

Asymptotic von Neumann measurement strategy for solid-state qubits

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(Received 19 May 2003; published 19 August 2003)

A measurement on a macroscopic quantum system does not, in general, lead to a projection of the wave function in the basis of the detector as predicted by von Neumann's postulate. Hence, it is a question of fundamental interest, how the preferred basis onto which the state is projected is selected out of the macroscopic Hilbert space of the system. Detector-dominated von Neumann measurements are also desirable for both quantum computation and verification of quantum mechanics on a macroscopic scale. The connection of these questions to the predictions of the spin-boson model is outlined. I propose a measurement strategy, which uses the entanglement of the qubit with a weakly damped harmonic oscillator. It is shown that the degree of entanglement controls the degree of renormalization of the qubit and identify that this is equivalent to the degree to which the measurement is detector dominated. This measurement very rapidly decoheres the initial state, but the thermalization is slow. The implementation in Josephson quantum bits is described and it is shown that this strategy also has practical advantages for the experimental realization.

DOI: 10.1103/PhysRevB.68.060503

PACS number(s): 03.67.Lx, 03.65.Ta, 03.65.Yz, 74.50.+r

The field of quantum computation¹ has been experimentally pioneered in quantum optics, atomic physics, and nuclear magnetic resonance (NMR). In these quantum-mechanical systems with few degrees of freedom and strong quantum coherence, the measurement devices ("meters") are well described and can be classified into two types. In atomic physics, e.g., "strong" measurements can be performed, which satisfy von Neumann's measurement postulate,² i.e., the state of the system is projected onto the eigenstate of the *meter* corresponding to the measurement result. In NMR, on the other hand, the meter couples weakly to each individual spin and decoheres it only weakly. In order to still obtain enough signal and information, the measurement is performed on an ensemble of qubits.

These qubits cannot be easily integrated to large-scale circuits. Thus, solid-state qubits, which can be lithographically manufactured, are a promising alternative. Solid-state systems consist of many degrees of freedom, hence quantum coherence can so far only be maintained over very short times.^{3,4} It was proposed that superconducting Josephson circuits in the charge^{3,5} or flux^{4,6} regime could act as solid-state qubits with appreciable coherence times. In these cases, the measurement apparatus is permanently close to the qubit, although the interaction may effectively be switched off.^{7,8} The measurement process in this system can be described within the spin-boson^{9,10} or related models.^{7,11,12}

From a density-matrix description, we can obtain detailed (although incomplete) information about the dynamics of the measurement: After a dephasing time τ_ϕ , the density matrix is brought into an incoherent mixture, and after the relaxation time τ_r it thermalizes and the information about the initial state is lost.⁷ In order to render τ_r long enough, usually³⁻⁵ the meter is only weakly coupled to the qubit. This makes it necessary to ensemble average by repeating the measurement. Theoretical research^{7,11,12} shows that an optimization of these weak measurements allows for single-shot measurements without averaging, by waiting longer than the dephasing time. These are *optimized weak measurements* or *qubit dominated* measurements: They completely decohere

the state of the qubit, however, the final state is *not* an eigenstate of the measured observable, but of the qubit. Qubit and apparatus do *not* get strongly entangled. It has also been shown theoretically,⁷ that detector-dominated *strong* measurements of superconducting qubits are possible, on the expense of τ_R being very short, which sets a strong experimental challenge. It is a fundamental question, under which conditions a measurement performed on a potentially macroscopic object follows the postulates of quantum mechanics and how, in general, the preferred observable basis is selected out of the large Hilbert space of the system and the detector.¹³ This question should be addressed using specific models which describe actual detectors. Moreover, there are practical issues: (i) The theoretical signal-to-noise ratio of a weak measurement is limited to 4 (Ref. 12), (ii) and Efficient quantum algorithms such as error correction¹⁴ or the test of Bell-type inequalities¹⁵ rely on strong measurements.

In this paper, I am going to connect the abstract notions of quantum measurement theory to the concepts of the spin-boson model, in particular, the issue of entanglement will be connected to scaling of the tunnel matrix element. I will outline a method how to perform genuine detector-dominated measurements in this context.

