Holographic Superfluids

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References

C. P. H., P. K. Kovtun, D. T. Son, “Holographic model of superfluidity,” PRD [arXiv:0809:4870 [hep-th]].

C. P. H. and S. S. Pufu, “The Second Sound of SU(2),” JHEP, arXiv:0902.0409 [hep-th].

C. P. H. and A. Yarom, “Sound modes in holographic superfluids,” arXiv:0906:4810 [hep-th].


In case of emergency
break the glass!

Superfluids from Holography

Superfluids are an old and well studied subject.

- Experimental realizations: He^4, He^3, atomic gases, neutron stars (?).
- Perturbation theory: A superfluid as a Bose-Einstein condensate.
- Mean/effective field theory using the order parameter.
- Monte Carlo approaches and computer simulations.

What can AdS/CFT add to the story?

- AdS/CFT is intrinsically a strong coupling approach. It works where perturbation theory doesn't, mapping a strongly interacting field theory to a classical gravitational description.
- It works for real time physics and at nonzero density, unlike many numerical lattice approaches.
- Given a stringy embedding of the gravity dual, one can in principle understand exactly what field theory one is solving (c.f. S. Pufu's talk).

Outline

- Two models of a holographic superfluid:
  - scalar order parameter
  - vector order parameter
- Phase diagram and a scalar order parameter, probe limit.
- Second sound and a scalar order parameter.
- Analytic results and a vector order parameter, probe limit.

Holographic Phase Transitions

Goal: To have a simple holographic model of a (classical) phase transition where we can calculate the phase diagram and transport coefficients.

$$S = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g}(R - 2\Lambda) - \frac{1}{4g^2} \int d^{d+1}x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}$$

- Einstein-Hilbert produces correlators of the stress tensor $T^{\mu\nu}$ in the boundary theory.
- Maxwell produces correlators of a global current J^{μ} in the boundary.
- To model a (classical) phase transition, we need something that will serve as an order parameter.

Two Choices of Order

- We can add a charged scalar field

$$-\int d^{d+1}x\sqrt{-g}\left(|(\partial - iqA)\Psi|^2 + V(|\Psi|)\right) \ .$$

  The order parameter is the boundary value of Ψ.

- We can promote the Abelian gauge field to an SU(2) gauge field

$$F_{\mu\nu} \to F^a_{\mu\nu} \ .$$

  We find a vector order parameter which is the boundary value of A^a_μ.

Motivating the Action from String Theory

- Ammon, Erdmenger, Kaminski, Kerner and Basu, He, Mukherjee, Shieh: an $SU(2)_F$ theory from maximally supersymmetric $SU(N)$ Yang-Mills theory with two hypermultiplets via a D3- and D7-brane construction.
- Denef and Hartnoll: the scalar field theory in 2+1 dimensions from a consistent truncation of 11 dimensional supergravity
- Gubser, Herzog, Pufu, and Tesileanu: a proposal in 3+1 dimensions involving a consistent truncation of type IIB supergravity and condensation of a gluino bilinear (c.f. Pufu's talk).

In the (first and third) cases, the action is a bit different than what we propose to study.

Dyonic Black Holes and the Normal Phase

One solution to our scalar action with $\psi = 0$ is a dyonic black hole in AdS_4. The dyonic black hole is also a solution to the SU(2) action. Dyonic black holes have electric and magnetic charge.

- The Hawking temperature of the black hole is the temperature T of the field theory.
- The magnetic field of the black hole is the magnetic field B in the field theory.
- The electric field of the black hole becomes the charge density ρ of the field theory.

One can freely tune the temperature and charges of the black hole.

An instability for the scalar action

Assuming $V(\Psi) = m^2|\Psi|^2$, Gubser observed an instability for the scalar to condense when ρ gets too large:

$$m_{\text{eff}}^2 = m^2 + g^{tt} A_t^2$$

where

$$g_{tt} = -g(r) \; ; \quad A_t = \frac{\rho}{r r_+}(r - r_+) \; .$$

The effective mass becomes tachyonic and the scalar condenses in a narrow region of radial coordinate r.

There is no need for a Ψ^4 term!

For the case $B = 0$, there is only one other scale in the problem, the temperature, so large ρ corresponds to small T.

The SU(2) instability

The SU(2) action has a similar instability. Let the τ^i generate $\mathfrak{su}(2)$. For an electrically charged black hole in the τ^3 direction, there is an instability to generate a nonzero $A_x^1 = w$:

$$A = \phi\,\tau^3\,dt + w\,\tau^1\,dx \ .$$

The nonzero w corresponds to a nonzero current in the boundary field theory!

