hat{X}^{\hat{\mu}})(\partial_\nu\hat{X}^{\hat{\nu}})\right)$$

3 Evaluate scaling dimensions at Fixed Points

Results

Matter in Asymptotic Safe Gravity

1 Find the Fixed Points

$$\Gamma_k = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G_N(k)}(R - 2\Lambda(k)) + \frac{1}{2}\delta_{AB}g^{\mu\nu}\partial_\mu\varphi^A\partial_\nu\varphi^B\right) + \text{Additional matter}$$

Matter content	N_S	N_D	N_V
SM (type II)	4	45/2	12
SM (type I)	4	45/2	12
SM + SF (type II)	5	45/2	12
SM + 3 ν (type II)	4	24	12

$\lambda_\star$	$g_\star$
-1.11626	0.834855
-3.58874	2.59505
-3.79874	2.77608
-4.83385	3.19355

Asymptotic Safety

Results

2 Compute the flow of the observables

$$\Gamma_k^{\text{rel.}} = \int \mathrm{d}^4x\, \tilde{e} \left(\alpha_0(k) + \alpha_R(k) R + \alpha_1(k) \delta_{\hat{\mu}\hat{\nu}} g^{\mu\nu} (\partial_\mu \hat{X}^{\hat{\mu}})(\partial_\nu \hat{X}^{\hat{\nu}}) \right)$$

$$S = \begin{pmatrix} 4 & \frac{g(\eta_\phi - 4)}{\pi(1-2\lambda)^2} & -\frac{1}{6\pi^2} \\ 0 & 6 + \gamma_g + \frac{g(10\eta_\phi\lambda + 4\eta_\phi - 30\lambda - 21)}{9\pi(2\lambda-1)^3} & -\frac{1}{24\pi^2} \\ 0 & -\frac{g^2(\eta_\phi - 4)}{2(2\lambda-1)^3} & 8 + \gamma_g + \frac{g(\eta_\phi - 3)}{6\pi(1-2\lambda)^2} - \frac{5g}{24\pi} \end{pmatrix}$$

Asymptotic Safety

Results

2 Compute the flow of the observables

$$\Gamma_k^{\text{rel.}} = \int \mathrm{d}^4x\, \tilde{e} \left(\alpha_0(k) + \alpha_R(k) R + \alpha_1(k) \delta_{\hat{\mu}\hat{\nu}} g^{\mu\nu} (\partial_\mu \hat{X}^{\hat{\mu}})(\partial_\nu \hat{X}^{\hat{\nu}}) \right)$$

$$S = \begin{pmatrix} 4 & \frac{g(\eta_\phi - 4)}{\pi(1-2\lambda)^2} & -\frac{1}{6\pi^2} \\ 0 & 6 + \gamma_g + \frac{g(10\eta_\phi\lambda + 4\eta_\phi - 30\lambda - 21)}{9\pi(2\lambda-1)^3} & -\frac{1}{24\pi^2} \\ 0 & -\frac{g^2(\eta_\phi - 4)}{2(2\lambda-1)^3} & 8 + \gamma_g + \frac{g(\eta_\phi - 3)}{6\pi(1-2\lambda)^2} - \frac{5g}{24\pi} \end{pmatrix}$$

$$\theta_0 = -4 \qquad \theta_R = -6 \qquad \theta_1 = -8$$

Gaussian Fixed Point

The critical exponents reduce to the canonical dimensions of the corresponding operators.

Results

Matter in Asymptotic Safe Gravity

3 Evaluate scaling dimensions at Fixed Points

$$\Gamma_k^{\text{rel.}} = \int \mathrm{d}^4x\, \tilde{e} \left(\alpha_0(k) + \alpha_R(k) R + \alpha_1(k) \delta_{\hat{\mu}\hat{\nu}} g^{\mu\nu} (\partial_\mu \hat{X}^{\hat{\mu}})(\partial_\nu \hat{X}^{\hat{\nu}}) \right)$$

Non-Gaussian Fixed Point

Matter content	θ_0	θ_R	θ_1
SM (type II)	-4	-5.97643	-7.92358
SM (type I)	-4	-5.97467	-7.8177
SM + SF (type II)	-4	-5.97505	-7.80603
SM + 3 ν (type II)	-4	-5.98015	-7.78084