For definiteness, it is assumed that the variable of the quantum bit which is measured is described by the pseudospin operator $\hat{\sigma}_z$. When the measurement apparatus is coupled to the qubit, the same term experiences a fluctuating force, which is assumed to be Gaussian and be modeled by a bath of harmonic oscillators. Consequently, we end up with the spin-boson Hamiltonian.^{9,16} After integrating out high frequencies, its pseudospin part reads

$$\hat{H}_{\text{eff}} = \hbar \left(\frac{\epsilon}{2} \hat{\sigma}_z + \frac{\Delta_{\text{eff}}}{2} \hat{\sigma}_x \right), \quad (1)$$

where the off-diagonal term Δ_{eff} is in general rescaled due to the environment as compared to the original splitting Δ of an isolated qubit. The spin-boson model generally predicts¹⁶ the dynamics described in the preceding section. In particular,

after the dephasing time, the density matrix is projected onto a mixture of *eigenstates of H_{eff}* . Usually, in the weak-coupling regime,⁹ Δ_{eff} is close to the bare Δ of the qubit and consequently the eigenstates of H_{eff} are far from being eigenstates of $\hat{\sigma}_z$. Consequently, the state of the qubit will not be projected onto the measured variable, i.e., the measurement is qubit dominated. A detector-dominated measurement would be realized for $\Delta_{\text{eff}}=0$,¹⁷ i.e., when $\hat{H}_{\text{eff}}=(\epsilon/2)\hat{\sigma}_z$ and hence commutes with the coupling to the meter. A number of schemes allow to directly suppress Δ using an external control parameter.^{5,6,18} In what follows, I want to describe, using a generic model, how this is accomplished by the measurement apparatus itself, in agreement with the usual understanding of quantum measurements.

Consider a qubit coupled to a single (big) harmonic oscillator, which experiences linear friction, which is in turn described quantum mechanically through a bath of oscillators. The Hamiltonian of this system then reads

$$\hat{H} = \hbar \left(\frac{\epsilon}{2} \hat{\sigma}_z + \frac{\Delta}{2} \hat{\sigma}_x \right) + \frac{\hat{P}^2}{2M} + \frac{M}{2} \Omega^2 (\hat{X} - q \hat{\sigma}_z)^2 + \sum_i \left(\frac{\hat{p}_i^2}{2m_i} + \frac{m_i}{2} \omega_i^2 (\hat{x}_i - (c_i/m_i \omega_i^2) \hat{X})^2 \right), \quad (2)$$

where the displacement q characterizes the coupling of the qubit to the big oscillator. The oscillator bath is characterized through an ohmic spectral density $J(\omega) = \sum (\pi c_i^2 / (2m_i \omega_i)) \delta(\omega - \omega_i) = M\Gamma\omega$, where $\Gamma/2$ is the friction coefficient of the damped big oscillator. It was shown¹⁹ that this system is equivalent to the spin-boson model, with an effective spectral density

$$J_{\text{eff}}(\omega) = 2\pi\alpha\omega \frac{\Omega^4}{(\omega^2 - \Omega^2)^2 + 4\Gamma^2\omega^2}, \quad (3)$$

where $\alpha = 2Mq^2\Gamma/h$ is a dimensionless dissipation coefficient, which here is assumed to be small, $\alpha \ll 1$. From now on, we want to concentrate on the case $\epsilon, \Delta \ll \Omega$.

