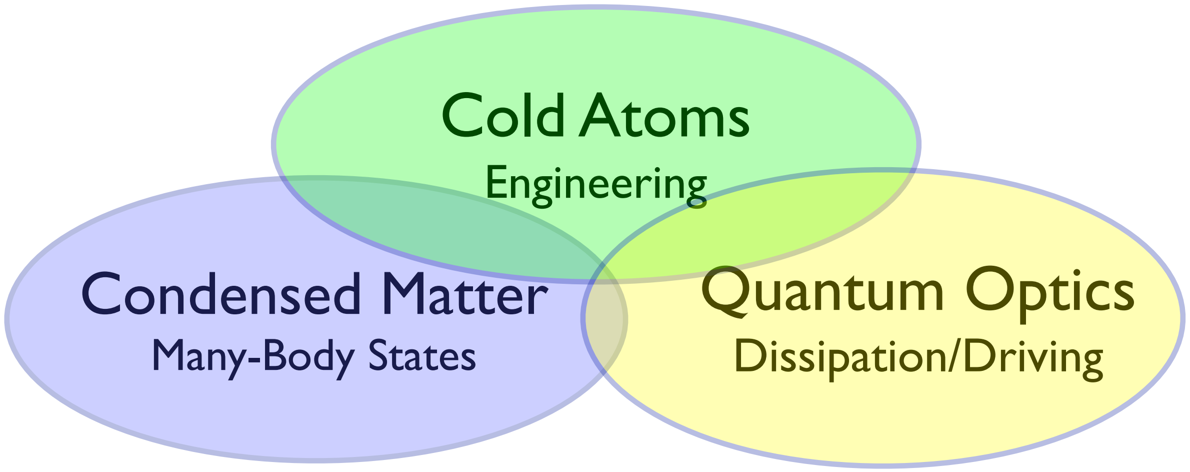
ASC Lectures, October 14/15 2009,
Arnold Sommerfeld Center, Munich

Quantum States and Phases in Dissipative Many-Body Systems with Cold Atoms



UNIVERSITY OF INNSBRUCK

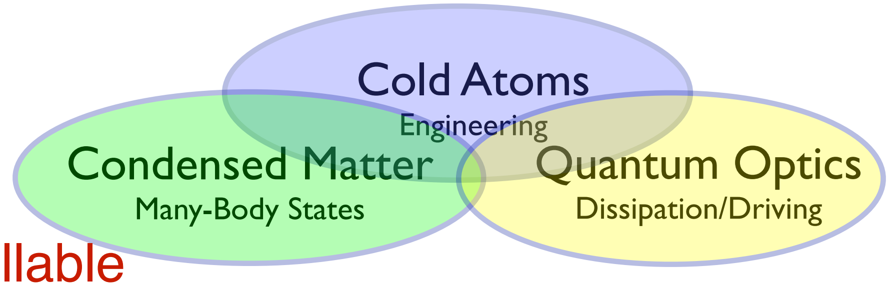
IQOQI
AUSTRIAN ACADEMY OF SCIENCES

SFB
Coherent Control of Quantum Systems

Sebastian Diehl

IQOQI Innsbruck

Lecture Overview



Main theme:
Dissipation can be turned into a favorable, controllable tool in cold atom many-body systems.

Part I: Quantum State Engineering in Driven Dissipative Many-Body Systems

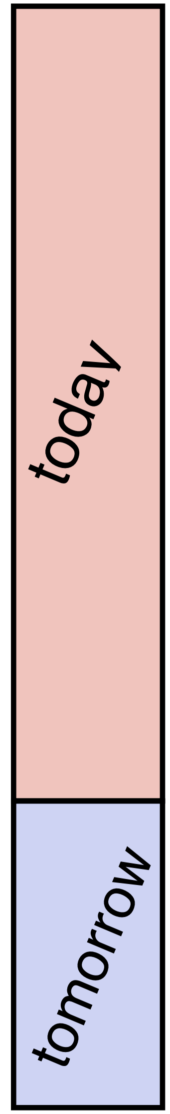
- Proof of principle: Driven Dissipative BEC
- Application I: Nonequilibrium phase transition from competing unitary and dissipative dynamics
- Application II: Cooling into antiferromagnetic and d-wave states of fermions

- Collaboration: H. P. Büchler, A. Daley, A. Kantian, B. Kraus, A. Micheli, A. Tomadin, W. Yi, P. Zoller

Part II: Dissipative Generation and Analysis of 3-Body Hardcore Models

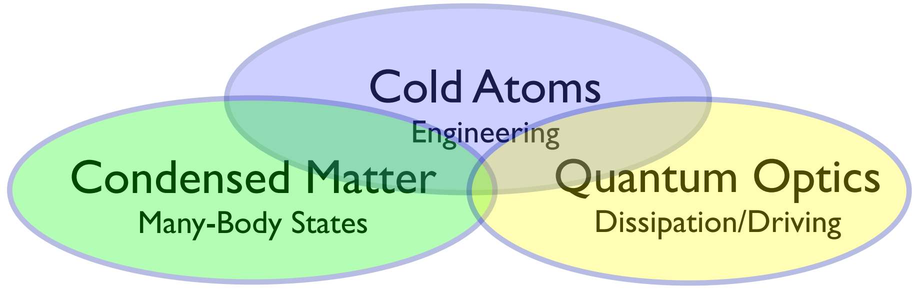
- Mechanism
- Experimental prospects, ground state preparation
- Application I: phase diagram for attractive 3-hardcore bosons
- Application II: atomic color superfluid for 3-component fermions

- Collaboration: M. Baranov, A. J. Daley, M. Dalmonte, A. Kantian, J. Taylor, P. Zoller

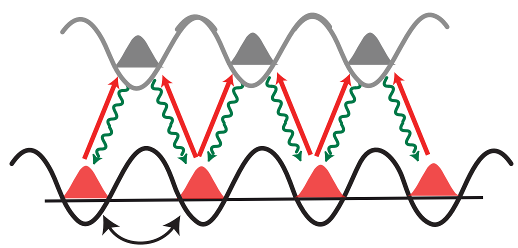


Outline Part I:

Quantum State Engineering in Driven Dissipative Many-Body Systems



- Introduction: Open Systems in Quantum Optics
- Driven Dissipative BEC:
  - Mechanism for pure DBEC: Many-Body Quantum Optics
  - Physical Implementation of DBEC: Reservoir Engineering, Bogoliubov bath
- Application I: Competition of unitary vs. dissipative dynamics
  - first look: weak interactions
  - strong interactions: nonequilibrium phase transition
- Application II: Targeting pure fermion states
  - An excited many-body state: η-condensate
  - Antiferromagnetic and d-wave fermion states



References:
SD, A. Micheli, A. Kantian, B. Kraus, H.P. Büchler, P. Zoller, Nature Physics 4, 878 (2008);
B. Kraus, SD, A. Micheli, A. Kantian, H.P. Büchler, P. Zoller, Phys. Rev. A 78, 042307 (2008)

F. Verstraete, M. Wolf, I. Cirac, Nature Physics 5, 633 (2009)

Quantum State Engineering in Many-Body Systems

- **thermodynamic equilibrium**
  - standard scenario of condensed matter & cold atom physics

$$H\,|E_g\rangle = E_g\,|E_g\rangle \qquad \rho \sim e^{-H/k_B T} \xrightarrow{T\to 0} |E_g\rangle\langle E_g|$$

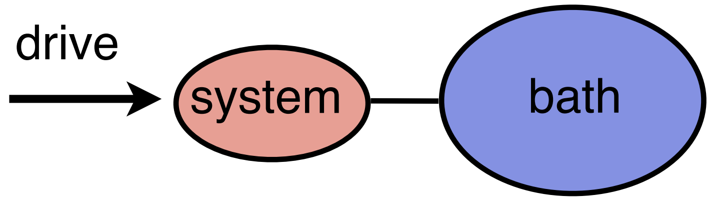
Hamiltonian (many body)

cooling to ground state

Hamiltonian Engineering:

✓interesting ground states
✓quantum phases

- **driven / dissipative dynamical equilibrium**
  - quantum optics



$$\frac{d\rho}{dt} = -i\,[H,\rho] + \mathcal{L}\rho$$

competing dynamics

master equation

$$\rho(t) \xrightarrow{t\to\infty} \rho_{ss}$$

mixed state

$$\stackrel{!?}{=} |D\rangle\langle D|$$

pure state (“dark state”)

steady state

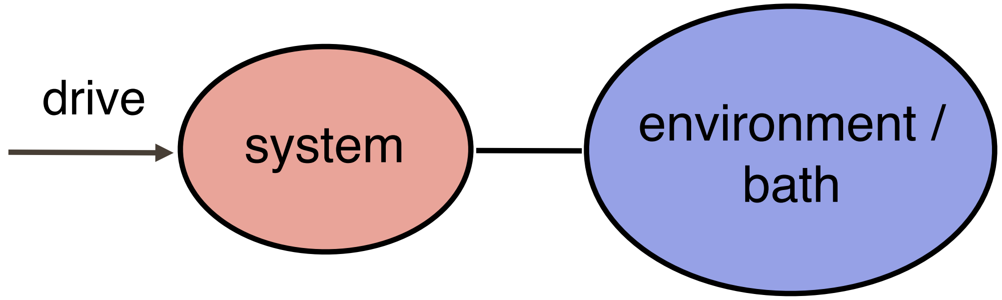
Liouvillian Engineering:

✓many body pure states / driven quantum phases
✓mixed states ~ “finite temperature”
✓useful an interesting fermion states

Open Quantum Systems

Open Quantum Systems

$$H = H_S + H_B + H_{\text{int}}$$



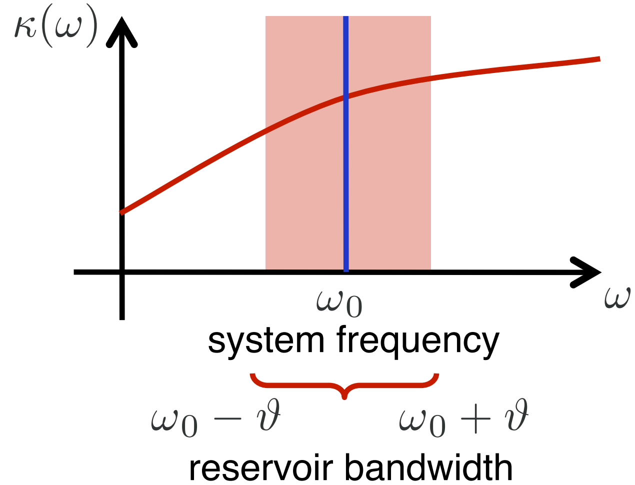
$$H_B = \int d\omega\, \omega b_\omega^\dagger b_\omega$$ continuum bath of harmonic oscillators

$$H_{\text{int}} = i \int d\omega \kappa(\omega) \left[b_\omega^\dagger J - b_\omega J^\dagger \right]$$

quantum jump operators
polynomial in system operators

linear bath operator coupling to the system

$\kappa(\omega) \to \kappa_0$



Three approximations:

(1) Born approximation: $\kappa(\omega)/\omega_0 \ll 1$

(2) Markov approximation: $\kappa(\omega) \approx$ const. $\Rightarrow \kappa(t-t') \sim \delta(t-t')$

