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ScienceDirect
Physica E 40 (2008) 1142–1144
PHYSICA E
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# Local current distribution at large quantum dots (QDs): A self-consistent screening model

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Available online 7 September 2007

**Abstract**

We report the implementation of the self-consistent Thomas–Fermi screening theory, together with the local Ohm's law to a quantum dot system in order to obtain local current distribution within the dot and at the leads. We consider a large dot (size $>700$ nm) defined by split gates, and coupled to the leads. Numerical calculations show that the non-dissipative current is confined to the incompressible strips. Due to the non-linear screening properties of the 2DES at low temperatures, this distribution is highly sensitive to external magnetic field. Our findings support the phenomenological models provided by the experimental studies so far, where the formation of the (direct) edge channels dominate the transport.
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*PACS:* 73.20.Dx; 73.40.Hm; 73.50.−h; 73.61.−r

*Keywords:* Edge states; Quantum Hall effect; Screening; Quantum dots

## 1. Introduction

For the last two decades, the electron transport through quantum dots (QDs) has been a central question. A wide range of different approaches, including independent electron picture, constant interaction model and scattering matrix theory, have provided mechanisms for explaining the transport properties of QDs. The QDs are constructed within a two-dimensional electron system (2DES), by (split-) gates and/or chemical etching. In the presence of a strong perpendicular magnetic field, it has been shown that semi-metallic (compressible) and semi-insulating (incompressible) regions are formed due to Coulomb interaction [1]. Despite some limited quantum mechanical treatments, a microscopic model describing the current distribution is still not available for large QDs ($d>700$ nm) [2]. In this work we implement the self-consistent (SC) Thomas–Fermi (TF) theory of screening [3–5] together with the local version of the Ohm's law [4,5] to obtain the local current distribution inside the QD and the leads. We use a conductivity model [4,6] based on Gaussian broadened density of states (following Ref. [7]). Our model calculations show that the non-dissipative current is, in fact, confined to the incompressible strips. Due to the non-linear screening properties of the 2DES at low temperatures, this distribution is highly sensitive to the external magnetic field and our findings support the phenomenological models provided by the experimental groups [2].

## 2. Model

Our calculation procedure starts with generating the (external) potential $V_{\rm ext}(\mathbf{r})$ landscape from the metallic surface gates, which are kept at the potential $V_{\rm g}$, where $\mathbf{r}=(x,y)$. The calculation of $V_{\rm ext}(\mathbf{r})$ is based on the solution of the 2D Laplace's equation [6,8,9]. The screened potential at zero field and zero temperature is obtained by

$$V_{\rm scr}(\mathbf{r}) = F^{-1}[F[V_{\rm ext}(\mathbf{r})]/\varepsilon(q)], \quad (1)$$

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doi:10.1016/j.physe.2007.08.055

where $F$ presents the Fourier transformation, $\varepsilon(q)$ is the TF dielectric function, $\varepsilon(q) = 1 + 2(me^2)/(\bar{\kappa}\hbar^2|q|)$, and $\bar{\kappa}$ is the static dielectric constant ($\sim$12.4 for GaAs). Including a magnetic ($B$) field perpendicular to the 2DES, one has to solve the Poisson's equation for the given initial potential and electron distribution in a SC way, that is

$$n_{\rm el}(\mathbf{r}) = \int \mathrm{d}E\, D(E) f([E + V(\mathbf{r}) - \mu^\star(\mathbf{r})]/k_{\rm B}T) \tag{2}$$

and

$$V(\mathbf{r}) = V_{\rm ext}(\mathbf{r}) + \frac{2e^2}{\bar{\kappa}} \int_A \mathrm{d}\mathbf{r}'\, K(\mathbf{r},\mathbf{r}')\, n_{\rm el}(\mathbf{r}'). \tag{3}$$

Here, $f(\xi)$ is the Fermi function, $\mu^\star(\mathbf{r})$ position dependent chemical potential, $T$ temperature, $D(E)$ is the Gaussian broadened Landau density of states (DOS), $V(\mathbf{r})$ the total potential energy, $K(\mathbf{r},\mathbf{r}')$ is the Poisson kernel satisfying periodic boundary conditions. For zero external current, $\mu^\star(\mathbf{r})$ is a constant at the thermal equilibrium, otherwise modified by the driving electric field as

$$E(\mathbf{r}) = \nabla\mu^\star(\mathbf{r})/e = \hat{\rho}(\mathbf{r}).j(\mathbf{r}), \tag{4}$$