At $\Gamma=0$, the low-energy Hilbert space is spanned by $|\pm\rangle_{\text{eff}} = |\pm\rangle|L/R\rangle$ where $|\pm\rangle$ are the basis states of the qubit, $\sigma_z|\pm\rangle = \pm|\pm\rangle$, and $|L/R\rangle$ are coherent states of the harmonic oscillator centered around $X = \pm q$, see Fig. 1 (left). So in a general low-energy state $|\psi\rangle = a|+\rangle_{\text{eff}} + b|-\rangle_{\text{eff}}$, $|a|^2 + |b|^2 = 1$, qubit and oscillator are entangled²⁰ and the oscillator states are pointers onto the qubit states.¹³ In this low-energy basis, the Hamiltonian acquires form (1), with $\Delta_{\text{eff}} = \Delta\langle L|R\rangle = \Delta e^{-\eta}$ where $\eta = M\Omega q^2/\hbar$. Under an appropriate choice of parameters, we can achieve $\eta > 1$ and $\Delta_{\text{eff}} \ll \Delta$. This choice corresponds to the condition of almost [meaning here and henceforth “up to an error of $O(e^{-\eta})$ ”] *orthogonal* states in the environment, which has been identified as the condition for an ideal von Neumann measurement.^{13,21}

For finite Γ , this system can be analyzed using adiabatic renormalization.^{9,22} One finds²³ $\Delta_{\text{eff,damp}}(\alpha) = \Delta e^{-\eta/(1-\alpha)} (\Delta/\Omega)^{\alpha/(1-\alpha)}$. Thus, finite dissipation $0 < \alpha \ll 1$ scales Δ down even slightly further.

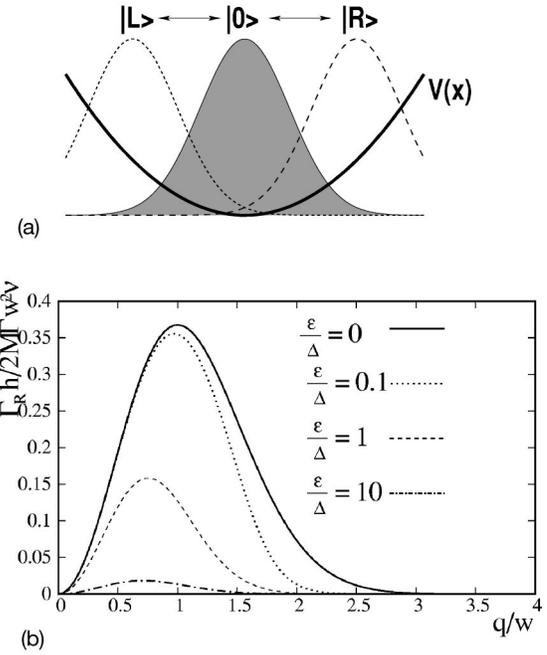


FIG. 1. Left: Visualization of the ground state $|0\rangle$ and the coherent pointer states $|L\rangle$ and $|R\rangle$ of the oscillator in the potential $V(x)$; right: Relaxation rates as a function of the coupling q/w for different energy biases. w is the width of the ground-state wave function of the pointer, $w = \sqrt{\hbar/M\Omega}$.

The coherence properties of our system can at $\epsilon, \Delta, T \ll \Omega$ be studied using a systematic weak damping approximation¹⁶ of the spin-boson model. The relaxation and dephasing rates $\Gamma_{r/\phi} = \tau_{r/\phi}^{-1}$ are given by

$$\Gamma_r = \pi\alpha \frac{\Delta_{\text{eff}}^2}{\nu_{\text{eff}}} \coth\left(\frac{\nu}{2T}\right), \quad \Gamma_\phi = \frac{\Gamma_r}{2} + 2\pi\alpha k_B \frac{\epsilon^2}{\nu_{\text{eff}}} T/\hbar, \quad (4)$$

where $\nu_{\text{eff}} = \sqrt{\Delta_{\text{eff}}^2 + \epsilon^2}$. In our case, if $\eta > 1$, Δ_{eff} is exponentially reduced compared to Δ , transitions between the basis states are suppressed leaving relaxation very slow, i.e., the state becomes almost localized or “frozen,” see Fig. 1 (right). The second contribution to Γ_ϕ in Eq. (4) reflects dephasing processes which do not change the qubit energy and are consequently not frozen.