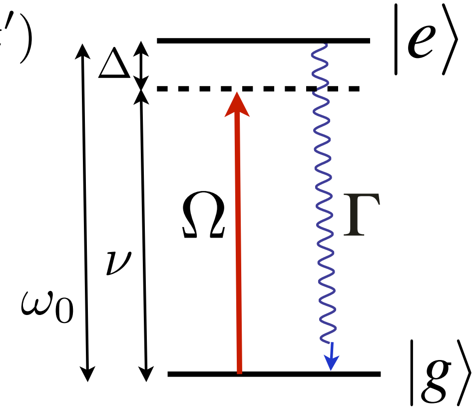
(3) Rotating wave approximation: $\dfrac{\omega_0 - \nu}{\omega_0 + \nu} \ll 1$

$$\omega_0 - \nu = \Delta$$

detuning

$$J_\alpha = |g\rangle\langle e| = \sigma^-$$

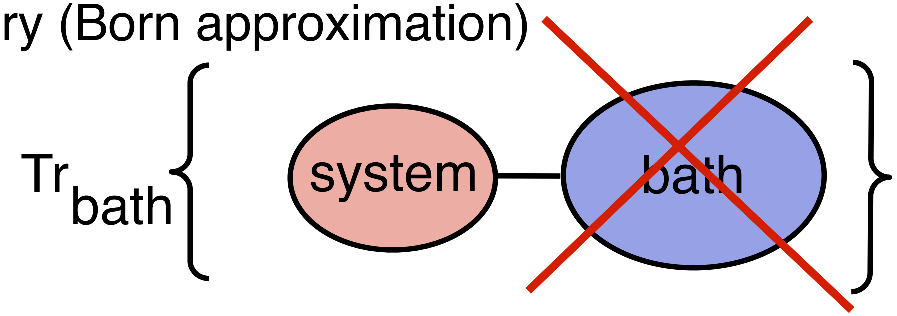
$$H = (|e\rangle, |g\rangle) \begin{pmatrix} \Delta & \Omega \\ \Omega & 0 \end{pmatrix} \begin{pmatrix} \langle e| \\ \langle g| \end{pmatrix}$$



Open Quantum Systems

$$\partial_t \rho_{\text{tot}} = -i[H_S + H_B + H_{\text{int}}, \rho_{\text{tot}}]$$

➡ Eliminate bath degrees of freedom in second order time-dependent perturbation theory (Born approximation)



quantum jump operators

effective system dynamics from Master Equation (zero temperature bath)

$$\partial_t \rho = -i[H_S, \rho] + \underbrace{\kappa \sum_\alpha J_\alpha \rho J_\alpha^\dagger - \tfrac{1}{2}\{J_\alpha^\dagger J_\alpha, \rho\}}_{\mathcal{L}[\rho]}$$

$\mathcal{L}[\rho]$ Liouvillian operator in Lindblad form

- Structure: second order perturbation theory
- mnemonic: norm conservation $\partial_t \mathrm{tr}\rho = 0$
- but: $\partial_t \mathrm{tr}\rho^2 \neq 0$

pure state: $\mathrm{tr}\rho = \mathrm{tr}\rho^2 = 1$

$\Rightarrow \mathrm{tr}\rho^2$ -- "purity"

➡ Purity is not conserved

➡ go for $\partial_t \mathrm{tr}\rho^2 < 0$

Open Quantum Systems

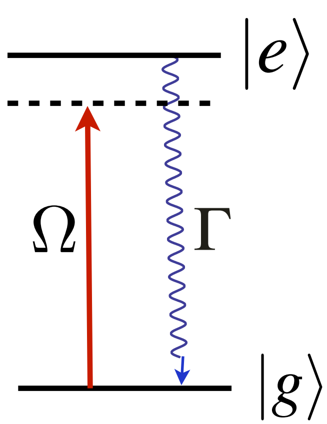
- Stochastic Interpretation: Quantum Jumps

$$\partial_t \rho = -i[H,\rho] + \kappa \sum_\alpha J_\alpha \rho J_\alpha^\dagger - \tfrac{1}{2}\{J_\alpha^\dagger J_\alpha, \rho\}$$

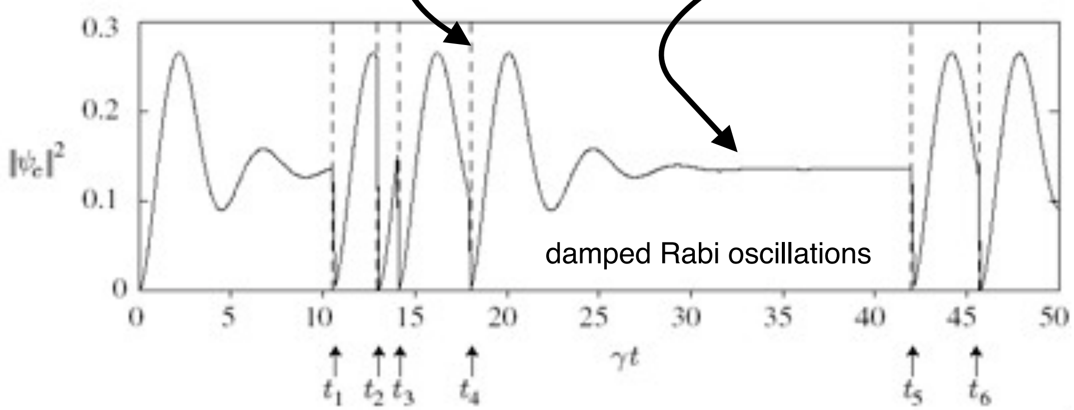
$$= -i[H_{\text{eff}},\rho]^* + \kappa \sum_\alpha J_\alpha \rho J_\alpha^\dagger \qquad H_{\text{eff}} = H - i\kappa/2 \sum_\alpha J_\alpha^\dagger J_\alpha$$

quantum jump operators

decay



$J_\alpha = |g\rangle\langle e| = \sigma^-$



time evolution of upper state population of driven dissipative two-level system (single run)

- Averaging over "quantum trajectories" generates all correlation functions

➡ Engineer the jump operators J_α

$$[A,B]^* := AB - B^\dagger A^\dagger$$

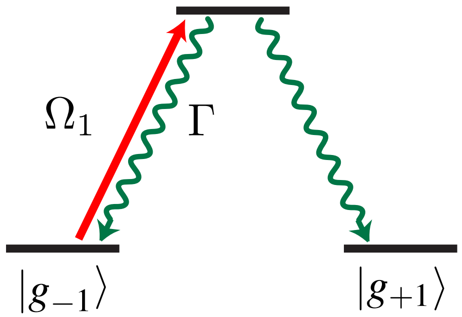
Driven Dissipative BEC

Dark States in Quantum Optics

- Goal: pure BEC as steady state solution, independent of initial density matrix:

$$\rho(t) \longrightarrow |BEC\rangle\langle BEC| \text{ for } t \to \infty$$

- Such situation is well-known quantum optics (three level system): optical pumping
  (Kastler, Aspect, Cohen-Tannoudji; Kasevich, Chu; ...)



$$\rho(t) \xrightarrow{t\to\infty} |g_+\rangle\langle g_+|$$

➡ Driven dissipative dynamics "purifies" the state

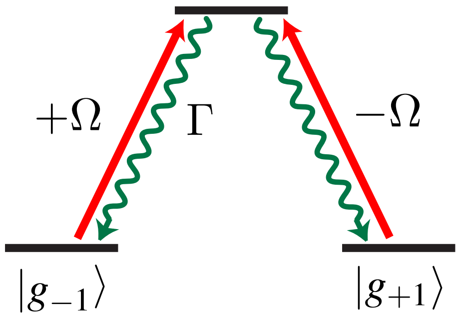
➡ $|g_+\rangle$ is a "dark state" decoupled from light

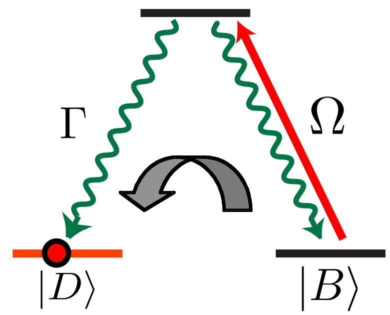
$$c_\alpha|g_+\rangle = 0$$

➡ Dark state is Eigenstate of jump operators with zero Eigenvalue
➡ Time evolution stops when system is in DS: pure steady state

An Analogy

- Λ-system: three electronic levels (VSCPT by Aspect, Cohen-Tannoudji; Kasevich, Chu)

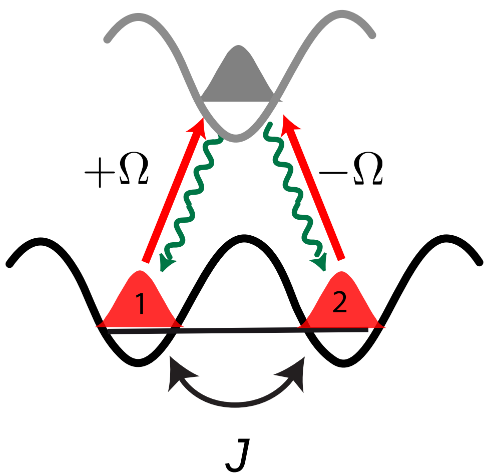




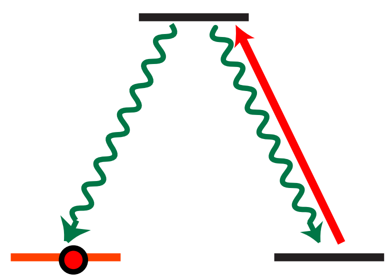
dark state $\quad$ bright state

$|D\rangle \sim |g_{+1}\rangle + |g_{-1}\rangle \quad |B\rangle \sim |g_{+1}\rangle - |g_{-1}\rangle$

- 1 atom on 2 sites



~ dissipative Josephson junction



$(a_1^\dagger + a_2^\dagger)\,|\mathrm{vac}\rangle \quad (a_1^\dagger - a_2^\dagger)\,|\mathrm{vac}\rangle$

symmetric "in-phase" $\quad$ anti-symmetric "out-of-phase"

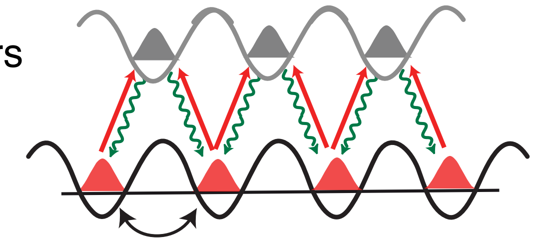
pumping into symmetric state

➡ "phase locking": like a BEC

Driven Dissipative lattice BEC

- Consider jump operator:

nearest neighbours



$$c_{ij} = (a_i^\dagger + a_j^\dagger)(a_i - a_j)$$

(1) BEC state is a dark state: $|BEC\rangle = \frac{1}{N!}\Big(\sum_\ell a_\ell^\dagger\Big)^N |vac\rangle$

$$c_{ij}|BEC\rangle = 0 \;\forall i \qquad (a_i - a_j)\sum_\ell a_\ell^\dagger = \sum_\ell a_\ell^\dagger (a_i - a_j) + \sum_\ell \delta_{i\ell} - \delta_{j\ell}$$

