Proof of a Universal Lower Bound on η/s

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+MEDVED 0908.1473

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0808.3498,0810.2193, 0901.2191

+GORBONOS, HADAD 0712.3206

+G 0902.1553, +H 0903.0823

+DVALI, VENEZIANO 0907.5516

- Quadratic, 2-derivatives effective action for perturbations ➔ brane thermodynamics & hydrodynamics (linear) for generalized theories of gravity

- Geometry ➔ entropy density calibrated to Einstein value
- Unitarity ➔ preferred direction on the space of couplings

Outline

- KSS bound: $\eta / s \geq 1/4\pi$
- Both η, s couplings of 2-derivatives effective action → no apparent reason for directionality of deviation from Einstein gravity

Preferred direction

Idea 1: Entropy is always a geometric quantity → The entropy density can be calibrated to its Einstein value

Idea 2: (non-trivial) Extensions of Einstein can only increase the number of gravitational DOF →

Unitarity forces increase of couplings (including η)

Hydro of generalized gravity

$$S=\int dtdrd^{p}x\sqrt{-g}\,L\left(g_{\mu\nu},R^{\mu\nu}{}_{\rho\sigma},\phi,\nabla\phi,\ldots\right)$$

$$L=\frac{1}{16\pi G_E}R+\lambda L_{corr}$$

Expand to quadratic order in gauge invariant fields Z_i and to quadratic order derivatives, diagonalize

$$g_{\mu\nu}=\bar{g}_{\mu\nu}+h_{\mu\nu},\ \phi=\bar{\phi}+\varphi$$

$$S^{(2)}=\int dtdrd^{p}x\frac{1}{(\kappa_{eff}^{2}(r))_{i}}(\bar{\nabla}Z_{i})^{2}$$

$$Z_i\sim e^{i\Omega t-iQz}$$

$$q=\frac{Q}{2\pi T},\ w=\frac{\Omega}{2\pi T}$$

Liu+Iqbal:0809.3808 Einstein gravity
R.B+MEDVED:generalized gravity
0808.3498,0810.2193, 0901.2191

The effective gravitational coupling: Einstein gravity

$$\frac{1}{16\,\pi\,G_{\mathrm{E}}}\sqrt{-g}\,R = \frac{1}{16\,\pi\,G_{\mathrm{E}}}\sqrt{-\bar{g}}\left(\overline{R} + \mathscr{L}^{(2)}_{EH}\right)$$

$$g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(x) + \kappa h_{\mu\nu}$$

$$\kappa^2 = 32\pi G_E$$

$$\mathscr{L}^{(2)}_{EH} = \frac{1}{2}\overline{\nabla}_\alpha h_{\mu\nu}\overline{\nabla}^\alpha h^{\mu\nu} - \frac{1}{2}\overline{\nabla}_\alpha h\overline{\nabla}^\alpha h + \overline{\nabla}_\alpha h\overline{\nabla}_\beta h^{\alpha\beta}$$
$$-\overline{\nabla}_\alpha h_{\mu\beta}\overline{\nabla}^\beta h^{\mu\alpha} + \overline{R}\left(\frac{1}{2}h^2 - \frac{1}{2}h_{\mu\nu}h^{\mu\nu}\right)$$
$$+\overline{R}^{\mu\nu}\left(2\,h^\alpha{}_\mu h_{\nu\alpha} - h\,h_{\mu\nu}\right)$$

$$h = h^\lambda{}_\lambda$$

The effective gravitational coupling: Generalized gravity

$$I = \int d^D x \sqrt{-g}\, \mathscr{L}\left(R_{\rho\mu\lambda\nu}, g_{\mu\nu}, \nabla_\sigma R_{\rho\mu\lambda\nu}, \phi, \nabla\phi, \ldots\right)$$

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + h_{\mu\nu}$$

Contributions to the graviton kinetic terms must appear through factors of the Riemann tensor (or it derivatives) in the action.

$$\delta I = \int d^D x \sqrt{-g}\, \frac{\delta \mathscr{L}}{\delta R_{\rho\mu\lambda\nu}} \delta R_{\rho\mu\lambda\nu}$$

