Exercise 1

Compute the Hawking temperature of the AdS-Schwarzschild black hole.

$$ds^2 = -\frac{r^2}{R^2}(-f dt^2 + d\mathbf{x}^2) + \frac{R^2}{r^2 f}\,, \qquad f = 1 - \frac{r_0^4}{r^4} \tag{1}$$

by going to Euclidean space.

Exercise 2

Show that the entropy of an AdS-Schwarzschild black hole is proportional to T^3.

Exercise 3

Compute the retarded Green function the stress tensor in the shear channel within hydrodynamics. Assume $q = (\omega, 0, 0, k)$, compute $\langle T^{tx}T^{tx}\rangle$, $\langle T^{tx}T^{zx}\rangle$, $\langle T^{zx}T^{zx}\rangle$.

Hint:

First write down the hydrodynamic equation in curved space

$$\nabla_\mu T^{\mu\nu} = 0 \tag{2}$$

Now consider small metric perturbations around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{3}$$

In the shear channel, we turn on the following perturbations:

$$h_{tx} = h_{tx}(t, z), \qquad h_{zx} = h_{zx}(t, z) \tag{4}$$

The response of the medium on the perturbation is parameterized by

$$u^\mu = (1, u^x, 0, 0) \tag{5}$$

Write down the linearized equation for u^μ.

Exercise 4

At finite temperatures, what is the behavior of the imaginary part of $\langle T^{xy}T^{xy}\rangle$ at large ω? ($\omega \gg T$)?

Exercise 5

Define the correlation functions by differentiating the partition function with respect to the metric tensor,

$$\langle T^{\mu\nu}T^{\alpha\beta}\rangle = 4\frac{\delta^2 \ln Z}{\delta g_{\mu\nu}(x)\delta g_{\alpha\beta}} \tag{6}$$

Assuming that the partition function is diffeomorphism and Weyl invariant,

$$Z[g_{\mu\nu}] = Z[g_{\mu\nu} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu] \tag{7}$$

$$Z[g_{\mu\nu}(x)] = Z[e^{2\omega}(x) g_{\mu\nu}(x)] \tag{8}$$

derive the Ward identities

$$\nabla_\mu \langle T^{\mu\nu} T^{\alpha\beta} \rangle = \cdots \tag{9}$$

$$g_{\mu\nu} \langle T^{\mu\nu}(x) T^{\alpha\beta}(0) \rangle = \cdots \tag{10}$$

(reference: Policastro, Son, Starinets hep-th/02102220).

Exercise 6

The equation for the minimally coupled scalar in AdS-Schwarzschild metric is

$$\left(\frac{f}{z^3} \phi' \right)' + \frac{1}{z^3} \left(\frac{\omega^2}{f} - q^2 \right) \phi = 0 \tag{11}$$

Find the solution to this equation for small ω and q, with the boundary condition $\phi(r, x) \to \phi_0(x)$ at $r \to \infty$ and with incoming-wave boundary condition at the horizon.

Use this solution to compute the retarded Green's function of the operator dual to ϕ.

Exercise 7: Schrödinger symmetry

Suppose $\psi(t, \mathbf{x})$ satisfies the Schrödinger equation in free space. Show that one can perform the following transformations to get new solutions:

1. Galilean boosts: $t \to t$, $\mathbf{x} \to \mathbf{x} + \mathbf{v}t$,

i.e., show that there exists a another solution to the Schrödinger equation of the form:

$$\tilde{\psi}(t, \mathbf{x}) = C_{\mathbf{v}}(t, \mathbf{x}) \psi(t, \mathbf{x} + \mathbf{v}t) \tag{12}$$

and find $C(t, \mathbf{x})$, which is independent of the choice of ψ.

Similarly, find the new solutions related to

2. Proper conformal transformation:

$$t \to t' = \frac{t}{1 - \lambda t}, \qquad \mathbf{x} \to \mathbf{x}' = \frac{\mathbf{x}}{1 - \lambda t} \tag{13}$$