(2) BEC state is the only dark state:

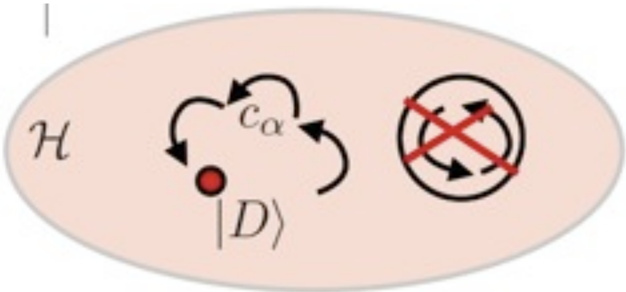
- $(a_i^\dagger + a_j^\dagger)$ has no eigenvalues
- $(a_i - a_j)$ has unique zero eigenvalue

$$(a_i - a_j)\;\forall i \longrightarrow (1 - e^{i\mathbf{q}\mathbf{e}_\lambda})a_{\mathbf{q}}\;\forall \mathbf{q}$$

Driven Dissipative lattice BEC

(3) Uniqueness: IBEC> is the only stationary state (sufficient condition)

If there exists a stationary state which is not a dark state, then there must exist a subspace of the full Hilbert space which is left invariant under the set $\{c_\alpha\}$



(4) Compatibility of unitary and dissipative dynamics

$|D\rangle$ be an eigenstate of H, $H|D\rangle = E|D\rangle$

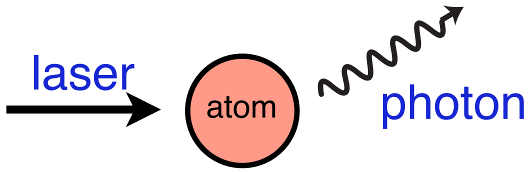
$$\rho(t) \xrightarrow{t\to\infty} |D\rangle\langle D|$$

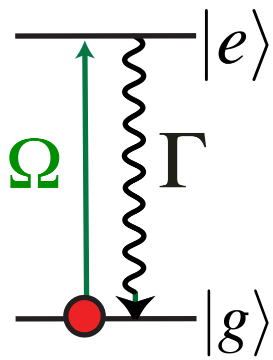
- Long range order in many-body system from quasi-local dissipative operations
- Uniqueness: Final state independent of initial density matrix
- Criteria are general: jump operators for AKLT states (spin model), eta-states (fermions), d-wave states (fermions, next lecture)

A. Griessner, A. Daley et al. PRL 2006; NJP 2007
(noninteracting atom)

Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission





- reservoir: vacuum modes of the radiation field (T=0)
- $\omega \sim 2\pi \times 10^{14} Hz$

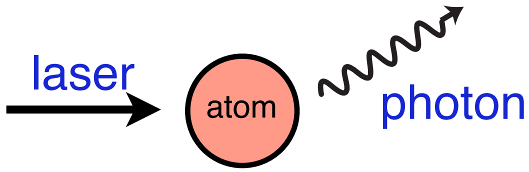
Quantum optics ideas/techniques

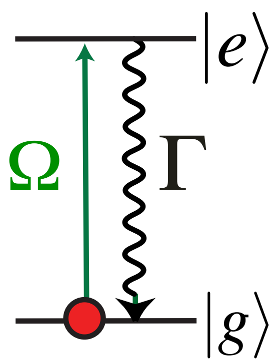
- much lower energy scales...

A. Griessner, A. Daley et al. PRL 2006; NJP 2007
(noninteracting atom)

Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission





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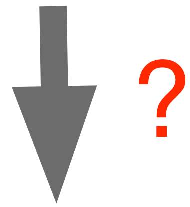


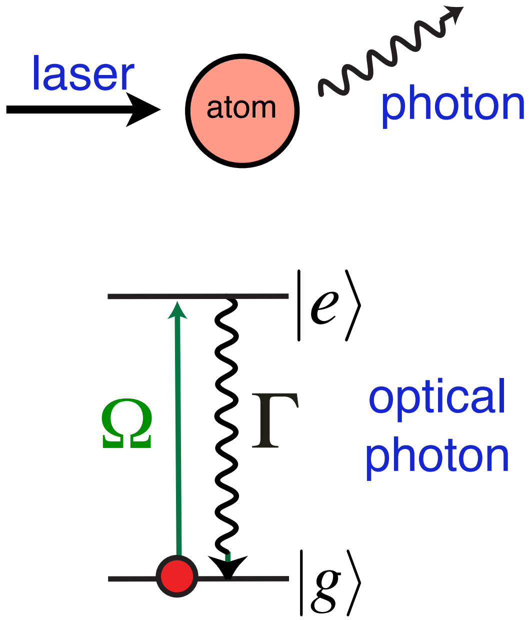
(many body) cold atom systems

- much lower energy scales...

A. Griessner, A. Daley et al. PRL 2006; NJP 2007
(noninteracting atom)

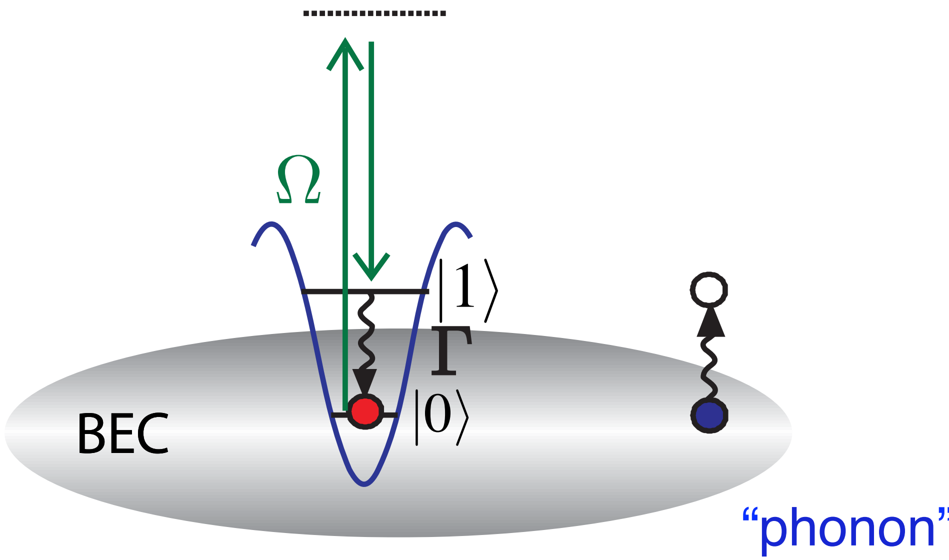
Physical Realization: Reservoir Engineering

- driven two-level atom + spontaneous emission



- reservoir: vacuum modes of the radiation field (T=0)
- $\omega \sim 2\pi \times 10^{14} Hz$

- trapped atom in a BEC reservoir



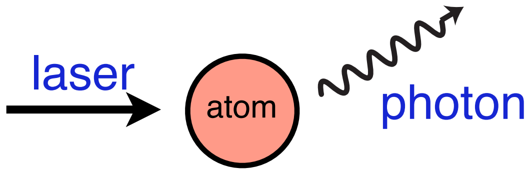
laser assisted atom + BEC collision

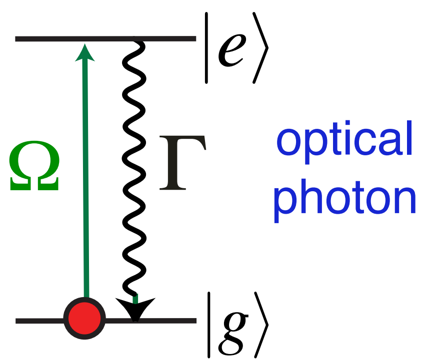
$\omega_{bd} \sim 2\pi \times kHz$

A. Griessner, A. Daley et al. PRL 2006; NJP 2007
(noninteracting atom)

Physical Realization: Reservoir Engineering

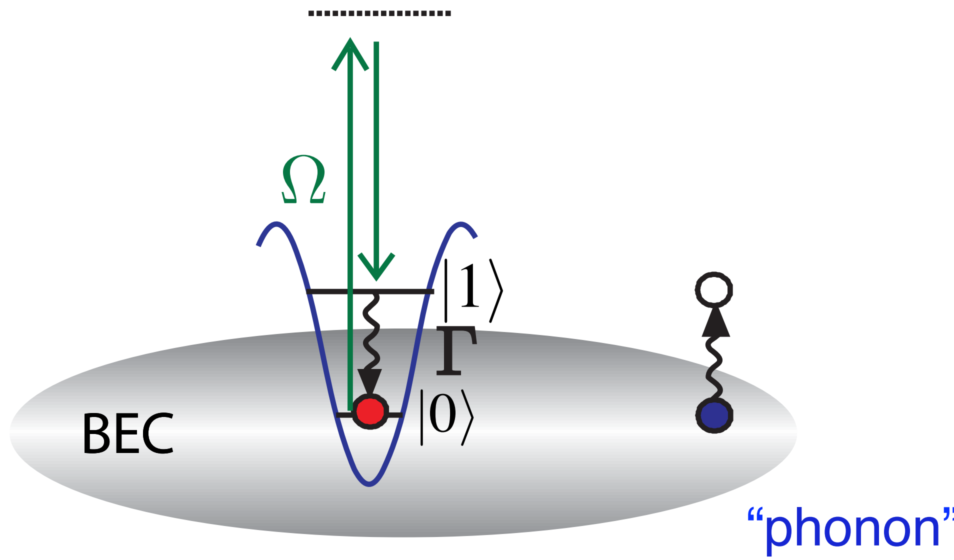
- driven two-level atom + spontaneous emission





- reservoir: vacuum modes of the radiation field (T=0)
- $\omega \sim 2\pi \times 10^{14} Hz$

- trapped atom in a BEC reservoir

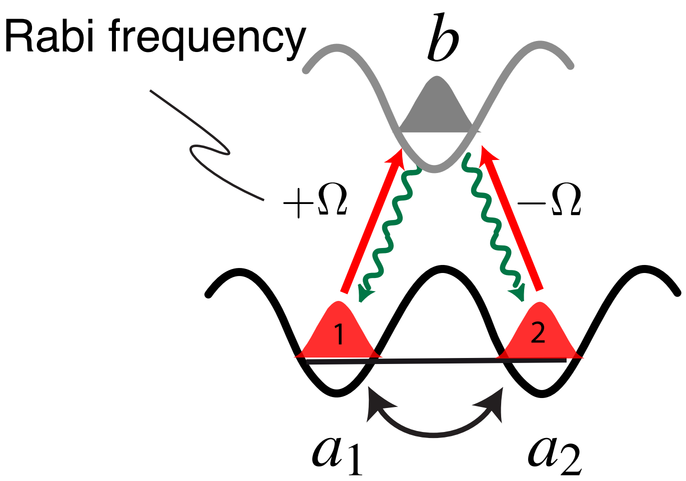


laser assisted atom + BEC collision

- reservoir: Bogoliubov excitations of the BEC (at temperature T)
- $\omega_{bd} \sim 2\pi \times kHz$