The use of a weak damping approximation for $\Gamma_{r,\phi}$ is appropriate, although $J(\omega)$ can be large at the peak and in fact the down scaling of Δ_{eff} is essentially a nonperturbative effect. However, decoherence is mostly probing the $J_{\text{eff}}(\omega)$ around $\omega = \nu_{\text{eff}} \ll \Omega$, where the weak damping condition holds. This is supported by two observations: i) if we project the full Hamiltonian onto its low-energy Hilbert space spanned by $|\pm\rangle_{\text{eff}}$, we find an effective ohmic model leading to Eq. 4; (ii) a full nonperturbative calculation²³ based on the noninteracting blip approximation (NIBA)⁹ reproduces both the scaling and Γ_r within the known¹⁶ limitations of NIBA.

The measurement can now be performed as follows: As a first step, q is adiabatically ramped from $q=0$ to a finite q_0 where $\eta > 1$ and $\Delta \rightarrow \Delta_{\text{eff}} \ll \Delta$. The adiabatic theorem pre-

dicts, that the state of the system evolves as $(\alpha|+\rangle + \beta|-\rangle) \otimes |0\rangle \rightarrow (\alpha^{\text{eff}}|+\rangle|L\rangle + \beta^{\text{eff}}|-\rangle|R\rangle)$, where

$$\begin{pmatrix} \alpha^{\text{eff}} \\ \beta^{\text{eff}} \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{\theta^{\text{eff}} - \theta}{2}\right) & \sin\left(\frac{\theta^{\text{eff}} - \theta}{2}\right) \\ -\sin\left(\frac{\theta^{\text{eff}} - \theta}{2}\right) & \cos\left(\frac{\theta^{\text{eff}} - \theta}{2}\right) \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad (5)$$

and $\tan \theta^{\text{(eff)}} = \epsilon / \Delta^{\text{(eff)}}$. The condition for adiabaticity is $dq/dt \ll v_{\text{eff}}^2 / 2qM\Omega\Delta_{\text{eff}}$, i.e., for small q the ramping can be very fast.

When $\eta > 1$, the matrix element is scaled down and the state is ‘‘premeasured’’ by entanglement with well-separated pointer states L and R .¹³ Only now, we start the measurement, by coupling the oscillator to the meter and decohering the state is projected onto the eigenstates of \hat{H}_{eff} , which are close to the ones of $\hat{\sigma}_z$. We can then read off the position of the big oscillator serving as a pointer and switch off the meter (or q) again way before τ_r without destroying information by relaxation.

In practice, it will usually not be possible to switch the coupling between oscillator and meter separately. Thus, before the entanglement is established, the relaxation rate (4) does *not* profit from the reduction of Δ_{eff} , see Fig. 1 (right). In order not to lose the information to be measured, the *maximum* relaxation rate $\Gamma_{r,\text{max}}$ reached at $\eta = 1/2$ (i.e., $q = q_c = \sqrt{\hbar/2M\Omega}$) should be slow enough, such that by the time τ_{ent} it takes to ramp above q_c , the information is not lost. In practice, this can be achieved by switching q very fast, at a time $\tau_{\text{ent}} \ll \Gamma_{r,\text{max}}^{-1}$, to q_c and slower afterwards, when the actual measurement occurs.

In the ohmic spin-boson model,^{9,16} i.e., for $J_{\text{eff}} = 2\pi\alpha\omega e^{-\omega/\omega_c}$, a scaling of Δ_{eff} to zero can be achieved through a dissipative phase transition at strong coupling to the bath ($\alpha > 1$).^{9,17,24,25} This transition is driven by the entanglement with a *collective* state involving the whole oscillator bath. Ramping α to large values increases $J_{\text{eff}}(\omega)$ at all frequencies, which leads to rapid relaxation *before* the scaling is established. Moreover, it is not known, how long it will take for the system to go through this phase transition. Here, according to the adiabatic theorem, this time is set through the inverse level spacing of the coupled system, which is infinite for the dense Ohmic spectrum. On the contrary, the model studied in the present paper provides strong scaling of Δ_{eff} with predictably *slow* relaxation and gives a clear prediction for the time scale of the entanglement set by the finite level spacing.