for a given resistivity tensor $\hat{\rho}(\mathbf{r})$ and current density $j(\mathbf{r})$. We calculate the conductivity using the Gaussian broadened DOS given by

$$D(E) = \frac{1}{2\pi l^2} \sum_{n=0}^{\infty} \frac{\exp(-[E_n - E]^2/\Gamma^2)}{\sqrt{\pi}\,\Gamma}, \tag{5}$$

where $\Gamma$ is the impurity parameter yielding the Landau level (LL) broadening. The Landau energy is given by $E_n = \hbar\omega_c(n + 1/2) = E_{\rm F}\Omega(n + 1/2)$ where $E_{\rm F}$ is the Fermi energy. Beyond the linear response (i.e. when the current is large enough to modify $\mu^\star(\mathbf{r})$), one has to insert the modified chemical potential into Eq. (2) and repeat the SC calculation until convergence is achieved.

## 3. Results and discussion

We define the QD by split gates (dark (blue) regions in Fig. 1a) on the surface of the GaAs/AlGaAs heterostructure, 85 nm above the 2DES and considering a unit cell of size $2100 \times 2100\,\mathrm{nm}^2$, with an average Fermi energy 12.75 meV, corresponding a bulk electron density $3 \times 10^{11}\,\mathrm{cm}^{-2}$ (indicated by the thick-dashed (red) contour line). The metallic gates are biased (negatively) such that no electrons can reside below. The QD has a rectangular shape ($\sim 1200 \times 600\,\mathrm{nm}^2$), whereas the openings are approximately 300 nm. Furthermore, the QD is coupled electrically to the leads, where the capacitive and tunneling effects are negligible. In Fig. 1, we plot the bare confinement (b) and screened (c) potentials obtained from Eq. (1). Following the contour lines, one observes that $V_{\rm ext}(\mathbf{r})$ is smooth, i.e. there are no potential variations within and near the QD, whereas $V_{\rm scr}(\mathbf{r})$ exhibits a local extremum, due to the strong non-linear screening (for a recent review see Ref. [8]). Two interesting potential cross-sections are highlighted in Fig. 1d, indicated by horizontal lines (solid, depicting the bare and dashed the screened potential at the opening and center of the QD) shown in the contour plots. We observe that, the screened potential is suppressed compared to the bare potential at the leads of the QD, implying that more states are allowed to pass through the barrier. More interestingly, depending on the dot size, a local maximum develops at the very center of the QD surrounded by a minimum close to the edges where the high $q$ components dominate the screening. It is clear that, if the dot is smaller

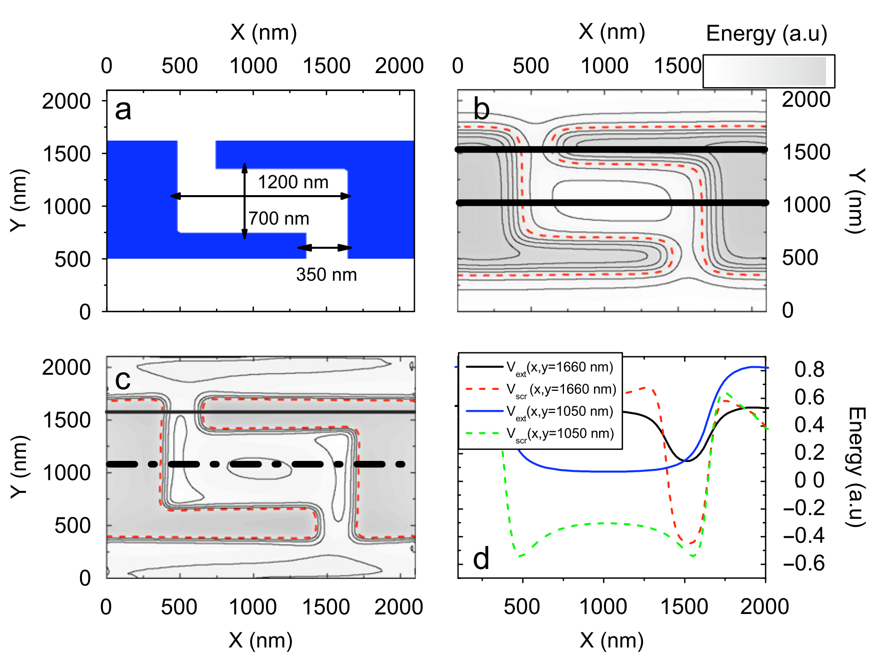


Fig. 1. (a) The split gate defined Qdot. Equipotential lines of the bare confinement (b) and screened potential (c). Cross-section of the potential profiles at the opening and at the center of the dot (d).