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The effective gravitational coupling: General action

$$\delta I^{(2)} = \int d^D x \left\{ \left(\sqrt{-g}\,\frac{\delta \mathscr{L}}{\delta R_{\rho\mu\lambda\nu}} \right)^{(0)} \delta R^{(2)}_{\rho\mu\lambda\nu} + \cancel{\left(\sqrt{-g}\,\frac{\delta \mathscr{L}}{\delta R_{\rho\mu\lambda\nu}} \right)^{(1)} \delta R^{(1)}_{\rho\mu\lambda\nu}} \right\}$$

$$\delta R_{\rho\mu\lambda\nu} = \nabla_\lambda \delta\Gamma_{\nu\mu\rho} - \nabla_\nu \delta\Gamma_{\lambda\mu\rho}$$

$$\delta I^{(2)} = \int d^D x \sqrt{-\bar{g}}\,\frac{1}{2} \left(\frac{\delta \mathscr{L}}{\delta R_{\rho\mu\lambda\nu}} \right)^{(0)} \left(\overline{\nabla}_\delta h_{\lambda\mu} \overline{\nabla}^\delta h_{\nu\rho} + 2 \overline{\nabla}^\delta h_{\lambda\rho} \overline{\nabla}_\mu h_{\nu\delta} \right)$$

Kinetic matrix, coupling constant matrix

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Unitarity forces increase of couplings (including η)

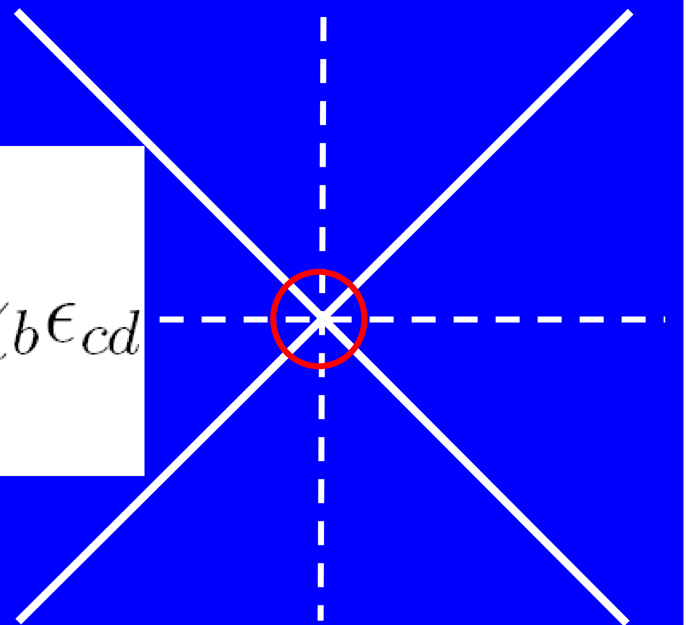
Wald's entropy

Wald '93

$$I = \int d^D x \sqrt{-g}\, \mathscr{L}\left(R_{\rho\mu\lambda\nu}, g_{\mu\nu}, \nabla_\sigma R_{\rho\mu\lambda\nu}, \phi, \nabla\phi, \ldots\right)$$

- Stationary BH solutions with bifurcating Killing horizons

$$S_W = -\frac{1}{T}\oint_\Sigma \left(\frac{\delta \mathscr{L}}{\delta R_{abcd}}\right)^{(0)} \nabla_a \chi_b \epsilon_{cd}$$



$$S_W = \frac{1}{4}\frac{A}{G_{eff}}$$

$$\delta I^{(2)} = \int d^D x \sqrt{-\overline{g}}\, \frac{1}{2} \left(\frac{\delta \mathscr{L}}{\delta R_{\rho\mu\lambda\nu}} \right)^{(0)} \left(\overline{\nabla}_\delta h_{\lambda\mu} \overline{\nabla}^\delta h_{\nu\rho} + 2 \overline{\nabla}^\delta h_{\lambda\rho} \overline{\nabla}_\mu h_{\nu\delta} \right)$$

$$S_W = -2\pi \oint_\Sigma \left(\frac{\delta \mathscr{L}}{\delta R_{abcd}} \right)^{(0)} \hat{\epsilon}_{ab} \overline{\epsilon}_{cd}$$

