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T VI: Soft Matter and Biological Physics
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Problem set 4

Problem 4.1 *weakly bending rod*

In the weakly bending rod approximation the propagator for the tangents $G(\vec{t}_\perp, s|\vec{0}, 0) \equiv Z(\vec{t}_\perp, s)$ of a worm-like chain fulfills a Schrödinger-type equation reminiscent of a harmonic oscillator

$$\frac{\partial}{\partial s} Z(\vec{t}_\perp, s) = \left[\frac{1}{2\ell_p} \nabla_\perp^2 + \frac{1}{k_B T} f_\parallel \left(1 - \frac{1}{2} \vec{t}_\perp^2 \right) + \frac{1}{k_B T} \vec{f}_\perp \cdot \vec{t}_\perp \right] Z(\vec{t}_\perp, s) \, .$$

Here the tangent vector $\vec{t} = (\vec{t}_\perp, t_\parallel)$ has been decomposed into parallel, $t_\parallel$, and perpendicular components, $\vec{t}_\perp$, and a similar decomposition is used for the force $\vec{f} = (\vec{f}_\perp, f_\parallel)$ acting on the polymer. By assumption, the deviations from directing to the north pole are small, $|\vec{t}_\perp| \ll 1$, $t_\parallel \simeq 1 - \vec{t}_\perp^2/2$. The Schrödinger equation has to be supplemented by the initial condition for the propagator

$$G(\vec{t}_\perp, s = 0|\vec{0}, 0) = Z(\vec{t}_\perp, s = 0) = \delta(\vec{t}_\perp) \, .$$

To simplify part the algebra, only the case of vanishing perpendicular force $\vec{f}_\perp = 0$ shall be considered.

1. Solve for the propagator using a gaussian ansatz

$$Z(\vec{t}_\perp, s) = \exp\left(-\frac{M(s)}{2} \vec{t}_\perp^2 + \Gamma(s) \right) ,$$

   with unspecified functions $M(s), \Gamma(s)$. Show that this ansatz transforms the partial differential equation (Schrödinger equation) into a *closed* set of ordinary differential equations. Formulate appropriate conditions for the unknown functions in the limit $s \to 0$. Recall that for small s the forces are irrelevant and the Schrödinger equation reduces to a diffusion problem.

2. Use the correspondence of the generating function of the end-to-end distance for the polymer problem without force and $G(\vec{t}_\perp, L|\vec{0}, 0)$ to determine the average *stored length* $L - \langle r_\parallel(L) \rangle$ and the mean-square perpendicular fluctuations $\langle \vec{r}_\perp(L)^2 \rangle$ in the weakly bending rod approximation.