Kerr-CFT and Gravitational Perturbations

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(See also: Amsel et al arxiv:0906.2376)

Introduction

- Brown-Henneaux: 3d AdS gravity
  - boundary conditions on metric fall-off
  - asymptotic symmetry group (diffeos preserving bcs modulo trivial diffeos): Virasoro x Virasoro
- Strominger: AdS3 quantum gravity is a 2d CFT
  - can calculate entropy of BTZ statistically

Kerr black hole

- Black hole uniqueness theorem: Kerr is unique time-independent vacuum black hole
- 2 parameters: mass M, angular momentum J
- RxU(1) isometry group
- Kerr bound: $GM^2 \geq |J|$, saturated by *extreme Kerr*

Near-horizon extreme Kerr

Bardeen & Horowitz 99

$$ds^2 = 2GJ\Omega(\theta)^2 \left[-(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + d\theta^2 + \Lambda(\theta)^2 (d\phi + r dt)^2 \right]$$

$$\Omega(\theta) = \frac{1}{2}(1+\cos^2\theta) \qquad \Lambda(\theta) = \frac{2\sin\theta}{1+\cos^2\theta}$$

- Surfaces of constant θ are S^1 fibred over AdS_2 (warped AdS_3)
- Isometry group SL(2,R)xU(1)
- Geodesically complete, timelike infinity at r=±∞
- Can Brown-Henneaux method be used here?

Kerr-CFT

Guica, Hartman, Song & Strominger 08

- Invent boundary conditions for NHEK quantum gravity
- Asymptotic symmetry group R x Virasoro: chiral CFT $c_R = 12J/\hbar$
- Frolov-Thorne vacuum state gives $T_L = 0,\, T_R = \frac{1}{2\pi}$
- Cardy formula $$S = \frac{\pi^2}{3} c_R T_R = \frac{2\pi J}{\hbar} = S_{BH}$$

GHSS fall-off conditions

$$\{t, r, \theta, \phi\}$$

$$h_{\mu\nu} \sim \begin{pmatrix} \mathcal{O}(r^2) & \mathcal{O}(\frac{1}{r^2}) & \mathcal{O}(\frac{1}{r}) & \mathcal{O}(1) \\ & \mathcal{O}(\frac{1}{r^3}) & \mathcal{O}(\frac{1}{r^2}) & \mathcal{O}(\frac{1}{r}) \\ & & \mathcal{O}(\frac{1}{r}) & \mathcal{O}(\frac{1}{r}) \\ & & & \mathcal{O}(1) \end{pmatrix}$$

- Some components O(1) relative to background
- Does this lead to well-defined initial value problem?

Zero energy constraint

- GHSS: don't want to consider non-extreme excitations so impose $E_L = Q_{\partial/\partial t}[g] = 0$
- Makes sense only if E_L non-negative
- NHEK ergoregion: $g_{tt} > 0$ for large r, $\theta \approx \pi/2$
- Ergoregion: energy in test matter fields unbounded below Friedman 78
- Outgoing energy at infinity ⇒ instability?

Motivation

- Do GHSS fall-off conditions lead to well-defined initial value problem?
- Does GHSS zero energy condition make sense?
- Is NHEK stable?
- Investigate these problems through study of gravitational perturbations of NHEK

Teukolsky equation Teukolsky 72

- Kerr (and NHEK) are Petrov type D spacetimes
- Miracle 1: massless spin-s perturbations of type D vacuum spacetime can be decoupled to obtain single wave equation for complex scalar $\Psi^{(s)}$
- Miracle 2: this equation is separable

Teukolsky in NHEK

- Separable Ansatz: $\Psi^{(s)} = \Phi(t,r)e^{im\phi}S_{lm}(\theta)$
- ODE for θ-dependence, quantization of separation constant Λ_{lm} in terms of integer l, numerical solutions
- $\Phi(t,r)$ obeys charged Klein-Gordon eq in AdS_2 with homogeneous electric field
- *complex* charge and mass^2

$$q = m - is \qquad \mu^2 = q^2 + \Lambda_{lm}$$

Behaviour of solutions Bardeen& Horowitz 99

- Assume $\Phi(t,r) = e^{-i\omega t}R(r)$
- Asymptotically: $R(r) \sim r^{-1/2 \pm \eta/2}$ $\eta = \sqrt{1 + 4\Lambda_{lm}}$
- η is real for small |m|, imaginary for $|m| \approx l$
- Real **η**: normalizable and non-normalizable modes, former fill out highest-weight reps of SL(2,R) Strominger 98

$$\omega = \pm(n + 1/2 + \eta/2), n = 0, 1, 2, \ldots$$

- Imaginary **η**: solutions oscillate: “traveling waves”