Wald's prescription picks a specific polarization and location - horizon

$$\left(\frac{\delta \mathscr{L}}{\delta R_{\rho\mu\lambda\nu}} \right)^{(0)} = \frac{1}{"\kappa^2"}$$

Spherically symmetric, static BHs → κ_{rt}

Calibration of entropy density

$$S_E \;=\; V_\perp r_h^p/(4G_E)$$

Fixed geometry
Fixed charges

$$S_X \;=\; \frac{V_\perp r_h^p}{4G_X} \;=\; \frac{V_\perp r_h^p}{4G_E + \lambda\delta G} \;+\; \mathcal{O}[\lambda^2]$$

ps)
n between r_h
der in λ

Transform to (leading order) “Einstein frame”
Can be done order by order in λ

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = e^{\Omega} g_{\mu\nu}$$

$$\Omega = -\tfrac{2}{p}\lambda\delta G/G_E$$

rescale

$$G_X \rightarrow G_{\tilde{X}} = G_E$$

To preserve the form of leading term

$$(16\pi G)^{-1}\sqrt{-g}\,\mathcal{R}$$

Change the units of G_X such that its numerical value is equal to G_E
The value of r_h = largest zero of $|g_{tt}/g_{rr}|$ is not changed

$$S_{\tilde{X}} = \frac{\widetilde{V}_\perp\, r_h^p}{4G_{\tilde{X}}} = \frac{\left(V_\perp - \lambda\frac{\delta G}{G_E}\right) r_h^p}{4G_E} + \mathcal{O}[\lambda^2]$$

r_h largest zero of $|g_{tt}/g_{rr}|$ ➔ not changed

$$S_{\tilde{X}}/S_E = \widetilde{V}_\perp/V_\perp \Longrightarrow s_{\tilde{X}} = s_E$$

Preferred direction determined by η only!

- Entropy invariant to field redefinitions $S_{\tilde{X}}=S_X$
- Entropy density not invariant
- Calibration possible: entropy⇔geometry

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Unitarity forces increase of couplings (including η)

The graviton propagator in Einstein gravity

$$[\mathcal{D}(q^2)]_{\mu\ \alpha}^{\ \nu\ \beta} \equiv \langle h_\mu^{\ \nu}(q) h_\alpha^{\ \beta}(-q) \rangle$$

Gravitons exchanged between two conserved sources

One graviton exchange approximation h's << 1

In Einstein gravity: a single massless spin-2 graviton

$$[\mathcal{D}(q^2)]_{\mu\ \alpha}^{\ \nu\ \beta} = \rho_E(q\ )\left[\delta_\mu^{\ \beta}\delta_\alpha^{\ \nu} - \frac{1}{2}\delta_\mu^{\ \nu}\delta_\alpha^{\ \beta}\right]\frac{G_E}{q^2}$$

$$\rho_E(0) = 1$$

Vectors do not contribute to exchanges between conserved sources

First appearance of a preferred direction

$$[\mathcal{D}(q^2)]_{\mu\ \alpha}^{\ \nu\ \beta} \quad = \rho_E(q^2)\left[\delta_\mu^{\ \beta}\delta_\alpha^{\ \nu} - \frac{1}{2}\delta_\mu^{\ \nu}\delta_\alpha^{\ \beta}\right]\frac{G_E}{q^2}$$

$$\rho_E(0) = 1$$

Spectral decomposition $$\frac{\rho_E(q^2)}{q^2} = \int \frac{\rho_E(s)}{q^2 - s} ds$$