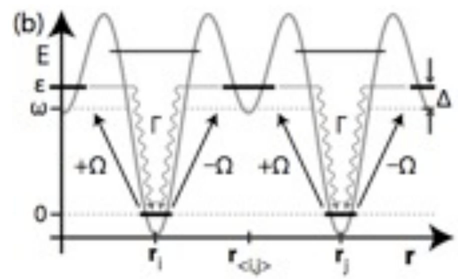
Physical Realization

Schematic

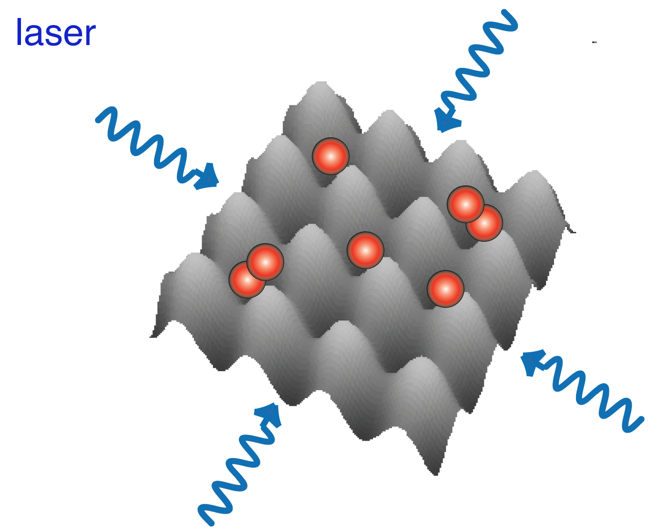


In practice

- level structure: optical superlattice



- coherent excitation: Raman laser



(1) Coherent excitation with opposite sign of Rabi frequency

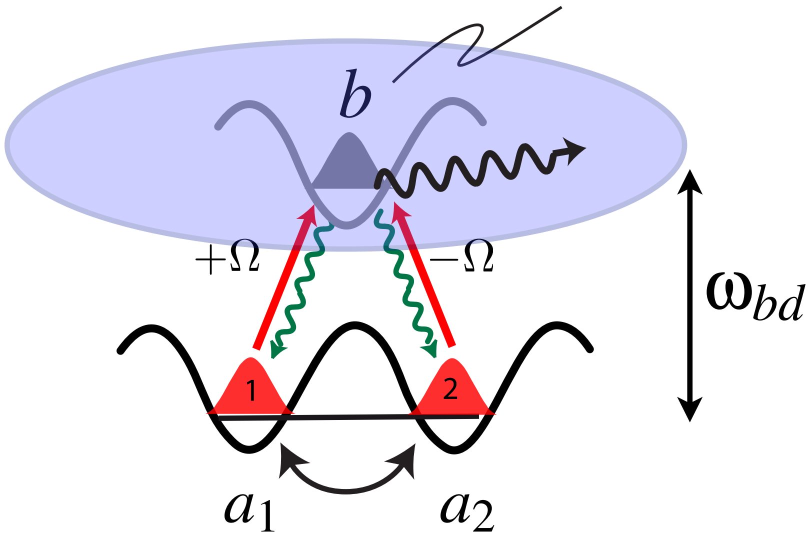
$$\Omega\, b^{\dagger}(a_1 - a_2) + h.c.$$

antisymmetric

$$c_{ij} = (a_i^{\dagger} + a_j^{\dagger})(a_i - a_j)$$

Physical Realization

Schematic



(2) Dissipative decay back:

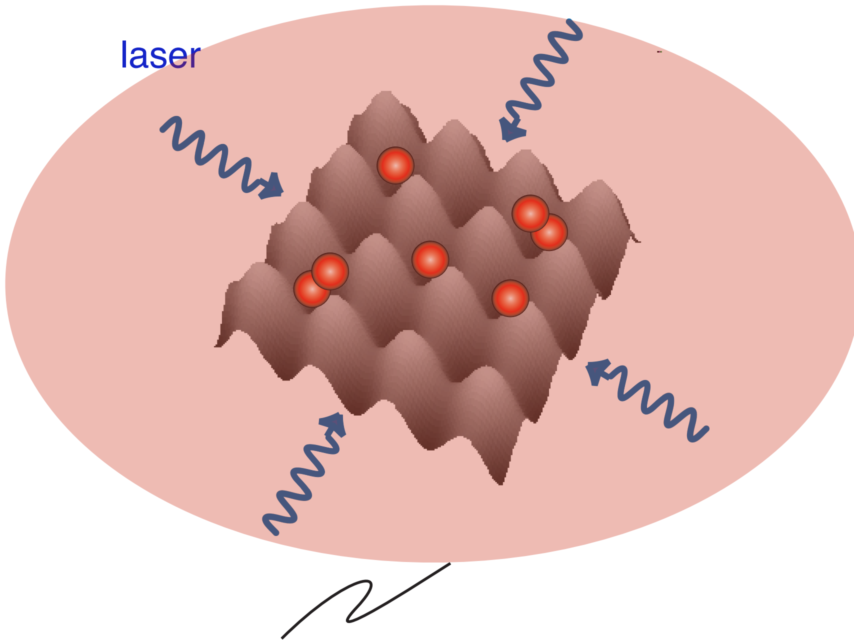
coupling of upper level to reservoir

$$\kappa(a_1^\dagger + a_2^\dagger) b \sum_{\mathbf{k}} (r_{\mathbf{k}} + r_{\mathbf{k}}^\dagger)$$

symmetric

$$c_{ij} = (a_i^\dagger + a_j^\dagger)(a_i - a_j)$$

In practice



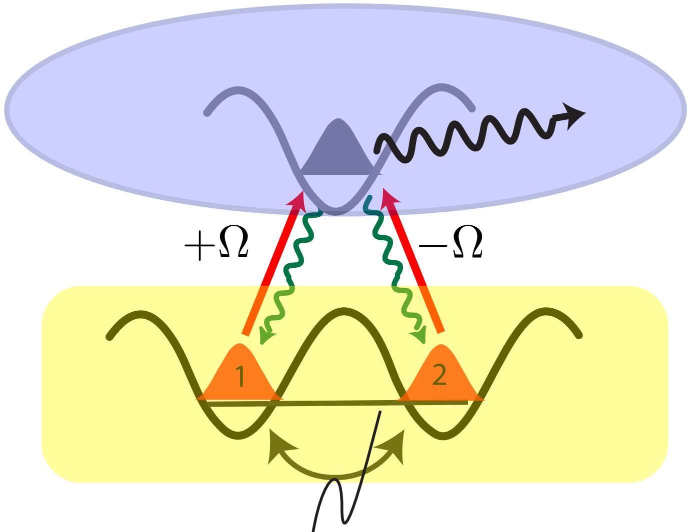
BEC = reservoir of Bogoliubov excitations

➡ $T_{BEC} \ll \omega_{bd}$ effective

zero temperature reservoir

- coupling to system: interspecies interaction
- short coherence length in bath provides quasi-local dissipative processes, but not mandatory for our setup to work

Physical Realization

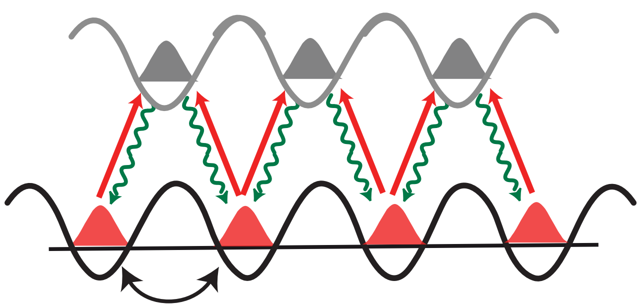


(3) adiabatic elimination of auxiliary level, trace out the bath

Effective single band jump operators

$$c_{12} = (a_1^\dagger + a_2^\dagger)(a_1 - a_2)$$

Many sites: Array of dissipative junctions



Comments:

- Long range phase coherence from quasi-local dissipative operations
- - Coherent drive: locks phases

  - Dissipation: randomizes

  - Conspiracy: purification
- The coherence of the driving laser is mapped on the matter system
- Setting is therefore robust

Competition of unitary vs. dissipative dynamics

Effects of finite interactions

$$H = -J \sum_{<i,j>} a_i^\dagger a_j + U \sum_i a_i^{\dagger 2} a_i^2$$

dissipative dynamics favors pure BEC state

interacting Hamiltonian dynamics not compatible

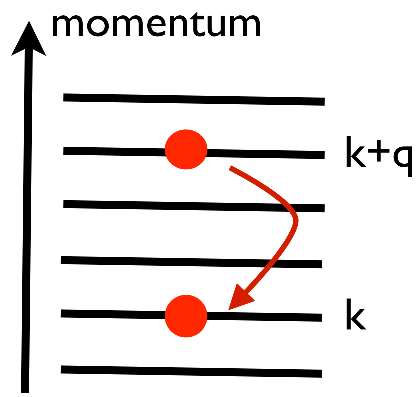
Competition

$$\frac{d\rho}{dt} = -i\left[H, \rho\right] + \mathcal{L}\rho$$

treating interactions in

- weak coupling
  - 3D: true (depleted) condensate, fixed phase: Bogoliubov theory
  - 1,2D: phase fluctuations destroy long range order: Luttinger theory
- Strong coupling, 3D
  - mixed state Gutzwiller Ansatz

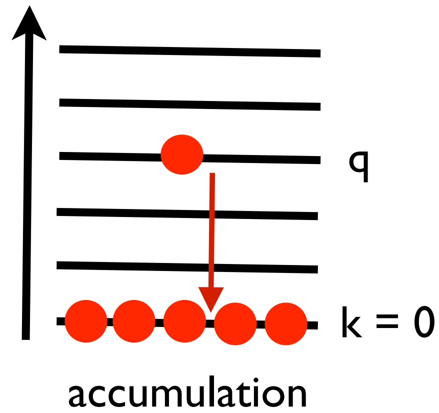
Weak Coupling: Linearized jump operators



- momentum space jump operators are nonlocal nonlinear objects

$$c_{\mathbf{q},\lambda} = \frac{1}{M^{d/2}} \sum_{\mathbf{k}} (1 + \mathrm{e}^{\mathrm{i}\mathbf{k}\mathbf{e}_\lambda})(1 - \mathrm{e}^{-\mathrm{i}(\mathbf{k}+\mathbf{q})\mathbf{e}_\lambda}) a^\dagger_{\mathbf{k}} a_{\mathbf{k}+\mathbf{q}}$$

- In a linearized theory the reduce to (any dimension)