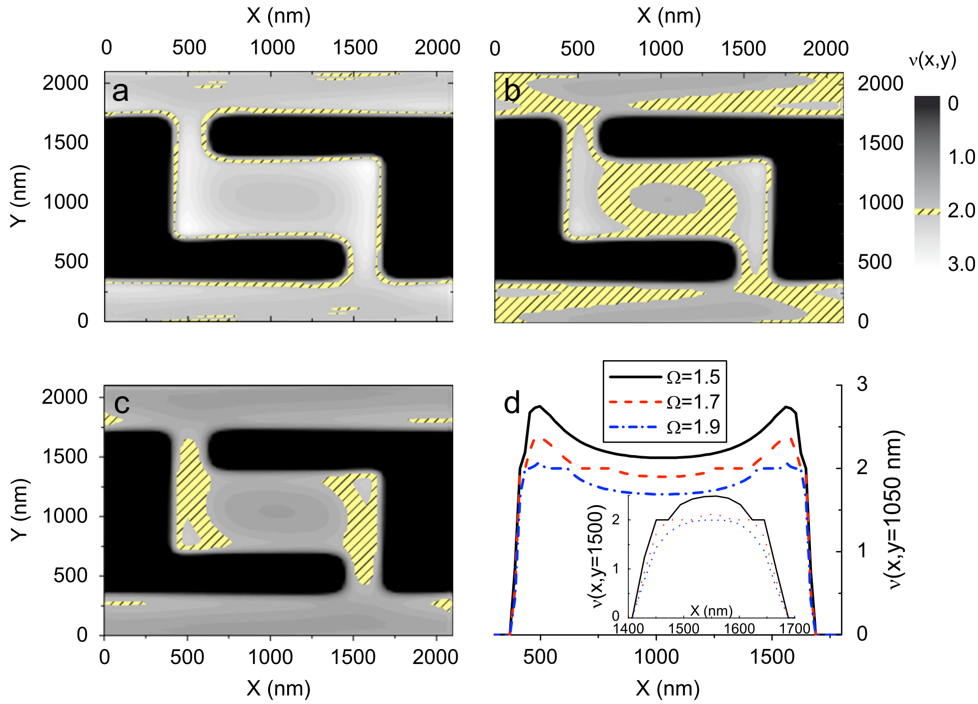


Fig. 2. (a–c) The gray scale plot of $\nu(x,y)$ for three $B$ values calculated at $kT/\hbar\omega_c = \frac{1}{40}$. (d) Density cross-section again at the center (plot) and at the opening (inset).

such potential inhomogeneities will disappear, similar to what happens at the opening, however the dot will become more confining compared to the non-interacting models. We should also note that, if the 2DES is buried deeper, due to exponential decay of the short range oscillations, such local minima will not be seen even considering smaller dot sizes. We continue our discussion of screening now also considering an external $B$ field. In Fig. 2, we show a sequence of local filling factors while changing the $B$ field. At the highest $B$ only the lowest LL is occupied therefore the system is compressible almost everywhere except near the gates, where local minimum is observed (see Fig. 2d). In this situation, we see that the current is distributed all over the sample, similar to a metal. From the edge state picture point of view, a direct channel already exists and conduction is quantized. Lowering the magnetic field results in the formation of incompressible regions where $E_F$ falls in between two LLs. We observe that, an incompressible ring is formed within the dot which is connected to the leads again by incompressible strips. Since, the current flows within the incompressible edge states, this is the most interesting case, due to the opening of a direct channel, which is coupled to a compressible lake inside the dot, separated by an incompressible region. At the lowest $B$ shown in figure, we see that the center of the dot becomes compressible surrounded by incompressible regions, due to the local minimum near the gates. The leads remain compressible all over, therefore act as a metal and current is directly proportional to the local electron density. To conclude, we have provided an explicit calculation of the spatial distribution of the incompressible edge states considering a split gate defined (large) QD. We have shown that depending on the sample geometry and $B$ field applied, a direct channel can emerge, connecting the source to the drain. More interestingly, a dot-in-dot structure is obtained for a certain range of parameters.

## Acknowledgments

The authors acknowledge the support of the Marmaris Institute of Theoretical and Applied Physics (ITAP), TUBITAK Grant 105T110, SFB631 and DIP.

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