semiclassical unitarity $$\rho_E(s) \geq 0$$

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DVALI: MANY PAPERS

➔ $\rho_E(q^2)$ can only increase towards the UV
➔ Mass screening is not allowed

The graviton propagator in generalized gravity

- massless spin-2 graviton
- massive spin-2 graviton
- scalar gravitons

$$
\begin{aligned}
[\mathcal{D}(q^2)]_{\mu\ \alpha}^{\ \nu\ \beta} &= \left(\rho_E(q^2)+\rho_{NE}(q^2)\right)\boxed{\left[\delta_\mu^{\ \beta}\delta_\alpha^{\ \nu}-\frac{1}{2}\delta_\mu^{\ \nu}\delta_\alpha^{\ \beta}\right]\frac{G_E}{q^2}} \\
&+ \sum_i \rho_{NE}^i(q^2)\boxed{\left(\delta_\mu^{\ \beta}\delta_\alpha^{\ \nu}-\frac{1}{3}\delta_\mu^{\ \nu}\delta_\alpha^{\ \beta}\right)\frac{G_E}{q^2-m_i^2}} \\
&+ \sum_j \widetilde{\rho}_{NE}^j(q^2)\boxed{\delta_\mu^{\ \nu}\delta_\alpha^{\ \beta}\frac{G_E}{q^2-\widetilde{m}_j^2}}.
\end{aligned}
$$

Gravitons exchanged between two conserved sources
One graviton exchange approximation h's << 1
Vectors do not contribute to exchanges between conserved sources

Gravitational couplings of generalized gravity can **only** increase compared to Einstein gravity

Spectral decomposition
$$\frac{\rho_{NE}(q^2)}{q^2} = \int \frac{\rho_{NE}(s)}{q^2 - s} ds$$

semiclassical unitarity
$$\rho_{NE}(s) \geq 0 \Longrightarrow \rho_{NE}(q^2) \geq 0$$

$$\begin{aligned}
[\mathcal{D}(q^2)]_{\mu\ \alpha}^{\ \nu\ \beta} \quad = \quad & \left(\rho_E(q^2) + \rho_{NE}(q^2)\right) \boxed{\left[\delta_\mu^{\ \beta}\delta_\alpha^{\ \nu} - \frac{1}{2}\delta_\mu^{\ \nu}\delta_\alpha^{\ \beta}\right]\frac{G_E}{q^2}} \\
& + \sum_i \rho_{NE}^i(q^2) \boxed{\left(\delta_\mu^{\ \beta}\delta_\alpha^{\ \nu} - \frac{1}{3}\delta_\mu^{\ \nu}\delta_\alpha^{\ \beta}\right)\frac{G_E}{q^2 - m_i^2}} \\
& + \sum_j \widetilde{\rho}_{NE}^j(q^2) \boxed{\delta_\mu^{\ \nu}\delta_\alpha^{\ \beta}\ \frac{G_E}{q^2 - \widetilde{m}_j^2}}.
\end{aligned}$$

The shear viscosity as a gravitational coupling

$$S = \int dtdrd^p x \sqrt{-g} L\left(g_{\mu\nu}, R^{\mu\nu}{}_{\rho\sigma}, \phi, \nabla\phi, \ldots\right)$$

$$g_{\mu\nu} = \bar{g}_{xy} + h_{xy}$$

$$L = \frac{1}{16\pi G_E} R + \lambda L_{corr}$$

$$S^{(2)} = \int dtdrd^p x \frac{1}{(\kappa_{eff}^2(r))_{xy}} (\bar{\nabla} Z_x{}^y)^2$$

$$Z_x{}^y \sim e^{i\Omega t - iQz}$$

In the hydro limit

$$q = \frac{Q}{2\pi T}, \quad w = \frac{\Omega}{2\pi T} \ll 1$$

$$\left\langle Z_x{}^y Z_y{}^x \right\rangle \sim \eta$$

See also 0811.1665
CAI, NIE, SUN

The shear viscosity of generalized gravity (and its FT dual) can **only** increase compared to its Einstein gravity value

$$\frac{\eta_X}{\eta_E} = \left[\frac{\langle h_x{}^y h_y{}^x \rangle_X}{\langle h_x{}^y h_y{}^x \rangle_E}\right] = 1 + \frac{1}{\rho_E(0)} \sum_i \rho^i_{NE}(0)$$

$$\rho_E(0) = 1,\ \rho_{NE}(0) \geq 0 \Rightarrow \frac{\eta_X}{\eta_E} \geq 1$$

Only spin-2 particles with $q = \dfrac{Q}{2\pi T},\ w = \dfrac{\Omega}{2\pi T},\ \dfrac{m}{T} \ll 1$ contribute to the sum