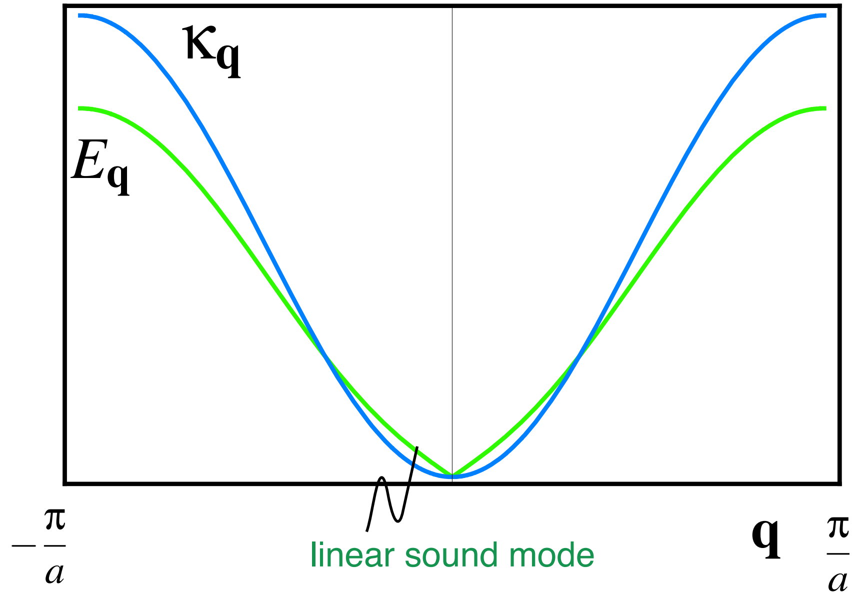
$$c_{\mathbf{q},\lambda} = f_{\mathbf{q},\lambda} a_{\mathbf{q}} \qquad f_{\mathbf{q},\lambda} = 2\sqrt{n}(1 - e^{-\mathrm{i}\mathbf{q}\mathbf{e}_\lambda})$$

- Interpretation:
    - bosonic mode operators: depopulation of momentum q in favor of condensate
    - zero mode explicit: $f_{\mathbf{q}=0,\lambda} = 0$
    - lead to momentum dependent decay rate

$$\kappa_{\mathbf{q}} = \sum_\lambda \kappa |f_{\mathbf{q},\lambda}|^2 \sim \mathbf{q}^2$$

Many-Body Master Equation

- Interpretation: How close are we to the GS of the Hamiltonian?
  - Diagonalize H
  - consider equation for single mode



Bogoliubov / hydrodynamic excitation

$$\partial_t \rho = \boxed{-i\frac{E}{2}[d^\dagger d, \rho]}$$

$$+2\kappa(\boxed{u^2 d\rho d^\dagger} + \boxed{v^2 d^\dagger \rho d} - \boxed{uv(d^\dagger \rho d^\dagger + d\rho d)})$$

+ anticommutator term

“cooling” “heating” squeezing

$v_{\mathbf{q}}^2,\ u_{\mathbf{q}}^2 = v_{\mathbf{q}}^2 + 1$ generalized Bogoliubov coefficients

$N, \qquad N+1$ cf. thermal reservoir

➡ Intrinsic heating/cooling, though reservoir is at T = 0

Characterization of Steady State: Density Operator

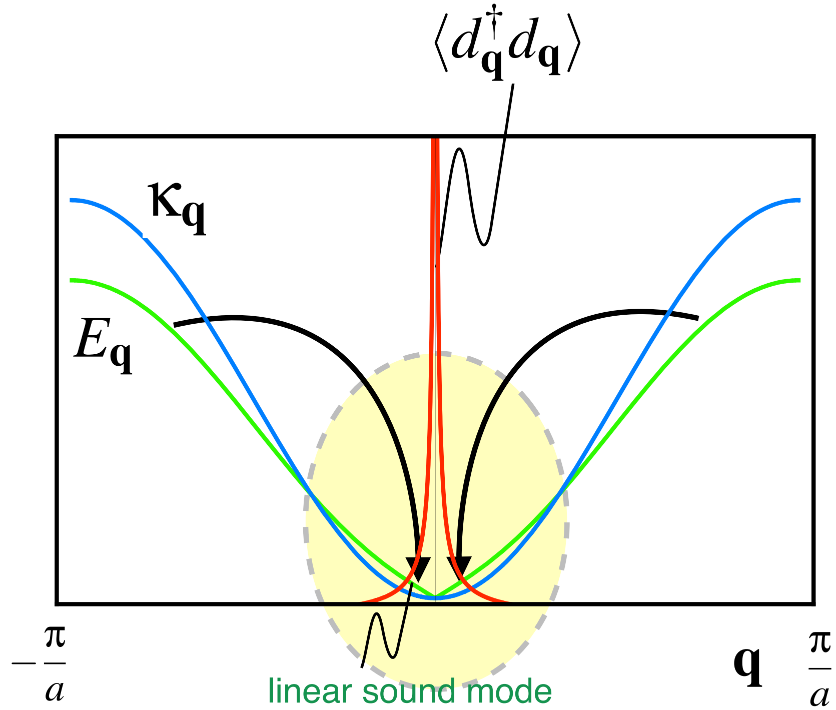
- linearized ME exactly solvable: Gaussian density operator for each mode expressible as

$$\rho_{\mathbf{k}} = \exp\left(-\beta_{\mathbf{k}} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}}\right)$$

with squeezed operators b (Bogoliubov transformation)

➡ mixed state with

$$\coth^2(\beta_{\mathbf{k}}/2) = \frac{\kappa_{\mathbf{k}}^2 + (\epsilon_{\mathbf{k}} + Un)^2}{\kappa_{\mathbf{k}}^2 + E_{\mathbf{k}}^2}$$



- at low momenta, resemblance to thermal state:

$$\beta_{\mathbf{k}} \approx \frac{E_{\mathbf{k}}}{T_{\text{eff}}}, \quad T_{\text{eff}} = \frac{Un}{2}$$

‣ role of temperature played by interaction

Correlations in various dimension: 3D

- Steady state: condensate depletion:

$$n_{\mathrm{D}} = n - n_0 = \frac{1}{2}\int \frac{d\mathbf{q}}{v_0} \frac{(Un)^2}{\kappa_q^2 + E_q^2}$$

  - small depletion justifies Bogoliubov theory
  - squeezing and mixing effects tied to interaction strength (unlike th. equilibrium)

- Approach to the steady state:

$$n_{0,eq} - n_0(t) \sim \sqrt{\frac{Un}{8J}}\frac{1}{2\kappa n}\, t^{-1}$$

  - power-law: Many-body effect due to mode continuum
  - sensitive probe to interactions: cf. for noninteracting system

$$n_{0,eq} - n_0(t) \sim t^{-3/2}$$

  - universal at late times

Correlations in various dimension: 1/2D

- Steady State: quasi-condensates in low "temperature" phase

$$\langle a_x^\dagger a_0 \rangle \sim \langle \exp i(\phi_x - \phi_0) \rangle \sim \begin{cases} \mathrm{e}^{-\frac{T_{\mathrm{eff}}}{8Jn} x}, & d = 1 \\ (x/x_0)^{-T_{\mathrm{eff}}/4T_{\mathrm{KT}}}, & d = 2 \end{cases}$$

$$T_{\mathrm{KT}} = \pi J n \gg T_{\mathrm{eff}} \qquad T_{\mathrm{eff}} = Un/2 \qquad x_0 = 2\kappa n (T_{\mathrm{eff}} J)^{-1/2}$$

Kosterlitz-Thouless temperature of 2D quasi-condensate

Dissipative coupling: only sets cutoff scale

- steady state well understood as thermal Luttinger liquid
- similar results for temporal correlations (from ME via quantum regression theorem)
- weak effect of dissipation on phase fluctuations:

$$E_{\mathbf{q}} \sim |\mathbf{q}|, \kappa_{\mathbf{q}} \sim \mathbf{q}^2$$

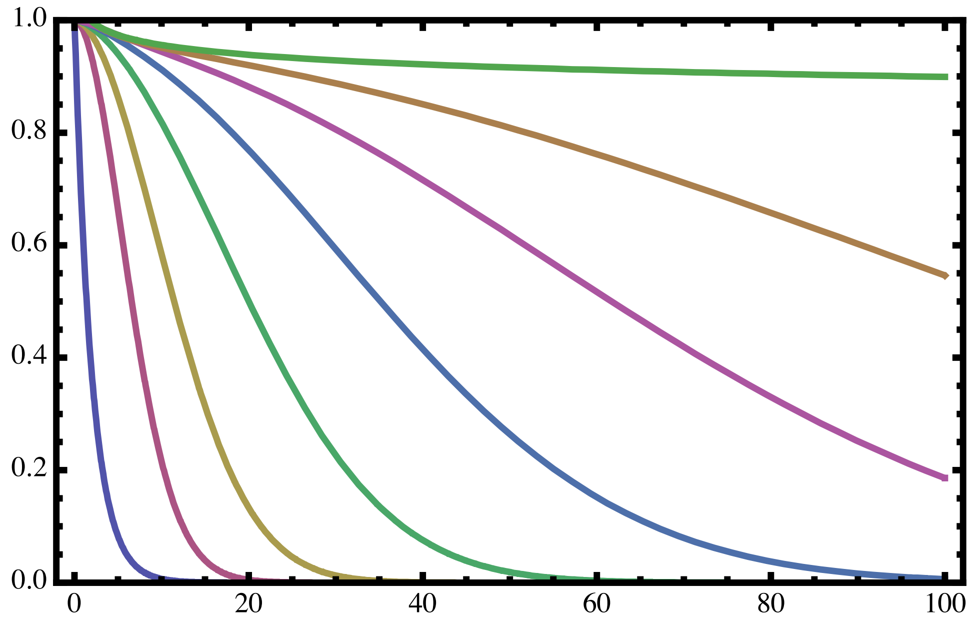
2D: Real Time Evolution

- Buildup of spatial correlations from disordered state

$$\Psi_t(x,0) \sim \begin{cases} e^{-|x|/\xi} & t = 0 \\ (x/x_0)^{-\frac{T_{\rm eff}}{4T_{\rm KT}}}\, e^{-\frac{x^2}{4\xi\sqrt{\pi\kappa n t}}} & t \longrightarrow \infty \end{cases}$$

broadening of Gaussian governed by time-dependent length scale

$$x_t = 2(\pi\xi^2\kappa n t)^{1/4}$$



with A. Tomadin

Strong Coupling: Nonequilibrium Phase Transition

- Analogy to Mott insulator / Superfluid quantum phase transition :

• enhancement of superfluidity:	Hopping J	driven dissipation κ
• suppression of superfluidity:	interaction U	interaction U

➡ Expect phase transition as function of J/U κ/U

- Differences:

➡ Competition of two unitary evolutions vs. competition of unitary and dissipative evolution

✓ phase transition (temperature T)

✓ quantum phase transition (g)

Reminder: Mott Insulator-Superfluid Phase Transition

$$H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j - \mu \sum_i \hat{n}_i + \tfrac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

- Hopping J favors delocalization in real space:
- Condensate (local in momentum space!)
- Fixed condensate phase: Breaking of phase rotation symmetry

$$\langle b_i \rangle \sim e^{\mathrm{i}\varphi}$$

- Interaction U favors localization in real space for integer particle numbers:
- Mott state with quantized particle no.
- no expectation value: phase symmetry intact (unbroken)