Traveling waves

- Phase and group velocity have same sign near one boundary, opposite sign at other boundary
- Energy flux follows phase velocity but group velocity governs physical propagation Bardeen & Horowitz 99
- Outgoing group velocity ⇒ frequency quantization: *stable* quasi-normal modes, decay with time
- No instability because energy flux not positive

Qualitative picture

- Impose "normalizable-outgoing" boundary conditions
- Initial data consists of superposition of normal modes and traveling waves
- Traveling waves disperse, leaving normal modes
- How does this compare with GHSS boundary conditions? Need to know metric fall-off!

Reconstructing the metric perturbation Cohen & Kegeles, Chrzanowski 75, Wald 78

- Vacuum, type D spacetime
- Components of metric perturbation obtained from *Hertz potential,* satisfies Teukolsky eq with $s \longrightarrow -s$ (why?)
- Given solution of Teukolsky eq can read off a solution of linearized Einstein eq

Metric fall-off

$$\eta = \sqrt{1 + 4\Lambda_{lm}}$$

$$h_{\mu\nu} \sim r^{\frac{3}{2} \pm \frac{1}{2}\eta} \qquad \{t, r, \theta, \phi\} \qquad \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\frac{1}{r^2}) & \mathcal{O}(\frac{1}{r}) & \mathcal{O}(\frac{1}{r}) \\ & \mathcal{O}(\frac{1}{r^4}) & \mathcal{O}(\frac{1}{r^3}) & \mathcal{O}(\frac{1}{r^3}) \\ & & \mathcal{O}(\frac{1}{r^2}) & \mathcal{O}(\frac{1}{r^2}) \\ & & & \mathcal{O}(\frac{1}{r^2}) \end{pmatrix}$$

- GHSS fall-off: $h_{tr} = \mathcal{O}(1/r^2) \quad h_{t\theta} = \mathcal{O}(1/r)$
- Requires real **η**: excludes traveling waves
- Requires **η**≥3: excludes some normal modes, e.g.

$$l = 4, |m| = 3 \Rightarrow \eta = 2.74$$

Initial value problem(s)

- Traveling waves, and some normal modes excluded by GHSS fall-off conditions: a restriction on allowed values of (l,m) for *individual modes*
- Initial data of compact support satisfies GHSS fall-off but contains dangerous modes ⇒ evolution of initial data violates GHSS fall-off. Initial value problem looks sick!
- Can’t restrict (l,m) at nonlinear level

Conserved charges

- For test field, define conserved charge associated with Killing field of background

$$Q_\xi[\Phi] = -\int_\Sigma \star J, \qquad J_\mu = T_{\mu\nu}\xi^\nu$$

- E_L is conserved charge associated to $\xi = \partial/\partial t$
- Angular momentum/U(1) charge associated to $\xi = -\partial/\partial\phi$

Scalar field charges

- Traveling waves: infinite charges
- Normal modes: use eqs of motion to obtain

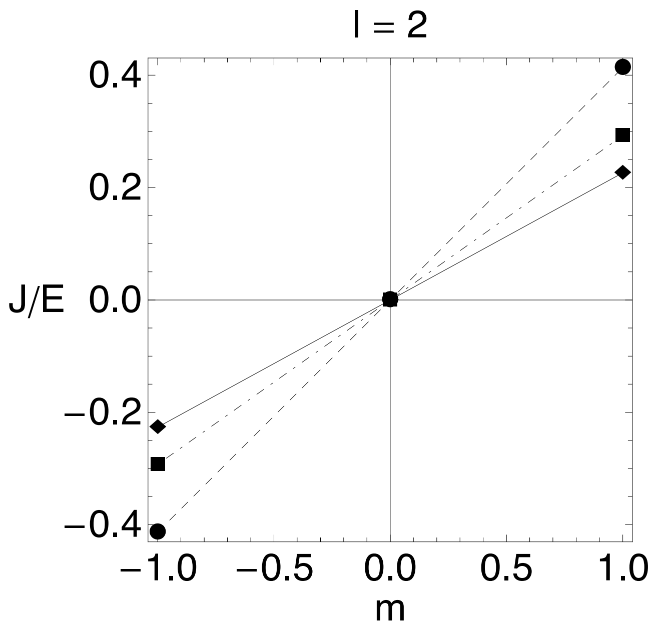
$$\mathcal{E}_{nlm} \equiv 4\pi M^2 \omega_{nlm} \int_0^{\pi} d\theta \sin\theta |S_{lm}(\theta)|^2 \int_{-\infty}^{+\infty} dr |R_{nlm}(r)|^2 \frac{\omega_{nlm} + mr}{1+r^2}$$

