Problem sheet 1 Tuesday, Oct 22, 2013

Basic definitions

Problem 1.1: Show that from the definition of a group it follows that

- The left inverse is also a right inverse: $g^{-1}g = e \Rightarrow gg^{-1} = e$.
- The identity is unique
- The inverse g^{-1} is unique
- $gG = G$

Problem 1.2: Show that from the definition of a subgroup it follows that

- Every subgroup must contain the identity, i.e. $e_H = e$.
- A subset $H \subset G$ is a subgroup if and only if for all $h_1, h_2 \in H$ we have $h_1 h_2^{-1} \in H$.

Problem 1.3: Show that

- A coset is a subgroup if and only if $g \in H$.
- Two cosets $g_1 H$ and $g_2 H$ are either equal or disjoint.
- All cosets have the same number of elements. (What does "same number of elements" mean for infinite group?)

Problem 1.4: Show that conjugacy is an equivalence relation

- reflexive: $g \sim g$
- symmetric: $g_1 \sim g_2 \Rightarrow g_2 \sim g_1$
- transitive: $g_1 \sim g_2$ and $g_2 \sim g_3 \Rightarrow g_1 \sim g_3$.