➡ Competition gives rise to a quantum phase transition as a function of

$$U/J$$

Reminder: Gutzwiller Ansatz

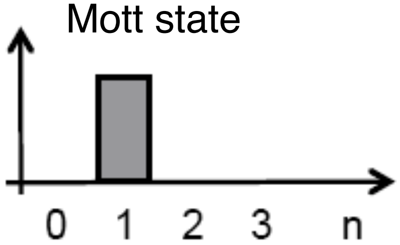
- Interpolation scheme encompassing the full range J/U.
  - Main ingredient: product wave function ansatz

$$|\psi\rangle = \prod_i |\psi\rangle_i, \quad |\psi\rangle_i = \sum_n f_n^{(i)} |n\rangle_i, \quad {}_i\langle\psi|\psi\rangle_i = 1\,\forall i$$

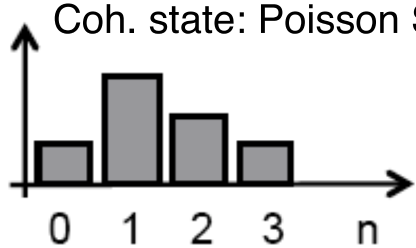
complex amplitudes　　wave function normalization

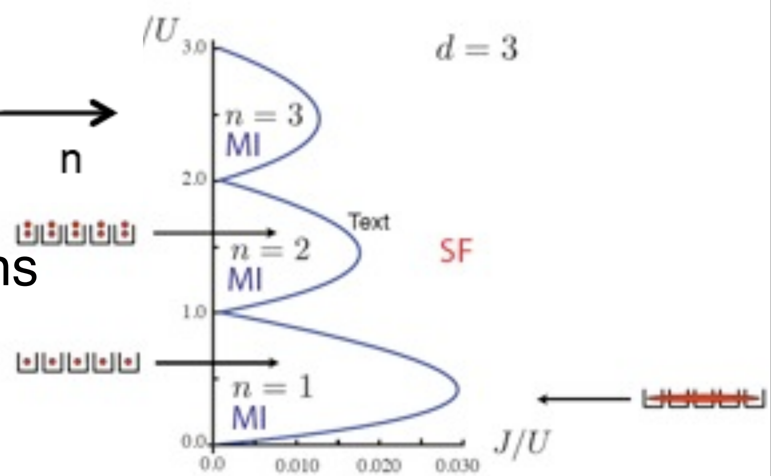
  - Limiting cases (homogeous, drop site index, amplitudes chosen real):
    - Mott state with particle number m: $f_n = \delta_{n,m}$
    - coherent state: $f_n = \sqrt{N/n!}e^{-N/2}$

Mott state



Coh. state: Poisson Statistics





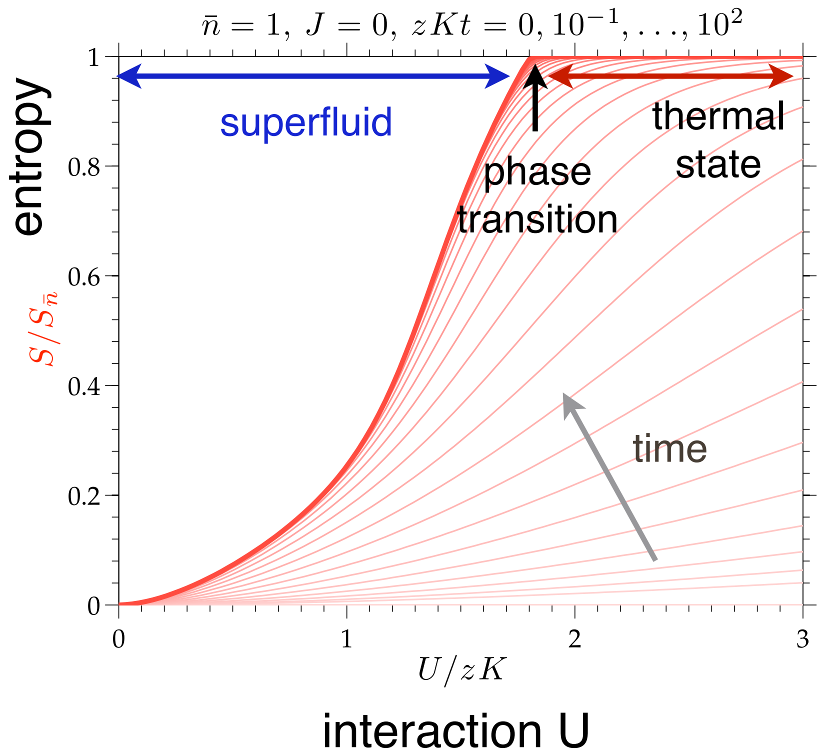
- Validity: approximation neglects all spatial correlations
  - becomes exact in infinite dimensions
  - reasonable in d=2,3 (T=0)

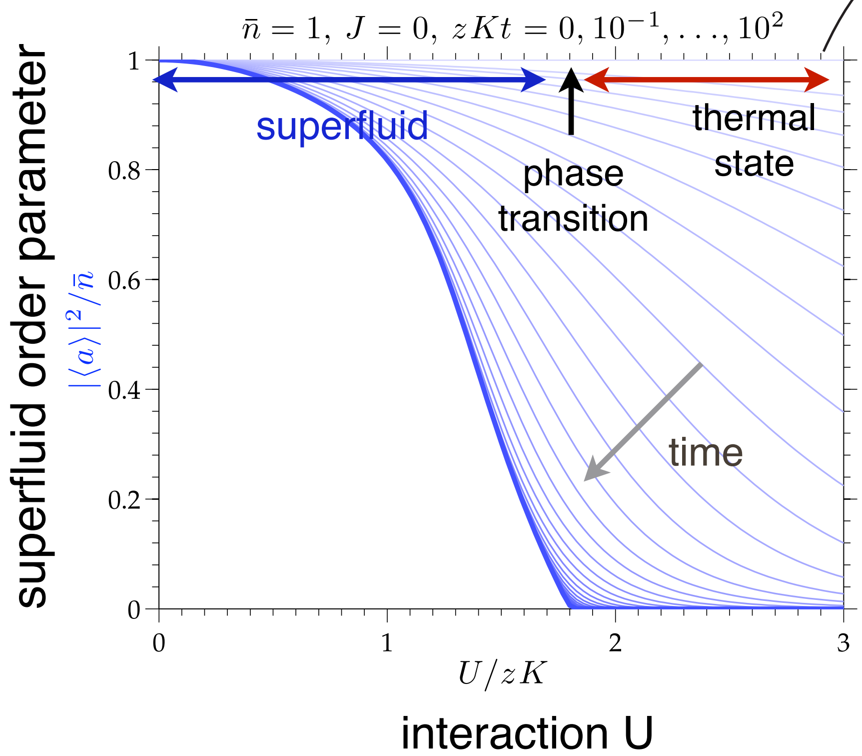
Some slides taken out

Driven Dissipative Phase Transition

$$\rho_{n,n} = \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}}$$

- Dynamic generation of the phase transition from initial coherent state

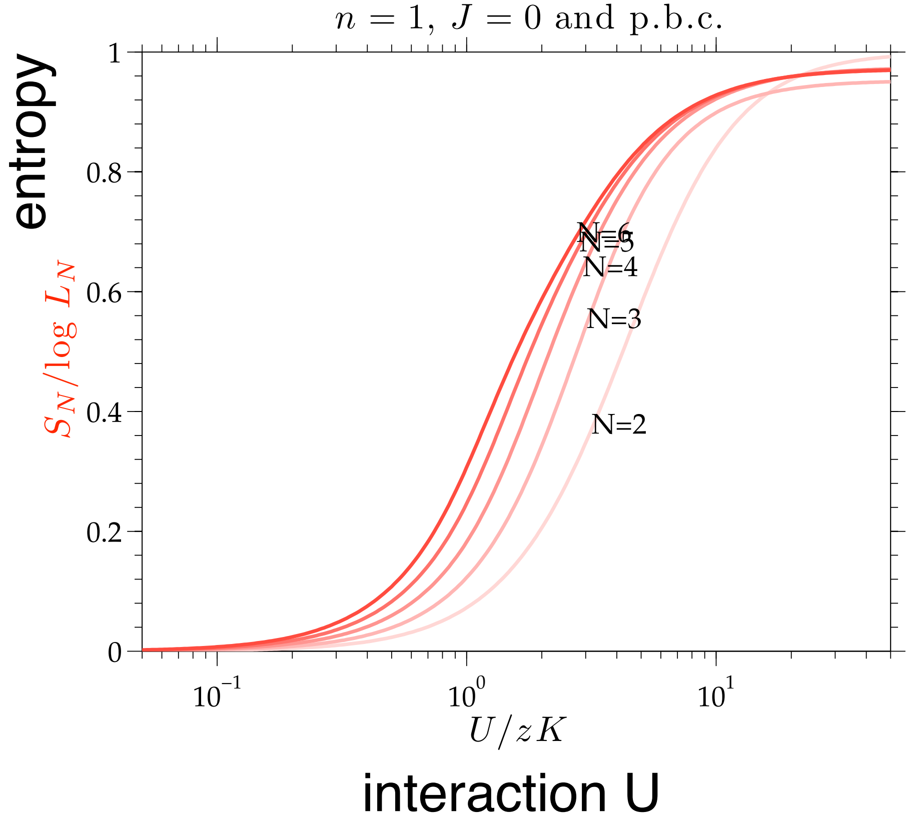


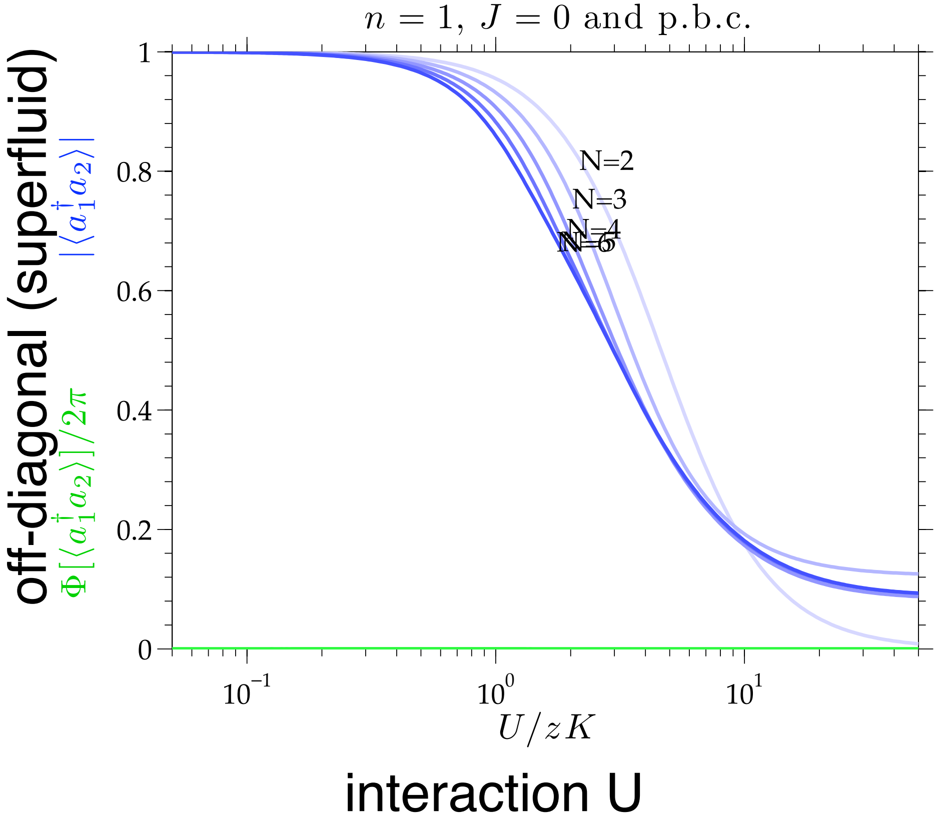


- $U \rightarrow 0$ pure coherent state solution
- Phase transition: Non-analyticity develops for $t \longrightarrow \infty$
- above critical point: thermal state: “fixed temperature” given by mean particle density N; no other scale appears
- No signatures of Mott physics due to strong mixing effect of U: unlike Bose-Hubbard case of two unitary tendencies at T=0:

A. Tomadin

Exact calculations for N=6 sites

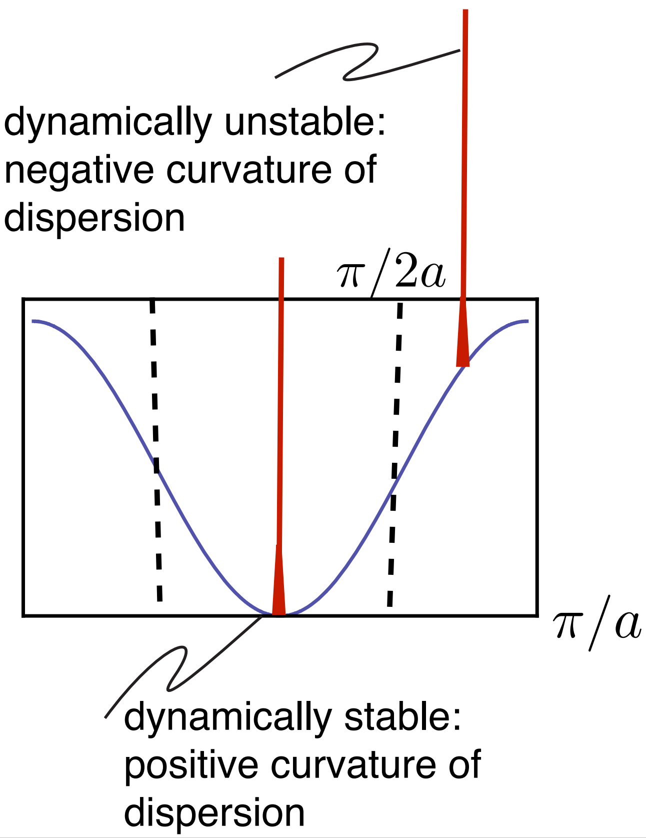


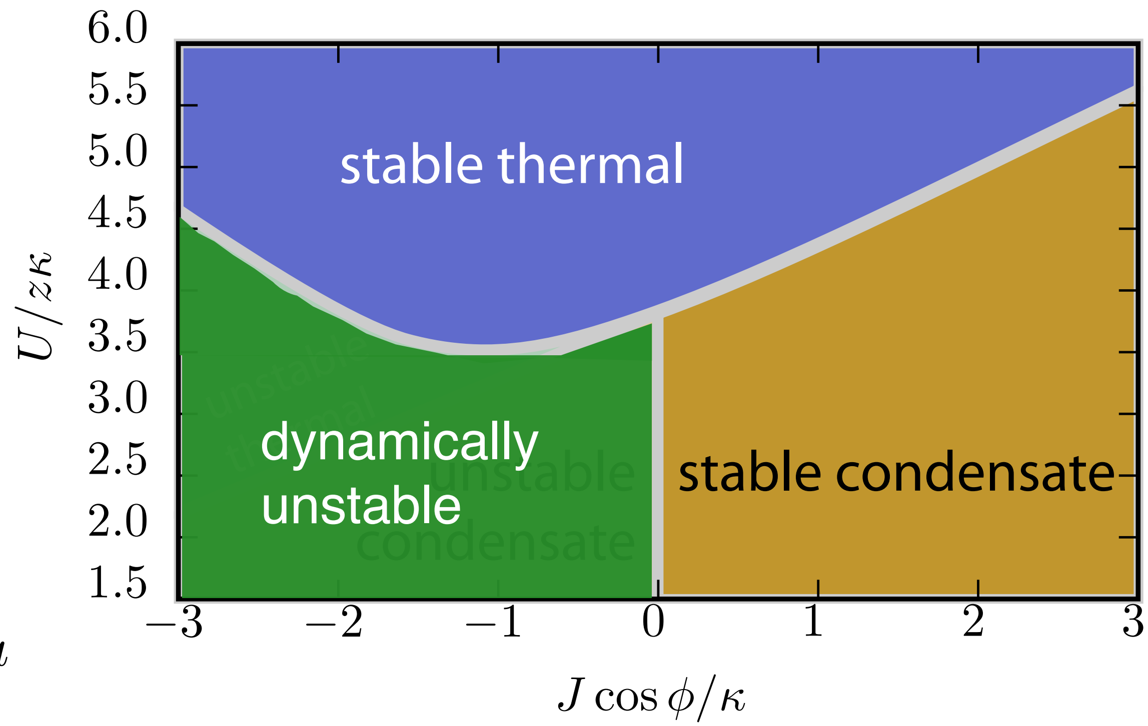


Nonequilibrium Phase Diagram

Classification?

- U/K transition:
  - interaction driven (like quantum PT)
  - terminates in thermal state (like classical finite temperature PT)
- Add negative J (via phase imprinting): further competition through dynamical instability
  - no stable equilibrium state (no dynamical fixed point)
  - dynamical limit cycle?





- Initialization: Coherent state, U=J=0
- follow time evolution of the system

Dissipative Driving of Fermions

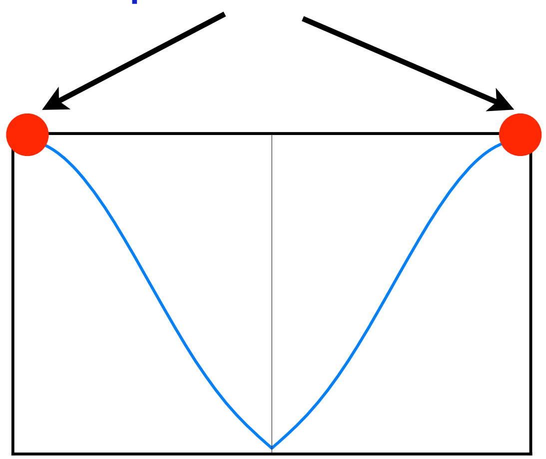
- Excited states: η Condensate
- Cooling into Antiferromagnetic and d-Wave States

Cooling to Excited States: η-Condensate

- η-state: exact excited (i.e. metastable) eigenstate of the two-species Fermi Hubbard Hamiltonian in d dimensions [Yang '89]

$$H = -J \sum_{\langle i,j\rangle,\sigma} f_{i\sigma}^\dagger f_{j\sigma} + U \sum_i f_{i\uparrow}^\dagger f_{i\downarrow}^\dagger f_{i\downarrow} f_{i\uparrow}$$

- local "doublon" $\eta_i^\dagger = f_{i\uparrow}^\dagger f_{i\downarrow}^\dagger$



- checkerboard superposition η-particle

$$\eta^\dagger = \frac{1}{M^{d/2}} \sum_i \phi_i \eta_i^\dagger \qquad \phi_i = \pm 1$$

- N-η-condensate:

$$H(\eta^\dagger)^N|0\rangle = NU(\eta^\dagger)^N|0\rangle$$

exact eigenstate,
off-diagonal long range order

Cooling to Excited States: η-Condensate

- Small scale simulations (open BC) demonstrate η condensation for jumps

$$c_{ij}^{(1)} = (\eta_i^\dagger - \eta_j^\dagger)(\eta_i + \eta_j)$$

$$c_{ij}^{(2)} = n_{i\uparrow} f_{i\downarrow}^\dagger f_{j\downarrow} + n_{j\uparrow} f_{j\downarrow}^\dagger f_{i\downarrow}$$

- Interpretation: Quantum Jump picture

  - H generates spin-up and down configurations on each pair of sites (for any initial density matrix)
  - $c_{ij}^{(2)}$ associates into local doublons
  - $c_{ij}^{(1)}$ creates checkerboard superposition: η condensate

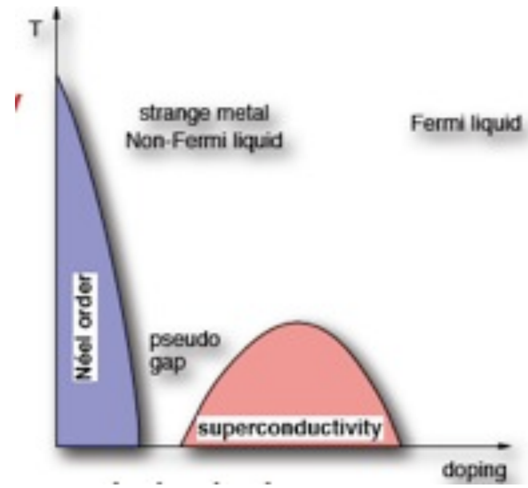
➡ May be conceptually interesting

➡ However, these jump operators are two-body: difficult to engineer

Motivation: Cooling Fermion Systems

- High temperature superconductivity
  - discovered in 1986 (Müller, Bednorz): cuprates show superconductivity at unconventionally high temperature
  - riddle: attraction from repulsion
    - microscopically, strong Coulomb onsite repulsion
    - still, observe pairing of fermions with d-wave symmetry

Experimental phase diagram for cuprates



- Minimal model: 2d Fermi-Hubbard model

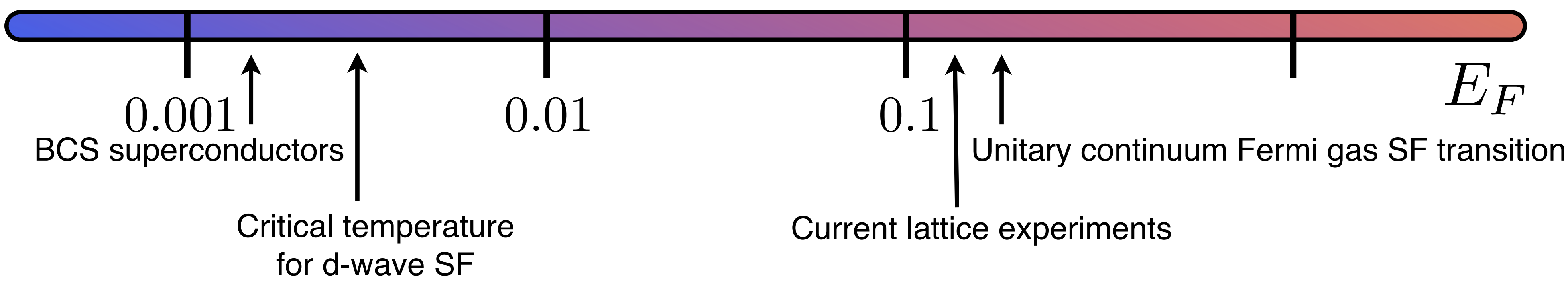
$$H_{\mathrm{FH}} = -J \sum_{\langle i,j\rangle,\sigma} c^\dagger_{i,\sigma} c_{j,\sigma} + U \sum_i \hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow} \qquad U \approx 10J$$

- realistic for cuprate high-temperature superconductors?
- hard to solve: strongly interacting fermion theory
  - no controlled analytical approach available
  - numerically (classical computer) intractable

➡ Quantum simulation of the Fermi-Hubbard model in optical lattices?