Phase Diagram, Scalar Order Parameter, Probe Limit

- We now study the holographic model with a scalar order parameter in the probe limit (in 2+1 dimensions).
- The probe limit decouples the metric degrees of freedom from the gauge and scalar degrees of freedom. It's the weak gravity limit.
- But first we recall some facts from Landau-Ginzburg mean field theory.

A Traditional Approach to the Phase Diagram

Landau and Ginzburg mean field theory

$$\mathcal{L} = -\left(|\nabla\phi|^2 + \alpha|\phi|^2 + \beta|\phi|^4 + \gamma|\phi|^6\right) \ .$$

- Assuming $\alpha(T) = (T - T_c)\alpha_0$ and $\alpha_0, \beta > 0$, a second order phase transition occurs at $T = T_c$. Note that for $T \lesssim T_c$,

$$|\phi| \sim (T_c - T)^{1/2} \ .$$

- If $\phi = |\phi|e^{i\xi x}$, then $|\nabla\phi|^2 = \xi^2|\phi|^2$, lowering T_c. Such a phase gradient gives rise to a nonzero current.
- If $\beta < 0$ for $T < T_1$, then the phase transition can become first order provided $\gamma > 0$.

The holographic phase transition

Given $V = m^2 L^2 |\psi|^2 = -2|\psi|^2$ (above the BF bound), we can choose a scalar in the field theory with scaling dimension one or two.

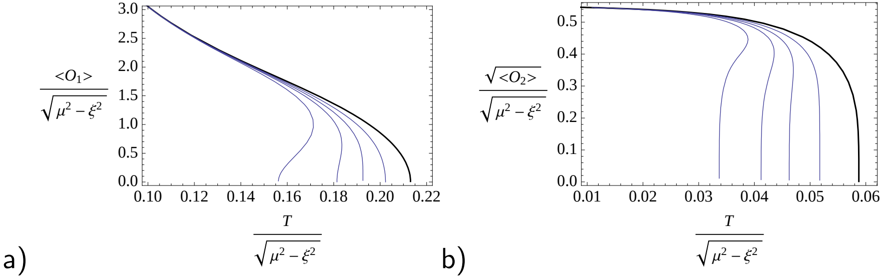


Figure: The condensate as a function of temperature for operators of conformal dimension (a) one and (b) two. The curves in the plots, from right to left, are for $\xi/\mu = 0$, $1/4$, $1/3$, $2/5$, and $1/2$.

Numerically, we find that for $T \lesssim T_c$, $\langle O_i \rangle \sim (T - T_c)^{1/2}$.

First and Second Order Phase Transitions

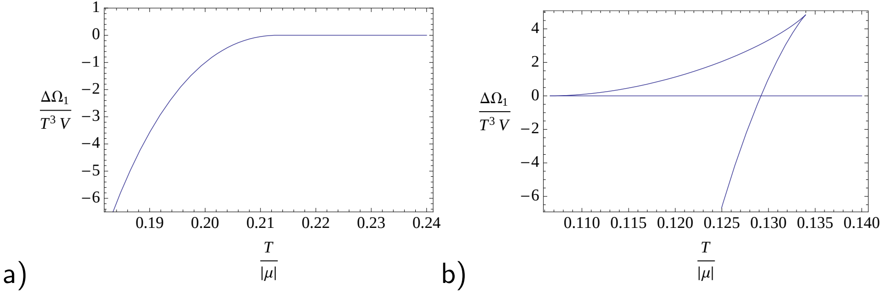
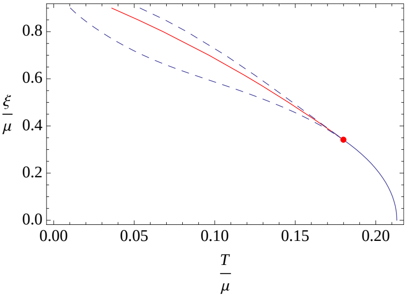
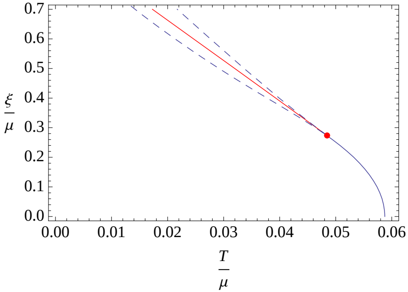


Figure: The difference in free energy $\Delta\Omega_1$ between the phase with a scalar condensate and without one as a function of T/μ: a) $\xi = 0$ and b) $\xi/\mu = 4/7$.

The Phase Diagram, Probe Limit



a)



b)

Figure: The phase diagrams for the theory with a scalar with a) conformal dimension one and b) conformal dimension two. The solid blue line indicates a second order phase transition while the solid red line (in between the dashed lines) indicates a first order phase transition. The dashed lines are spinodal curves, while the red dot indicates the tricritical point.