This model does not generally predict the efficiency of the detection. In order to do so, I chose a specific realization of the model, a superconducting quantum bit.^{5,6} In this case, the readout device is a Josephson junction, whose critical current I_0 is influenced by the state of the qubit, either a superconducting single-electron transistor⁵ or a dc superconducting quantum interference device (SQUID),⁶ see Fig. 2. We study the junction on the superconducting branch at low bias current I_B . We assume the tunable junction to be shunted only by a very large resistor R and an external capacitance C_x and consequently underdamped. This system has been studied in

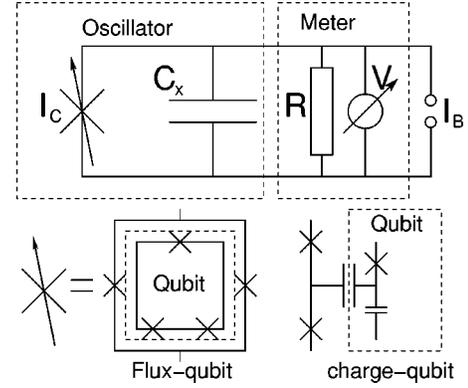


FIG. 2. Underdamped readout devices for superconducting flux (left) and charge (right) quantum bits.

the weak damping regime in Ref. 8 and resembles the one experimentally used in (Ref. 4). The oscillator in our model represents the plasma resonance²⁶ of the tunable junction at $\Omega = \sqrt{2eI_0/\hbar C_x}(1 - I_B^2/I_0^2)^{1/4}$. We can identify $M = C_x(\hbar/2e)^2$, $X = \phi$ (the Josephson phase), and $q = (\delta I_c/I_c)(I_B/\sqrt{I_0^2 - I_B^2})$, where $\delta I_c/I_c$ is the difference in critical current induced by the two fundamental states of the qubit. The damping is provided by the resistor and leads to $\alpha = \hbar q^2/2e^2 R = q^2 11.8k\Omega/R$, and the scaling exponent reads $\eta = q^2 \sqrt{C_x I_c \hbar}/8e^3$.

When ramping I_B , the junction switches to a finite voltage at $I_{\text{sw}} < I_0$, which provides a measure for I_0 . This switching is a stochastic process, so, if the measurement is repeated, one finds a histogram of switching currents^{27,28} centered around $I_{\text{sw},0}$, whose width δI_{sw} limits the resolution of this detector. In our case, the switching is predominantly due to thermal activation, where we can express $I_{\text{sw},0}/I_0 = 1 - [\ln(\omega_T/\Gamma_s)/u_0]^{2/3}$ and $\delta I/I_0 = [u_0^2 \ln(\omega_T/\Gamma_s)]^{-1/3}$ through the dimensionless height of the barrier at zero bias $u_0 = (4\sqrt{2}/3)(\hbar I_c/2ekT)$, the activation frequency $\omega_T = 2\Omega/\pi$, and the ramp rate $\Gamma_s = d(q/q_{\text{max}})/dt$. The current can be switched within a time $\tau_{\text{sw}} = \Omega^{-1}$, i.e., the ramp rate is limited by $\Gamma_{s,\text{max}} = \Omega$.

In a flux qubit, one can realize⁴ $I_c = 1 \mu\text{A}$, shunt with $C_x = 100 \text{ pF}$ and $R = 10 \text{ k}\Omega$, and $q_{\text{max}} = 0.05$ at a typical switching current level. We will assume $\Delta_0 = 1 \text{ GHz}$ and $\epsilon = 1 \text{ GHz}$ for the qubit. These parameters are accessible by doubling the size of the sample studied in (Ref. 4). This leads to $\Omega = 2 \text{ GHz}$, $\alpha = 0.003$, and $\eta_{\text{max}} = 3.5$, i.e., $\Delta_{\text{eff}} = 0.03\Delta_0$. Entanglement sets in at $q_c = 0.015$, where the relaxation time is $\tau_{r,\text{min}} = \Gamma_{r,\text{max}}^{-1} = 5 \mu\text{s}$. For 1% error, the first switch over this point has to be done at 700 ns, which is way above $\Gamma_{s,\text{max}} = 500 \text{ ps}$ and the adiabatic condition $qdq/dt < (500 \text{ ns})^{-1}$. Close to the measuring point $I_{\text{sw},0}$, we find $\tau_r = 120 \mu\text{s}$ and $\tau_\phi = 100 \text{ ns}$, which leaves a huge measurement window.