Quantum Simulation of Fermion Hubbard model

- Clean realization of fermion Hubbard model possible
  - Detection of Fermi surface in 40K (M. Köhl et al. PRL 94, 080403 (2005))
  - Fermionic Mott Insulators (R. Jördens et al. Nature 455, 204 (2008); U. Schneider et al., Science 322, 1520 (2008))
- Cooling problematic: small d-wave gap sets tough requirements



➡ Still need to be 10-100x cooler

- Existing proposal: Adiabatic quantum simulation (S. Trebst et al. PRL 96, 250402 (2006))
  - Start from a pure initial state of noninteracting model
  - Adiabatically transform to unknown ground state of interacting model
  - Concrete scheme: find path protected by large gaps:
    - prepare RVB ground state on isolated 2x2 plaquettes
    - couple these plaquettes to arrive at many-body ground state

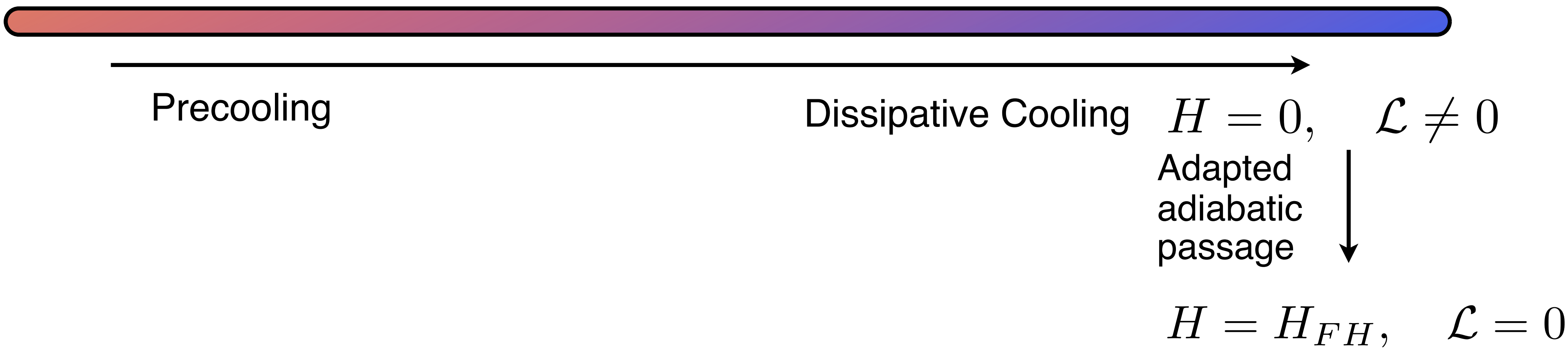
Dissipative Quantum State Engineering Approach

- Roadmap:

(1) Precool the system (lowest Bloch band)

(2) Dissipatively prepare pure (zero entropy) state close to the expected ground state:
  - energetically close
  - symmetry-wise close
  - spin-wise close

(3) Adapted adiabatic passage to the Hubbard ground state
  - switch dissipation off
  - switch Hamiltonian on



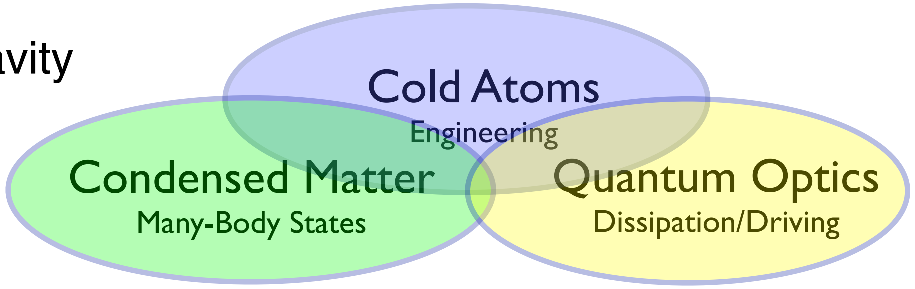
Some slides taken out

Summary Part I

By merging techniques from quantum optics and many-body systems:

Driven dissipation can be used as controllable tool in cold atom systems.

- Pure states with long range correlations from quasilocal dissipation
  - Many-body dark state, independent of initial density matrix
  - Laser coherence mapped on matter system
  - System steady state has zero entropy
- Nonequilibrium phase transition driven via competition of unitary and dissipative dynamics
  - driven by interactions (like quantum phase transition)
  - terminates into thermal state (like classical phase transition)
- Strong potential applications for fermionic quantum simulation
  - cool into zero entropy d-wave state as intial state for Fermi-Hubbard model
  - single particle operations due to Pauli blocking
  - realistic setting using earth alkaline atoms in a cavity



Optical Lattices

- AC-Stark shift
  - Consider an atom in its electronic ground state exposed to laser light at fixed position $\vec{x}$.
  - The light be far detuned from excited state resonances: ground state experiences a second-oder *AC-Stark shift*

$$\delta E_g = \alpha(\omega) I$$

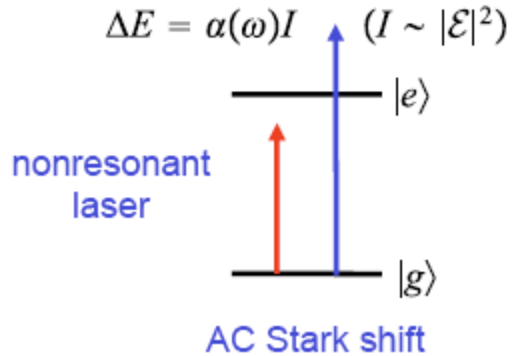
with $\alpha(\omega)$ - dynamic polarizability of the atom for laser frequency ω, $I \propto \vec{E}^2$ - light intensity.
  - Example: two-level atom $\{|g\rangle, |e\rangle\}$.

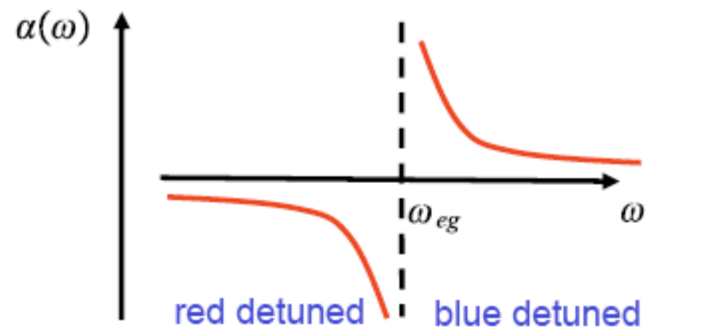
Rabi frequency

$$\delta E_g = \hbar \frac{\Omega^2}{4\Delta}$$

detuning from resonance $\Delta = \omega - \omega_{eg}$

$\Omega \ll \Delta$

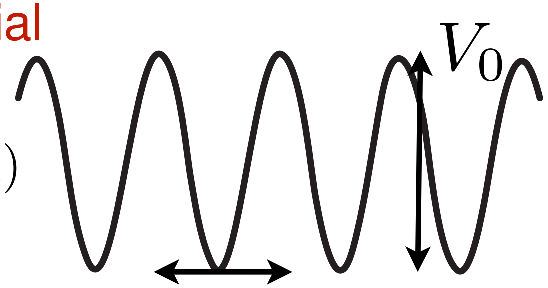




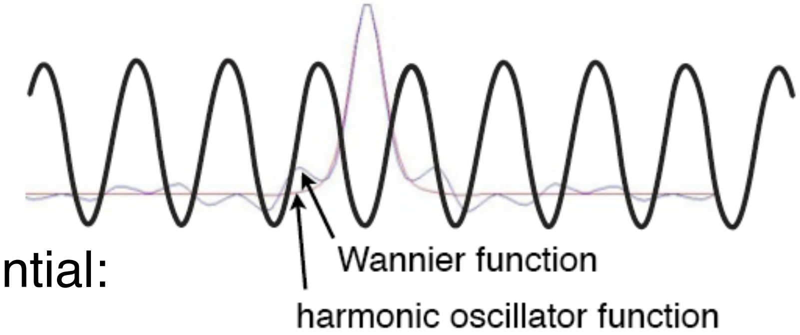
$\Delta < 0,\ \delta E_g < 0$ $\Delta > 0,\ \delta E_g > 0$

- For standing wave laser configuration $\vec{E}(\vec{x}, t) = \vec{e}\mathcal{E} \sin kx\, e^{-i\omega t} + \text{h.c.}$, AC-Stark shift is a function of position: It generates an optical potential

$$V_{\text{opt}}(\vec{x}) \equiv \delta E_g(\vec{x}) = \hbar \frac{\Omega^2(\vec{x})}{4\Delta} \equiv V_0 \sin^2 kx \qquad (k = 2\pi/\lambda)$$



Effective Lattice Hamiltonian



- Start from our model Hamiltonian, add optical potential:

$$H = \int_{\mathbf{x}} [a^\dagger_{\mathbf{x}} (- \frac{\triangle}{2m} - \mu + V(\mathbf{x}) + V_{\text{opt}}(\mathbf{x})) a_{\mathbf{x}} + g \hat{n}^2_{\mathbf{x}}]$$

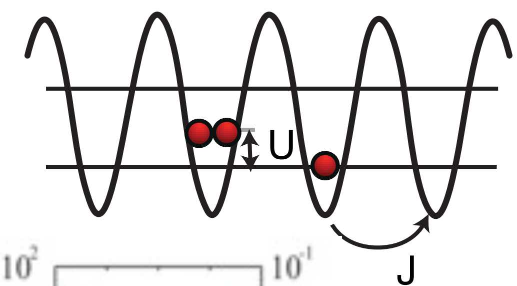
- Periodicity of the optical potential suggests expansion of field operators into localized lattice periodic Wannier functions (complete set of orthogonal functions)

$$a_{\mathbf{x}} = \sum_{i,n} w_n(\mathbf{x} - \mathbf{x}_i) b_{i,n}$$

band index   minimum position

- For low enough energies (temperature), we can restrict to lowest band:

$$T, U, J \ll \sqrt{4 V_0 E_R}, E_R = k^2/(2m) \rightarrow n = 0$$



- Then we obtain the single band Bose-Hubbard model

$$H = -J \sum_{\langle i,j \rangle} b^\dagger_i b_j - \mu \sum_i \hat{n}_i + \sum_i \epsilon_i \hat{n}_i + \tfrac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

$$\hat{n}_i = b^\dagger_i b_i$$

$$J = - \int dx w^*_0(x) (-\hbar^2/2m\triangle - V_{\text{opt}}(x)) w_0(x - \lambda/2)$$

$$U = g \int dx |w_0(x)|^4$$