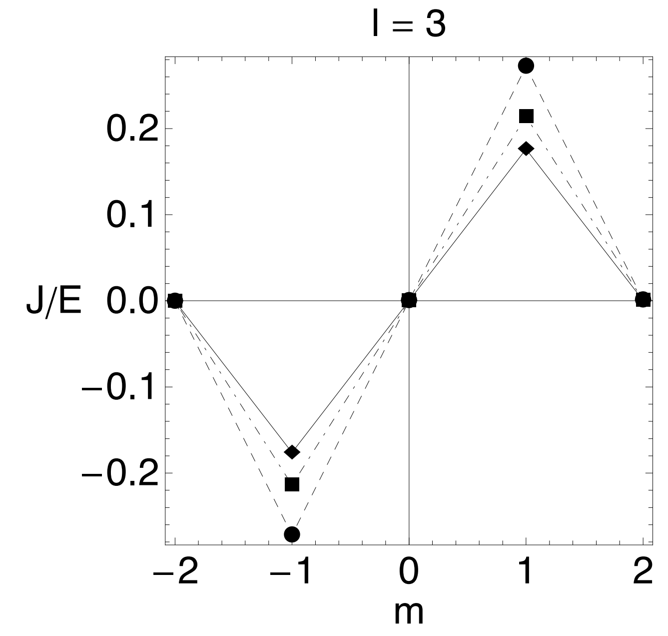
- radial integral evaluated numerically: positive in all cases checked
- Angular momentum: $\dfrac{\mathcal{J}_{nlm}}{\mathcal{E}_{nlm}} = \dfrac{m}{\omega_{nlm}}$