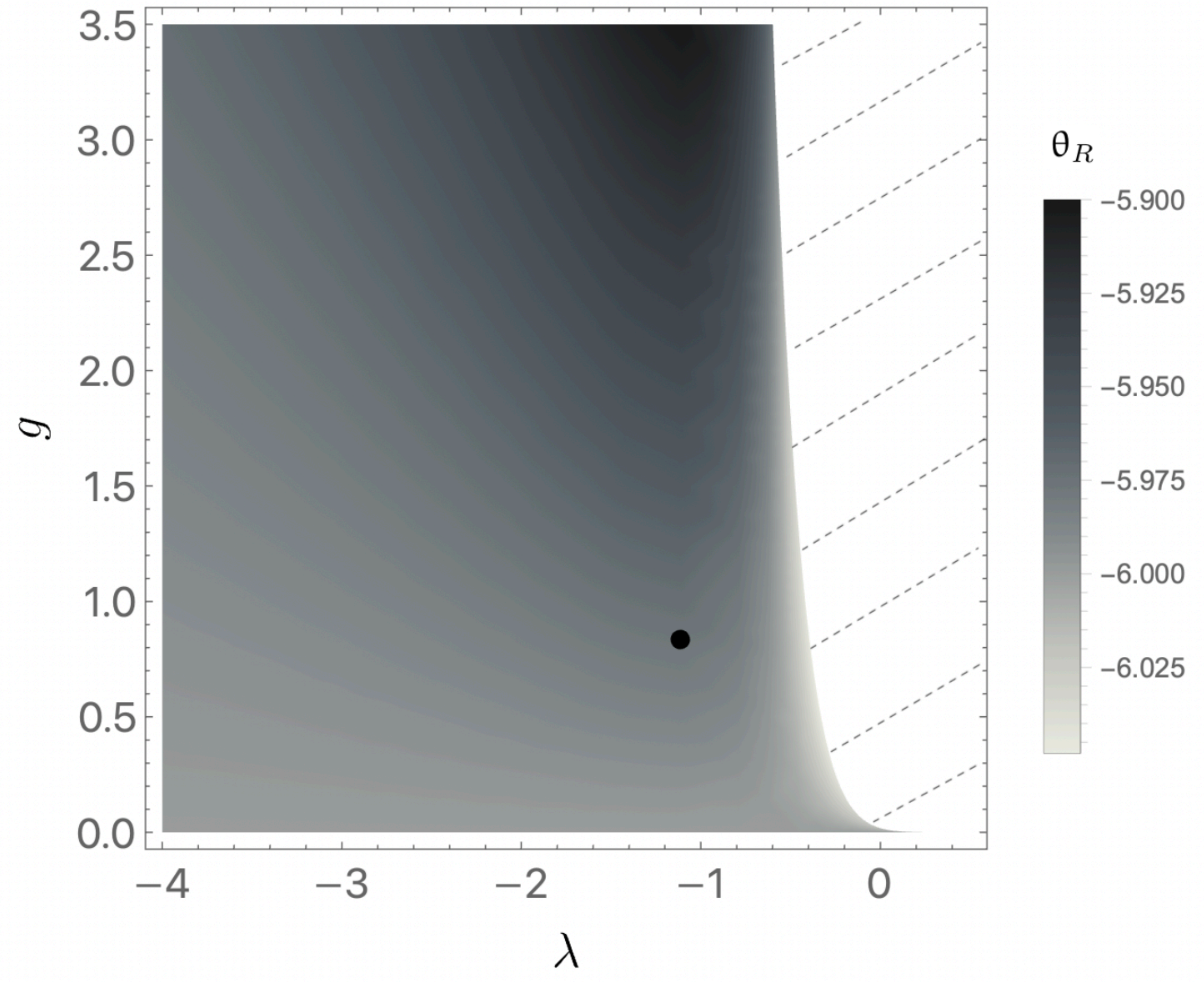
Small quantum corrections

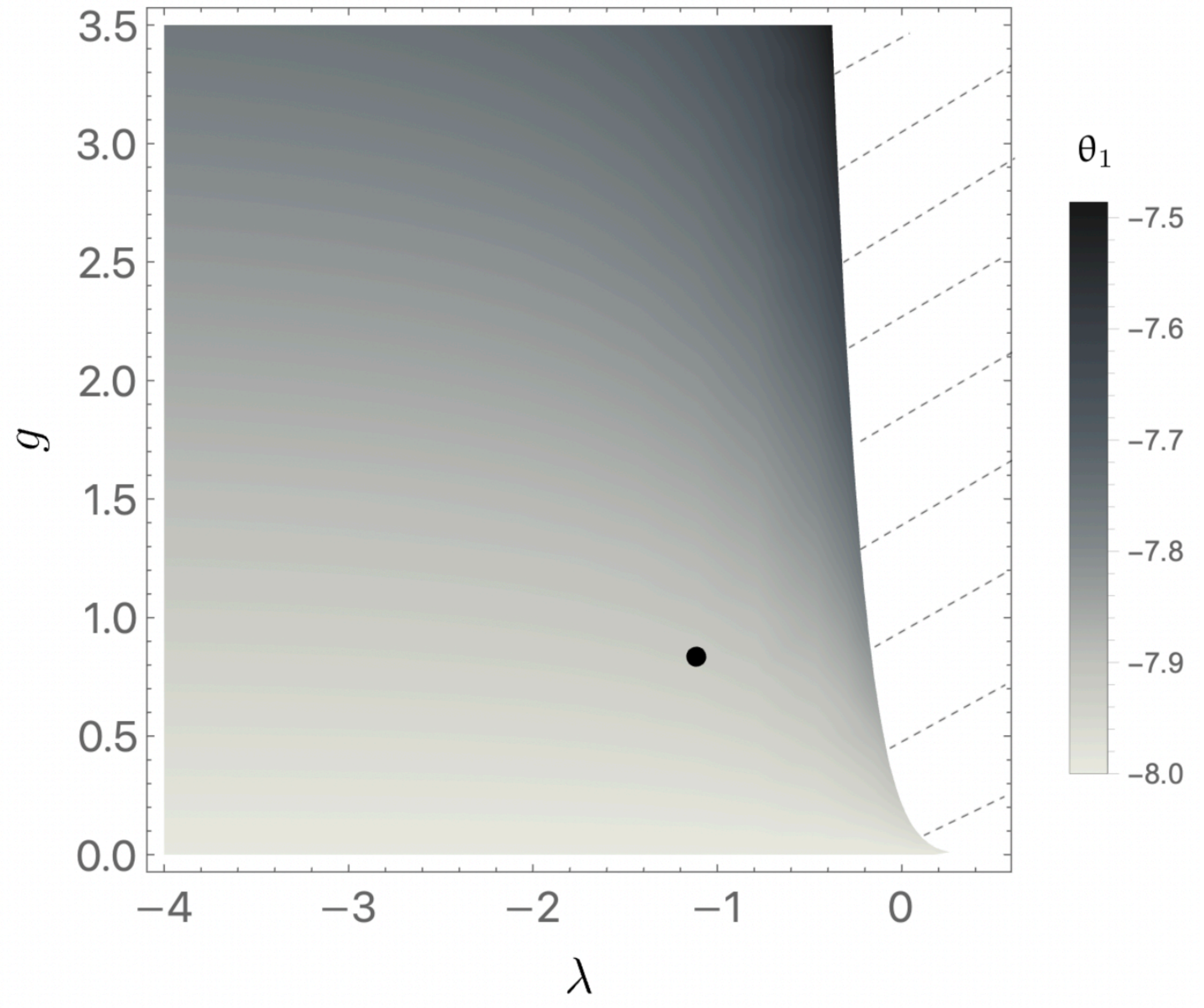
More relevant

Asymptotic Safety

Results

$$\Gamma_k^{\text{rel.}} = \int \mathrm{d}^4x\, \tilde{e} \left(\alpha_0(k) + \alpha_R(k) R + \alpha_1(k) \delta_{\hat{\mu}\hat{\nu}} g^{\mu\nu} (\partial_\mu \hat{X}^{\hat{\mu}})(\partial_\nu \hat{X}^{\hat{\nu}}) \right)$$