Sound and the Scalar Order Parameter

- We consider the scalar order parameter with back reaction (in 3+1 dimensions).
- As $T \to 0$, the order parameter gets large. The probe limit makes less physical sense here.
- A principal goal will be to analyze the speeds of sound in the $T \to 0$ limit

Different Kinds of Sound

Superfluids have two components which means there is more than one kind of propagating collective motion.

- first sound: the usual sound, sourced by pressure oscillations, where the components move in phase.
- second sound: the components move out of phase, sourced by temperature oscillations
- third sound: involves surface waves on a thin film, not important for today.
- fourth sound: waves in a capillary tube packed with a powder that immobilizes the normal component.

Sound Speeds

From a hydrodynamic analysis, we calculate the speed of sound from thermodynamic quantities.

- Vanishing of the trace of the stress tensor implies

$$c_1^2 = \left(\frac{\partial P}{\partial \epsilon}\right)_s = \frac{1}{d}$$

- For the speed of second sound, we find

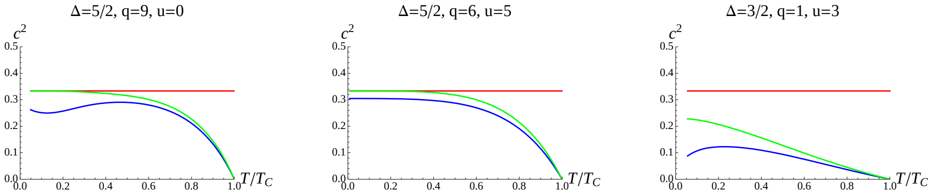
$$c_2^2 = \frac{\sigma^2 \rho_{\rm s}}{w} \frac{1}{(\partial \sigma / \partial T)_\mu} \ ,$$

  where $w = \epsilon + P = sT + \mu \rho_{\rm n}$ and $\sigma = s/\rho$.

- A nice formula for the speed of fourth sound is

$$c_4^2 = \frac{\mu \rho_{\rm s}}{sT + \mu\rho} c_1^2 + \frac{w}{sT + \mu\rho} c_2^2 \ .$$

Typical Sound Speeds



The red line is ordinary sound. The green line is fourth sound. The blue line is second sound. Here $m^2L^2 = \Delta(\Delta - 4)$,

$$V(\psi) = m^2|\psi|^2 + u|\psi|^4 \ ,$$

and q is the charge of the scalar field.

Second Sound of Helium-4 from Khalatnikov's Book

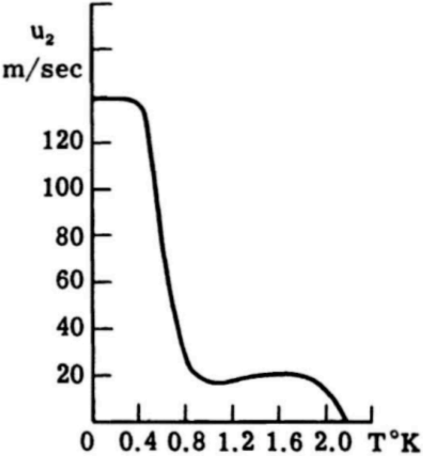


Figure 4. The temperature dependence of the velocity of second sound.

Second Sound (3+1 dimensions)

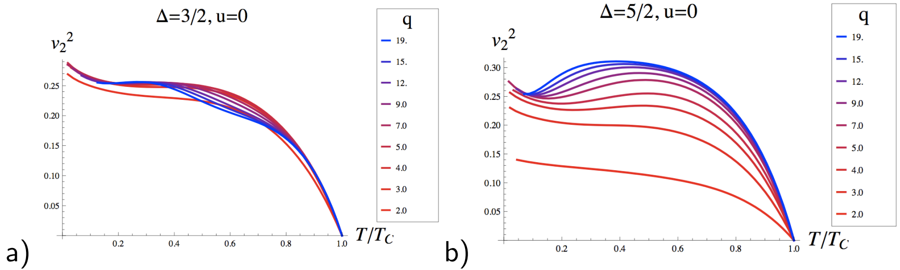


Figure: The speed of second sound as a function of T/T_c, computed by evaluating thermodynamic derivatives: a) $O_{3/2}$ scalar, b) $O_{5/2}$ scalar for a 3+1 dimensional field theory. The speed of second sound vanishes as $T \to T_c$. $u = 0$ means no quartic term in V.

Universality at low T

Landau predicted that

$$\lim_{T \to 0} c_2^2 = \frac{c_1^2}{d} \ ,$$

i.e. second sound becomes a sound wave propagating in a gas of phonons. Reasonable for helium-4.