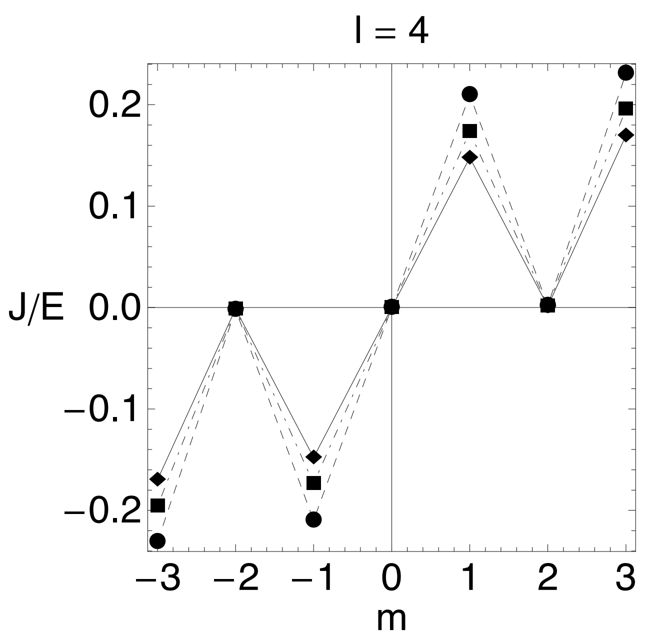
Linearized gravitational field

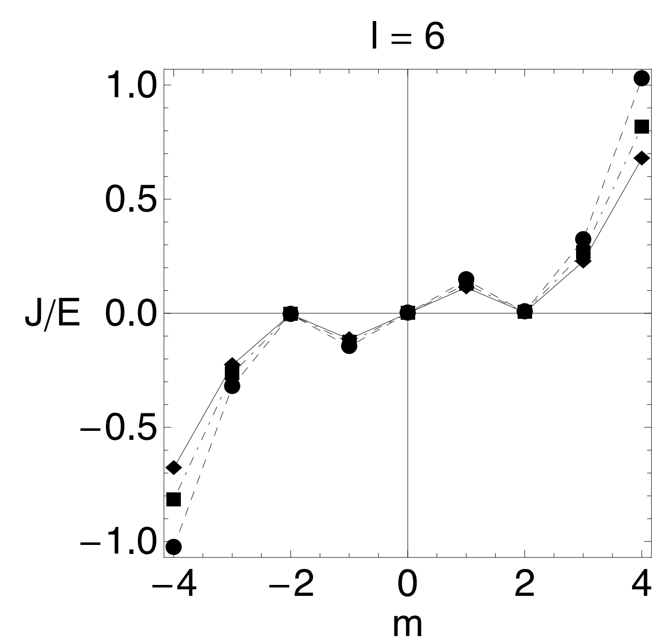
- Use Landau-Lifshitz stress tensor: 2nd order in derivatives of metric perturbation
- Metric perturbation 2nd order in derivatives of Hertz potential ⇒ conserved charges *6th order* in derivatives
- Use eqs of motion and Mathematica to reduce to 2nd order expressions, evaluate numerically
- Energy positive for all normal modes checked

Numerical results









Summary so far

- Energy of normal modes is always positive
- Can still construct compactly supported initial data of negative energy (must involve traveling waves)
- GHSS fall-off conditions do not lead to well-defined initial value problem for linearized fields
- Make further progress by considering 2nd order perturbations

2nd order perturbations

- 1st order metric perturbation h^1 sources 2nd order perturbation h^2
- Conserved charges given as bulk integral quadratic in h^1 or (difference of) boundary integrals linear in h^2
- Consider initial data h^1 of compact support $\Rightarrow h^2$ satisfies linearized eqs of motion near infinity
- Puzzle: no linearized solution discussed so far decays at correct rate to contribute to boundary integrals for charges!

Missing modes

- Same problem arises for Kerr black hole
- Resolution: Teukolsky/Hertz potential formalism misses modes that preserve type D property i.e. modes corresponding to changes in M or J
- For NHEK, can change J: gives a perturbation h^2 that contributes to angular momentum boundary integral but violates GHSS fall-off...
- But what modes contribute to energy integral?

Energy carrying modes

- Is there a finite energy deformation of NHEK?
- Take decoupling limit of near-extreme Kerr keeping temperature (and J) fixed: resulting geometry is isometric to NHEK (cf Reissner-Nordstrom Maldacena & Strominger 98)
- Subleading term in decoupling limit is a solution of linearized equations, and contributes to surface integral for energy...but violates GHSS fall-off

Zero energy condition

- Any initial data for h^1 with non-zero energy or angular momentum will excite h^2 that violates GHSS fall-off
- GHSS fall-off *implies* zero energy condition

Zero charge data

- Consider compactly supported initial data h^1 with *zero* energy and angular momentum
- Must involve traveling waves ⇒ evolution of h^1 violates GHSS fall-off conditions ⇒ badly posed initial value problem?
- More likely: h^1 still excites h^2 s.t. boundary integrals non-zero but equal ⇒ initial data violates GHSS fall-off at 2nd order

Conclusion

- Appears that only solution of the GHSS fall-off conditions is NHEK itself: there is “no dynamics in NHEK”
- No simple modification of GHSS fall-off will change this
- Can *prove* uniqueness of NHEK among stationary, axisymmetric solutions, although with stronger fall-off than GHSS Amsel et al 09

Origin of chiral CFT Balasubramanian et al 09

- Near-horizon limit of extreme BTZ: locally AdS_3, SL(2,R) x U(1) symmetry

$$ds^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + (d\phi + r dt)^2$$

- DLCQ of non-chiral CFT dual to AdS_3: chiral CFT
- SL(2,R) acts trivially on CFT ⇒ no dynamics associated with AdS_2 (cf Maldacena & Strominger 98)
- Is NHEK CFT the DLCQ of a non-chiral parent CFT?

Other developments

- Scattering by extreme Kerr: reproduces CFT correlation functions Bredberg et al 09
- Near-extreme Kerr-CFT? Matsuo et al 09, Castro & Larsen 09