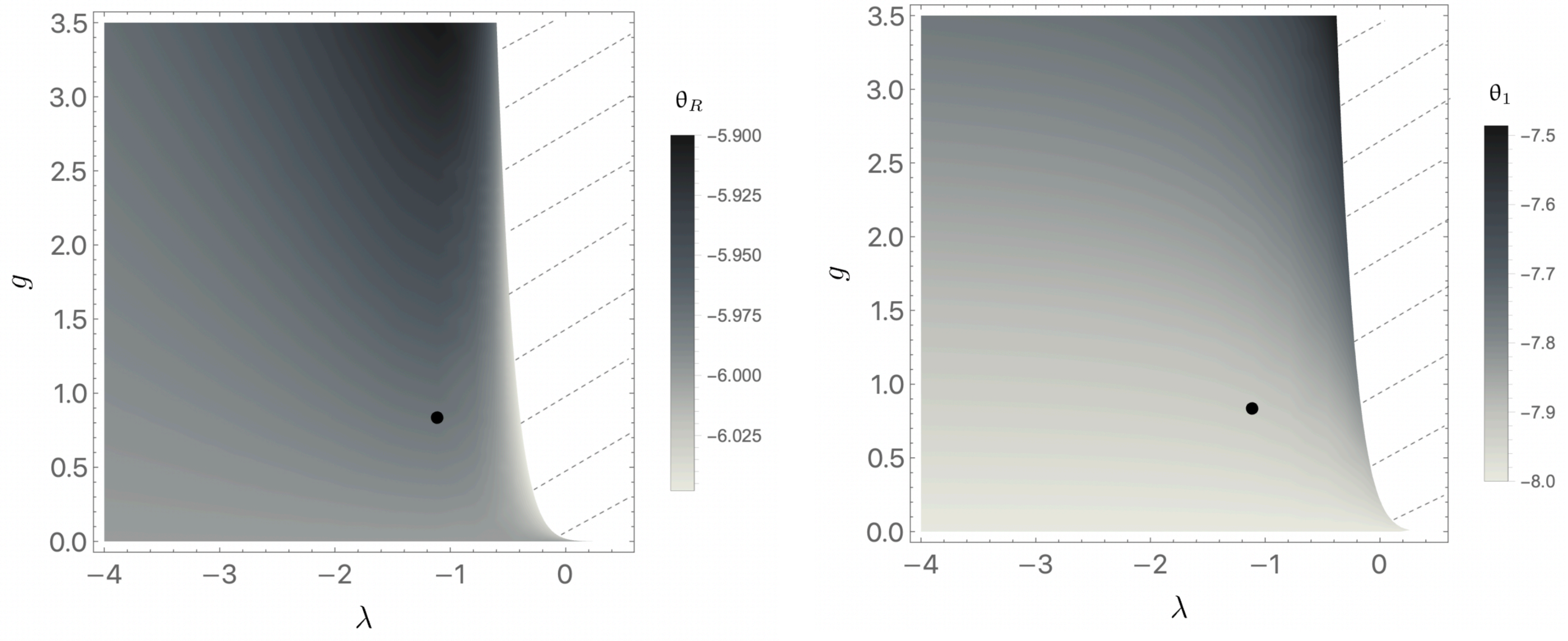




Asymptotic Safety

Results

$$\Gamma_k^{\text{rel.}} = \int d^4x\, \tilde{e} \left(\alpha_0(k) + \alpha_R(k) R + \alpha_1(k) \delta_{\hat{\mu}\hat{\nu}} g^{\mu\nu} (\partial_\mu \hat{X}^{\hat{\mu}})(\partial_\nu \hat{X}^{\hat{\nu}}) \right)$$



Related to the universal scaling of the observables.

Open questions

Contact with different approaches of quantum gravity? CDT, Tensor Field Theory

Meaning of the critical exponents

Scaling of correlation functions?

Fixed points: scale invariance!

?

Which are good observables?

Is there a good truncation?

Suitable physical coordinate system

Locality - microcausality?

Relational locality/
Relational microcausality

The RG flow is furnishing a natural set of subsystems?

Relational Observables in Asymptotically Safe Gravity

Thank you
for your attention.

Quantum gravity, hydrodynamics and emergent cosmology
LMU München, Dec 9th 2022

JOHANNES GUTENBERG UNIVERSITÄT MAINZ