For definiteness, we set the temperature to $T = 200 \text{ mK}$ and find, using $\Gamma_s = (15 \mu\text{s})^{-1}$, that $I_{\text{sw},0}/I_0 = 0.96$ and $\delta I/I_0 = 0.35\%$, so, because $q = 5\%$, we have a signal-to-noise ratio of about 14. Hence, a single-shot von Neumann measurement appears to be feasible within a gradual improvement of technology.

For the readout for a charge qubit³ by a superconducting single electron transistor (S-SET), one can achieve values of $q=0.5$ within a charging energy $E_{C,SET}=2K$, corresponding to a capacitance scale of $C=1$ fF. We take the critical current of the SET to be $I_c=10$ nA and a shunt of $R=10$ k Ω and $C_x=1$ pF shunt capacitance. This leads to $\eta=3.5$, $\Omega=2$ GHz, and damping $\alpha=0.25$. Assuming $\Delta=1$ GHz, and $\epsilon=1$ GHz, we find $\tau_{R,min}=60$ ns, so for 1% error we have to switch to q_c in about 10 ns, which is close to the limit of $\Gamma_{s,max}=500$ ps. However, this may pose some challenge for the limiting time scales which are *not* due to the on-chip circuitry. For the readout step, we find $\tau_R=25$ μ s and $\tau_\phi=15$ ns. Applying the histogram theory as above at $T=200$ mK and $\Gamma_S=(3 \mu\text{s})^{-1}$, we end up with $I_{sw,0}/I_0=40\%$ and $\delta I_{sw}/I_c=8\%$, which can resolve our large signal of $q=30\%$ at signal-to-noise ratio of 4. It has been shown^{29,30} that experimentally SET's *can* reach signal-to-noise figures comparable to the quantum limit, hence even though the resolution is slightly less favorable than above, von Neumann measurements appear to be possible. For qubits³¹ operating in the regime of $E_J/E_c \approx 1$, more favorable parameters should be accessible.

The readout of the detector by switching is only one, and not necessarily the optimum alternative. Measurements could also be performed by detecting the kinetic inductance using the same parameters.

A similar circuit, a *normal conducting* SET with *out* the shunt capacitor has been thoroughly studied in Refs. 5,7. In that case, the measurement is started by rapidly switching the gate and monitoring the current. It has been shown that in

this way weak as well as strong measurements can be performed. As a consequence of the direct coupling of a dense spectrum of normal electrons to the qubit, the entanglement and the decoherence are not as strongly separated as in our case. Typical⁵ mixing times during the whole measurement are of the order of 1 μ s, i.e., one has to be able to monitor the current through the SET on the scale of 100 ns. In our case, one has to make the first entanglement switch on a similar scale, but has to monitor the voltage only afterwards, when mixing times are on the scale of 10 μ s. These numbers clearly indicate an advantage of the entanglement with the intermediate oscillator.

I have proposed a strategy for performing detector-dominated von Neumann measurements on qubits, using entanglement with coherent states of an harmonic oscillator. This system has been quantitatively analyzed using the spin-boson model and it has been shown that it has very favorable coherence and relaxation properties. A connection between the familiar scaling of the tunnel matrix element and the degree of entanglement with the environment has been established. Realistic superconducting circuitry, which could perform such measurements within present-day technology, has been proposed.

I thank J. von Delft, M. Governale, M. Grifoni, A.C.J. ter Haar, P. Hadley, P. Hakonen, C.J.P.M. Harmans, S. Kleff, L. Levitov, S. Lloyd, A. Lupascu, J.E. Mooij, T.P. Orlando, A. Shnirman, and C.H. van der Wal for clarifying discussions, as well as acknowledge support by the EU through TMR Supnan and Quiprocone and through ARO under Contract No. P-43385-PH-QC.

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