We do not find this result. We believe the reason is that the low temperature limit of this system is not a gas of phonons.

$$\lim_{T \to 0} \frac{C_\mu}{sd} \neq 1 \ , \qquad \lim_{T \to 0} \frac{sT}{sT + \mu \rho_{\mathrm{n}}} \neq c_1^2$$

What exactly is it?

Analytic Results, Vector Order Parameter, Probe Limit

Starting with an electrically charged black hole at small μ/T where $A = \phi\,\tau^3\,dt$, there is a critical chemical potential at which an instability appears, characterized by a zero mode for A_x^1 (Basu, He, Mukherjee, Shieh, 0810.3970 [hep-th]):

$$\partial_z^2 A_x^1 + \left(\frac{f'}{f} - \frac{1}{z}\right)\partial_z A_x^1 = -\frac{\phi^2}{f^2} A_x^1$$

where $\phi = (1-z)\mu/\pi T$ and $f = 1 - z^4$:

$$A_x^1 = \epsilon \frac{z^2}{(1+z^2)^2} \qquad \text{where} \qquad \frac{\mu_c}{\pi T} = 4 \ .$$

We find a solution as a power series in ϵ.

The Phase Transition, Analytically

- The vector order parameter $\langle j_x^1 \rangle \sim \epsilon$.
- We find
$$\mu/\pi T = 4 + \frac{71}{6720}\epsilon^2 + \mathcal{O}(\epsilon^4) \ ,$$
from which we infer $\langle j_x^1 \rangle \sim (T_c - T)^{1/2}$.
- We can also introduce a superfluid velocity, $\xi_\parallel$ and $\xi_\perp$. (The phase transition breaks rotational symmetry). We find a phase separation line
$$\mu \approx 4\pi T + \frac{1}{6\pi T}\xi_\parallel^2 \ , \quad \mu \approx 4\pi T + \frac{1}{3\pi T}\xi_\perp^2 \ .$$

Speed of Second Sound

Because of the broken rotational symmetry, there are actually two speeds of second sound

$$c_\perp^2 \approx \frac{140}{281}\left(\frac{\mu}{\pi T} - 4\right) \ ,$$
$$c_\parallel^2 \approx \frac{70}{281}\left(\frac{\mu}{\pi T} - 4\right) \ .$$

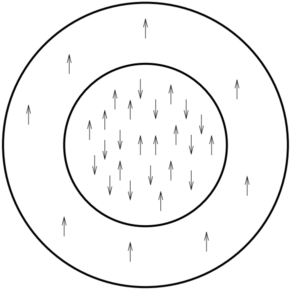
We were able to see these results in two ways:

- From the thermodynamic identity mentioned above.
- From poles in the current-current correlation functions.

NB: These results are valid only near T_c.

Fulde-Ferrell

- That the order parameter $\langle j_x^1 \rangle$ is a current is strange.
- Reminiscent of an idea by Fulde and Ferrell (also Larkin and Ovchinnikov) — BCS in a magnetic field

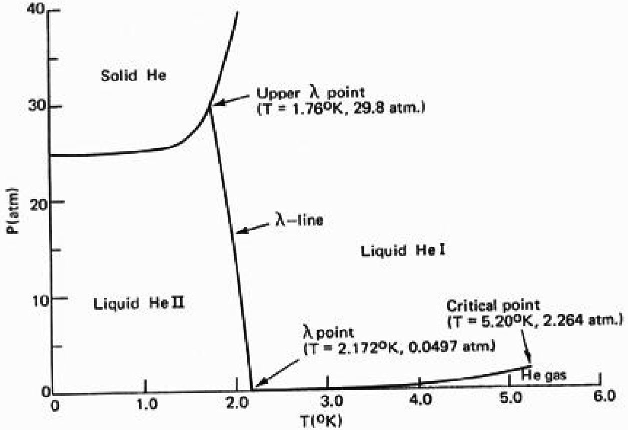


$$k_{F\uparrow} - k_{F\downarrow} > 0$$

Remarks and Plans for the Future

- Tried to convince you that AdS/CFT is a useful tool for studying strongly interacting field theories — equations of state, correlation functions, transport properties.
- The hope is that these field theories may be relevant for understanding real world condensed matter systems.
- We saw that AdS/CFT can be used to study the superfluid phase transition.
- How do we gain control over the $T \to 0$ limit?
  - Stringy issues: Is this limit stable in string theory? What supergravity modes do we include?
  - Numerical issues: Why are the numerics difficult in this limit?
  - Conceptual issues: Can we learn anything new in this limit? Beyond Landau?

Phase Diagram for Helium-4



The phase diagram of He^4.

Phase Diagram for Helium-3