KSS bound

$$\frac{\eta_X}{\eta_E} \geq 1 \Rightarrow \frac{\eta_{\tilde{X}}}{\eta_E} \geq 1$$

True also for r →infinity =AdS boundary
→ valid for FT dual of bulk theory

$$s_{\tilde{X}} = s_E$$

Valid for FT by the gauge-gravity duality

$$\left(\frac{\eta}{s}\right)_X \geq \left(\frac{\eta}{s}\right)_E = \frac{1}{4\pi}$$

Example: KK model

Einstein gravity D=4+n compactified on an n-torus of radius R.

Two regimes: Hawking thermal wavelength >> R $T R \ll 1$

Hawking thermal wavelength << R $T R \gg 1$

$$[\mathcal{D}_{KK}]^{y\ \ x}_{x\ \ y} \sim R^2 G_E \sum_{i=1}^{n} \sum_{k_i=1}^{\infty} \frac{\rho_{k_i}(q^2)}{R^2 q^2 - k_i^2}$$

$$m_{k_i} \sim k_i / R$$

$$\rho_{k_i}(q^2) \geq 0$$

$$m \sim 1/R \ll T$$

qR>>1

$$D_{KK} \sim qR \frac{G_E}{q^2} \gg \frac{G_E}{q^2}$$

KK contribution dominates ➔ $\eta \sim TR\, \eta_E \gg \eta_E$

Example: KK model

Entropy density remains the same!

$$s_{4+n} \sim \frac{A_{2+n}}{G_{4+n}} \sim \frac{r_h^{\;2} R^n}{G_4 R^n} \sim \frac{r_h^{\;2}}{G_4} \sim s_4$$

For TR >> 1:

$$\left(\frac{\eta}{s}\right)_{KK} \sim TR\left(\frac{\eta}{s}\right)_E \gg \left(\frac{\eta}{s}\right)_E$$

Example: 5D GB gravity

$$\mathcal{L} = \mathcal{R} + 12 + \lambda G_E \left[\mathcal{R}_{abcd}\mathcal{R}^{abcd} - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}^2\right]$$

$$\mathcal{L}_{kin} = \left[(1-8\lambda)\left(h_{ab}\Box h^{ab} - h\Box h\right) + 4\lambda \sum_{a,b\neq a}^{\{t,r,x,y,z\}} \mathcal{R}^{ab}{}_{ab}\left(h_{ab}\Box h^{ab} - h^a_a\Box h^b_b\right)\right]$$

$$-g_{tt} = g^{rr} = r^2\left[1 - \frac{r_h^4}{r^4}\right] \qquad g_{xx} = g_{yy} = g_{zz} = r^2 \qquad r_h = \pi T$$

On horizon (for example)

$$\mathcal{L}_{kin} = \left[h_{ab}\Box h^{ab} - h\Box h - 8\lambda\left(\sum_{a,b}^{\{x,y,z\}} + 4\sum_{a}^{\{r,t\}}\sum_{b}^{\{x,y,z\}}\right)\left(h_{ab}\Box h^{ab} - h^a_a\Box h^b_b\right)\right]$$

No contributions from rt gravitons => no corrections to entropy density

Example: 5D GB gravity

$$\mathcal{L} = \mathcal{R} + 12 + \lambda G_E \left[\mathcal{R}_{abcd}\mathcal{R}^{abcd} - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}^2\right]$$

xy gravitons on horizon
(similarly elsewhere)

$$\mathcal{L}_{kin} = \sum_{a,b\neq a}^{\{x,y,z\}} \left[(1-8\lambda)\, h_{ab}\Box h^{ab}\right]$$

$$\eta_{GB} = (1-8\lambda)\eta_E$$

Entropy density remains the same as in Einstein
Violation of the KSS bound ⇔ ghost contribution

Summary & conclusions

$$\eta / s \geq 1/4\pi$$

For extensions of Einstein gravity and their FT duals

- Unitarity
- Entropy $\Leftrightarrow$ Geometry

- Is it possible to do better?